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Facility Location Decisions Based on Driving Distances on Spherical Surface

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Abstract

Facility location problems are concerned with the location of one or more facilities in a way that optimizes a certain objective such as minimizing transportation cost, providing equitable service to customers, capturing the largest market share, etc. Many facility location decisions involving distance objective functions on Spherical Surface have been approached using algorithmic, metaheuristic algorithms, branch-and-bound algorithm, approximation algorithms, simulation, heuristic techniques, and decomposition method. These approaches are most based on Euclidean distance or Great circle distance functions. However, if the location points are widely separated, the difference between driving distance, Euclidean distance and Great circle distance may be significant and this may lead to significant variations in the locations of the corresponding optimal source points. This paper presents a framework and algorithm to use driving distances on spherical surface and explores its use as a facility location decision tool and helps companies assess the optimal locations of facilities.

Keywords

Facility Location, Spherical Surface, Euclidean Distance, Great Circle Distance, Clustering, Heuristic Method

1. Introduction

The facility location problem, also known as location analysis or *k* center problem, is a branch of operations research and computational geometry concerned with the optimal placement of facilities to minimize transportation costs plus facilities costs while considering factors like avoiding placing hazardous materials near housing and competitors' facilities. Poor facility location decisions can result in high transportation costs, inadequate supplies of raw materials and labor, loss of competitive advantage, and financial loss (Reid and Sanders [1]).

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The facility location problem on general graphs is NP-hard to solve optimally (Wikipedia [2]). In the past, many facility location decisions involving distance objective functions on Spherical Surface have been approached using algorithmic, metaheuristic algorithms (Brimberg *et al.* [3], branch-and-bound algorithm (Tcha and Lee [4]), heuristic techniques (Francis and White [5], Love *et al.* [6] and Farahani and Masoud [7]), approximation algorithm (Vygen [8] and Shmoys *et al.* [9]), simulation, and decomposition method (Iyigun and Ben-Israel [10]). Ballou [11] presented an exact Center of Gravity method for facility location problems; he used iterative method to solve the facility location problems based on Euclidean distance. Several papers present general problem formulations which are involving the distance functions. The spherical facility location model studied by Katz and Cooper [12] and by Drezner and Wesolowsky [13] is a more realistic model than the Euclidean planar facility location model, especially for large regional facility location problems. Xue [14] has presented a gradient algorithm for solving the spherical facility location problem and proved a global convergence theorem for the algorithm as well as a hull property for the spherical facility location problem. Mwemezi and Huang [15] stated that current practices are dominated by Euclidean and Rectilinear models which are best suited to planar surfaces and presented an alternative distance measurement based on "great circle distance" which represents the shortest path on spherical surface. Sullivan and Peters [16] presented a new location-allocation algorithm that clusters points in a two-dimensional region into mutually exclusive groups, such that the sum of the value of a user-specified objective function calculated for each group is minimized over the whole study region. The squared distance, L-shaped distance and straight line distance functions were selected for the objective function in their example. Bespamyatnikh *et al.* [17] presented efficient algorithms for two problems of facility location, the first to seek a location that *maximizes* a weighted distance function between the facility and the sites, and the second to find a location that *minimizes* the sum (or sum of the squares) of the distances of k of the sites from the facility. Levin and Ben-Israel [18] presented a heuristic method to solve large-scale multi facility location problems; the method is using the authors' single facility location method (Levin and Ben-Israel [19]) as a parallel subroutine, and reassigning customers to facilities using the heuristic of *Nearest Center Reclassification*. Rodríguez-Chía and Franco [20] presented a procedure to solve the classical location median problem where the distances are measured with p-norms with $p > 2$. The global convergence of the sequence generated by this iterative scheme is proved by considering an approximated problem. Kotian *et al.* [21] developed a procedure that will solve contingent on a necessary and sufficient optimality condition, the planar k-centra location problem seeks to find a location that minimizes the sum of the Euclidean distances to the k furthest existing facilities. Iyigun and Ben-Israel [10] proposed a probabilistic decomposition method for the K facilities location problem. The problem is relaxed using probabilistic assignments, depending on the distances to the facilities. The probabilities, that decompose the problem into K single-facility location problems, are updated at each iteration together with the facility locations.

These approaches are most based on Euclidean distance or Great circle distance functions. So far, there seem to be no published papers that present the optimal or near optimal facility location decisions involving driving distance objective functions.

2. Motivation

In **Figure 1**, the driving distance, Euclidean distance and Great circle distance between Detroit, MI (42.331427, −83.0457538) and Cleveland, OH (41.4994954, −81.6954088) are calculated as:

Euclidean distance = 90.23 Miles as shown in **Figure 2**.

Great circle distance = 90.251 Miles (SAS Inc. [22], KSU [23] and The Math Forum [24]) as shown in **Figure 3**.

Driving distance = 170 Miles (Google Inc. [25]) as shown in **Figure 4**.

From **Figures 2-4**, it is clear that the Euclidean distance and Great circle distance are not much different, the driving distance are much longer than Euclidean distance and Great circle distance. Moreover, in the U.S, as a whole, road distances between major cities are about 18% greater than the straight-line distances (Love *et al.* [6]). The Euclidean distance is the "ordinary" distance between two points defined as the square root of the sum of the squares of the differences between the corresponding coordinates of the points. In Euclidean three-space (Wikipedia [26] and Wiktionary [27]), the distance between points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

Figure 1. Points of Detroit, MI and Cleveland, OH.

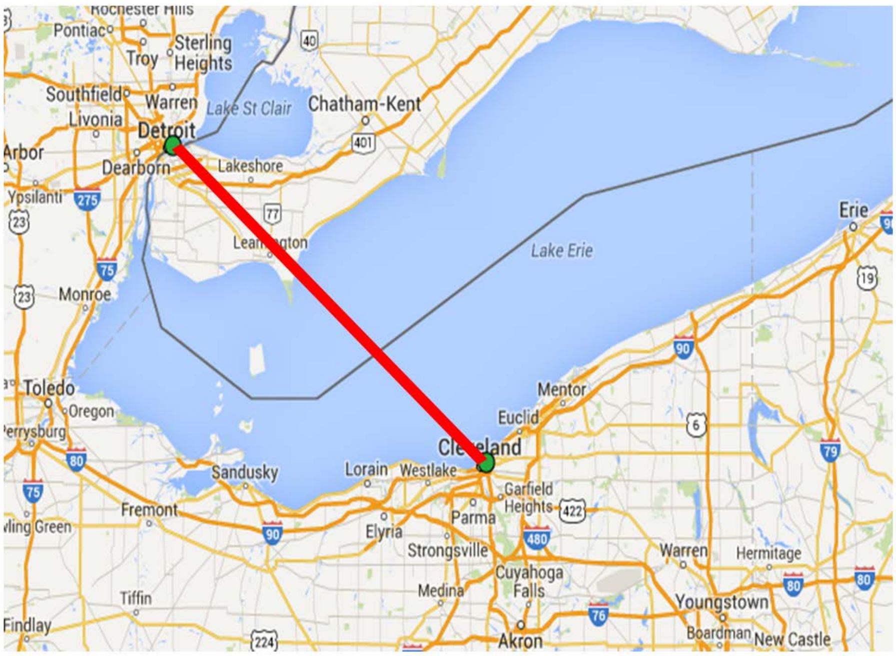

Figure 2. Euclidean distance between Detroit, MI and Cleveland, OH.

The great-circle distance is the shortest distance between two points on the surface of a sphere, measured with "Haversine" formula as (Wikipedia [28] and Movable Type Ltd [29]).

Haversine: $a = \sin^2(\Delta\varphi/2) + \cos\varphi_1 * \cos\varphi_2 * \sin^2(\Delta\lambda/2)$

Great circle distance: $d = 2 * R * a\tan 2\left(\sqrt{a}, \sqrt{(1-a)}\right)$

where φ is latitude, λ is longitude, R is earth's radius (mean radius = 6371 km);

$\Delta\varphi$ Is difference between two latitudes radians

$\Delta\lambda$ Is difference between two longitudes radians

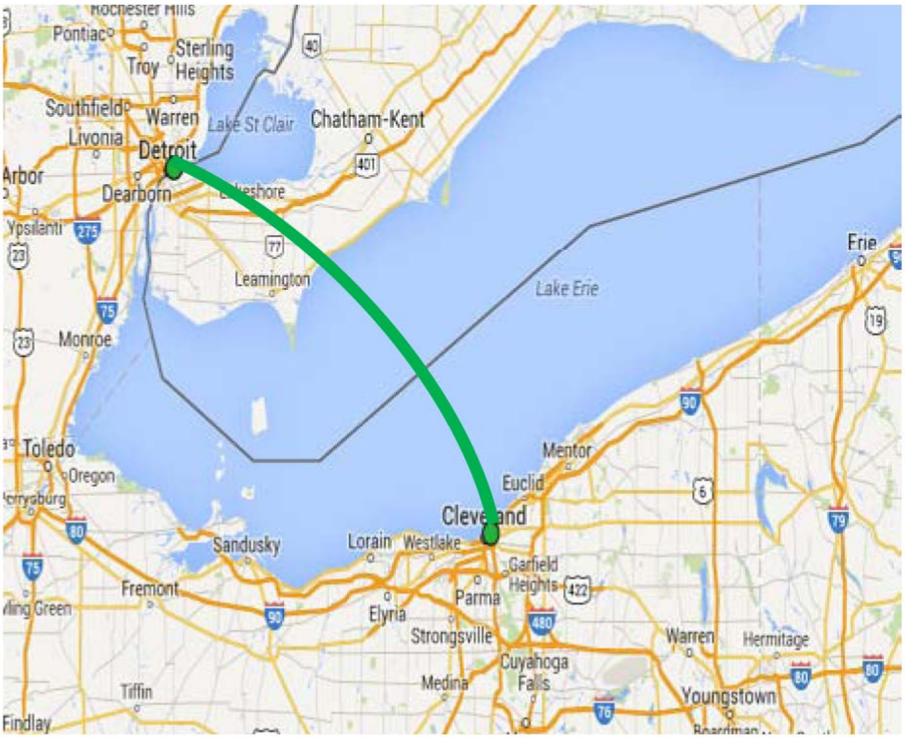

Figure 3. Great circle distance between Detroit, MI and Cleveland, OH.

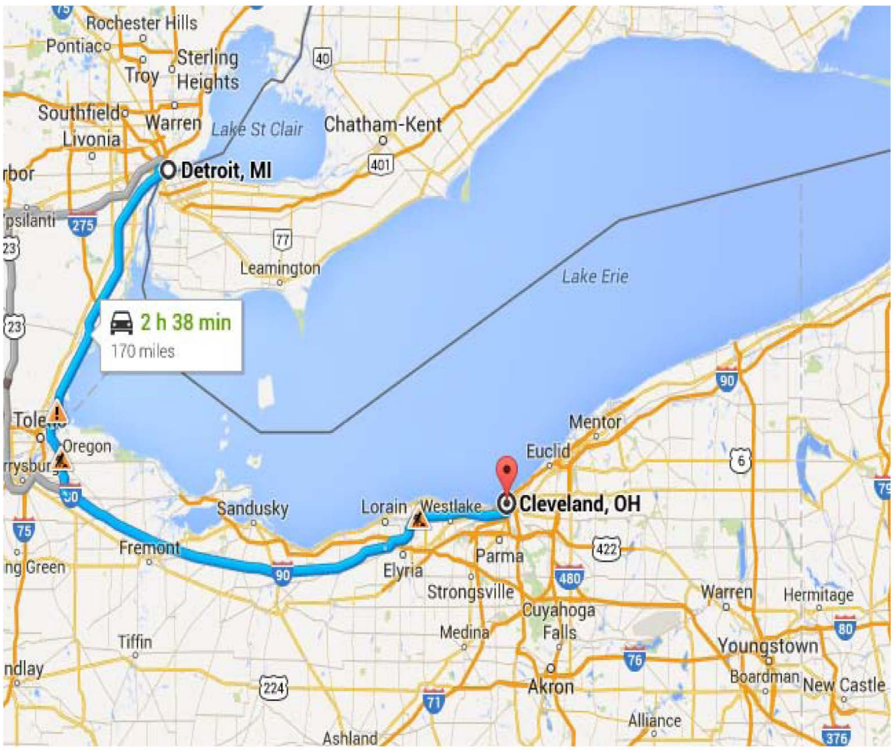

Figure 4. Driving distance between Detroit, MI and Cleveland, OH.

where $a\tan 2$ is the arctangent function with two arguments. In terms of the standard arctan function, whose range is $(-\pi/2, \pi/2)$, it can be expressed as follows:

$$a\tan 2(y,x)=\begin{cases}\arctan\left(\dfrac{y}{x}\right) & \text{if } x>0,\\ \arctan\left(\dfrac{y}{x}\right)+\pi & \text{if } x<0 \text{ and } y\geq 0,\\ \arctan\left(\dfrac{y}{x}\right)-\pi & \text{if } x<0 \text{ and } y<0,\\ +\dfrac{\pi}{2} & \text{if } x=0 \text{ and } y>0,\\ -\dfrac{\pi}{2} & \text{if } x=0 \text{ and } y<0,\\ \text{undefined} & \text{if } x=0 \text{ and } y=0.\end{cases}$$

Microsoft C# uses Math. Atan2(*y*, *x*) function to calculate Atan2(*y*, *x*)

Several papers have applied Weiszfeld [30] iterative method to solve spherical facility locations problems involving Euclidean distance and Great circle distance. Katz and Cooper [12] proposed the following closed form iteration formula for solving the spherical facility location problems. There are no any iteration formulas in relation to driving distances.

For Euclidean distance $$\overline{x^{k+1}}=\frac{\sum_{j=1}^{N}\frac{w_j}{\left|\overline{x^k}-\overline{x_j}\right|}\overline{x_j}}{\left|\sum_{j=1}^{N}\frac{w_j}{\left|\overline{x^k}-\overline{x_j}\right|}\overline{x_j}\right|}$$

For Great circle distance

$$\overline{x^{k+1}}=\frac{\sum_{j=1}^{N}\left\{\frac{w_j}{\left[\left|\overline{x^k}-\overline{x_j}\right|\left(1-\left(\frac{\left|\overline{x^k}-\overline{x_j}\right|}{2}\right)^2\right)^{1/2}\right]}\right\}\overline{x_j}}{\left|\sum_{j=1}^{N}\left\{\frac{w_j}{\left[\left|\overline{x^k}-\overline{x_j}\right|\left(1-\left(\frac{\left|\overline{x^k}-\overline{x_j}\right|}{2}\right)^2\right)^{1/2}\right]}\right\}\overline{x_j}\right|}$$

So far, there seem to be no published papers that have applied iterative method based on driving distances. Katz and Cooper [12] stated that if points are widely separated, the difference between Euclidean and Great circle distances may be significant, and this may lead to significant variations in the location of the corresponding optimal source points. Aykin [31] stated that the assumption that the earth can be considered as a plane is not appropriate and introduces error if the demand points are spread over a relatively large region.

These facts motivate me to do research and propose an approach for facility location decisions based on driving distances on spherical surface using heuristic technique and helps companies assess the optimal locations of facilities. In particular, the following problem is worth pursuing:

Given:

- The location of each destination in terms of their co-ordinates
- The requirement at each destination
- A set of shipping costs for the region of interest

To determine:

- The optimal number of facilities
- The location of each facility

The optimal facilities locations are those that minimize the driving distances related costs plus facilities costs.

The implementation method proposed in this paper is carried out by using heuristic iterative approach. This paper is organized as follows: Sections 1 is Introduction. Section 2 states the motivation for researching this paper and proposes a framework and algorithm. In Section 3, the facility locations decisions implementation of the proposed framework and algorithm is carried out using a hypothetical case study. In conclusion, Section 4 summarizes this study and outlines its contributions and future research.

3. Implementation Methodology for Facility Location Decisions

3.1. Algorithm

The algorithm of this paper is to develop a method to determine the optimal facility location to minimize the sum of facilities cost and the sum of the volume of goods at a destination multiplied by the transportation rate to ship to the destination multiplied by the Google maps driving distance based on the following assumptions:

1) The good of every destination points can be transported in one time, and the velocity is not changed.
2) The one destination point is only served by one warehouse.
3) The cost is related the length from the warehouse to the destination point, the transport conditions are not considered. Transportation cost is related to the distance only. The transportation cost equals the distances traveled times a fixed price per unit, distance.
4) The warehouse locations are located at populated places (cities/towns).
5) All service facilities are identical.
6) Each destination point wishes to minimize the cost of acquiring the product.
7) The company treats each cluster independently.

The strategy of the algorithm involves the following steps:

Step 1. Generate Google maps driving distance matrix from the set of destination locations' latitudes/longitudes.

Step 2. Perform K means clustering based on the destination locations driving distance matrix to generate K clusters.

Step 3. Calculate starting point of facility location for each cluster using Center of Gravity method and sets as current facility location.

Step 4. Construct the circle whose radius is the maximal Google maps driving distance from current facility location.

Step 5. Query the populated cities within the circle constructed in Step 4.

Step 6. Calculate the cost from each queried city to all destination locations, the queried city with the smallest cost will become new current point.

Step 7. Repeat steps 4 to 6 until no new current point can be found.

Step 8. Randomly select several cities points as current points and perform steps 4 to 6 to show that the local optimal facility location is the global optimal or nearly global optimal facility location.

Step 9. Calculate the total cost for each set of clusters.

Step 10. Perform another set of clustering if desired and repeat Steps 3 to 9 until no clustering is desired.

Step 11. Compare the cost of each set of clusters; select the set of clusters with the minimal total cost and optimal facility locations.

Figure 5 shows the flowchart of Facility location decisions process.

In **Figure 5**, the implementation details are described as follows and a lot of C# programs have been written for the implementation (Source codes are available from the author).

Step 1. A Google maps driving distance matrix is generated as a distance data set and used as input in SAS

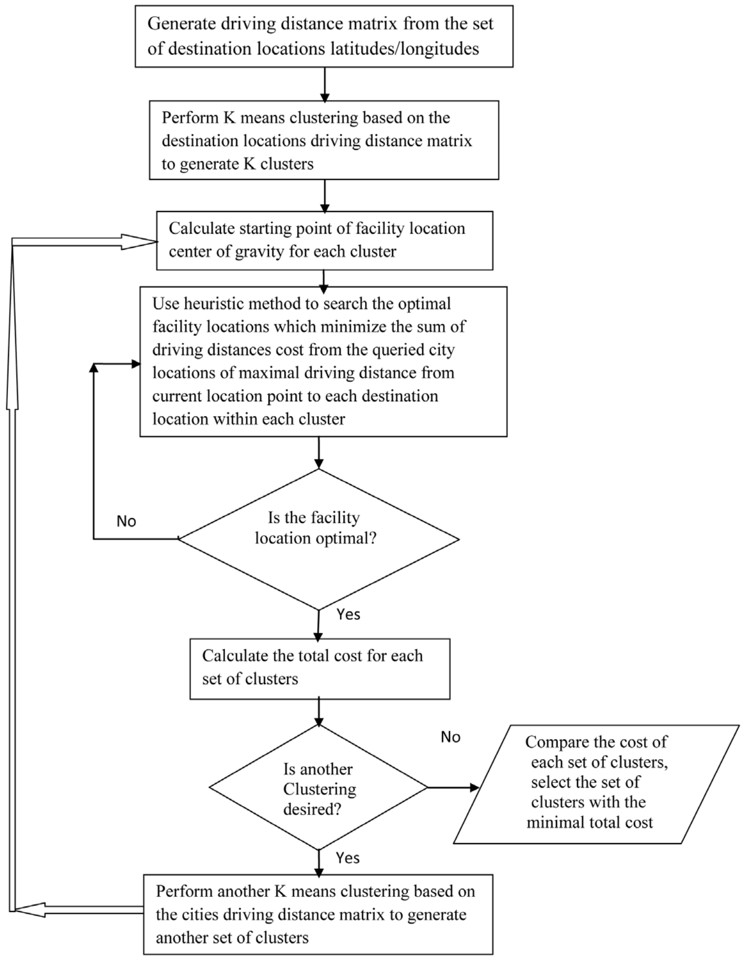

Figure 5. Flowchart of facility location decisions process.

