

A Note on Standard Goal Programming with Fuzzy Hierarchies: A Sequential Approach

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Abstract

In the paper [Standard goal programming with fuzzy hierarchies: a sequential approach, *Soft Computing*, First online: 22 March 2015], it was assumed that the normalized deviations should lie between zero and one. In some cases, this assumption may not be valid. Therefore, additional constraints must be incorporated into the model to ensure that the normalized deviations should not exceed one. This modification is illustrated by the given numerical example.

Keywords

Fuzzy Goal Programming, Imprecise Hierarchy, Normalized Deviations

1. Introduction

The problem of fuzzy goal programming when the importance relation between the fuzzy goals is vague has initially been investigated by Aköz and Petrovic [1] and followed by Li and Hu [2] and Cheng [3]. A suggested sequential approach in fuzzy goal programming, when the importance hierarchy of the goals is imprecise, has been presented by Arenas-Parra *et al.* [4]. In their article, the model of goal programming with fuzzy hierarchy (GPFH) is given as

$$\text{Maximize } \lambda \sum_{i=1}^k \left(1 - \frac{n_i}{m_i - f_i} \right) + (1 - \lambda) \sum_{\substack{(i,j)=1 \\ i \neq j}}^k \sum_{r=1}^3 b_{\tilde{R}_r(i,j)} \mu_{\tilde{R}_r(i,j)}$$

subject to:

$$\begin{aligned}
 & f_i(x) + n_i - p_i = m_i, \quad i = 1, \dots, k, \\
 & 1 - \left(\frac{n_i}{m_i - \underline{f}_i} - \frac{n_j}{m_j - \underline{f}_j} \right) \geq \mu_{\tilde{R}_1(i,j)}, \quad \text{if } b_{\tilde{R}_1(i,j)} = 1, \\
 & \frac{1 - \left(\frac{n_i}{m_i - \underline{f}_i} - \frac{n_j}{m_j - \underline{f}_j} \right)}{2} \geq \mu_{\tilde{R}_2(i,j)}, \quad \text{if } b_{\tilde{R}_2(i,j)} = 1, \\
 & \frac{n_j}{m_j - \underline{f}_j} - \frac{n_i}{m_i - \underline{f}_i} \geq \mu_{\tilde{R}_3(i,j)}, \quad \text{if } b_{\tilde{R}_3(i,j)} = 1, \\
 & 0 \leq \mu_{\tilde{R}_r(i,j)} \leq 1, \quad r = 1, 2, 3, \\
 & n_i, p_i \geq 0, \quad n_i \times p_i = 0, \quad i = 1, \dots, k, \\
 & x \in X,
 \end{aligned} \tag{1}$$

where $0 \leq \lambda \leq 1$, and $f_i(x)$ is an i^{th} linear function of an x vector of decision variables, $i = 1, \dots, k$. Also, n_i and p_i are the negative and positive deviations, respectively, where m_i is the aspiration level and \underline{f}_i is the anti-ideal value for the i^{th} fuzzy goal constraint. Moreover, $b_{\tilde{R}_r(i,j)}$ ($r = 1, 2, 3$) is a binary variable associated with the membership function of the r^{th} importance relation (slightly, moderately, significantly) of the i^{th} goal more than the j^{th} goal; while $\mu_{\tilde{R}_r(i,j)}$ is the membership function of the r^{th} imprecise relation between the i^{th} and the j^{th} fuzzy goals. Finally, X is a set of system constraints which define the feasible set of the problem.

This model is implemented for each class of *Phase I*. Hence, it is assumed that the normalized deviation for the i^{th} fuzzy goal constraint must lie between zero and one *i.e.*,

$$0 \leq n_i / (m_i - \underline{f}_i) \leq 1. \tag{2}$$

This assumption may be violated, especially when the anti-ideal value is close to the aspiration level. In this case, $n_i / (m_i - \underline{f}_i)$ may exceed one, due to a small denominator value, which means that the value of the achieved goal is worse than the anti-ideal value of that goal. Accordingly, for each class, the following constraints should be incorporated in the GPFH model:

$$n_i \leq m_i - \underline{f}_i, \tag{3}$$

if the negative deviation is required to be minimized for the i^{th} fuzzy goal constraint, *i.e.*, if $f_i(x) \geq m_i$; or

$$p_i \leq \underline{f}_i - m_i, \tag{4}$$

if the positive deviation is required to be minimized for the i^{th} fuzzy goal constraint, *i.e.*, if $f_i(x) \leq m_i$.

Notably, constraints (3) and (4) correspond to the non-negativity of the membership functions of the fuzzy goal constraints given by Aköz and Petrovic [1].

