

# Reliability Measures for Tele-Communication System with Redundant Transferring Machine by Using Algebraic Method

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## Abstract

There are so many existing methods to obtain system reliability like re-generating point function technique, supplementary variables technique etc., but all these techniques are full of complicated calculations. Keeping above these facts in mind, the authors in this paper have evaluated some reliability parameters for tele-communication system by using Boolean functions technique and algebraic method. Reliability of considered tele-communication system has been evaluated by considering the fact that failures follow arbitrary time distribution. In particular, the reliability expression has also been calculated for Exponential and Weibull distributions. Further, an important reliability parameter namely M.T.T.F. (mean time to failure) has also been calculated. A numerical example with graphical illustrations has been appended at the end to highlight the important results and practical utility of the model.

## Keywords

Boolean Function Technique, Algebra of Logics, Weibull Distribution, Exponential Distribution

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## 1. Introduction

The whole tele-communication system has been divided into three parts, e.g., telephone instrument, main exchange and radar. The work of telephone instrument is to convert the human voice into radio waves and vice-versa. The main exchange receives these waves and transfers to their respective destinations (dialed by instrument) through the function of transferring machine and radar. In this paper, the author has assumed that, there are two identical transferring machines in both the main exchanges and working

in parallel redundancy. The system configuration has been shown in **Figure 1**. The author has been used algebra of logics and method of Boolean functions to obtain some important reliability parameters under the following assumptions:

1. Initially, all components are in operable condition.
2. The state of each component and of the whole system is either good or fail.
3. There is no repair facility.
4. The states of all components of the system are statistically independent.
5. The reliability of every component of the system is known in advance.
6. The failure times of all components are arbitrary.
7. Supply between any two components of the system is fully reliable.
8. On failure of any one transferring machine, the system can work with the help of other.

The notations used for this study are as follows:

$x_1, x_7$	:	States of telephone instruments dialer & receiver.
$x_2, x_3$	:	States of transferring machines of first main exchange.
$x_5, x_6$	:	States of transferring machines of second main exchange.
$x_4$	:	State of radar.
$x_i (i = 1, 2, \dots, 7)$	:	$\begin{cases} 0, & \text{in bad state;} \\ 1, & \text{in good state.} \end{cases}$
$x'_i (i = 1, 2, \dots, 7)$	:	Negation of $x_i$ .
$\wedge / \vee$	:	Conjunction, disjunction respectively.
$R_i (i = 1, 2, \dots, 7)$	:	Reliability of the $i^{\text{th}}$ component of considered system corresponding to system state $x_i$ .
$R_s$	:	Reliability of the system as a whole.
$R_{sw}(t), R_{se}(t)$	:	Reliability functions, in case, failures follow Weibull, exponential time distribution, respectively.