Clustering procedure. A C# program is written using the Google Maps API (Google Inc. [25]) to create a driving distance matrix that will be used as input in SAS Clustering procedure.

Step 2. After driving distance matrix is generated, the next step is to cluster the cities based on Driving distances. This paper will use SAS Clustering procedure (SAS Inc. [32]) to cluster for the cities. TYPE = DISTANCE data set can be used as an input data set to PROC MODECLUS or PROC CLUSTER (SAS Inc. [32]). PROC CLUSTER ignores the upper triangular portion set and assumes that all main diagonal values are zero, even if they are missing. PROC MODECLUS uses the entire distance matrix and does not require the matrix to be symmetric. Since the driving distance matrix data set is not necessary symmetric, the MODECLUS Procedure is used in my paper to cluster the cities.

Step 3. Calculate the initial starting facility location centers for each cluster. Several quantitative techniques (Heizer and Render [33], Reid and Sanders [1] and Pearson [34]) have been developed to assist initial site selection. This paper uses Center of Gravity method (Ballou [11], Geo Midpoint [35], Chen and He [36], Heizer and Render [33] and Kuo and White [37]) to calculate the starting point of facility location. The Center of Gravity method assumes that the cost is directly proportional to distance and volume shipped, inbound and outbound transportation costs are equal, and it does not include special shipping costs for less than full loads. Using latitude and longitude coordinates might be helpful (Heizer and Render [33] and Chase *et al.* [38]) to calculate the initial facility location centers for each cluster. This paper assumes a spherical earth and sea-level points, the following formula is used to perform spherical coordinate conversion from latitude/longitudes to Cartesian coordinates for each destination location. The formula is much more complex for ellipsoidal earth (Ligas and Banasik [39]).

$$x_i = r * \cos(radianlat_i) * \cos(radianlon_i) \tag{1}$$

$$y_i = r * \cos(radianlat_i) * \sin(radianlon_i) \tag{2}$$

$$z_i = r * \sin(radianlat_i) \tag{3}$$

where r is the earth's radius.

Next, compute weight volume of good multiple by shipping cost for each location

$$w_i = v_i * r_i \tag{4}$$

where v_i is the volume of good to be shipped to location *i*, r_i is the shipping rate per unit of good to location *i*.

The initial center of gravity X_C, Y_C and Z_C are calculated as

$$X_C = \sum_i^n x_i w_i \Big/ \sum_i^n w_i \tag{5}$$

$$Y_C = \sum_i^n y_i w_i \Big/ \sum_i^n w_i \tag{6}$$

$$Z_C = \sum_i^n z_i w_i \Big/ \sum_i^n w_i \tag{7}$$

Then, the initial starting Cartesian coordinates are converted to latitude/longitudes points as input for Step 4.

$$lon = a\tan 2(Y_C, X_C) \tag{8}$$

$$hyp = sqrt(X_C * X_C + Y_C * Y_C) \tag{9}$$

$$lon = a\tan 2(Z_C, hyp) \tag{10}$$

Step 4. Calculate the driving distances from the current point calculated in Step 3 to each destination location of each cluster using the Google Maps API (Google Inc. [25]).

Step 5. Search the optimal facility locations. All distances are calculated using the Google maps driving distances. Let the starting point be the current point calculated in Step 3. Use the maximal Google maps driving distance calculated from Step 4 as radius of current point. The cities are queried in a circular pattern around the current point at a maximal driving distance. This paper utilizes a function in Map Suite WinForms Desktop free trial Edition (Thinkgeo LLC [40]) and a populated places database of Natural earth data source (Natural Earth [41]) to query all cities with the maximal driving distance of current location point.

Step 6. Calculate the total cost between each queried city to all destinations locations within each cluster. Compare the cost of each queried city and find the city which has minimal cost. If any of the queried cities has a new minimal cost, then that city will become new current point and go back to repeat steps 4 to 6 until no new smallest cost can be found. Otherwise, If none of the queried cities has a new minimal cost, then the current city becomes the optimal facility location for that cluster.

Step 7. Randomly select several points (Hillier and Lieberman [42]) and perform Steps 5 to 6 to show that the local optimal facility location obtained in Step 6 is the global optimal or nearly global optimal facility location.

Step 8. Calculate the total cost for all clusters in the cluster set.

Step 9. Generate other clusters if desired and repeat steps 3 to 8 to calculate the total cost for all clusters in the cluster set until no clustering is desired.

Step 10. Select the set of clusters which has the minimal total cost, the optimal facilities locations will be the cities found in Step 7.

3.2. Methodology Implementation

Based on the formulas from several papers (Rodriguez-Chia and Valero-Franco [20], Mwemezi and Huang [15], Katz and Cooper [12] and Litwhiler and Aly [43]), this paper formulates the model as follows:

$$\text{Min } TC = \sum_{k=1}^{m}\sum_{i=1}^{n_k} V_{ki} R_{ki} d_{ki} + \sum_{k=1}^{m} Y_k = \sum_{k=1}^{m}\left(\sum_{i=1}^{n_k} V_{ki} R_{ki} d_{ki} + Y_k\right) \tag{11}$$

Subject to: $m > 0$

$$n_k > 0 \quad \forall k, k = 1, \cdots, m$$

$$V_{ki} > 0 \quad \forall k, k = 1, \cdots, m; \forall i, i = 1, \cdots, n_k$$

$$R_{ki} > 0 \quad \forall k, k = 1, \cdots, m; \forall i, i = 1, \cdots, n_k$$

$$d_{ki} > 0 \quad \forall k, k = 1, \cdots, m; \forall i, i = 1, \cdots, n_k$$

$$Y_k > 0 \quad \forall k, k = 1, \cdots, m$$

$$\sum_{i=1}^{k} n_k = \text{total \# of destinations}$$

where TC = Total cost for m clusters

n_k = destination locations for each cluster

V_{ki} = Volume at destination *i* in cluster *k*

R_{ki} = Transportation rate to destination *i* in cluster *k*

d_{ki} = Driving distance from the facility to be located to destination *i* in cluster *k*

Y_k = facility cost at each cluster *k*

Given the assumptions described in Section 3.1. We search the optimal facility to minimize the sum of facilities cost and the sum of the volume at a destination multiplied by the transportation rate to ship to the destination multiplied by the driving distance for the cluster.

3.3. Hypothetical Case Study

ABC Company has 88 market cities located in Midwest, USA; the company seeks to build several warehouse centers to ship their goods to those markets to serve their customers. Assume that travel originates and ends at the warehouse centers. The coordinates (Latitude/Longitudes) of each market location and volume of goods and transportation rate for each market location are provided. The facility cost is 100,000.00. The goal is to minimize the sum of facilities costs and the sum of transportation costs. **Figure 6** is the graphical map of 88 major market cities.

Appendix A is the coordinates (Latitude/Longitudes) of each market location and volume of goods and transportation rate for each market location. The facility cost is 100,000.00.

Step 1. Converting major cities data to driving distance matrix

Use coordinates (Latitude/Longitudes) provided in **Appendix A** as inputs to create a driving distance matrix that will be used as inputs to MODECLUS clustering procedure in SAS. **Appendix B** is part of Google maps distance matrix of 88 Midwest major market cities.

Step 2. Clustering techniques

Set the driving distance matrix in **Appendix B** as input, the SAS MODECLUS Procedure is used to perform clustering for the cities. **Table 1** is cluster output from SAS code:

Proc modeclus data = work. Driving Data (TYPE = DISTANCE) list m = 1 k = 3; id location; run;

Figure 7 is the map of 14 clusters for the 88 major market cities located in Midwest, USA.

The following steps 3 to 7 are performed for cluster 1.

Step 3. Using equations from (1) to (7), the initial center of gravity for cluster 1 is calculated as in **Table 2** and shown in **Figure 8**.

Using equations (8) to (10), the Latitude and Longitude of Center of gravity is calculated as 41.5995134613, −96.62241872.

Step 4. **Table 3** shows the driving distances from current point to each market city location.

Step 5. The surrounding cities/towns with radius of maximum driving distance (157 miles) from Center of gravity are queried with Map suite tools and Natural populated placed data, the queried results are show in **Table 4** and the map is shown in **Figure 9**.

Step 6. **Table 5** shows the costs from each city to all market cities locations. From **Table 5**, the Depot 15 has the smallest cost, and then Depot 15 (Omaha, NEB) will become new current point. The driving distances from new current point to each market city location are calculated as in **Table 6**. The maximum driving distance is 190 miles.

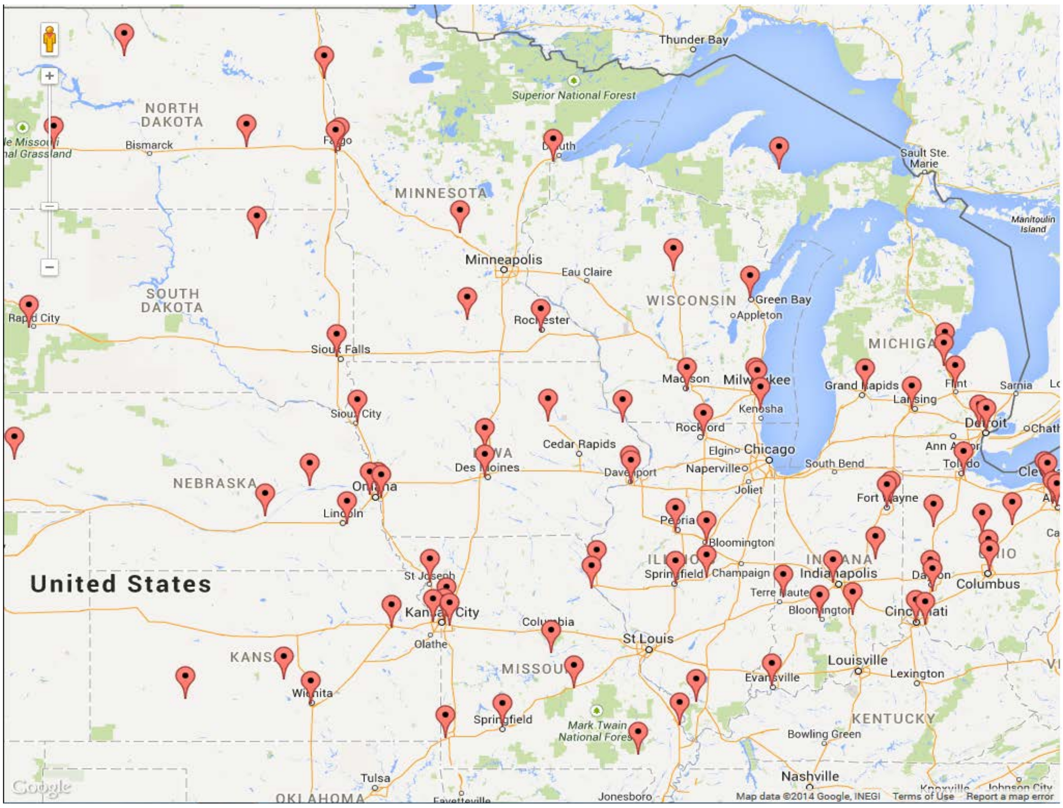

Figure 6. Map of 88 major market cities located in Midwest, USA.

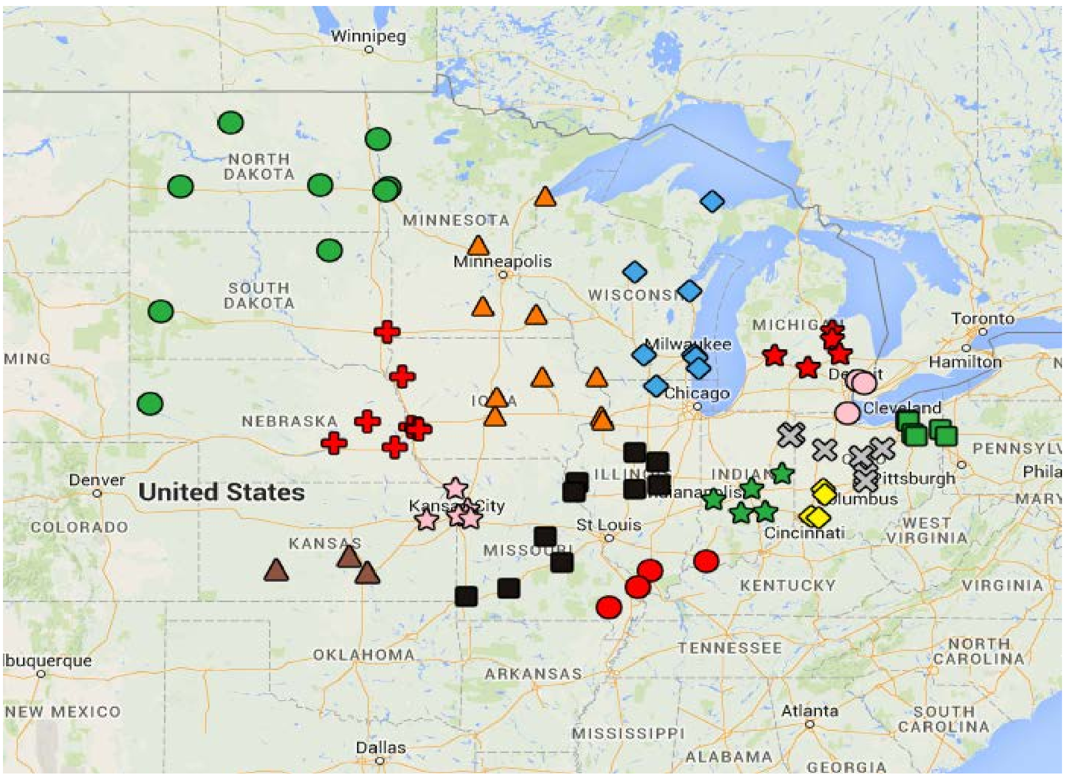

Figure 7. Map of 14 clusters for the 88 major market cities in Midwest, USA.

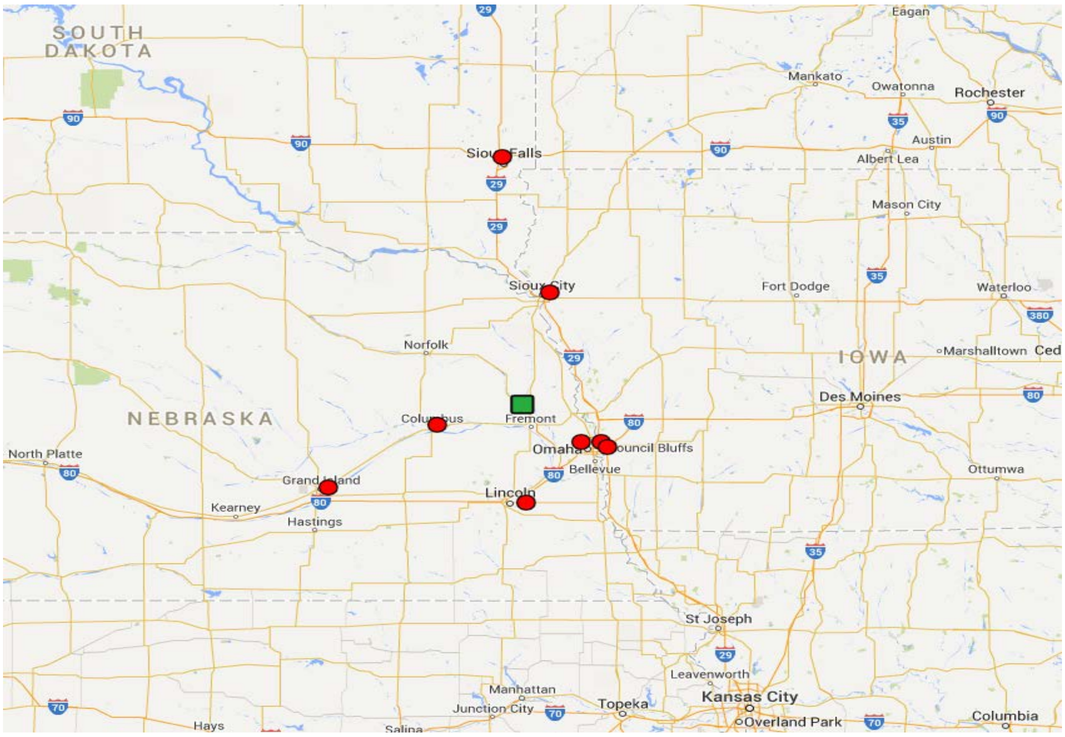

Figure 8. Cluster 1 with initial starting point- center of gravity (Scribner, NE—Green square).

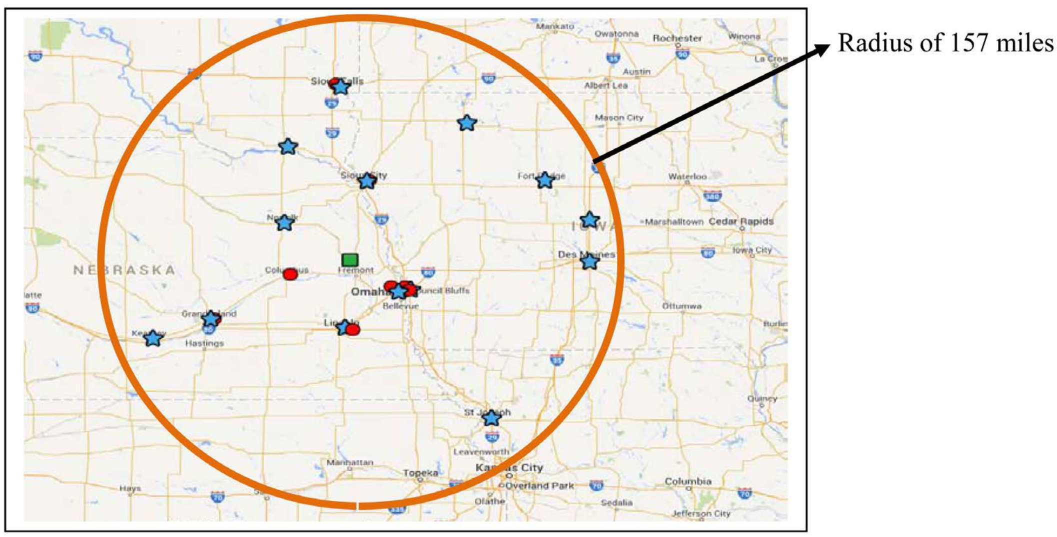

Figure 9. Cluster1 with Cities (Blue stars) of radius of maximum driving distance of current point (Scribner, NE—Green square).

