

A Linear Programming Approach for Parallel Cell Scheduling with Sequence-Dependent Setup Times

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Abstract

In this study, we consider the problem of scheduling a set of jobs with sequence-dependent setup times on a set of parallel production cells. The objective of this study is to minimize the total completion time. We note that total customer demands for each type should be satisfied, and total required production time in each cell cannot exceed the capacity of the cell. This problem is formulated as an integer programming model and an interface is designed to provide integrity between data and software. Mathematical model is tested by both randomly generated data set and real-world data set from a factory that produce automotive components. As a result of this study, the solution which gives the best alternative production schedule is obtained.

Keywords

Production Scheduling, Total Completion Time, Sequence Dependent Setup Times

1. Introduction

Production scheduling is one of the major issues in production planning as it has an effect on productivity. Each combination of production sequence could have different completion time, in this point to find optimum combination is very important for a company in today's competitive market place, and if company has a lot of different product types with different processing time and/or if each of them needs setup time, to find optimum sequence could be difficult or impossible with estimation or intuition. Stoop and Wiers have mentioned that the task of scheduling production units can become very complicated. Humans are not well equipped to control or optimize large and complex systems. Due to these reasons, techniques and information systems are commonly

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The first systematic approach to scheduling problems was undertaken in the mid-1950s. After this approach, a lot of papers on different scheduling problems have appeared in the literature, and most of them assumed that the setup time can be ignored or reckoned as the job processing time. This assumption simplifies the analysis, however it affects the solution quality of scheduling that requires setup times [2].

The setup times in scheduling problems were considered as separate after mid-1960s. Yang and Liao extensively researched all types of scheduling problems with setup times [3]. Cheng *et al.* reviewed flow shop scheduling problems [4]. Potts and Kovalyov surveyed scheduling problems with batching [5]. After these studies there has been a significant increase in publication of scheduling problems with setup times.

Allahverdi *et al.* classified scheduling problems with setup times as batch and non-batch. A batch setup time occurs when jobs are processed in batches (pallets, containers, boxes) and a setup of a certain time or cost precedes the processing of each batch. In a non-batch processing environment, a setup time is incurred prior to the processing of each job. The corresponding model can also be viewed as a batch setup time model in which each family consists of a single job. And also they classified batch and non-batch scheduling problems as sequence-dependent and sequence-independent. It is sequence-dependent if its duration depends on the families of both the current and the immediately preceding batches, and is sequence-independent if its duration depends solely on the family of the current batch to be processed. All these classifications are valid in both a single machine and parallel machines problems [2].

Bigras *et al.* worked on scheduling problems on a single machine in a sequence dependent setup times. They extended time-dependent travelling salesman problem to single machine scheduling problems with sequence dependent setup times [6]. Ng *et al.* studied a problem of scheduling n jobs in a single machine in batches. They assumed that a batch is a set of jobs processed contiguously and completed together when the processing of all jobs in the batch is finished and processing of a batch requires a machine setup time dependent on the position of this batch in the batch sequence [7]. Gagne *et al.* compared several heuristics for solving a single machine scheduling problem with sequence-dependent setup times. They describe an ant colony optimization algorithm, genetic algorithm, a simulated annealing approach, a local search method and a branch-and-bound algorithm [8].

Haung *et al.* addressed the problem of scheduling on parallel machines in which the setup time is sequencedependent. They formulated the problem as an integer program and for the general cases they developed a hybrid genetic algorithm [9]. Tahar *et al.* suggest a heuristic algorithm for the problem of scheduling a set of jobs with sequence-dependent setup times on a set of parallel machines by using a linear programming modeling with setup times and job splitting [10]. Gacias *et al.* proposed a branch-and-bound method and heuristics based on discrepancy-based search methods for the parallel machine scheduling problem with precedence constraints and setup times [11].

This paper can contribute both academic researches and business life. A developed mathematical model was used to find an optimum solution to a real-world problem. Parallel machine scheduling with sequence dependent setup times and minimization of the sum of the completion time is considered.

2. Problem Description and Mathematical Formulation

In our problem, customer demands d_m for each product type m $(m = 1, \dots, n)$ are produced in monthly basis without exceeding cell capacity h_l for each cell l $(l = 1, \dots, w)$ and a type batch cannot split, because each job splitting requires additional setup time.