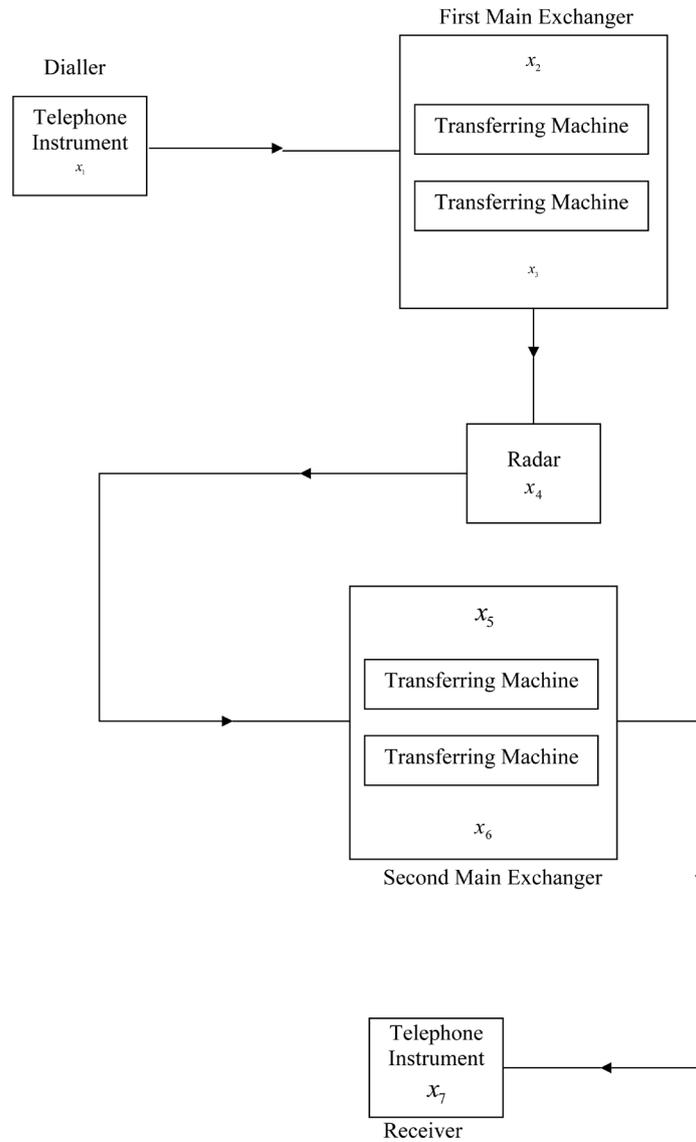
## 2. Material and Methods

This study was conducted at Department of Mathematics, K.E.C., Sahibabad, Ghaziabad, India during April 2015. The results obtained are studied there during June 2015.

In this study, the authors have been used Boolean function technique [1] to formulate mathematical model of the considered system. Various paths of successful operations of the system have been found. The reliability of considered system and MTTF [2] has been evaluated. These results can also be used to get various reliability parameters for the system having similar configurations.

Using algebra of logics, the conditions of capability for the successful operation of the considered system in terms of logical matrix has given as below:

$$F(x_1, x_2, \dots, x_7) = \begin{bmatrix} x_1 & x_2 & x_4 & x_5 & x_7 \\ x_1 & x_2 & x_4 & x_6 & x_7 \\ x_1 & x_3 & x_4 & x_5 & x_7 \\ x_1 & x_3 & x_4 & x_6 & x_7 \end{bmatrix} \tag{1}$$



**Figure 1.** System configuration.

Using algebra of logics, Equation (1) can be written as

$$F(x_1, x_2, \dots, x_7) = x_1 x_4 x_7 \cdot \begin{bmatrix} x_2 & x_5 \\ x_2 & x_6 \\ x_3 & x_5 \\ x_3 & x_6 \end{bmatrix} = x_1 x_4 x_7 \cdot G \quad (2)$$

where,

$$G = \begin{bmatrix} x_2 & x_5 \\ x_2 & x_6 \\ x_3 & x_5 \\ x_3 & x_6 \end{bmatrix} \quad (3)$$

Now, substituting

$$A_1 = [x_2 \quad x_5] \tag{4}$$

$$A_2 = [x_2 \quad x_6] \tag{5}$$

$$A_3 = [x_3 \quad x_5] \tag{6}$$

and

$$A_4 = [x_3 \quad x_6] \tag{7}$$

in Equation (3), we obtain

$$G = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} \tag{8}$$

Now using the theorem of summation of probabilities of compatible events, viz;

$$P_r \left\{ \bigcup_{i=1}^n A_i \right\} = \sum_{i=1}^n P_r \{A_i\} - \sum_i \sum_j P_r \{A_i \cap A_j\} + \sum_i \sum_j \sum_K P_r \{A_i \cap A_j \cap A_K\} - \dots + (-1)^{n-1} P_r \{A_1 \cap A_2 \cap \dots \cap A_n\}$$

or,

$$P_r \left\{ \bigvee_{i=1}^n A_i \right\} = \sum_{i=1}^n P_r \{A_i\} - \sum_i \sum_j P_r \{A_i \wedge A_j\} + \sum_i \sum_j \sum_K P_r \{A_i \wedge A_j \wedge A_K\} - \dots + (-1)^{n-1} P_r \{A_1 \wedge A_2 \wedge \dots \wedge A_n\} \tag{9}$$

The probability of successful operation of the function  $G$  is given by

$$\begin{aligned} P_r \{G = 1\} &= P_r \left\{ \bigvee_{i=1}^4 A_i \right\} = P_r \{A_1\} + P_r \{A_2\} + P_r \{A_3\} + P_r \{A_4\} - P_r \{A_1 \wedge A_2\} \\ &\quad - P_r \{A_1 \wedge A_3\} - P_r \{A_1 \wedge A_4\} - P_r \{A_2 \wedge A_3\} - P_r \{A_2 \wedge A_4\} \quad [\text{By (9)}] \tag{10} \\ &\quad - P_r \{A_3 \wedge A_4\} + P_r \{A_1 \wedge A_2 \wedge A_3\} + P_r \{A_2 \wedge A_3 \wedge A_4\} \\ &\quad + P_r \{A_1 \wedge A_3 \wedge A_4\} + P_r \{A_1 \wedge A_2 \wedge A_4\} - P_r \{A_1 \wedge A_2 \wedge A_3 \wedge A_4\} \end{aligned}$$