Repeat Steps 4, 5, & 6. The surrounding cities/towns with radius of maximum driving distance (190 Miles in Table 6) from Depot 15 (Omaha, Nebraska, USA) are queried with Map Suite tools and Natural populated placed data, the queried results are show in Figure 10.

Table 7 is the queried cities within the radius of maximum driving distance (190 miles) from current point. Figure 10 is the map with queried cities of radius of maximum driving distance of current point. Table 8 shows the cost from each city to all market cities locations.

From Table 8, use Depot 23 as the current point (Omaha, Neb.) and repeat steps 4 to 6, there is no new smallest cost can be found.

Table 1. Modeclus cluster procedure with m = 1, K = 3.

Cluster	Location	Estimated Density	Same Cluster	Other Clusters	Total	Cluster	Location	Estimated Density	Same Cluster	Other Clusters	Total
Sums of Density Estimates Within Neighborhood											
1	Loc45	0.000179	0.000946	0	0.000946	8	Loc13	0.000361	0.0006687	0	0.000669
	Loc46	0.00084	0.002311	0	0.002311		Loc14	0.000306	0.0007238	0	0.000724
	Loc67	0.000106	0.000179	0.000151	0.00033		Loc15	0.00031	0.0007238	0	0.000724
	Loc1F	0.001495	0.001655	0	0.001655		Loc16	0.000363	0.0006711	0	0.000671
	Loc1G	0.000816	0.002335	0	0.002335	9	Loc19	0.000303	0.0006104	0	0.00061
	Loc1H	0.000299	0.002311	0	0.002311		Loc27	0.000303	0.0006099	0	0.00061
	Loc1I	0.000225	0.000988	0	0.000988		Loc28	0.000217	0.0003028	0.000275	0.000578
	Loc1J	0.000173	0.000524	0	0.000524		Loc29	0.000307	0.0006061	0	0.000606
2	Loc92	0.000744	0.00183	0	0.00183		Loc31	0.000216	0.0006099	0	0.00061
	Loc93	0.001086	0.001489	0	0.001489	10	Loc4	0.000266	0.0005507	0	0.000551
	Loc94	0.000648	0.00183	0	0.00183		Loc32	0.000283	0.0005344	0	0.000534
	Loc95	0.000304	0.001392	0	0.001392		Loc33	0.000268	0.000549	0	0.000549
	Loc1A	0.000744	0.00183	0	0.00183	11	Loc42	0.000167	0.0003166	0	0.000317
	Loc1B	0.000259	0.00183	0	0.00183		Loc43	0.00015	0.0003342	0	0.000334
3	Loc5	0.000541	0.001377	0	0.001377		Loc44	0.000167	0.0003686	0	0.000369
	Loc6	0.000685	0.001234	0	0.001234		Loc47	0.000201	0.0004421	0	0.000442
	Loc8	0.000513	0.001377	0	0.001377		Loc50	0.000228	0.0004151	0	0.000415
	Loc9	0.000693	0.001198	0	0.001198		Loc62	7.64E−05	0.0002726	0	0.000273
	Loc10	0.000354	0.000839	0	0.000839		Loc63	0.000151	0.000318	0	0.000318
	Loc11	0.000325	0.000867	0	0.000867		Loc64	0.000151	0.0002726	0	0.000273
4	Loc41	8.27E−05	0.000259	0	0.000259		Loc65	0.000122	0.0002273	0	0.000227
	Loc51	0.000636	0.001125	0	0.001125		Loc79	0.000214	0.00043	0	0.00043
	Loc52	0.000561	0.0012	0	0.0012	12	Loc66	0.000202	0.0004095	0	0.000409
	Loc53	0.000564	0.001197	0	0.001197		Loc68	9.42E−05	0.0003802	0	0.00038
	Loc54	0.000211	0.000754	0	0.000754		Loc69	7.41E−05	0.0001398	0	0.00014
	Loc55	0.000143	0.000677	0	0.000677		Loc70	0.000209	0.0004018	0	0.000402
	Loc56	0.000116	0.000354	0	0.000354		Loc71	0.0002	0.0004111	0	0.000411
	Loc78	0.000194	0.000776	0	0.000776		Loc72	0.000171	0.0004111	0	0.000411
5	Loc34	0.000363	0.000856	0	0.000856		Loc74	8.65E−05	0.000265	0	0.000265
	Loc35	0.000496	0.000724	0	0.000724		Loc75	9.42E−05	0.0002573	0	0.000257
	Loc36	0.00036	0.000859	0	0.000859		Loc1K	5.33E−05	7.411E−05	0.000173	0.000247
	Loc37	0.000254	0.000507	0	0.000507	13	Loc30	0.000143	0.0001705	0.000216	0.000386

Continued

	Loc40	0.000143	0.000617	0	0.000617		Loc86	0.00017	0.0003514	0	0.000351
6	Loc1	0.000341	0.000883	0	0.000883		Loc90	0.000208	0.0003068	0	0.000307
	Loc2	0.000473	0.000751	0	0.000751		Loc91	0.000136	0.0003786	0	0.000379
	Loc3	0.00041	0.000786	0	0.000786	14	Loc1C	0.000123	0.0001332	0.000259	0.000392
	Loc12	0.000313	0.000883	0	0.000883		Loc1D	0.000133	0.0002348	0	0.000235
	Loc18	0.000278	0.000684	0	0.000684		Loc1E	0.000112	0.0002558	0	0.000256
	Loc23	0.000275	0.000552	0	0.000552						
	Loc24	0.000274	0.000553	0	0.000553						
7	Loc80	0.000223	0.000611	0	0.000611						
	Loc81	0.000357	0.000619	0	0.000619						
	Loc83	0.000151	0.000417	0	0.000417						
	Loc84	0.000395	0.000611	0	0.000611						
	Loc85	0.000254	0.000752	0	0.000752						
	Loc89	0.000162	0.000405	0	0.000405						
	Loc96	0.000104	0.000158	0.000744	0.000902						
	Loc97	0.000159	0.00032	0	0.00032						
	Loc98	0.000158	0.000317	0	0.000317						
	Loc99	0.000158	0.000262	0	0.000262						

Table 2. Center of Gravity for cluster 1.

City Location	Latitude	Longitude	X Coordinate	Y Coordinate	Z Coordinate	Vi	Ri
IA Loc45	42.505046	−96.38003	−324.327336	−2900.569393	2674.918685	1100	0.03
IA Loc46	41.251631	−95.87447	−304.638558	−2960.828675	2610.434812	400	0.03
SD Loc67	43.577696	−96.80101	−339.639768	−2847.877667	2729.087508	1200	0.03
NE Loc1F	41.292321	−95.93427	−307.536885	−2958.664433	2612.547959	1500	0.04
NE Loc1G	41.296198	−96.11027	−316.604967	−2957.530006	2612.749234	800	0.04
NE Loc1H	40.798632	−96.59023	−343.959573	−2977.202028	2586.820615	1500	0.04
NE Loc1I	41.437838	−97.37192	−380.817694	−2943.427398	2620.09428	1200	0.04
NE Loc1J	40.922826	−98.33087	−433.420604	−2959.825426	2593.310824	500	0.04

City Location	Xi*Vi*Ri	Yi*Vi*Ri	Zi*Vi*Ri	Vi*Ri
IA Loc45	−10,702.80208861	−95,718.78997969	88,272.31661895	33.00
IA Loc46	−3,655.66269894	−35,529.94409800	31,325.21774265	12.00
SD Loc67	−12,227.03164169	−102,523.59600100	98,247.15027041	36.00
NE Loc1F	−18,452.21310819	−177,519.86597229	156,752.87753847	60.00
NE Loc1G	−10,131.35893869	−94,640.96019008	83,607.97548293	32.00
NE Loc1H	−20,637.57437751	−178,632.12169815	155,209.23690756	60.00
NE Loc1I	−18,279.24929086	−141,284.51511257	125,764.52543849	48.00
NE Loc1J	−8,668.41207663	−59,196.50851119	51,866.21648196	20.00
SUM	−102,754.30422113	−885,046.30156298	791,045.51648143	301.00
AVEAGE	X Coordinate	Y Coordinate e	Z Coordinate	
	−341.37642598	−2,940.35316134	2,628.05819429	

Table 3. Driving distances from current point to each market location.

Driving Distance	From Latitude	Longitude	To Latitude	Longitude	City Location
72.6	41.5995134613	−96.62241872	42.505046	−96.380030	IA Loc45
63.6	41.5995134613	−96.62241872	41.251631	−95.874470	IA Loc46
157	41.5995134613	−96.62241872	43.577696	−96.801010	SD Loc67
54.3	41.5995134613	−96.62241872	41.292321	−95.934270	NE Loc1F
43.6	41.5995134613	−96.62241872	41.296198	−96.110270	NE Loc1G
71.6	41.5995134613	−96.62241872	40.798632	−96.590230	NE Loc1H
61.8	41.5995134613	−96.62241872	41.437838	−97.371920	NE Loc1I
125	41.5995134613	−96.62241872	40.922826	−98.330870	NE Loc1J

Table 4. The queried cities within the radius of maximum driving distance from current point Radius of 157 miles.

Latitude	Longitude	City	State	Country
41.5995134613	−96.6224187157			
43.1452850536	−95.1471745217	Spencer	Iowa	USA
42.5068227326	−94.1802567970	Ft. Dodge	Iowa	USA
40.7007055858	−99.0811462827	Kearney	Nebraska	USA
40.9222682892	−98.3579862892	Grand Island	Nebraska	USA
42.5003890168	−96.3999921079	Sioux City	Iowa	USA
41.2622733755	−95.8608002134	Council Bluffs	Iowa	USA
42.0538529654	−93.6197225360	Ames	Iowa	USA
39.7690311880	−94.8463918475	St. Joseph	Missouri	USA
42.0287123810	−97.4335982684	Norfolk	Nebraska	USA
42.8820194693	−97.3924896665	Yankton	South Dakota	USA
40.8199747915	−96.6800008563	Lincoln	Nebraska	USA
43.5499890331	−96.7299978045	Sioux Falls	South Dakota	USA
41.5799800812	−93.6199809181	Des Moines	Iowa	USA
41.2400008332	−96.0099900734	Omaha	Nebraska	USA

Step 7. Randomly select several points (Hillier and Lieberman [42]) and perform Steps 5 to 6 to show that the local optimal facility location obtained in Step 6 is the global optimal or nearly global optimal facility location. **Tables 9-12** show the results of each randomly selected city to all market cities locations. It is concluded that the global optimal facility location is the city found in Step 6. The optimal facility location will be Omaha, Nebraska shown as in **Figure 11**.

Using the same steps for cluster 1, the optimal facility locations for clusters 2 to 14 are calculated and shown as from **Figures 12-24**.

The total cost for these 14 clusters is calculated using equation (11) and shown as follows:

Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	Cluster 6	Cluster 7	Cluster 8	Cluster 9	Cluster 10
21,222.6	5,593.8	3,392.8	38,896.4	11,857.5	11,339.2	57,779.0	1,660.4	9,898.2	1,805.6

Cluster 11	Cluster 12	Cluster 13	Cluster 14	# of Facility	Facility cost per unit	Facilities cost	Total Cost
92,088.5	72,057.0	12,892.2	8,700.0	14	100,000.00	1,400,000.00	**1,749,183.2**

Table 5. Cluster1 costs from queried cities to market locations-iteration1.

Center of Gravity and queried cities with Latitudes/Longitudes	Cities ID	Cost
41.5995864980917, −96.6224184226743; Center Of Gravity	Depot = 1	Cost = 23331
43.14528505360, −95.14717452170; Spencer, Iowa, USA	Depot = 2	Cost = 52554.1
42.50682273260, −94.18025679700; Ft. Dodge, Iowa, USA	Depot = 3	Cost = 57508
40.70070558580, −99.08114628270; Kearney, Nebraska, USA	Depot = 4	Cost = 54229
40.92226828920, −98.35798628920; Grand Island, Nebraska, USA	Depot = 5	Cost = 39933.2
42.50038901680, −96.39999210790; Sioux City, Iowa, USA	Depot = 6	Cost = 31472.9
41.26227337550, −95.86080021340; Council Bluffs, Iowa, USA	Depot = 7	Cost = 21732.4
42.05385296540, −93.61972253600; Ames, Iowa, USA	Depot = 8	Cost = 64183
39.76903118800, −94.84639184750; St. Joseph, Missouri, USA	Depot = 9	Cost = 56370
42.02871238100, −97.43359826840; Norfolk, Nebraska, USA	Depot = 10	Cost = 31357.3
42.88201946930, −97.39248966650; Yankton, South Dakota, USA	Depot = 11	Cost = 42640.6
40.81997479150, −96.68000085630; Lincoln, Nebraska, USA	Depot = 12	Cost = 25181
43.54998903310, −96.72999780450; Sioux Falls, South Dakota, USA	Depot = 13	Cost = 49678
41.57998008120, −93.61998091810; Des Moines, Iowa, USA	Depot = 14	Cost = 58027
41.24000083320, −96.00999007340; Omaha, Nebraska, USA	Depot = 15	Cost = 21222.6

Table 6. Driving distances from current point (Omaha, NE) to each market location.

Driving Distance	From Latitude	Longitude	To Latitude	Longitude	City Location
104	41.24000083320	−96.00999007340	42.505046	−96.380030	IA Loc45
11.4	41.24000083320	−96.00999007340	41.251631	−95.874470	IA Loc46
190	41.24000083320	−96.00999007340	43.577696	−96.801010	SD Loc67
9.5	41.24000083320	−96.00999007340	41.292321	−95.934270	NE Loc1F
10.6	41.24000083320	−96.00999007340	41.296198	−96.110270	NE Loc1G
50.1	41.24000083320	−96.00999007340	40.798632	−96.590230	NE Loc1H
83.5	41.24000083320	−96.00999007340	41.437838	−97.371920	NE Loc1I
143	41.24000083320	−96.00999007340	40.922826	−98.330870	NE Loc1J

3.4. Results: Driving Distance vs. Euclidean Distance and Great Circle Distance

With the data in **Table 13**, I use the steps described in Section 3.2 to search the optimal facility location based on driving distance vs. Euclidean distance and Great circle distance**.** As the results shown in **Figures 25-27**, the optimal facility location based on driving distance is different from the optimal facility location based on Euclidean distance and Great circle distance.

With the data in this case study, I use the steps described in Section 3.2 to search the optimal facility location based on Euclidean distance and Great circle distance. It is found out that the optimal facilities locations for Clusters 1 to 14 are the same when the distances are based on driving distance, Euclidean distance or Great circle distance. For examples, Cluster 3 (see **Figure 13** and **Figure 28**), Cluster 4 (see **Figure 14** and **Figure 29**), Cluster 5 (see **Figure 15** and **Figure 30**) and Cluster 10 (see **Figure 20** and **Figure 31**) have the same optimal facilities locations.

However, the above results reveal that if the driving distance is much longer than Euclidean distance or Great circle distance because there are lakes, rivers between locations (see **Figures 2-4** and **Figure 25**), the optimal facility location based on driving distance will be different from the optimal facility location based on Euclidean

Table 7. The cities within the radius of maximum driving distance from current point (Omaha, NE).

Latitude	Longitude	City	State	Country
41.2400008332	−96.0099900734			
41.0128829133	−92.4148090024	Ottumwa	Iowa	USA
43.1452850536	−95.1471745217	Spencer	Iowa	USA
42.5068227326	−94.1802567970	Ft. Dodge	Iowa	USA
39.1135805169	−94.6301463771	Kansas City	Kansas	USA
38.9597524201	−95.2552299416	Lawrence	Kansas	USA
39.1940275259	−96.5924351418	Manhattan	Kansas	USA
39.0911139110	−94.4152812111	Independence	Missouri	USA
40.7007055858	−99.0811462827	Kearney	Nebraska	USA
40.9222682892	−98.3579862892	Grand Island	Nebraska	USA
42.5003890168	−96.3999921079	Sioux City	Iowa	USA
41.2622733755	−95.8608002134	Council Bluffs	Iowa	USA
42.0538529654	−93.6197225360	Ames	Iowa	USA
38.8246702261	−97.6071794045	Salina	Kansas	USA
39.7690311880	−94.8463918475	St. Joseph	Missouri	USA
42.0287123810	−97.4335982684	Norfolk	Nebraska	USA
42.8820194693	−97.3924896665	Yankton	South Dakota	USA
40.8199747915	−96.6800008563	Lincoln	Nebraska	USA
39.0500053091	−95.6699849871	Topeka	Kansas	USA
39.1070885098	−94.6040942189	Kansas City	Missouri	USA
43.5499890331	−96.7299978045	Sioux Falls	South Dakota	USA
41.5799800812	−93.6199809181	Des Moines	Iowa	USA
41.2400008332	−96.0099900734	Omaha	Nebraska	USA

Table 8. Cluster1 costs from queried cities to market locations—iteration 2.