In this problem, *n* product types $(1, \dots, n)$ must be scheduled on $w(1, \dots, w)$ production cells. However, when a cell switches the production from a product type k $(k = 1, \dots, n)$ to a product type m $(m = 1, \dots, n, m \neq k)$, a set up time $a_{k,m} \ge 0$ is required. Without loss of generality, we set $a_{k,k} = 0$ $(k = 1, \dots, n)$ and to be able to start the production order we define a dummy product type, product type 0. With this assumption, we define a setup time for all product types which are produced immediately after product type 0. We assume that if a product type is scheduled at the beginning of the schedule, the average time setup time is taken.

All the processing times of each product type m $(m = 1, \dots, n)$ in cell l $(1, \dots, w)$ which are defined as $t_{m,l} > 0$ are our data that we use as inputs of this scheduling problem.

With the introduction of decision variable

$$y_{k,m,l} = \begin{cases} 1, \text{ if type } m \text{ is produced just type } k \text{ in cell } l \\ 0, \text{ otherwise} \end{cases}$$

and status variables

 $q_{k,m,l}$: the number of type *m* that produce just after type *k* in cell *l*

- $p_{m,l}$: total number of type *m* which is produced in cell *l*
- r_m : total number of type *m* which is produced in all cell
- $s_{k,m,l}$: producing time of $q_{k,m,l}$

 $b_{k,m,l}$: sum of producing time $q_{k,m,l}$ and setup time to produce type *m* just after type *k*

- $f_{m,l}$: total required time for type *m* for production and setup in cell *l*
- fc_l : total required time for all types which are assigned to cell l

the problem can be formulated as follows (M in the formulation is a large positive number):

$$Min \ z = \sum_{k=0}^{n} \sum_{m=1}^{n} \sum_{l=1}^{w} b_{k,m,l}$$

Subject to

$$\sum_{k=0}^{n} q_{k,m,l} = p_{m,l}, \forall l: 1 \cdots w, \forall m: 1 \cdots n$$
(1)

$$\sum_{l=1}^{w} p_{m,l} = r_m, \forall m : 1 \cdots n$$
⁽²⁾

$$r_m = d_m, \forall m : 1 \cdots n \tag{3}$$

$$q_{k,m,l} \le M \times y_{k,m,l}, \forall k : 0 \cdots n, \forall m : 1 \cdots n, \forall l : 1 \cdots w$$
(4)

$$\sum_{k=0}^{n} \sum_{l=1}^{w} y_{k,m,l} = 1, \forall m : 1 \cdots n$$
(5)

$$\sum_{l=1}^{w} \sum_{m=1}^{n} y_{k,m,l} = 1, \forall k : 1 \cdots n$$
(6)

$$\sum_{k=0}^{n} \sum_{m=1}^{n} y_{k,m,l} = \sum_{k=0}^{n} \sum_{m=1}^{n} y_{m,k,l}, \forall l : 1 \cdots w$$
(7)

$$y_{k,m,l} + y_{m,k,l} \le 1, \forall k : 1 \cdots n, \forall m : 1 \cdots n, \forall l : 1 \cdots w$$
(8)

$$\sum_{k=0}^{n} \sum_{m=1}^{n} y_{k,m,l} + \sum_{k=0}^{n} \sum_{m=1}^{n} y_{m,k,l} \le 2, \forall l : 1 \cdots w$$
(9)

$$\sum_{l=1}^{w} \sum_{m=1}^{n} y_{0,m,l} \le 11$$
(10)

$$q_{k,m,l} \times t_{m,l} = s_{k,m,l}, \forall k : 0 \cdots n, \forall m : 1 \cdots n, \forall l : 1 \cdots w$$
(11)

$$s_{k,m,l} + a_{k,m} \times y_{k,m,l} = b_{k,m,l}, \forall k : 0 \cdots n, \forall m : 1 \cdots n, \forall l : 1 \cdots w$$
(12)

$$\sum_{k=0}^{n} \sum_{m=1}^{n} b_{k,m,l} \le h_l, \forall l: 1 \cdots w$$
(13)

$$\sum_{k=0}^{n} b_{k,m,l} = f_{m,l}, \forall m : 1 \cdots n, \forall l : 1 \cdots w$$
(14)

$$\sum_{m=1}^{n} f_{m,l} = fc_l, \forall l: 1 \cdots w$$
(15)