Now, we have

$$\begin{aligned} A_1 \wedge A_2 &= [x_2 \quad x_5] \wedge [x_2 \quad x_6] = [x_2 \quad x_5 \quad x_6] \\ \therefore P_r \{A_1 \wedge A_2\} &= R_2 R_5 R_6 \end{aligned}$$

Similarly, we obtain the values of other terms appear in Equation (10) and get the following expression:

$$\begin{aligned} P_r \{G = 1\} &= R_2 R_5 + R_2 R_6 + R_3 R_5 + R_3 R_6 - R_2 R_5 R_6 - R_2 R_3 R_5 - R_2 R_3 R_5 R_6 \\ &\quad - R_2 R_3 R_5 R_6 - R_2 R_3 R_6 - R_3 R_5 R_6 + R_2 R_3 R_5 R_6 + R_2 R_3 R_5 R_6 \\ &\quad + R_2 R_3 R_5 R_6 + R_2 R_3 R_5 R_6 - R_2 R_3 R_5 R_6 \tag{11} \\ &= R_2 R_5 + R_2 R_6 + R_3 R_5 + R_3 R_6 + R_2 R_3 R_5 R_6 - R_2 R_5 R_6 \\ &\quad - R_2 R_3 R_5 - R_2 R_3 R_6 - R_3 R_5 R_6 \end{aligned}$$

From Equation (2), we obtain

$$R_s = P_r \{F = 1\} = P_r \{x_1 x_4 x_7\} \cdot P_r \{G = 1\}$$

or,

$$R_s = R_1 R_4 R_7 [R_2 R_5 + R_2 R_6 + R_3 R_5 + R_2 R_3 R_5 R_6 - R_2 R_5 R_6 - R_2 R_3 R_5 - R_2 R_3 R_6 - R_3 R_5 R_6] \quad (12)$$

### 3. Some Particular Cases

**Case I: If the reliability of each unit of system is R**

In this case, Equation (12) yields

$$R_s = 4R^5 - 4R^6 + R^7 \quad (13)$$

**Case II: When failures follow Weibull time distribution**

For this, let  $\lambda_i$  be the failure rate of the unit corresponding to system state  $x_i$ , and it follows Weibull time distribution, then

$$R_{sw}(t) = \sum_{i=1}^5 \exp\{-a_i t^p\} - \sum_{j=1}^4 \exp\{-b_j t^p\} \quad (14)$$

where,  $p$  is a real positive parameter and

$$a_1 = c + \lambda_2 + \lambda_5$$

$$a_2 = c + \lambda_2 + \lambda_6$$

$$a_3 = c + \lambda_3 + \lambda_5$$

$$a_4 = c + \lambda_3 + \lambda_6$$

$$a_5 = c + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6$$

$$b_1 = c + \lambda_2 + \lambda_5 + \lambda_6$$

$$b_2 = c + \lambda_2 + \lambda_3 + \lambda_5$$

$$b_3 = c + \lambda_2 + \lambda_3 + \lambda_6$$

$$b_4 = c + \lambda_3 + \lambda_5 + \lambda_6$$

and

$$c = \lambda_1 + \lambda_4 + \lambda_7$$

**Case III: When failures follow exponential time distribution**

Exponential time distribution is nothing but a particular case of Weibull distribution for  $p = 1$ . This is very useful in various practical problems. So, in this case, the reliability function for the considered tele-communication system, at any time “ $t$ ”, is given by:

$$R_{SE}(t) = \sum_{i=1}^5 \exp\{-a_i t\} - \sum_{j=1}^4 \exp\{-b_j t\} \quad (15)$$

Also, an important reliability parameter viz; mean time to failure for the considered system, in this case, is given by

$$M.T.T.F. = \int_0^{\infty} R_{SE}(t) dt = \sum_{i=1}^5 \left(\frac{1}{a_i}\right) - \sum_{j=1}^4 \left(\frac{1}{b_j}\right) \quad (16)$$

where,  $a_i$ 's and  $b_j$ 's have been mentioned earlier.