Current point and queried cities with Latitudes/Longitudes	Cities ID	Cost
41.24000083320, −96.00999007340; Current point	Depot = 1	Cost = 21222.6
41.01288291330, −92.41480900240; Ottumwa, Iowa, USA	Depot = 2	Cost = 83407
43.14528505360, −95.14717452170; Spencer, Iowa, USA	Depot = 3	Cost = 52554.1
42.50682273260, −94.18025679700; Ft. Dodge, Iowa, USA	Depot = 4	Cost = 57508
39.11358051690, −94.63014637710; Kansas City, Kansas, USA	Depot = 5	Cost = 71387
38.95975242010, −95.25522994160; Lawrence, Kansas, USA	Depot = 6	Cost = 76203
39.19402752590, −96.59243514180; Manhattan, Kansas, USA	Depot = 7	Cost = 61467
39.09111391100, −94.41528121110; Independence, Missouri, USA	Depot = 8	Cost = 75485
40.70070558580, −99.08114628270; Kearney, Nebraska, USA	Depot = 9	Cost = 54229
40.92226828920, −98.35798628920; Grand Island, Nebraska, USA	Depot = 10	Cost = 39933.2

Continued

42.50038901680, −96.39999210790; Sioux City, Iowa, USA	Depot = 11	Cost = 31472.9
41.26227337550, −95.86080021340; Council Bluffs, Iowa, USA	Depot = 12	Cost = 21732.4
42.05385296540, −93.61972253600; Ames, Iowa, USA	Depot = 13	Cost = 64183
38.82467022610, −97.60717940450; Salina, Kansas, USA	Depot = 14	Cost = 76040
39.76903118800, −94.84639184750; St. Joseph, Missouri, USA	Depot = 15	Cost = 56370
42.02871238100, −97.43359826840; Norfolk, Nebraska, USA	Depot = 16	Cost = 31357.3
42.88201946930, −97.39248966650; Yankton, South Dakota, USA	Depot = 17	Cost = 42640.6
40.81997479150, −96.68000085630; Lincoln, Nebraska, USA	Depot = 18	Cost = 25181
39.05000530910, −95.66998498710; Topeka, Kansas, USA	Depot = 19	Cost = 66331
39.10708850980, −94.60409421890; Kansas City, Missouri, USA	Depot = 20	Cost = 72022
43.54998903310, −96.72999780450; Sioux Falls, South Dakota, USA	Depot = 21	Cost = 49678
41.57998008120, −93.61998091810; Des Moines, Iowa, USA	Depot = 22	Cost = 58027
41.24000083320, −96.00999007340; Omaha, Nebraska, USA	Depot = 23	Cost = 21222.6

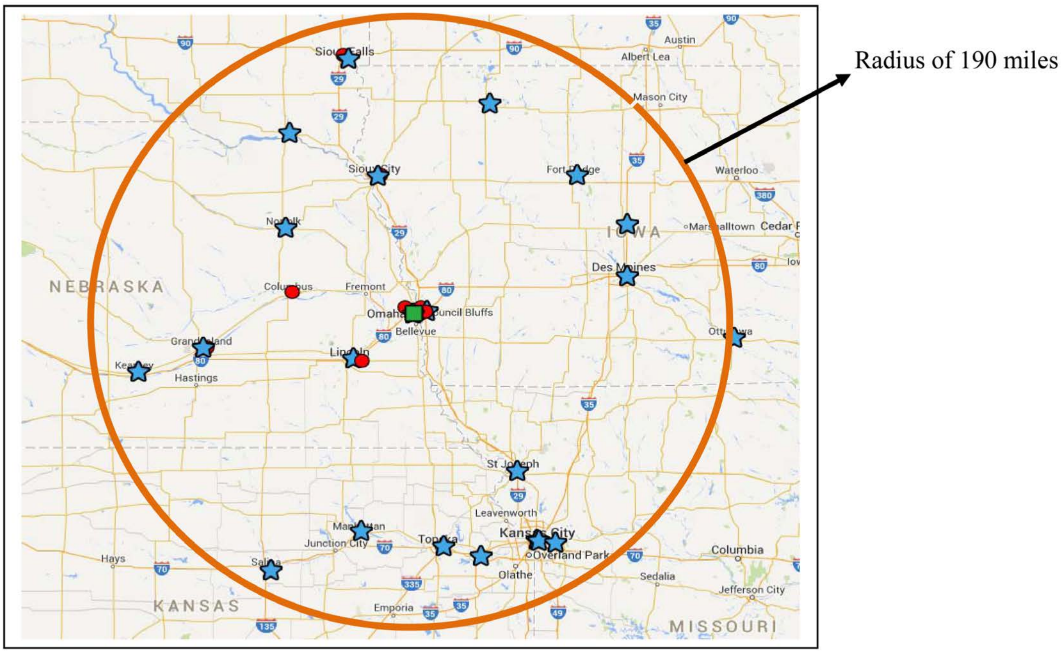

Figure 10. Cluster1 with cities (Blue stars) of radius of maximum driving distance of current point (Omaha, Neb.)

distance and Great circle distance. *The facility location decision based on driving distances is a practical approach. In regard to transportation cost, the driving distances in the presence of geographic barriers should be taken into consideration in facility location decisions*.

3.5. Compare the Total Cost of Different Set of Clusters

Now, this paper uses MODECLUS Procedure to get different set of clustering and select the best set of clusters.
Table 14 is clustering output of 11 clusters from SAS code with $m = 1$, $k = 4$.
Table 15 is clustering output of 9 clusters from SAS code with $m = 1$, $k = 5$.

From equation (11) $TC = \sum_{k=1}^{m}\sum_{i=1}^{n_k} V_{ki}R_{ki}d_{ki} + \sum_{k=1}^{m} Y_k = \sum_{k=1}^{m}\left(\sum_{i=1}^{n_k} V_{ki}R_{ki}d_{ki} + Y_k\right)$

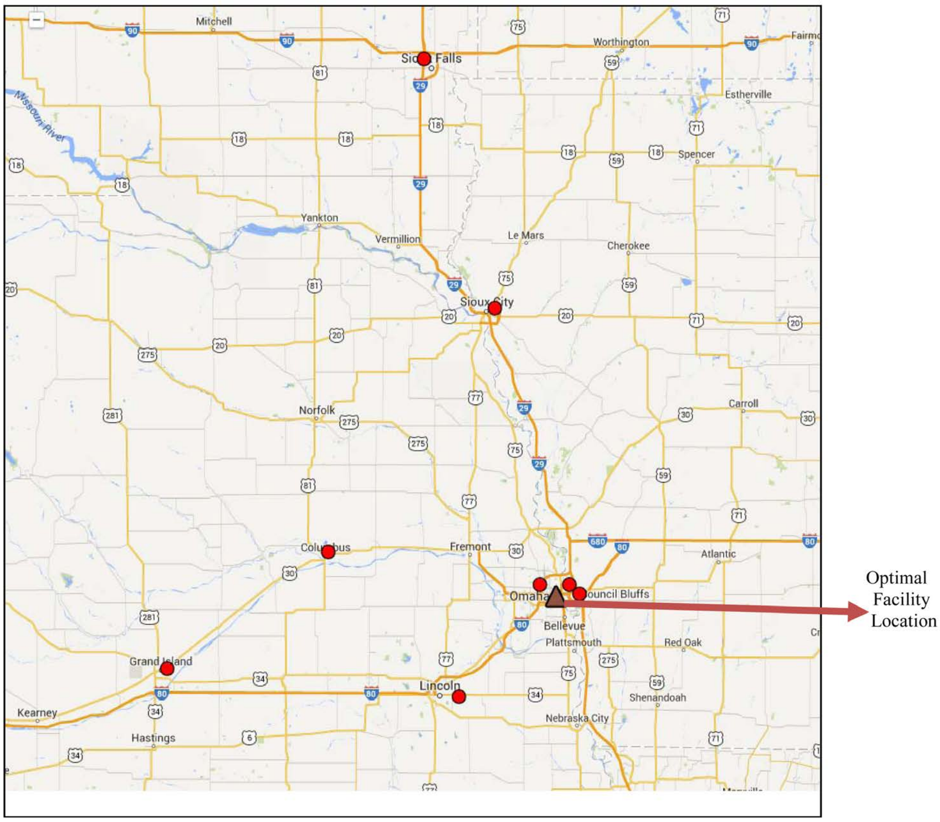

Figure 11. Cluster 1 with optimal facility location (Omaha, Neb.—Brown triangle).

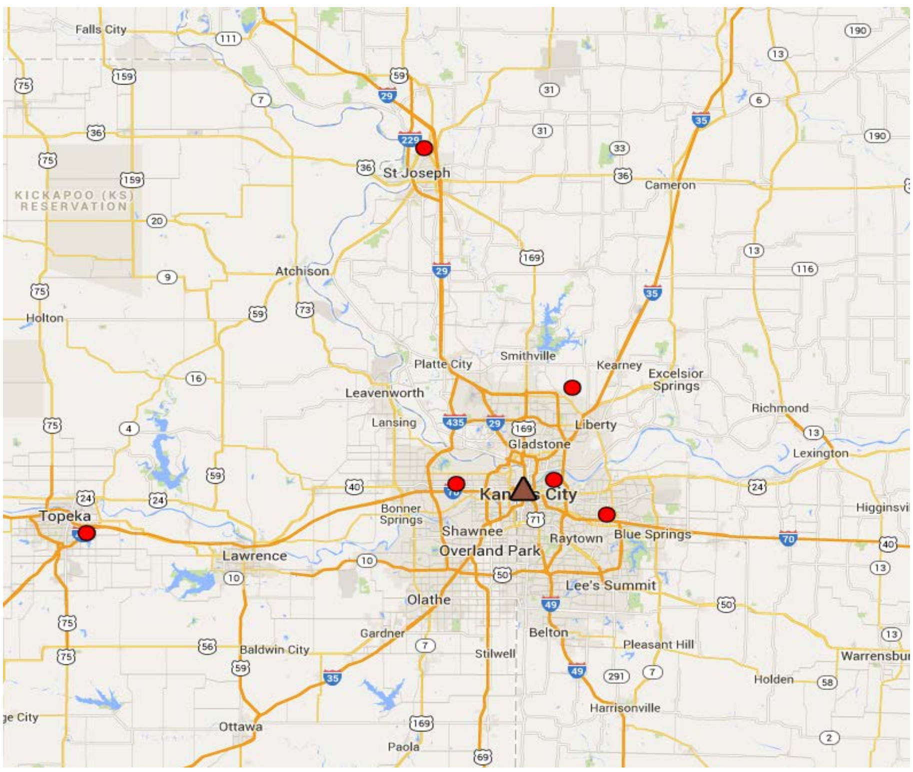

Figure 12. Cluster 2 with optimal facility location (Kansas City, MO—Brown triangle).

Figure 13. Cluster 3 with optimal facility location (Akron, OH—Brown triangle).

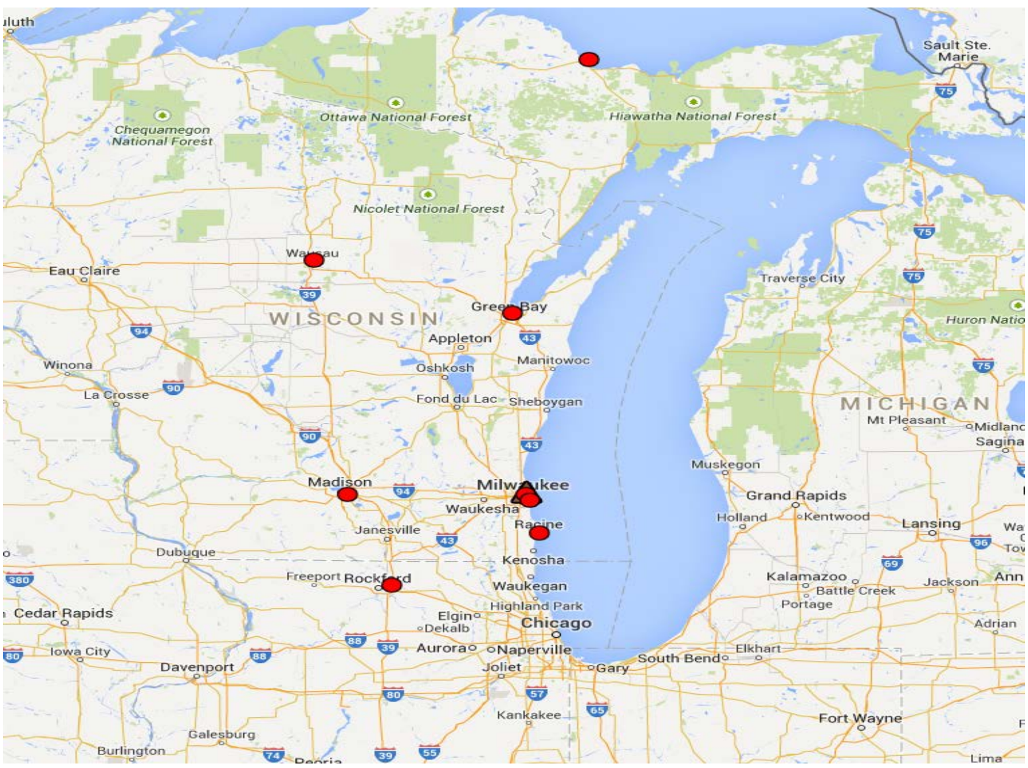

Figure 14. Cluster 4 with optimal facility location (Milwaukee, WI—Brown triangle).

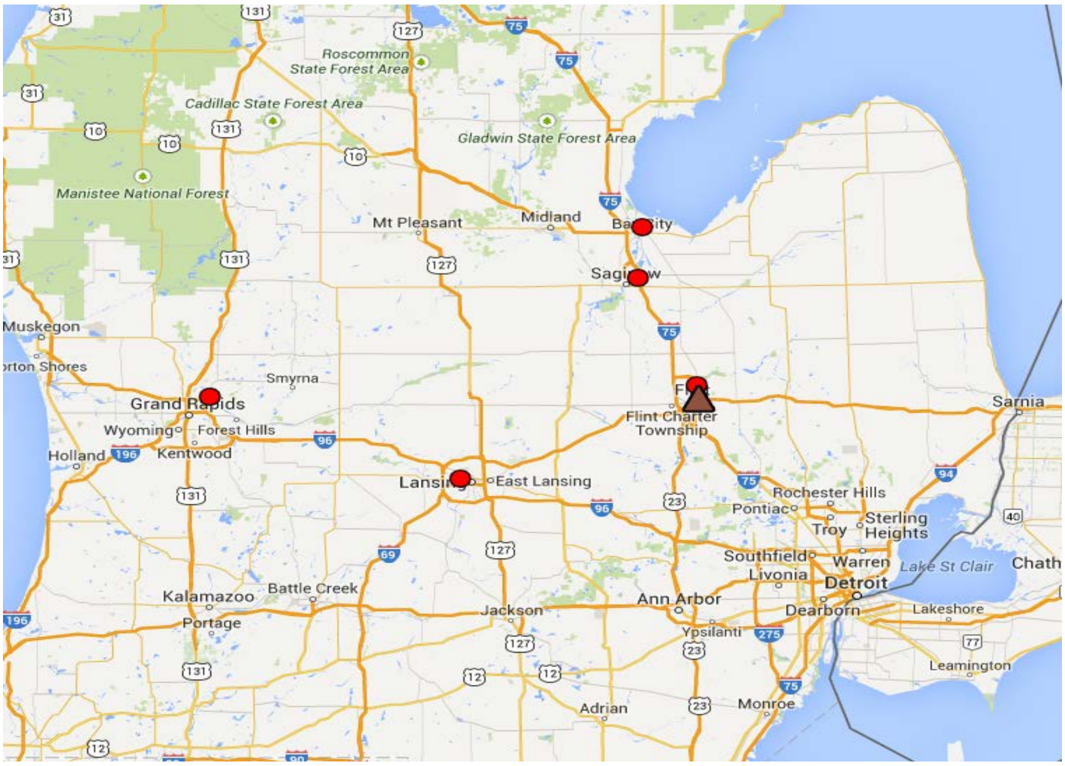

Figure 15. Cluster 5 with optimal facility location (Flint, MI—Brown triangle).

Figure 16. Cluster 6 with optimal facility location (Lima, OH—Brown triangle).

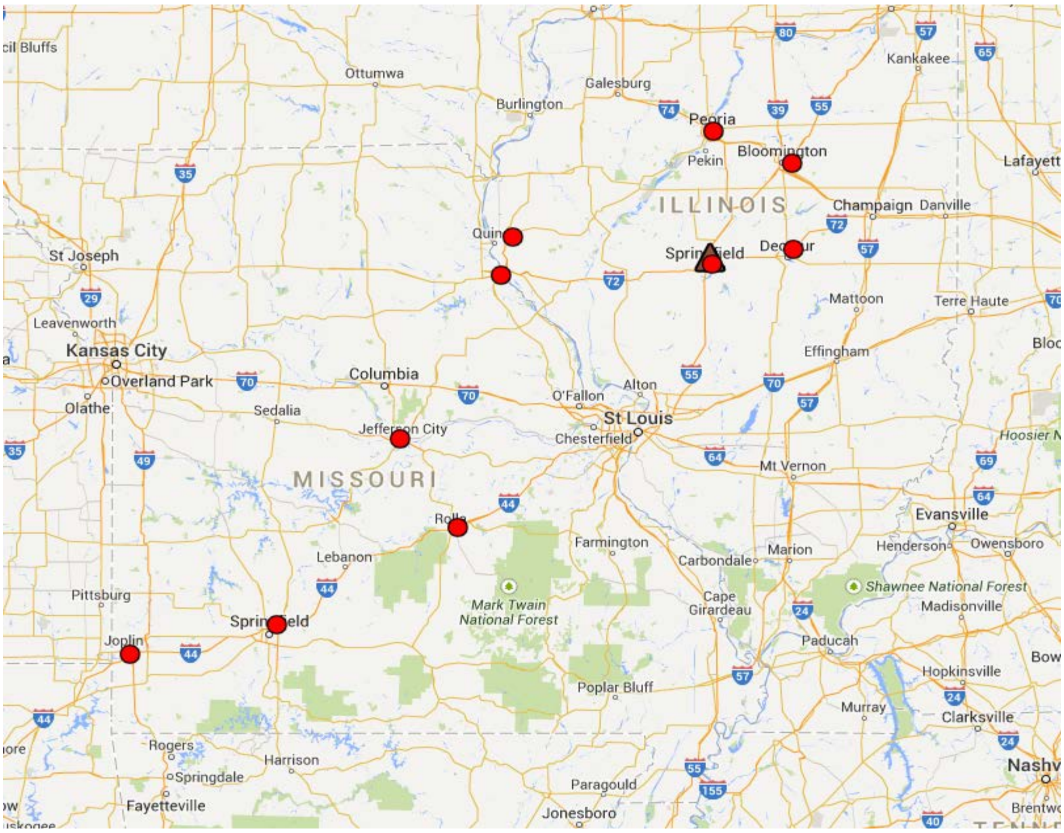

Figure 17. Cluster 7 with optimal facility location (Springfield, IL—Brown triangle).

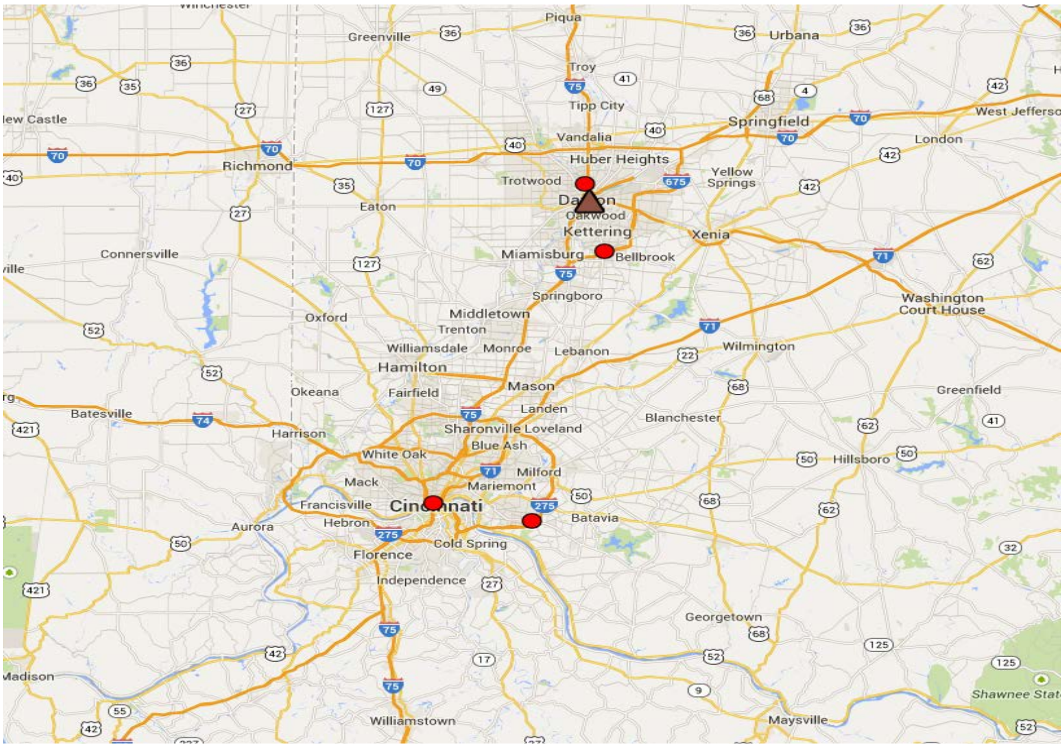

Figure 18. Cluster 8 with optimal facility location (Dayton, OH—Brown triangle).