The objective is to minimize the total production time in all cells. Constraint (1) ensures that total quantity of type m which are assigned to cell l is equal to total number of type m which are produced in cell l in any sequence combinations of type m with type k. Constraint (2) and (3) guarantee that all customer demands will be satisfied. Constraint (4) provides link between assignment and sequence. To see with which part type the schedule

starts we defined a dummy type number, type 0. With the help of constraint (5) the production schedule starts with type 0, so that the part type which comes just after type 0 is real type that production schedule starts with it. Constraint (6) and (7) ensure consecutive sequence among types. Constraint (8) guarantees that if a product type is assigned to a cell all parts of that type are produced in one sequence. Constraint (9) ensures that maximum one type can be produced after a part and maximum one type can be produced before the type. To meet the demand, Constraint (10) guarantees minimizing the cell number. To calculate production time of a product type which is assigned to a cell we defined constraint (11) and to add setup time to production time we defined constraint (12). Constraint (13) guarantees that total time of all parts assigned a cell cannot exceed the capacity of this cell. To see the total required time for a type m that is assigned to cell l, we defined constraint (15).

3. An Illustrative Example

In order to illustrate the method based on linear programming, we consider an example with 15 jobs and 11 production cells and the mathematical model for 15 types and 11 cells is solved with Mathematical Programming Language (MPL) software.

Demands based on product types are given in **Table 1**, capacities of each cell are given in **Table 2**, and setup time matrix between product types is represented in **Table 3**. For instance, if product type 2 is produced right after type 1, 3000 seconds needed to prepare a cell for type 2. The unit processing time matrix is given in **Table 4** and the results are transferred to Excel sheet by export command of MPL and the results are given in **Table 5**.

Table 1. Demands of j	product types.
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	Туре т														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Demand d _m	850	400	300	1300	240	2370	6220	650	1020	1710	3280	1280	600	640	5158

Table 2. Capacities of production cells.

	Cell No										
	Cell 1	Cell 2	Cell 3	Cell 4	Cell 5	Cell 6	Cell 7	Cell 8	Cell 9	Cell 10	Cell 11
Capacity h _l	1,555,200	1,652,400	1,458,000	1,555,200	1,458,000	1,691,280	1,535,760	1,477,440	1,594,080	1,613,520	1,574,640
(s/month)											

Table 3. Setup time matrix a_{km} (s).

	Type 0	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6	Type 7	Type 8	Type 9	Type 10	Type 11	Type 12	Type 13	Type 14	Type 15
Type 0	0	7200	7200	7200	7200	7200	7200	7200	7200	7200	7200	7200	7200	7200	7200	7200
Type 1	7200	0	3000	3000	10,200	3000	7200	10,200	7200	3000	3000	10,200	10,200	10,200	10,200	7200
Type 2	7200	3000	0	3000	10,200	3000	7200	10,200	7200	3000	3000	10,200	10,200	10,200	10,200	7200
Type 3	7200	3000	3000	0	10,200	3000	7200	10,200	7200	3000	3000	10,200	10,200	10,200	10,200	7200
Type 4	7200	10,200	10,200	10,200	0	10,200	7200	10,200	7200	10,200	10,200	10,200	7200	7200	7200	10,200
Type 5	7200	3000	3000	3000	10,200	0	7200	10,200	7200	3000	3000	10,200	10,200	10,200	10,200	7200
Type 6	7200	7200	7200	7200	7200	7200	0	10,200	600	10,200	10,200	10,200	7200	7200	7200	10,200
Type 7	7200	10,200	10,200	10,200	10,200	10,200	7200	0	10,200	10,200	10200	10,200	7200	7200	7200	10,200
Type 8	7200	7200	7200	7200	7200	7200	600	10,200	0	10,200	10200	10,200	7200	7200	7200	10,200
Type 9	7200	3000	3000	3000	10,200	3000	10,200	10,200	3000	0	3000	10,200	10,200	10,200	10,200	7200
Type 10	7200	3000	3000	3000	10,200	3000	7200	10,200	3000	3000	0	10,200	10,200	10,200	10,200	7200
Type 11	7200	10,200	10,200	10,200	10,200	10,200	7200	10,200	10,200	10,200	10200	0	7200	7200	7200	10,200
Type 12	7200	7200	7200	7200	7200	7200	7200	10,200	7200	7200	10200	7200	0	900	7200	10,200
Type 13	7200	7200	7200	7200	7200	7200	7200	10,200	7200	7200	10200	7200	600	0	7200	10,200
Type 14	7200	9000	9000	9000	9000	9000	9000	7200	9000	9000	9000	10,200	9000	9000	0	9000
Type 15	7200	7200	7200	7200	10,200	7200	9000	9000	10,200	7200	7200	10,200	10,200	10,200	9000	0