### 4. Numerical Computation

For a numerical computation, let us consider:

1)  $\lambda_i (i = 1, 2, \dots, 7) = \lambda = 0.02$  and  $p = 2$  (for Weibull distribution). Using these values in Equations (14) and (15), we get

$$R_{SW}(t) = 4e^{-0.1t^2} - 4e^{-0.12t^2} + e^{-0.14t^2} \tag{17}$$

and

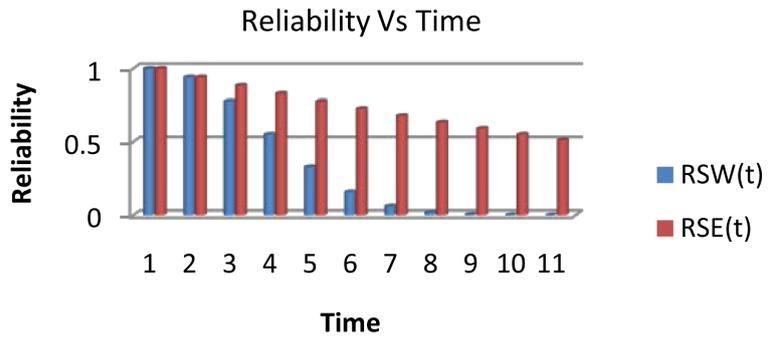
$$R_{SE}(t) = 4e^{-0.1t} - 4e^{-0.12t} + e^{-0.14t} \tag{18}$$

Now putting  $t = 0, 1, 2, \dots$  in Equations (17) and (18), we draw the graph has been shown in **Figure 2**.

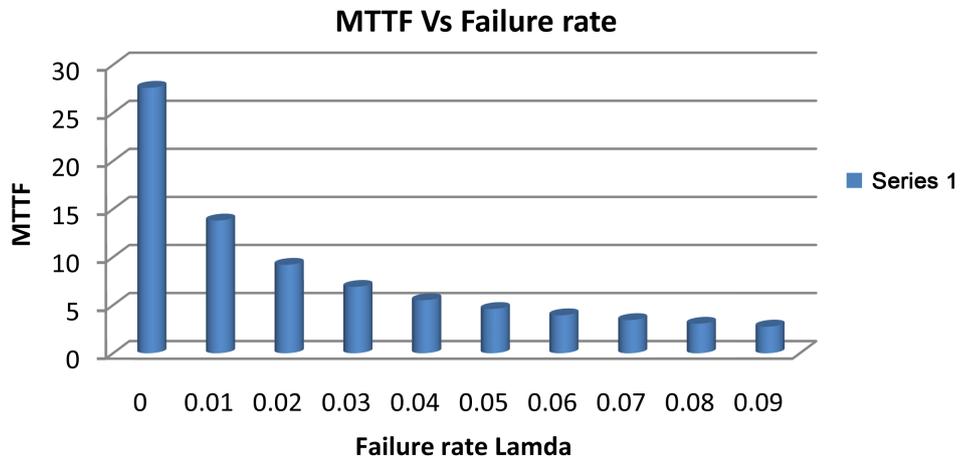
2)  $\lambda_i (i = 1, 2, \dots, 7) = \lambda$ . Using this Equation (16) gives

$$\text{M.T.T.F.} = \frac{0.2761904}{\lambda} \tag{19}$$

Now putting  $\lambda = 0.01, 0.02, \dots$  in Equation (19), we draw the graph has been represented in **Figure 3**.



**Figure 2.** Represent the way in which reliability of considered system decreases with time in case; failures follow Weibull and Exponential time distributions.



**Figure 3.** Represent the way in which mean time to failure of considered system decreases with increase in failure rate.

## 5. Results and Discussion

In this paper, we have evaluated reliability and mean time to failure [3] for the considered diesel tele-communication system by employing Boolean function technique [1]. Also, we have computed some particular cases to improve practical utility of the system. Gupta, P.P. *et al.* have done the reliability analysis of complex system but no care was given to structural redundancy [1]. On failure of main working unit, the whole system can also fail. So, for the better analysis we must have to consider the concept of structural redundancy. Thus we have done some better analysis of considered system of practical utility.

**Figure 2** shows the values of reliability functions, when failures follow Weibull and exponential time distributions. Analysis of **Figure 2** yields that as we make increase in time  $t$ , the values of  $R_{SE}(t)$  remains better as compared to  $R_{SW}(t)$ . So, we conclude that reliability function remains better in case of failures follow exponential time distribution

**Figure 3** gives the values of M.T.T.F. of considered system for different failure rates  $\lambda$ . Critical examination of **Figure 3** reveals that M.T.T.F. decreases slowly in the beginning but there after it decreases catastrophically with the increase in the value of failure rate  $\lambda$ .

## 6. Conclusion

In conclusion, author observes that we could improve system's overall performance by using the concept of redundancy. The main objective of the study was to obtain continuous reliability of the system, and there should be no sudden jumps in the value of reliability of the system, and at the end authors were successful to do so. Boolean function technique is easier [4] as compared with the other techniques to obtain reliability parameters. We have compared the values of reliability function, in case; failures follow Weibull and Exponential time distributions. By using the structural redundancy, we obtain the better values of reliability and mean time to failure by taking the system with parallel redundant transferring machine.

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