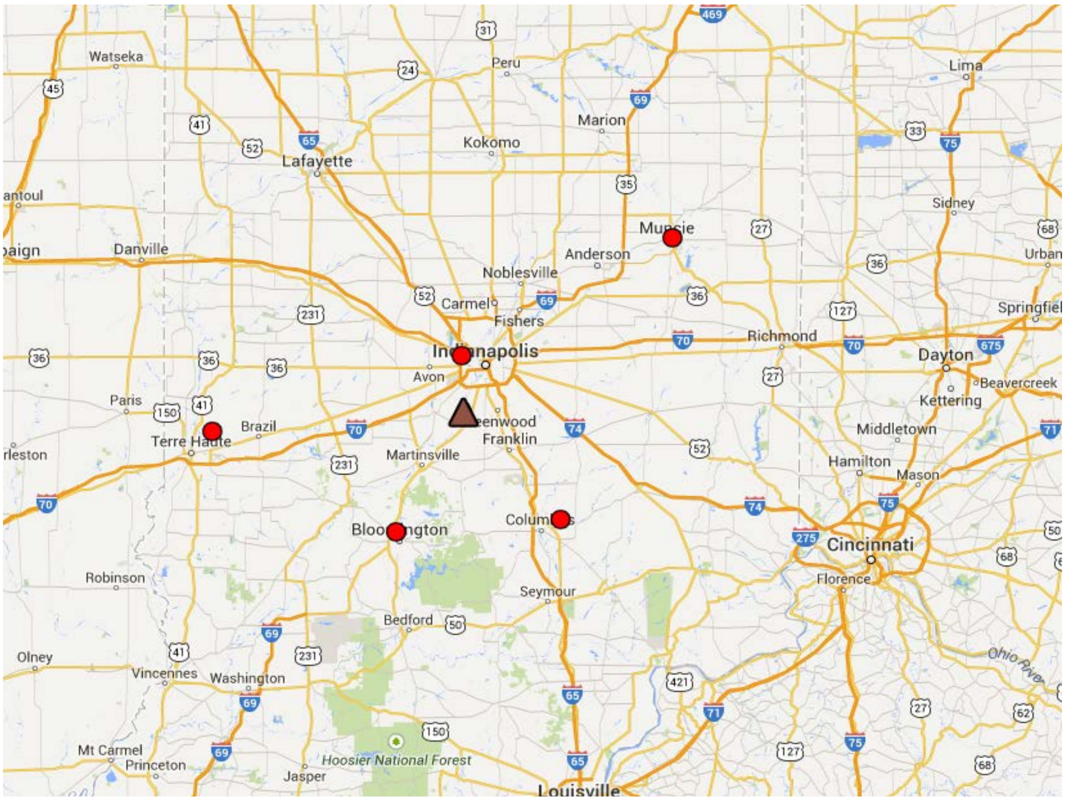

Figure 19. Cluster 9 with optimal facility location (Mooresville, IN—Brown triangle).

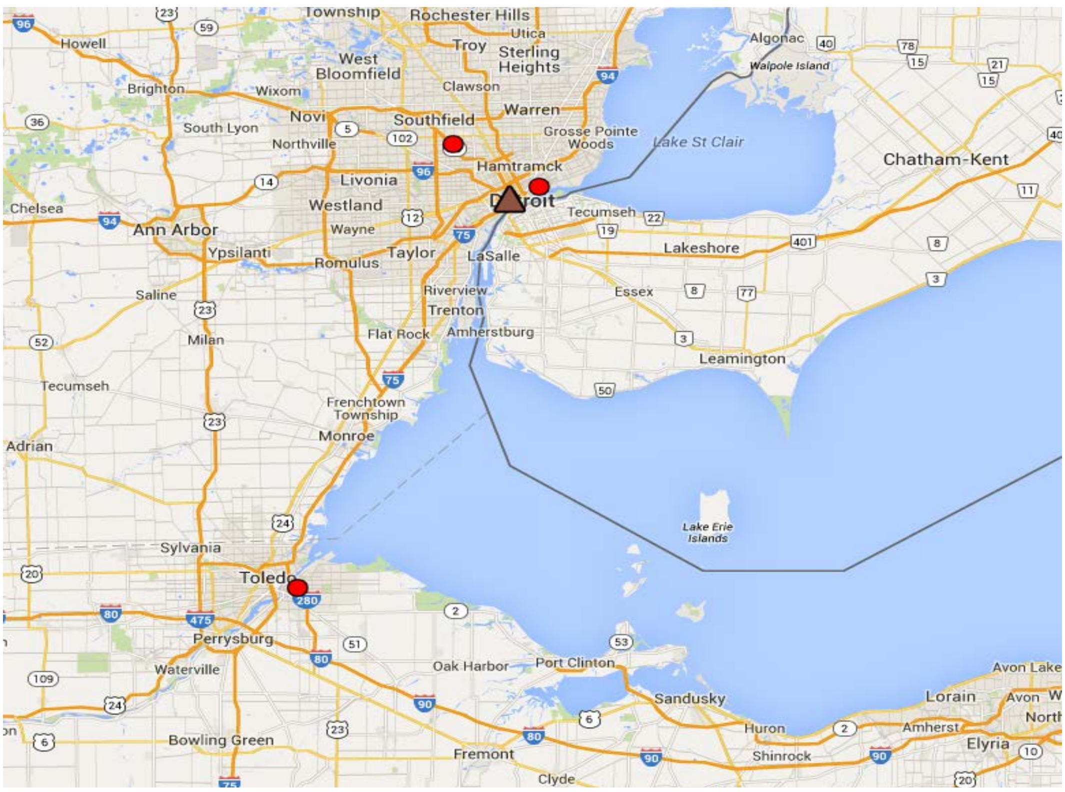

Figure 20. Cluster 10 with optimal facility location (Detroit, MI—Brown triangle).

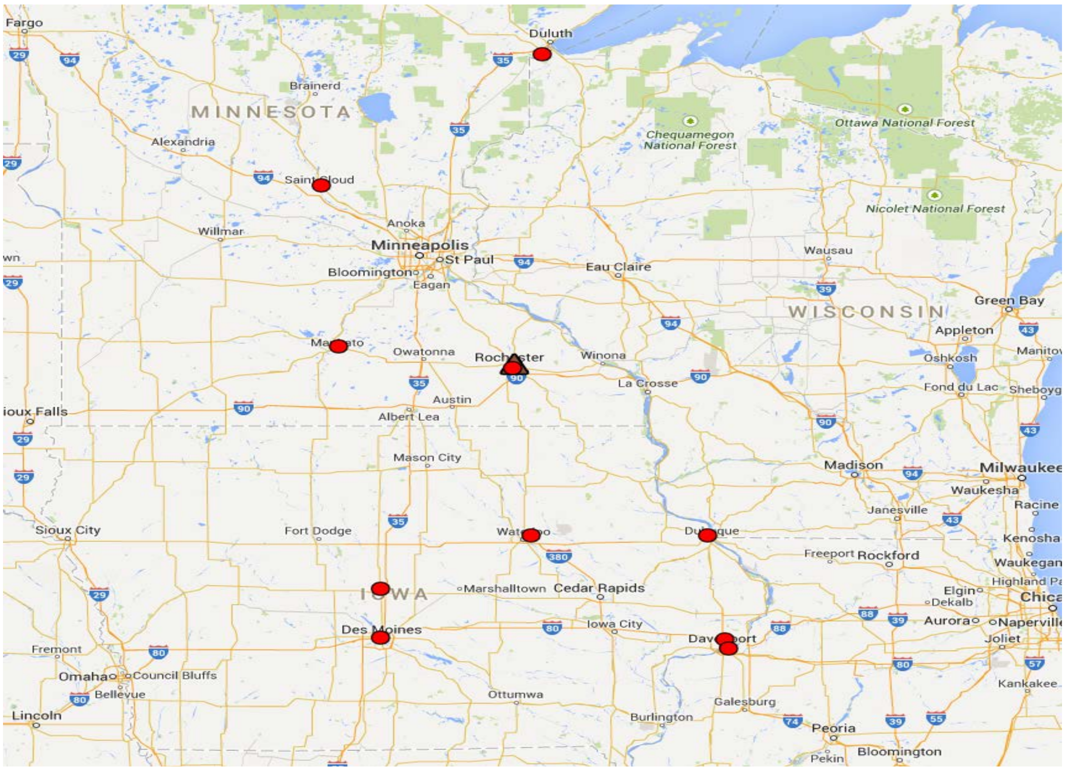

Figure 21. Cluster 11 with optimal location (Rochester, MN—Brown triangle).

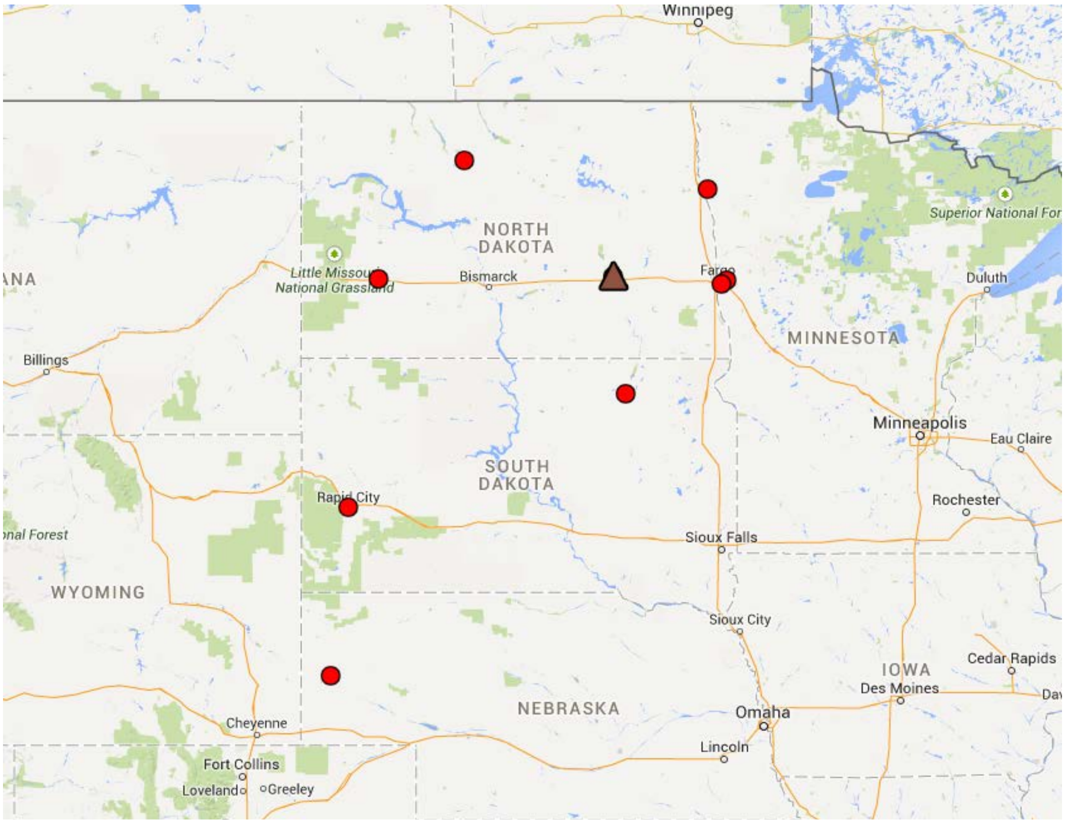

Figure 22. Cluster 12 with optimal facility location (Jamestown, ND—Brown triangle).

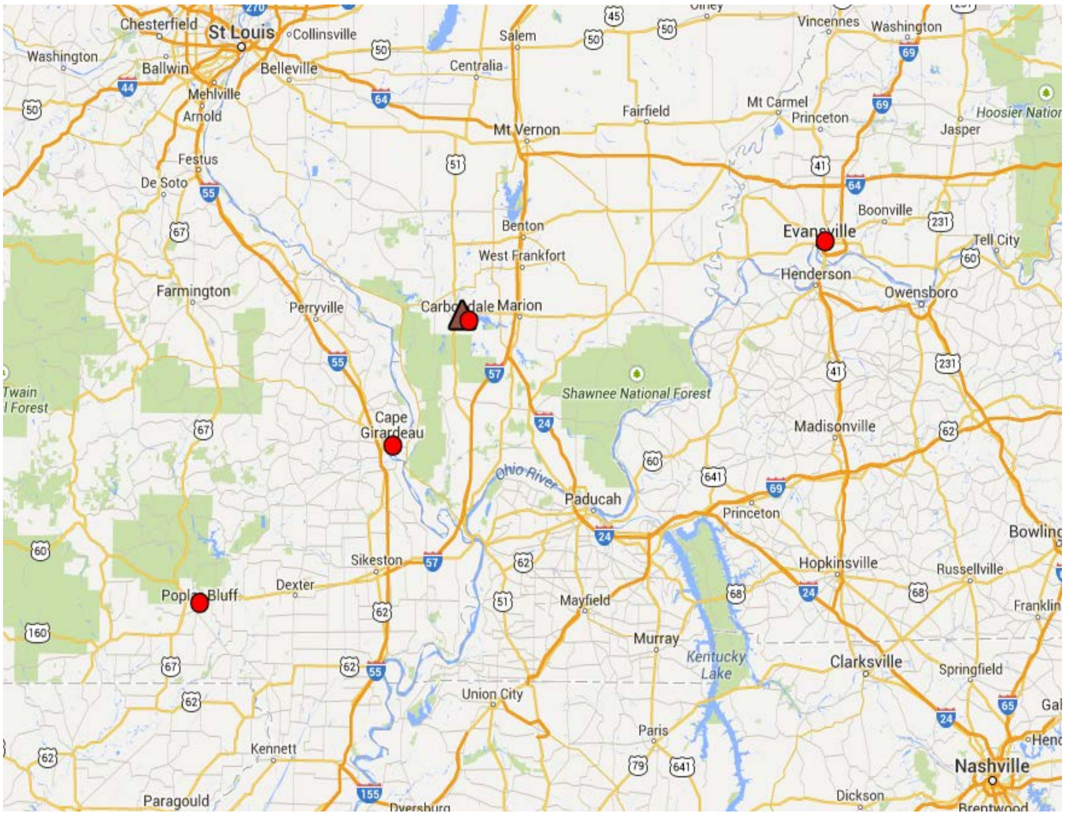

Figure 23. Cluster 13 with optimal location (Carbondale, IL—Brown triangle).

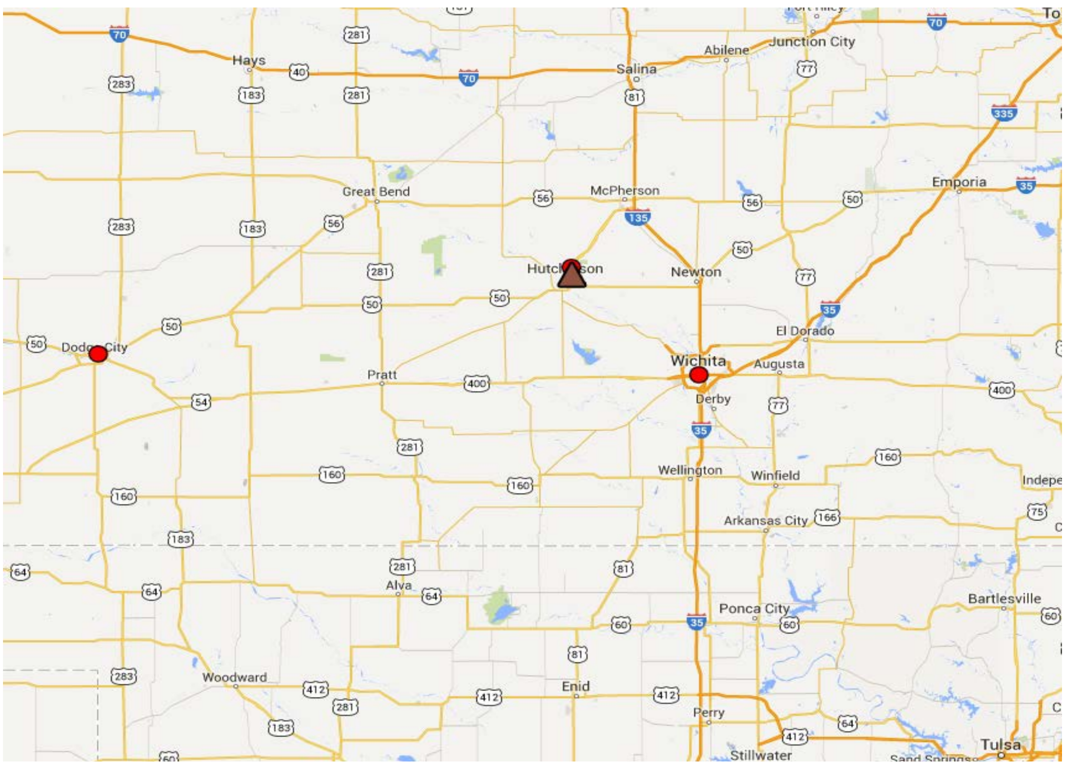

Figure 24. Cluster 14 with optimal facility location (Hutchinson, KS—Brown triangle).