Proposition: The normalized deviations constraints might limit the feasible set of the problem. This may worsen the value of the achievement function of each class, and therefore affect the results of the suggested sequential approach.

In the next section, this note is verified by the given illustrative example.

2. Illustrative Example

The GPFH model (*Phase I*) is solved using the following example that is given by Arenas-Parra *et al.* [4]:

$$\text{Goal 1: } 4x_1 + 2x_2 + 8x_3 + x_4 \leq 35$$

$$\text{Goal 2: } 4x_1 + 7x_2 + 6x_3 + 2x_4 \geq 100$$

Goal 3: $x_1 - 6x_2 + 5x_3 + 10x_4 \geq 120$

Goal 4: $5x_1 + 3x_2 + 2x_4 \geq 70$

Goal 5: $4x_1 + 4x_2 + 4x_3 \geq 40$

subject to:

$$\left. \begin{aligned} 7x_1 + 5x_2 + 3x_3 + 2x_4 &\leq 98, \\ 7x_1 + x_2 + 2x_3 + 6x_4 &\leq 117, \\ x_1 + x_2 + 2x_3 + 6x_4 &\leq 130, \\ 9x_1 + x_2 + 6x_4 &\leq 105, \\ x_i &\geq 0, i = 1, \dots, 4, \end{aligned} \right\} X$$

where *Class I* contains goals (1, 2, and 4). Accordingly, the assumed anti-ideal values for those goals are $\underline{f}_1 = 261.33$, $\underline{f}_2 = 0$, $\underline{f}_4 = 0$. Also, the GPFH model for *Class I* assumes that Goal 1 is *moderately more important than* Goal 2; and Goal 2 is *moderately more important than* Goal 4. Finally, the parameter λ_I is set equal to 0.8.

Thus, the model for *Class I* is as follows:

$$\text{Maximize } AF_I = \lambda_I \left(1 - \frac{P_1}{226.33} + 1 - \frac{n_2}{100} + 1 - \frac{n_4}{70} \right) + (1 - \lambda_I) \left[\mu_{\bar{R}_2(1,2)} + \mu_{\bar{R}_2(2,4)} \right]$$

subject to:

$$4x_1 + 2x_2 + 8x_3 + x_4 + n_1 - p_1 = 35,$$

$$4x_1 + 7x_2 + 6x_3 + 2x_4 + n_2 - p_2 = 100,$$

$$5x_1 + 3x_2 + 2x_4 + n_4 - p_4 = 70,$$

$$\frac{1 - \left(\frac{p_1}{226.33} - \frac{n_2}{100} \right)}{2} \geq \mu_{\bar{R}_2(1,2)},$$

$$\frac{1 - \left(\frac{n_2}{100} - \frac{n_4}{70} \right)}{2} \geq \mu_{\bar{R}_2(2,4)},$$

$$0 \leq \mu_{\bar{R}_2(1,2)} \leq 1, \quad 0 \leq \mu_{\bar{R}_2(2,4)} \leq 1,$$

$$n_k, p_k \geq 0, \quad n_k \times p_k = 0, \quad k = 1, 2, 4,$$

$$x \in X.$$

The given note is verified by just resolving the GPFH model for *Class I* in *Phase I*. Assume that the anti-ideal values of the first and the fourth fuzzy goal constraints \underline{f}_1 and \underline{f}_4 are 40 and 63 instead of 261.33 and 0, respectively. In this case, the normalized p_1 is $p_1/5$, while the normalized n_4 becomes $n_4/7$.

Then, the solution obtained is: $\mu_{\bar{R}_2(1,2)} = 0.463$, $\mu_{\bar{R}_2(2,4)} = 1$, $p_1 = 0.375$, $n_2 = 0$, $n_4 = 9$, $G_1 = 35.375$, $G_2 = 100$, $G_4 = 61$, $AF_I^* = 1.604$. Hence, $n_4/7 = 1.286$, which is greater than 1.

Accordingly, by incorporating the following three constraints:

$$p_1 \leq 5,$$

$$n_2 \leq 100,$$

$$n_4 \leq 7,$$

and by solving the model, the solution becomes: $\mu_{\bar{R}_2(1,2)} = 0.325$, $\mu_{\bar{R}_2(2,4)} = 1$, $p_1 = 1.750$, $n_2 = 0$, $n_4 = 7$, $G_1 = 36.750$, $G_2 = 105$, $G_4 = 63$, $AF_I^* = 1.585$.

It is realized that incorporating the normalized deviations constraints leads to a worse value of AF_I^* , which verifies the proposition.

3. Conclusion

The normalized deviations constraints must be included in the GPFH model in all classes of *Phase I* as well as in *Phase II* to ensure that the achieved value of each goal should never become worse than the anti-ideal value of that goal.

References

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