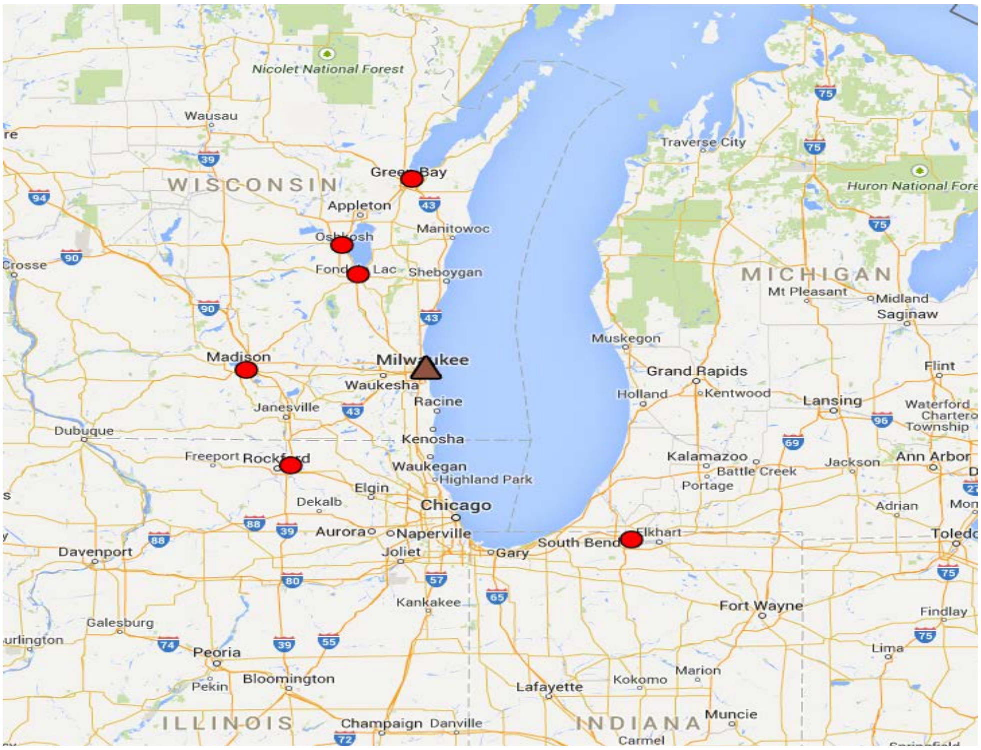

Figure 25. Optimal facility location (Milwaukee, WI) based on driving distance.

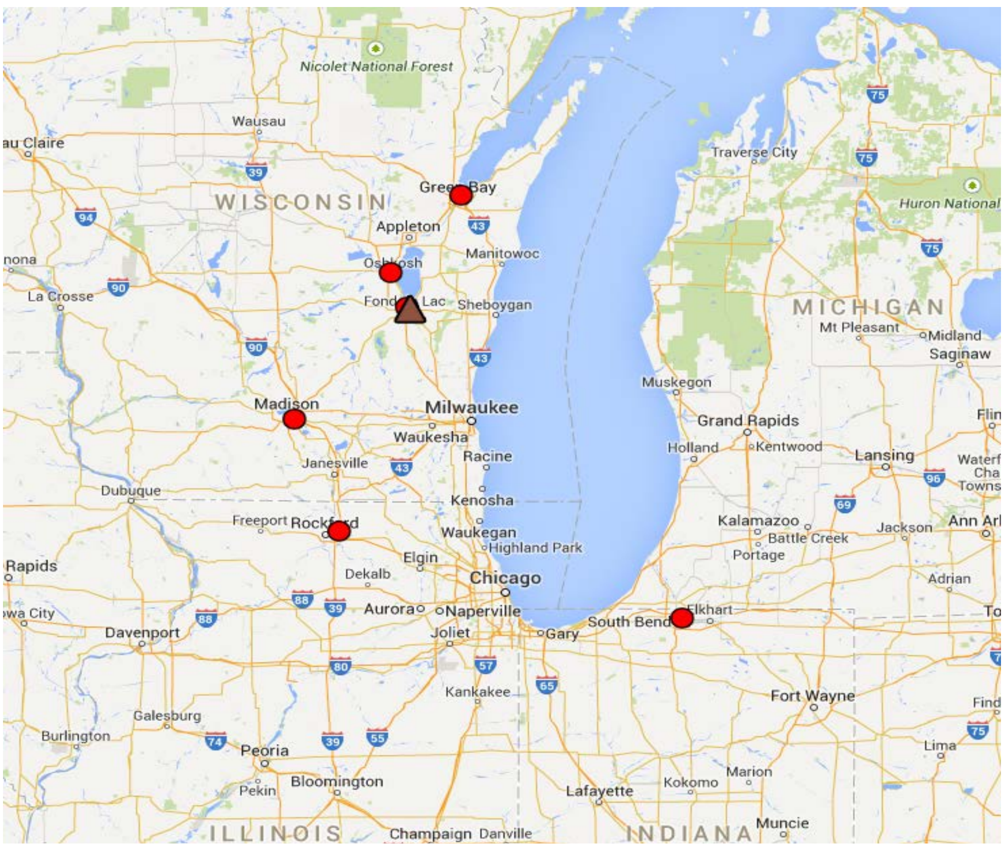

Figure 26. Optimal facility location (Fond du Lac, WI) based on Euclidean distance.

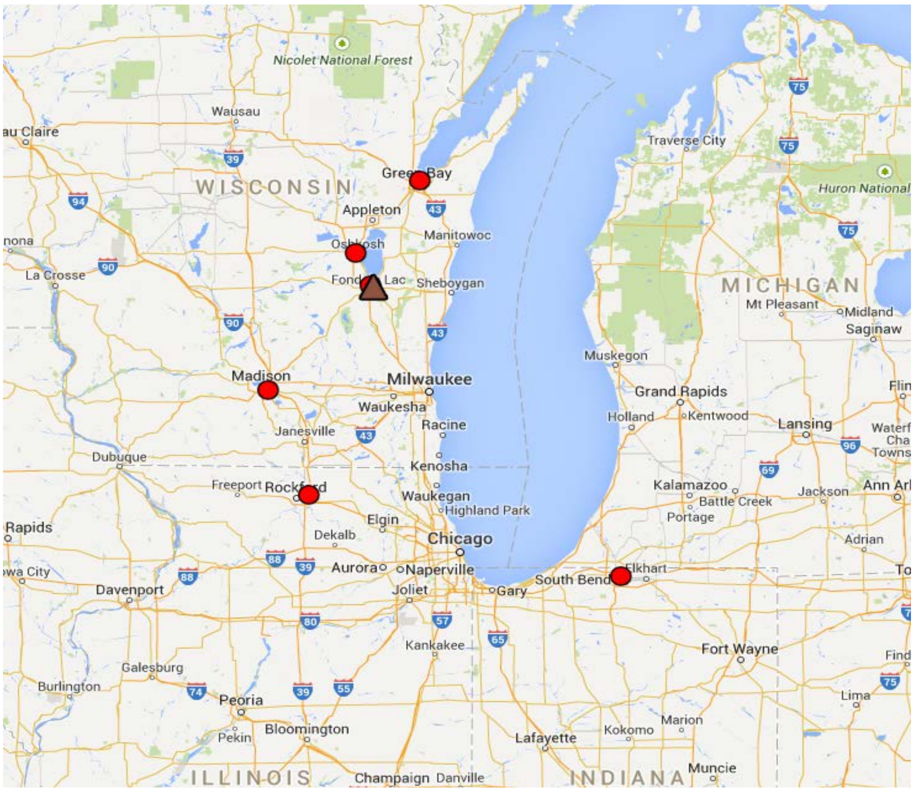

Figure 27. Optimal facility location (Fond du Lac, WI) based on Great circle distance.

Figure 28. Cluster 3 with optimal facility location (Akron, OH—Brown triangle) based on Euclidean distance or Great Circle distance.

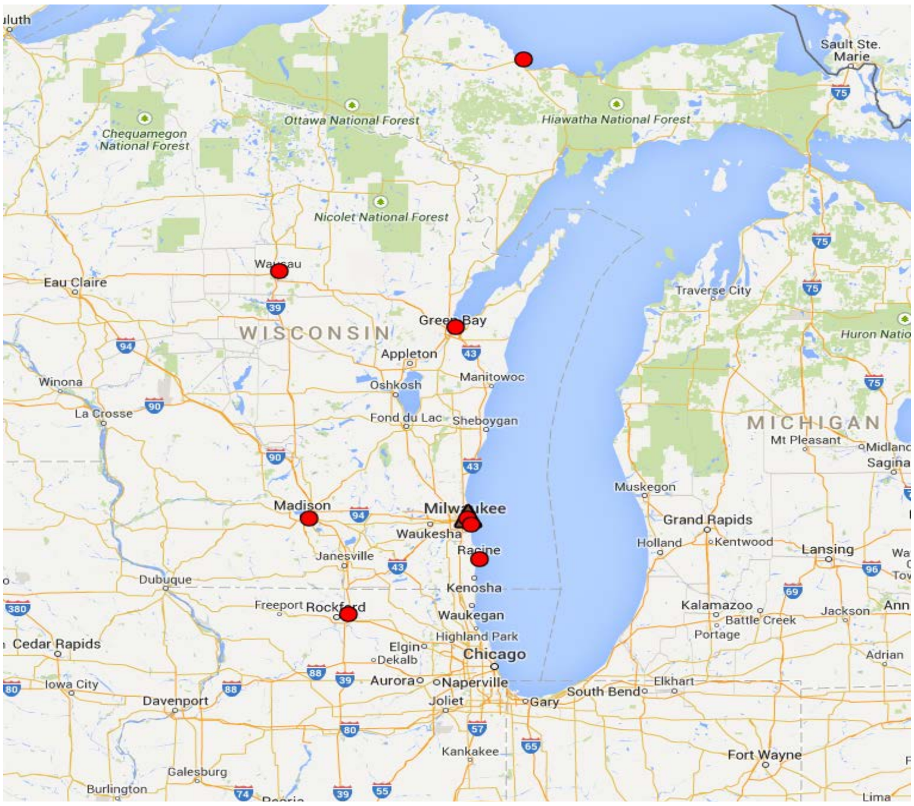

Figure 29. Cluster 4 with optimal facility (Milwaukee, WI—Brown triangle) based on Euclidean distance or Great Circle distance.

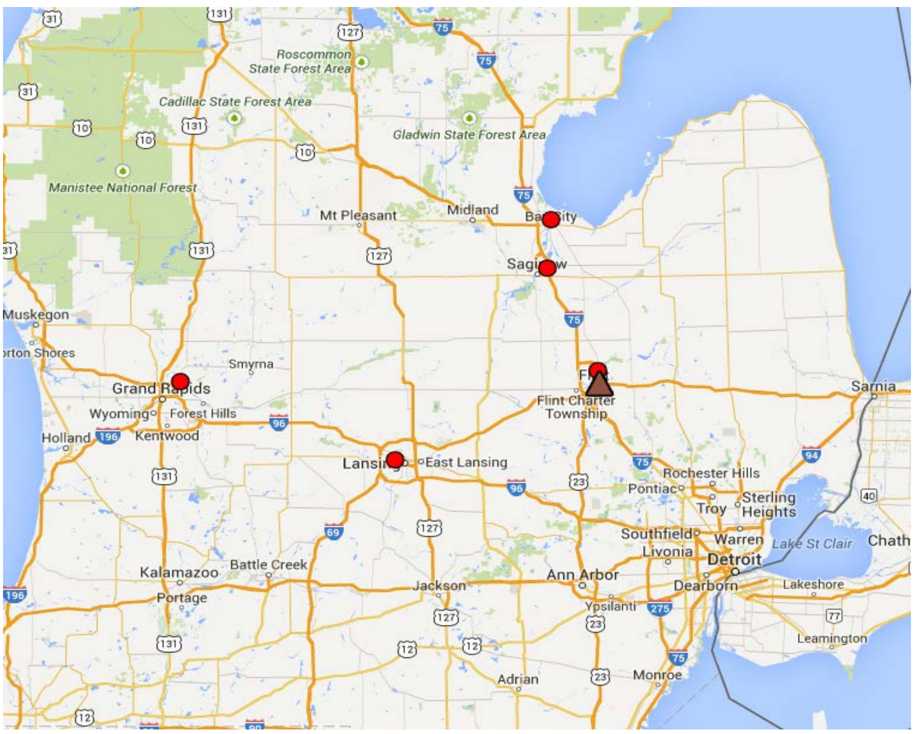

Figure 30. Cluster 5 with optimal facility location (Flint, MI—Brown triangle) based on Euclidean distance or Great Circle distance.

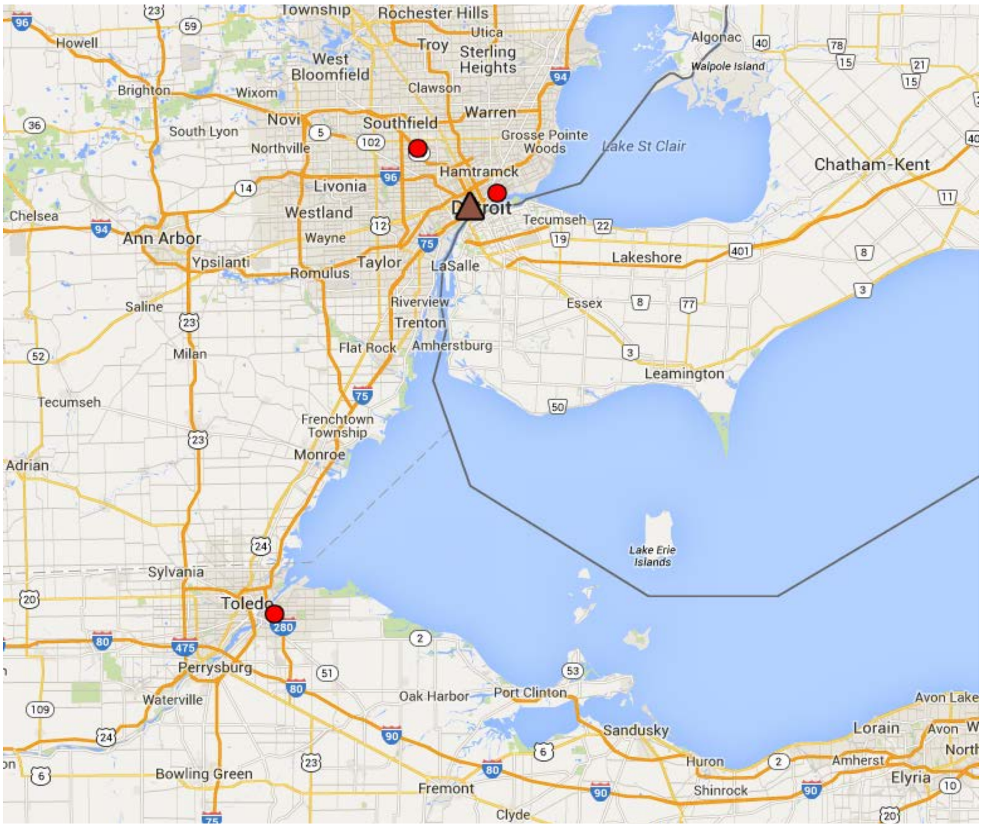

Figure 31. Cluster 10 with optimal facility (Detroit, MI—Brown triangle) based on Euclidean distance or Great Circle distance.

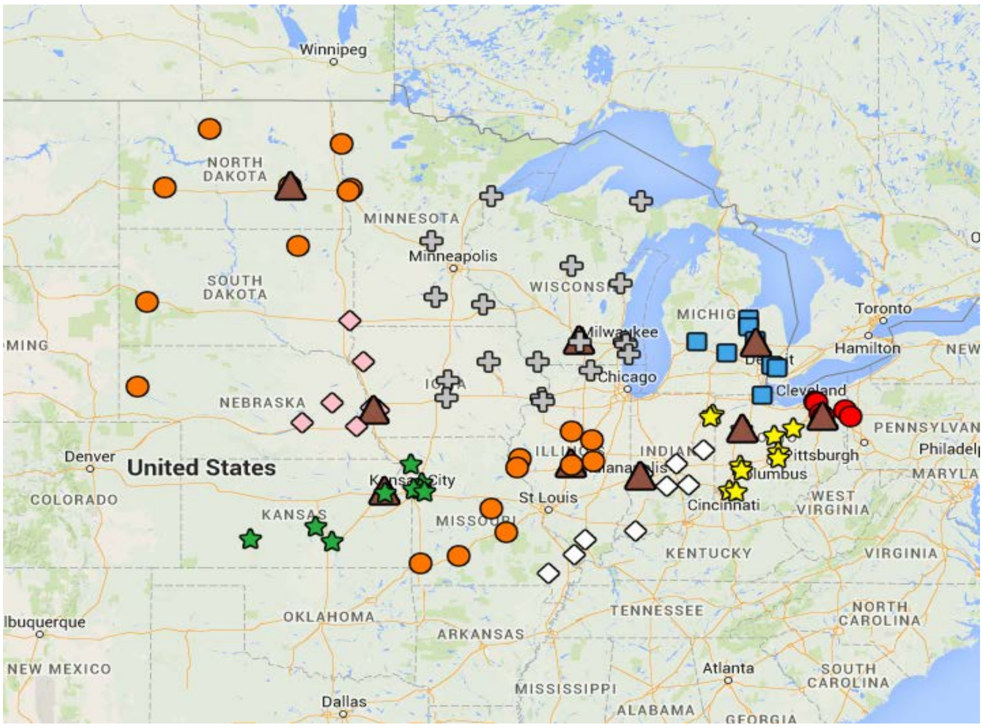

Figure 32. 9 clusters with 9 optimal facilities locations (Brown triangles).

Table 9. Cluster 1 costs from queried cities with Sioux City, Iowa as COG to market locations.

Latitudes/Longitudes of queried cities	Queried cities	Cities ID	Cost
42.50038901680, −96.39999210790	Center Of Gravity(Sioux City, Iowa, USA)	Depot = 1	Cost = 31469.3
44.16362082830, −93.99915674010	Mankato, Minnesota, USA	Depot = 2	Cost = 84665
43.64778668120, −93.36870426610	Albert Lea, Minnesota, USA	Depot = 3	Cost = 87730
43.14528505360, −95.14717452170	Spencer, Iowa, USA	Depot = 4	Cost = 52554.1
42.50682273260, −94.18025679700	Ft. Dodge, Iowa, USA	Depot = 5	Cost = 57508
40.70070558580, −99.08114628270	Kearney, Nebraska, USA	Depot = 6	Cost = 54229
40.92226828920, −98.35798628920	Grand Island, Nebraska, USA	Depot = 7	Cost = 39933.2
42.50038901680, −96.39999210790	Sioux City, Iowa, USA	Depot = 8	Cost = 31469.3
41.26227337550, −95.86080021340	Council Bluffs, Iowa, USA	Depot = 9	Cost = 21822.4
42.05385296540, −93.61972253600	Ames, Ames, Iowa, USA	Depot = 10	Cost = 64243
43.15401837060, −93.20083337580	Mason City, Iowa, USA	Depot = 11	Cost = 83813
42.02871238100, −97.43359826840	Norfolk, Nebraska, USA	Depot = 12	Cost = 31357.3
42.88201946930, −97.39248966650	Yankton, South Dakota, USA	Depot = 13	Cost = 42640.6
44.30676454580, −96.78803043800	Brookings, South Dakota, USA	Depot = 14	Cost = 65267
43.71429424950, −98.02619775570	Mitchell, South Dakota, USA	Depot = 15	Cost = 64908.4
40.81997479150, −96.68000085630	Lincoln, Nebraska, USA	Depot = 16	Cost = 25175
43.54998903310, −96.72999780450	Sioux Falls, South Dakota, USA	Depot = 17	Cost = 49678
41.57998008120, −93.61998091810	Des Moines, Iowa, USA	Depot = 18	Cost = 58147
41.24000083320, −96.00999007340	Omaha, Nebraska, USA	Depot = 19	Cost = 21222.6

Table 10. Cluster 1 costs from queried cities with Council Bluffs, Iowa as COG to market locations.

Latitudes/Longitudes of queried cities	Queried cities	Cities ID	Cost
41.26227337550, −95.86080021340	Center Of Gravity (Council Bluffs, Iowa, USA)	Depot = 1	Cost = 21822.4
41.01288291330, −92.41480900240	Ottumwa, Iowa, USA	Depot = 2	Cost = 83467
43.14528505360, −95.14717452170	Spencer, Iowa, USA	Depot = 3	Cost = 52554.1
42.50682273260, −94.18025679700	Ft. Dodge, Iowa, USA	Depot = 4	Cost = 57508
39.11358051690, −94.63014637710	Kansas City, Kansas, USA	Depot = 5	Cost = 71447
38.95975242010, −95.25522994160	Lawrence, Kansas, USA	Depot = 6	Cost = 74943
39.19402752590, −96.59243514180	Manhattan, Kansas, USA	Depot = 7	Cost = 61455
39.09111391100, −94.41528121110	Independence, Missouri, USA	Depot = 8	Cost = 75605
40.19368227040, −92.58280908380	Kirksville, Missouri, USA	Depot = 9	Cost = 101112
40.70070558580, −99.08114628270	Kearney, Nebraska, USA	Depot = 10	Cost = 54229
40.92226828920, −98.35798628920	Grand Island, Nebraska, USA	Depot = 11	Cost = 39933.2
42.50038901680, −96.39999210790	Sioux City, Iowa, USA	Depot = 12	Cost = 31469.3
41.26227337550, −95.86080021340	Council Bluffs, Iowa, USA	Depot = 13	Cost = 21822.4

Continued

42.05385296540, −93.61972253600	Ames, Iowa, USA	Depot = 14	Cost = 64243
43.15401837060, −93.20083337580	Mason City, Iowa, USA	Depot = 15	Cost = 83813
39.76903118800, −94.84639184750	St. Joseph, Missouri, USA	Depot = 16	Cost = 56430
42.02871238100, −97.43359826840	Norfolk, Nebraska, USA	Depot = 17	Cost = 31357.3
42.88201946930, −97.39248966650	Yankton, South Dakota, USA	Depot = 18	Cost = 42640.6
40.81997479150, −96.68000085630	Lincoln, Nebraska, USA	Depot = 19	Cost = 25175
39.05000530910, −95.66998498710	Topeka, Kansas, USA	Depot = 20	Cost = 65715
39.10708850980, −94.60409421890	Kansas City, Missouri, USA	Depot = 21	Cost = 72082
43.54998903310, −96.72999780450	Sioux Falls, South Dakota, USA	Depot = 22	Cost = 49678
41.57998008120, −93.61998091810	Des Moines, Iowa, USA	Depot = 23	Cost = 58147
41.24000083320, −96.00999007340	Omaha, Nebraska, USA	Depot = 24	Cost = 21222.6

Table 11. Cluster 1 costs from queried cities with Norfolk, Nebraska as COG to market locations.

Latitudes/Longitudes of queried cities	Queried cities	Cities ID	Cost
42.02871238100, −97.43359826840	Center Of Gravity (Norfolk, Nebraska, USA)	Depot = 1	Cost = 31357.3
43.14528505360, −95.14717452170	Spencer, Iowa, USA	Depot = 2	Cost = 52554.1
42.50682273260, −94.18025679700	Ft. Dodge, Iowa, USA	Depot = 3	Cost = 57508
40.70070558580, −99.08114628270	Kearney, Nebraska, USA	Depot = 4	Cost = 54229
40.92226828920, −98.35798628920	Grand Island, Nebraska, USA	Depot = 5	Cost = 39933.2
42.50038901680, −96.39999210790	Sioux City, Iowa, USA	Depot = 6	Cost = 31469.3
41.26227337550, −95.86080021340	Council Bluffs, Iowa, USA	Depot = 7	Cost = 21822.4
42.02871238100, −97.43359826840	Norfolk, Nebraska, USA	Depot = 8	Cost = 31357.3
41.13628623360, −100.77050053100	North Platte, Nebraska, USA	Depot = 9	Cost = 82006
42.88201946930, −97.39248966650	Yankton, South Dakota, USA	Depot = 10	Cost = 42640.6
44.30676454580, −96.78803043800	Brookings, South Dakota, USA	Depot = 11	Cost = 65267
43.71429424950, −98.02619775570	Mitchell, South Dakota, USA	Depot = 12	Cost = 64908.4
40.81997479150, −96.68000085630	Lincoln, Nebraska, USA	Depot = 13	Cost = 25175
43.54998903310, −96.72999780450	Sioux Falls, South Dakota, USA	Depot = 14	Cost = 49678
41.24000083320, −96.00999007340	Omaha, Nebraska, USA	Depot = 15	Cost = 21222.6

Table 16 is the summary of total cost for each set of clusters. From the result shown in **Table 16**, it is concluded that the set of 9 clusters with 9 optimal facilities locations are selected shown as in **Figure 32**.

4. Conclusions and Future Research

Facility location decisions play an important role in the strategic planning and design of logistics/supply chain network. Well-planned location decisions enable the efficient flow of materials through the distribution system,

Table 12. Cluster 1 costs from queried cities with Yankton, South Dakota as COG to market locations.

Latitudes/Longitudes of queried cities	Queried cities	Cities ID	Cost
42.88201946930, −97.39248966650	Center Of Gravity (Yankton, South Dakota, USA)	Depot = 1	Cost = 42,640.6
44.16362082830, −93.99915674010	Mankato, Minnesota, USA	Depot = 2	Cost = 84,665
43.64778668120, −93.36870426610	Albert Lea, Minnesota, USA	Depot = 3	Cost = 87,730
45.12188275050, −95.04330489270	Willmar, Minnesota, USA	Depot = 4	Cost = 92,692
43.14528505360, −95.14717452170	Spencer, Iowa, USA	Depot = 5	Cost = 52,554.1
42.50682273260, −94.18025679700	Ft. Dodge, Iowa, USA	Depot = 6	Cost = 57,508
40.70070558580, −99.08114628270	Kearney, Nebraska, USA	Depot = 7	Cost = 54,229
40.92226828920, −98.35798628920	Grand Island, Nebraska, USA	Depot = 8	Cost = 39,933.2
42.50038901680, −96.39999210790	Sioux City, Iowa, USA	Depot = 9	Cost = 31,469.3
41.26227337550, −95.86080021340	Council Bluffs, Iowa, USA	Depot = 10	Cost = 21,822.4
42.05385296540, −93.61972253600	Ames, Iowa, USA	Depot = 11	Cost = 64,243
43.15401837060, −93.20083337580	Mason City, Iowa, USA	Depot = 12	Cost = 83,813
42.02871238100, −97.43359826840	Norfolk, Nebraska, USA	Depot = 13	Cost = 31,357.3
41.13628623360, −100.77050053100	North Platte, Nebraska, USA	Depot = 14	Cost = 82,006
42.88201946930, −97.39248966650	Yankton, South Dakota, USA	Depot = 15	Cost = 42,640.6
44.30676454580, −96.78803043800	Brookings, South Dakota, USA	Depot = 16	Cost = 65,267
43.71429424950, −98.02619775570	Mitchell, South Dakota, USA	Depot = 17	Cost = 64,908.4
45.46511761380, −98.48640222350	Aberdeen, South Dakota, USA	Depot = 18	Cost = 105,103
40.81997479150, −96.68000085630	Lincoln, Nebraska, USA	Depot = 19	Cost = 25,175
43.54998903310, −96.72999780450	Sioux Falls, South Dakota, USA	Depot = 20	Cost = 49,678
41.57998008120, −93.61998091810	Des Moines, Iowa, USA	Depot = 21	Cost = 58,147
41.24000083320, −96.00999007340	Omaha, Nebraska, USA	Depot = 22	Cost = 21,222.6
44.36833701480, −100.35055200400	Pierre, Pierre, USA	Depot = 23	Cost = 108,595

Table 13. Six locations with volume and transportation rate.

Location	Latitude	Longitude	Volume	Rate
WI Loc1	43.037647	−89.39154	900.00	0.06
WI Loc2	44.530892	−88.04482	1,100.00	0.06
WI Loc3	44.018871	−88.61324	800.00	0.06
WI Loc4	43.785391	−88.48704	900.00	0.06
WI Loc5	42.280019	−89.03347	500.00	0.08
IN Loc6	41.683381	−86.25001	1,500.00	0.07

and lead to decreased costs and improved customer service. This paper has focused on the implementation of facility location decisions based on driving distances on a sphere surface. Two objectives have been achieved in-this paper. Given the location of each destination in terms of their coordinates, the requirement at each destination and shipping costs for the region of interest, the proposed methodology in this paper is able to determine the

Table 14. Modeclus cluster procedure with m = 1, K = 4.

Sums of Density Estimates Within Neighborhood											
Cluster	Location	Estimated Density	Same Cluster	Other Clusters	Total	Cluster	Location	Estimated Density	Same Cluster	Other Clusters	Total
1	Loc92	0.000878	0.0027064	0	0.002706	7	Loc4	0.000283	0.0006606	0.0003654	0.001026
	Loc93	0.00127	0.0023142	0	0.002314		Loc32	0.000313	0.0009782	0	0.000978
	Loc94	0.00075	0.0028338	0	0.002834		Loc33	0.000348	0.0009438	0	0.000944
	Loc95	0.000389	0.0027064	0	0.002706		Loc34	0.000348	0.0008367	0	0.000837
	Loc1A	0.000687	0.0028973	0	0.002897		Loc35	0.000277	0.0008376	0	0.000838
	Loc1B	0.00031	0.0028338	0	0.002834		Loc36	0.000212	0.000902	0	0.000902
	Loc1C	0.000149	0.0005339	0	0.000534		Loc37	0.000277	0.0008067	0	0.000807
	Loc1D	0.000141	0.0005413	0	0.000541		Loc40	0.000182	0.0008376	0	0.000838
	Loc1E	8.26E−05	0.0005998	0	0.0006	8	Loc80	0.00027	0.0009166	0	0.000917
2	Loc5	0.000571	0.0019546	0	0.001955		Loc81	0.000338	0.0008487	0	0.000849
	Loc6	0.000685	0.0018411	0	0.001841		Loc83	0.000188	0.0007759	0	0.000776
	Loc8	0.00056	0.0019658	0	0.001966		Loc84	0.000285	0.0009017	0	0.000902
	Loc9	0.00071	0.0018154	0	0.001815		Loc85	0.000293	0.0008936	0	0.000894
	Loc10	0.000402	0.0016097	0	0.00161		Loc89	0.000212	0.0006533	0	0.000653
	Loc11	0.000365	0.0016723	0	0.001672		Loc96	0.000137	0.0001684	0.0021472	0.002316
3	Loc1	0.000335	0.0012727	0	0.001273		Loc97	0.000172	0.0005614	0	0.000561
	Loc2	0.000414	0.001194	0	0.001194		Loc98	0.000161	0.0003405	0.0001612	0.000502
	Loc3	0.000454	0.0011543	0	0.001154		Loc99	0.000168	0.0004703	0	0.00047
	Loc12	0.000405	0.0008676	0.00071	0.001578	9	Loc41	8.95E−05	0.0003103	0.0001001	0.00041
	Loc18	0.000365	0.0009998	0	0.001		Loc51	0.000268	0.0008105	0	0.000811
	Loc23	0.00029	0.0006213	0.000283	0.000905		Loc52	0.000285	0.0007934	0	0.000793
	Loc24	0.000256	0.0006557	0.000283	0.000939		Loc53	0.000258	0.0008063	0	0.000806
4	Loc45	0.000238	0.000947	0	0.000947		Loc54	0.000268	0.0008063	0	0.000806
	Loc46	0.000391	0.0012231	0	0.001223		Loc55	0.000185	0.0006787	0	0.000679
	Loc67	0.000158	0.0010277	0.000142	0.00117		Loc56	0.000126	0.0007376	0	0.000738
	Loc1F	0.000398	0.0012163	0	0.001216		Loc78	0.000253	0.0007934	0	0.000793
	Loc1G	0.000438	0.0011764	0	0.001176	10	Loc42	0.000137	0.0003815	0.0003912	0.000773
	Loc1H	0.000387	0.0012271	0	0.001227		Loc43	0.00018	0.000338	0.0003912	0.000729
	Loc1I	0.000272	0.0009818	0	0.000982		Loc44	0.000201	0.0005398	0	0.00054
	Loc1J	0.000157	0.0010973	0	0.001097		Loc47	0.000256	0.000649	0	0.000649
5	Loc13	0.000412	0.0011216	0	0.001122		Loc50	0.000216	0.0004877	0.0002702	0.000758
	Loc14	0.000357	0.0011765	0	0.001177		Loc62	0.0001	0.0004334	0	0.000433

Continued

	Loc15	0.000357	0.0011765	0	0.001177		Loc63	0.000147	0.0004879	0	0.000488
	Loc16	0.000408	0.0011253	0	0.001125		Loc64	0.000142	0.0002914	0.0001578	0.000449
6	Loc19	0.000321	0.0009283	0	0.000928		Loc65	0.000145	0.0003888	0	0.000389
	Loc27	0.000267	0.0006996	0.000412	0.001111		Loc79	0.000231	0.0004727	0.0002702	0.000743
	Loc28	0.000283	0.0003215	0.000647	0.000969	11	Loc66	0.000228	0.0005871	0	0.000587
	Loc29	0.000378	0.0008006	0	0.000801		Loc68	0.00012	0.0006839	0	0.000684
	Loc30	0.000188	0.0007724	0	0.000772		Loc69	6.7E−05	0.0002781	0	0.000278
	Loc31	0.000212	0.0006996	0.000285	0.000985		Loc70	0.000235	0.0005802	0	0.00058
	Loc86	0.000182	0.0005102	0	0.00051		Loc71	0.000131	0.0006839	0	0.000684
	Loc90	0.000161	0.0005308	0	0.000531		Loc72	0.000221	0.0005835	0	0.000583
	Loc91	0.000161	0.000343	0.000161	0.000504		Loc74	9.88E−05	0.0003944	0	0.000394
							Loc75	0.000107	0.0004508	0	0.000451
							Loc1K	5.9E−05	6.704E−05	0.0004289	0.000496

Table 15. Modeclus cluster procedure with m = 1, K = 5.

Sums of Density Estimates Within Neighborhood											
Cluster	Location	Estimated Density	Same Cluster	Other Clusters	Total	Cluster	Location	Estimated Density	Same Cluster	Other Clusters	Total
1	Loc5	0.0004	0.0021	0	0.0021	6	Loc19	0.0004	0.0012	0	0.0012
	Loc6	0.0005	0.002	0	0.002		Loc27	0.0003	0.0009	0.0003	0.0013
	Loc8	0.0006	0.0019	0	0.0019		Loc28	0.0003	0.0007	0.0007	0.0014
	Loc9	0.0005	0.0015	0.0005	0.002		Loc29	0.0003	0.0013	0	0.0013
	Loc10	0.0005	0.002	0	0.002		Loc30	0.0002	0.0008	0	0.0008
	Loc11	0.0004	0.0021	0	0.0021		Loc31	0.0002	0.0008	0.0003	0.0011
2	Loc92	0.0004	0.002	0	0.002		Loc86	0.0002	0.0005	0.0003	0.0008
	Loc93	0.0005	0.0019	0	0.0019		Loc90	0.0002	0.0005	0.0002	0.0007
	Loc94	0.0005	0.0018	0	0.0018		Loc91	0.0001	0.0003	0.0003	0.0007
	Loc95	0.0004	0.002	0	0.002	7	Loc41	1E−04	0.0008	0	0.0008
	Loc1A	0.0005	0.0018	0	0.0018		Loc42	0.0002	0.0005	0.0006	0.001
	Loc1B	0.0004	0.002	0	0.002		Loc43	0.0002	0.0004	0.0006	0.001
	Loc1C	0.0002	0.0006	0.0002	0.0007		Loc44	0.0002	0.0008	0	0.0008
	Loc1D	0.0001	0.0011	0	0.0011		Loc47	0.0003	0.001	0	0.001
	Loc1E	1E−04	0.0006	0.0002	0.0008		Loc50	0.0002	0.0008	0.0003	0.0011
3	Loc1	0.0004	0.0017	0	0.0017		Loc51	0.0003	0.0012	0	0.0012
	Loc2	0.0003	0.0017	0	0.0017		Loc52	0.0003	0.0012	0	0.0012
	Loc3	0.0005	0.0016	0	0.0016		Loc53	0.0003	0.0012	0	0.0012

Continued

	Loc12	0.0005	0.0008	0.0011	0.0019		Loc54	0.0003	0.0012	0	0.0012
	Loc13	0.0003	0.001	0.0003	0.0014		Loc55	0.0002	0.0011	0	0.0011
	Loc14	0.0003	0.0011	0.0003	0.0014		Loc56	0.0001	0.0011	0	0.0011
	Loc15	0.0004	0.0014	0	0.0014		Loc62	0.0001	0.0006	0.0002	0.0008
	Loc16	0.0004	0.0014	0	0.0014		Loc63	0.0002	0.0009	0	0.0009
	Loc18	0.0004	0.0014	0	0.0014		Loc64	0.0001	0.0003	0.0004	0.0008
	Loc23	0.0003	0.0007	0.0006	0.0013		Loc65	0.0002	0.0005	0.0002	0.0006
	Loc24	0.0003	0.0007	0.0006	0.0013		Loc78	0.0003	0.0012	0	0.0012
4	Loc4	0.0003	0.0006	0.0009	0.0015		Loc79	0.0002	0.0008	0.0003	0.0011
	Loc32	0.0003	0.0014	0	0.0014	8	Loc80	0.0003	0.0008	0.0002	0.001
	Loc33	0.0003	0.0013	0	0.0013		Loc81	0.0002	0.0008	0.0002	0.0011
	Loc34	0.0004	0.0012	0	0.0012		Loc83	0.0002	0.001	0	0.001
	Loc35	0.0003	0.0014	0	0.0014		Loc84	0.0003	0.0008	0.0002	0.001
	Loc36	0.0002	0.0014	0	0.0014		Loc85	0.0003	0.001	0	0.001
	Loc37	0.0003	0.0013	0	0.0013		Loc89	0.0002	0.001	0	0.001
	Loc40	0.0002	0.001	0.0003	0.0013		Loc96	0.0002	0.0002	0.0014	0.0016
5	Loc45	0.0003	0.0012	0	0.0012		Loc97	0.0002	0.0008	0	0.0008
	Loc46	0.0003	0.0013	0	0.0013		Loc98	0.0002	0.0006	0.0001	0.0007
	Loc67	0.0002	0.0009	0.0001	0.0011		Loc99	0.0002	0.0005	0.0004	0.001
	Loc1F	0.0003	0.0013	0	0.0013	9	Loc66	0.0002	0.0005	0.0002	0.0006
	Loc1G	0.0004	0.0013	0	0.0013		Loc68	0.0001	0.0005	0.0002	0.0007
	Loc1H	0.0003	0.0013	0	0.0013		Loc69	8E−05	0.0003	0.0002	0.0005
	Loc1I	0.0003	0.0012	0	0.0012		Loc70	0.0002	0.0005	0.0002	0.0007
	Loc1J	0.0002	0.0014	0	0.0014		Loc71	0.0001	0.0006	0	0.0006
							Loc72	0.0002	0.0006	0	0.0006
							Loc74	1E−04	0.0005	0	0.0005
							Loc75	0.0001	0.0006	0	0.0006
							Loc1K	7E−05	8E−05	0.0009	0.0009

Table 16. Summary of total cost for each set of clusters.

14 clusters:

Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	Cluster 6	Cluster 7	Cluster 8	Cluster 9	Cluster 10
21,222.6	5,593.8	3,392.8	38,896.4	11,857.5	11,339.2	57,779.0	1,660.4	9,898.2	1,805.6

Cluster 11	Cluster 12	Cluster 13	Cluster 14	# of Facility	Facility cost per unit	Facilities cost	Total Cost
92,088.5	72,057.0	12,892.2	8,700.0	14	100,000.00	1,400,000.00	1,749,183.2

11 clusters:

Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	Cluster 6	Cluster 7	Cluster 8	Cluster 9	Cluster 10
46,194.0	3,392.8	11,339.2	21,222.6	1,660.4	46,711.6	19,995.0	57,779.0	38,896.4	92,088.5

Cluster 11	# of Facility	Facility cost per unit	Facilities cost	Total Cost
72,057.0	11	100,000.00	1,100,000.00	1,511,336.5

9 clusters:

Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	Cluster 6	Cluster 7	Cluster 8	Cluster 9
3,392.8	46,194.0	18,765.6	19,995.0	21,222.6	46,711.6	180,546.4	57,779.0	72,057.0

# of Facility	Facility cost per unit	Facilities cost	Total Cost
9	100,000.00	900,000.00	1,366,664.0

optimal location of each facility and helps companies assess the locations of facilities. The second object is to establish terminology and an analytical framework for locating optimal facility in perspective. In regard to transportation cost, the driving distance in the presence of geographic barriers should be taken into consideration in facility location decisions. The proposed method in this paper has shown very promising results. The potential benefits of my work include:

• The driving distance measure can be applied to transportation problems, travelling sales person problems, and etc., in Operations Research (OR) study.

• This proposed method can be extended to facility location decisions based on Euclidean distance and Great circle distance functions.

• The framework and methodology proposed in this paper can be applied to improve the management of logistics/supply chain system decisions and planning to make smooth flow throughout the supply chain.

This paper has focused on the facility locations in the region of Midwest, USA. Markets and competition are increasingly global, further research work could include investigation in the process of international facility locations around the world. Therefore, a global driving distance model within and/or outside the US is needed for further work. In addition to qualitative analysis, some factors affecting locations need to be considered, such as labor costs and availability, including wages, productivity, attitudes, age, distribution, unionization, and skills, State and local government fiscal policies (including incentives, taxes, unemployment compensation), proximity to customers, population distribution, quality-of-life issues (education, cost of living, health care, sports, cultural activities, housing, entertainment, religious facilities, etc.).

This paper assumes a spherical earth and uses equation (1) to (7) to calculate initial center of gravity and equation (8) to (10) to convert Cartesian coordinated to latitude/longitudes. Further research could be done on assumption of ellipsoidal earth. Also, further research work can be suggested to apply this proposed methodology on capacity facility location decisions problems.

Geographic Information Systems (GIS) also helps in location analysis; further research work can be approached using GIS instead of Google maps.

Finally, since uncertainty associated with future conditions exists in real world, facility location decisions must account for the inherent uncertainty. This is an area worthy of additional research too.

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Appendix A: Coordinates (Latitude/Longitudes) and Volume of Goods and Transportation Rate for Each Market City Location

Location	State	Latitude	Longitude	Volume	Rate
Loc1	OH	39.9718	−82.9688	800.00	0.02
Loc2	OH	40.146873	−82.98178	500.00	0.02
Loc3	OH	40.589747	−83.12173	1000.00	0.02
Loc4	OH	41.642549	−83.5104	900.00	0.02
Loc5	OH	41.480881	−81.80036	700.00	0.02
Loc6	OH	41.437018	−81.73488	400.00	0.02
Loc8	OH	41.089405	−81.53973	1500.00	0.02
Loc9	OH	41.144661	−81.61948	800.00	0.02
Loc10	OH	41.252748	−80.80773	1200.00	0.02
Loc11	OH	41.095658	−80.62652	1100.00	0.02
Loc12	OH	40.773556	−82.47612	1300.00	0.02
Loc13	OH	39.104127	−84.53067	500.00	0.02
Loc14	OH	39.066112	−84.32227	600.00	0.02
Loc15	OH	39.788458	−84.21111	1700.00	0.02
Loc16	OH	39.644609	−84.16931	1300.00	0.02
Loc18	OH	40.739517	−84.14468	1600.00	0.02
Loc19	IN	39.792993	−86.28575	500.00	0.03
Loc23	IN	41.071681	−85.15101	1100.00	0.03
Loc24	IN	41.138599	−85.05941	1700.00	0.03
Loc27	IN	39.232235	−85.86356	1200.00	0.03
Loc28	IN	40.192293	−85.38494	1600.00	0.03
Loc29	IN	39.188246	−86.56779	1300.00	0.03
Loc30	IN	37.974642	−87.57349	500.00	0.03
Loc31	IN	39.535285	−87.35222	1400.00	0.03
Loc32	MI	42.352837	−83.02449	800.00	0.05
Loc33	MI	42.427636	−83.19547	1100.00	0.05
Loc34	MI	43.062076	−83.69728	1600.00	0.05
Loc35	MI	43.431375	−83.93267	500.00	0.05
Loc36	MI	43.607523	−83.9162	1100.00	0.05
Loc37	MI	42.735535	−84.62698	700.00	0.05
Loc40	MI	43.022199	−85.61168	1000.00	0.05
Loc41	MI	46.554402	−87.42223	900.00	0.05
Loc42	IA	42.033235	−93.66741	800.00	0.03
Loc43	IA	41.587039	−93.67356	700.00	0.03

Continued

Loc44	IA	42.513636	−92.32418	1200.00	0.03
Loc45	IA	42.505046	−96.38003	1100.00	0.03
Loc46	IA	41.251631	−95.87447	400.00	0.03
Loc47	IA	42.50963	−90.75329	900.00	0.03
Loc50	IA	41.574581	−90.60303	1300.00	0.03
Loc51	WI	42.985465	−87.89998	500.00	0.06
Loc52	WI	43.037313	−87.93373	1800.00	0.06
Loc53	WI	42.714369	−87.82424	1000.00	0.06
Loc54	WI	43.037647	−89.39154	900.00	0.06
Loc55	WI	44.530892	−88.04482	1100.00	0.06
Loc56	WI	44.958382	−89.6693	700.00	0.06
Loc62	MN	46.674141	−92.2276	1200.00	0.07
Loc63	MN	43.991846	−92.49166	1000.00	0.07
Loc64	MN	44.184909	−94.04244	700.00	0.07
Loc65	MN	45.568516	−94.19251	1,100.00	0.07
Loc66	MN	46.861413	−96.75367	800.00	0.07
Loc67	SD	43.577696	−96.80101	1200.00	0.03
Loc68	SD	45.46932	−98.49646	700.00	0.03
Loc69	SD	44.052788	−103.34302	1300.00	0.03
Loc70	ND	46.812118	−96.839	600.00	0.04
Loc71	ND	47.938898	−97.08401	1100.00	0.04
Loc72	ND	46.906983	−98.72826	1000.00	0.04
Loc74	ND	46.878057	−102.8041	800.00	0.04
Loc75	ND	48.277024	−101.31942	1200.00	0.04
Loc78	IL	42.280019	−89.03347	500.00	0.08
Loc79	IL	41.4903	−90.56956	1500.00	0.08
Loc80	IL	40.678037	−89.62737	1200.00	0.08
Loc81	IL	40.459584	−88.96939	800.00	0.08
Loc83	IL	39.96008	−91.302633	1300.00	0.08
Loc84	IL	39.878041	−88.95637	500.00	0.08
Loc85	IL	39.7715	−89.63612	900.00	0.08
Loc86	IL	37.707763	−89.19246	1100.00	0.08
Loc89	MO	39.695798	−91.40084	500.00	0.03
Loc90	MO	37.291432	−89.54065	1200.00	0.03
Loc91	MO	36.759357	−90.41689	600.00	0.03
Loc92	MO	39.053255	−94.4061	1600.00	0.03

Continued

Loc93	MO	39.125304	−94.53234	1000.00	0.03
Loc94	MO	39.3173	−94.48655	700.00	0.03
Loc95	MO	39.812344	−94.84286	1100.00	0.03
Loc96	MO	37.047161	−94.51124	800.00	0.03
Loc97	MO	38.568287	−92.25329	1500.00	0.03
Loc98	MO	37.94384	−91.77105	1200.00	0.03
Loc99	MO	37.257053	−93.29015	600.00	0.03
Loc1A	KS	39.117009	−94.76614	1100.00	0.05
Loc1B	KS	39.013988	−95.65056	600.00	0.05
Loc1C	KS	37.669067	−97.36204	1500.00	0.05
Loc1D	KS	38.094945	−97.929	1200.00	0.05
Loc1E	KS	37.755267	−100.02632	900.00	0.05
Loc1F	NE	41.292321	−95.93427	1500.00	0.04
Loc1G	NE	41.296198	−96.11027	800.00	0.04
Loc1H	NE	40.798632	−96.59023	1500.00	0.04
Loc1I	NE	41.437838	−97.37192	1200.00	0.04
Loc1J	NE	40.922826	−98.33087	500.00	0.04
Loc1K	NE	41.885553	−103.65241	1000.00	0.04

Appendix B: Google Maps Distance Matrix of 88 Midwest Major Market Cities

Obs	Location	Loc 1	Loc 2	Loc 3	Loc 4	Loc 5	Loc 6	Loc 8	Loc 9	Loc 10	Loc 11	Loc 12	Loc 13	Loc 14	Loc 15	Loc 16	Loc 18
1	Loc1	0	14.4	50	143	136	136	125	121	175	173	67.8	110	111	73.9	80.8	95.5
2	Loc2	14.6	0	36	129	123	123	112	108	162	160	54.9	123	124	84.8	91.7	89.2
3	Loc3	50.1	36.2	0	95.6	112	112	101	96.7	151	149	41.6	143	156	90.4	97.3	57.3
4	Loc4	144	130	98.1	0	104	106	131	123	160	170	99.7	202	211	148	160	80.4
5	Loc5	136	122	111	104	0	5.9	39.8	31.5	63.4	79.6	70.8	244	245	206	213	151
6	Loc6	136	123	111	106	6	0	33.2	24.9	56.8	73	71.3	245	246	206	213	152
7	Loc8	125	112	100	131	40.6	33.2	0	9.9	49.1	52.8	60.3	234	235	196	202	157
8	Loc9	121	107	95.9	122	32	24.6	6.2	0	58.2	62.2	55.8	229	230	191	198	152
9	Loc10	170	156	145	160	63.1	56.5	48.2	59.7	0	22.3	105	278	279	240	247	201
10	Loc11	174	160	149	170	79.5	72.9	52.4	62.2	20.1	0	109	282	283	244	251	205
11	Loc12	67.8	54.5	41.8	99.8	71	71.4	60.4	56.1	110	108	0	176	177	138	145	98.1
12	Loc13	111	124	144	202	245	245	234	230	284	282	177	0	16.2	55.2	47.2	125
13	Loc14	111	124	156	210	245	245	234	230	284	282	177	15.4	0	63.7	55.7	134
14	Loc15	73.5	85.6	91.1	148	207	207	196	192	246	244	139	55	63.7	0	12.5	71.7
15	Loc16	80.2	92.2	97.8	163	213	214	203	198	252	251	146	47	55.7	16.7	0	86.7
16	Loc18	95.2	89.8	57.6	80	153	155	159	155	209	207	99.7	125	134	71	82.4	0
17	Loc19	183	195	201	234	329	317	306	302	356	354	249	121	137	117	133	164
18	Loc23	161	161	130	103	197	199	212	208	253	263	153	180	189	126	138	62
19	Loc24	161	161	130	101	196	197	212	208	251	261	153	184	193	130	142	62.1
20	Loc27	185	198	215	267	319	319	308	304	358	356	251	85.2	100	126	118	196
21	Loc28	147	159	140	181	275	277	249	245	299	297	190	103	114	80.2	96.6	92.1
22	Loc29	226	238	243	275	359	359	348	344	398	396	291	131	146	159	164	222
23	Loc30	327	339	372	395	461	461	450	446	500	498	393	218	227	273	265	343
24	Loc31	257	269	275	306	390	391	380	375	429	428	322	186	201	190	207	254
25	Loc32	204	191	159	60.3	163	164	190	181	218	228	158	263	272	209	221	141
26	Loc33	208	194	162	63.6	166	168	193	185	222	232	162	266	275	212	224	145
27	Loc34	250	236	205	118	222	224	249	241	278	288	228	309	317	255	266	187
28	Loc35	277	263	232	145	249	251	276	268	305	315	255	336	344	282	293	214
29	Loc36	290	276	245	158	262	264	289	281	318	328	267	349	357	295	306	227
30	Loc37	260	246	215	128	232	234	259	251	288	298	238	308	317	255	266	186
31	Loc40	323	309	278	191	295	297	322	314	351	361	301	354	362	300	311	232
32	Loc41	631	617	586	499	603	605	630	622	659	669	609	690	698	636	647	568
33	Loc42	675	687	638	588	681	683	708	700	737	747	660	611	626	609	625	570
34	Loc43	652	664	614	564	658	659	684	676	713	723	637	587	602	585	601	546
35	Loc44	617	629	580	530	623	625	650	642	679	689	603	553	568	551	567	512

Continued

36	Loc45	826	838	788	738	832	833	859	850	887	897	811	761	776	759	776	720
37	Loc46	778	790	740	691	784	786	811	802	839	850	763	713	729	711	728	673
38	Loc47	558	570	482	432	526	527	553	544	581	591	505	493	508	491	507	415
39	Loc50	489	501	452	402	495	497	522	514	551	561	475	424	440	422	439	384
40	Loc51	450	415	383	331	424	426	451	443	480	490	406	388	403	383	400	315
41	Loc52	454	418	386	335	428	430	455	446	483	494	409	391	407	387	403	319
42	Loc53	435	400	368	316	409	411	436	428	465	475	391	373	388	369	385	300
43	Loc54	503	475	444	394	487	489	514	506	543	553	467	441	456	436	453	376
44	Loc55	572	536	504	453	546	547	573	564	601	611	527	509	525	505	521	437
45	Loc56	637	601	569	518	611	613	638	629	666	677	592	574	590	570	586	502
46	Loc62	830	802	771	721	814	816	841	833	870	880	793	767	783	763	780	703
47	Loc63	706	678	647	597	690	692	717	709	746	756	669	644	659	639	656	579
48	Loc64	791	763	731	681	775	776	801	793	830	840	754	728	743	724	740	663
49	Loc65	833	805	773	723	817	818	843	835	872	882	796	770	785	766	782	705
50	Loc66	997	969	937	888	981	983	1008	999	1037	1047	960	934	950	930	946	870
51	Loc67	933	945	874	825	918	919	945	936	973	983	897	868	884	866	883	807
52	Loc68	1059	1031	1000	950	1043	1045	1070	1062	1099	1109	1022	996	1012	992	1008	932
53	Loc69	1268	1280	1216	1167	1260	1261	1287	1278	1315	1325	1239	1203	1218	1201	1217	1149
54	Loc70	1002	975	943	893	986	988	1013	1005	1042	1052	966	940	955	936	952	875
55	Loc71	1080	1052	1020	970	1064	1065	1090	1082	1119	1129	1043	1017	1032	1013	1029	952
56	Loc72	1094	1066	1035	985	1078	1080	1105	1097	1134	1144	1058	1032	1047	1027	1044	967
57	Loc74	1288	1260	1229	1179	1272	1274	1299	1291	1328	1338	1252	1226	1241	1221	1238	1161
58	Loc75	1270	1243	1211	1161	1254	1256	1281	1273	1310	1320	1234	1208	1223	1204	1220	1143
59	Loc78	442	414	382	333	426	427	453	444	481	491	405	379	395	375	391	315
60	Loc79	482	494	444	394	488	489	515	506	543	553	467	417	432	415	431	377
61	Loc80	387	399	405	385	478	480	505	497	534	544	453	323	338	321	337	353
62	Loc81	345	357	362	350	443	445	470	462	499	509	410	280	295	278	294	289
63	Loc83	496	508	513	497	591	592	618	614	646	666	561	431	446	429	445	462
64	Loc84	345	357	363	354	482	484	468	463	517	516	411	280	296	278	295	311
65	Loc85	385	397	403	418	512	513	508	503	567	556	451	320	336	318	335	351
66	Loc86	441	453	458	490	575	575	564	560	614	612	507	332	341	374	379	437
67	Loc89	488	500	505	521	614	616	610	606	670	658	553	423	438	421	437	454
68	Loc90	481	494	499	531	615	616	605	600	654	653	547	373	382	414	420	478
69	Loc91	530	542	570	602	664	664	653	649	703	701	596	421	430	485	468	549
70	Loc92	648	660	666	698	781	782	771	767	821	819	714	577	603	581	598	645

Continued

71	Loc93	660	672	677	709	793	793	782	778	832	830	725	589	614	593	609	657
72	Loc94	673	685	691	713	806	808	796	792	846	844	739	603	628	607	623	670
73	Loc95	679	691	697	713	806	808	802	798	861	850	745	615	630	613	629	645
74	Loc96	704	716	721	753	837	837	826	822	876	874	769	633	652	637	653	701
75	Loc97	548	560	565	597	681	681	670	666	720	718	613	477	502	481	497	545
76	Loc98	525	537	543	575	658	659	648	643	698	696	591	454	474	458	475	522
77	Loc99	631	643	649	681	764	765	754	749	804	802	697	560	580	564	581	628
78	Loc1A	670	682	688	719	803	804	793	788	842	841	735	599	625	603	620	667
79	Loc1B	720	732	738	770	854	854	843	839	893	891	786	650	675	654	670	717
80	Loc1C	854	866	872	903	987	988	977	972	1026	1025	919	783	809	787	804	851
81	Loc1D	875	887	892	924	1008	1008	997	993	1047	1045	940	804	829	808	824	872
82	Loc1E	993	1005	1011	1042	1126	1127	1116	1111	1165	1164	1059	922	948	926	943	990
83	Loc1F	786	798	748	698	791	793	818	810	847	857	771	721	736	719	735	680
84	Loc1G	795	807	757	707	800	802	827	819	856	866	780	730	745	728	744	689
85	Loc1H	812	824	797	747	841	842	867	859	896	906	820	747	762	745	761	729
86	Loc1I	857	869	819	769	862	864	889	881	918	928	842	792	807	790	806	751
87	Loc1J	907	919	890	840	933	935	960	952	989	999	913	842	857	840	856	822
88	Loc1K	1213	1225	1197	1147	1240	1242	1267	1259	1296	1306	1219	1148	1164	1146	1163	1129