fable 4. Unit Processing time matrix t_{ml} (s).											
T N						Cell No					
Type No	Cell 1	Cell 2	Cell 3	Cell 4	Cell 5	Cell 6	Cell 7	Cell 8	Cell 9	Cell 10	Cell 11
1	57	57	57	57	57	57	57	57	57	57	57
2	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5
3	57	57	57	57	57	57	57	57	57	57	57
4	57	57	57	57	57	57	57	57	57	57	57
5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5
6	57	57	57	57	57	57	57	57	57	57	57
7	57	57	57	57	57	57	57	57	57	57	57
8	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5
9	57	57	57	57	57	57	57	57	57	57	57
10	57	57	57	57	57	57	57	57	57	57	57
11	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5
12	57	57	57	57	57	57	57	57	57	57	57
13	57	57	57	57	57	57	57	57	57	57	57
14	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5	54.5
15	57	57	57	57	57	57	57	57	57	57	57

Table 5. The results of fifteen-job-example.

k	m	1	Production Time [s]	Demand	k	m	1	Production Time [s]	Demand
1	3	4	8700	300	9	8	6	15,350	650
2	5	4	7560	240	10	1	4	19,150	850
3	2	4	9800	400	11	13	3	18,600	600
4	14	6	18,720	640	12	11	3	69,520	3280
5	10	4	35,490	1710	13	12	3	22,360	1280
6	4	6	31,900	1300	14	7	6	112,940	6220
7	15	6	97,886	5158	15	9	6	24,540	1020
8	6	6	45,630	2370					

Table 5 indicates the sequence of part types based on cell numbers to provide the minimum total completion time and production times of each type. The results demonstrate that 15 parts can produced in cell 3, 4, and 6 that gives minimum total completion time. According to **Table 5**, the type sequence in cell 4 {1, 3, 2, 5, 10}, in cell 6 {4, 14, 7, 15, 9, 8, 6}, and in cell 3 {11, 13, 12} give the optimum results. In addition to these results the number of each product type demands can be seen on **Table 5**.

4. Real-World Problem

In this section, a real-world example is used to evaluate the performance of the model. To illustrate the effectiveness of the mathematical model, we compare the results with current production scheduling results.

This study is completed in an automotive injector factory, in body section. Body section has four main processes which are turning, milling, heat treatment, and special process. Within this project we studied in turning process which is the first step of all body section as a pilot. Turning process has 35 different part types and 11 production cells.

4.1. Current Situation

Demands and assignments in current situation are given in **Table 6**. With the data given in **Table 6**, we calculate processing times of each type and then we calculate total production time, sum of setup time and processing time, of all types. **Table 6** demonstrates that 2819 hours is needed to produce all types regarding to current sequence table.

T 11 (

i able o. Dema	nus, assignmen	ts and processing	g times of curre	int situation.				
Production Cell	Product Type	Demand/ Production Quantity (pcs)	Processing Time (s)	Production Cell	Product Type	Demand/ Production Quantity (pcs)	Processing Time (s)	
1	19 21	26,880 43,680	510,720 786,240		1	850	32,300	
2	20 18	46,000 26,000	874,000 494,000	9	4 10 13	1300 1710 600	49,400 64,125 22,500	
3	7 23	6220 41,000	105,740 697,000		33	1920	70,080	
4	26	41,522	788,918		2 5	400 240	15,000 11,040	
5	27 35	29,441 12,096	559,379 229,824	10	8 11	650 3280	26,650 131,200	
6	24 29	5376 25,600	96,768 484,400		14 17 25	640 4950 4032	24,960 188,100 167,328	
7	30 32 22	20,160 25,920 26,000	383,040 440,640 442,000	11	3 6 9 12	300 2370 1020 1280	11,400 92,430 38,250 48,000	
8	34 16	39,360 44,600	708,480 758,200		15 28 31	5158 1344 6048	193,425 57,120 257,040	
Total Processin	g Time (s) = 9,86	51,697						
Total Setup Tin	ne (s) = $285,600$							
Total Productio	on Time (h) $= 281$	9						

. 1

4.2. Proposed Situation

Demands and assignments in current situation are given in **Table 6**. With the data given in **Table 6**, we calculate processing times of each type and then we calculate total production time, sum of setup time and processing time, of all types. **Table 6** demonstrates that 2819 hours is needed to produce all types regarding to current sequence table.

The linear mathematical program with the same demands in current situation is run by MPL and results summarized as it is seen in Table 7. Production times in Table 7 consist of processing time and setup time.

According to **Table 7**, the sequence in cell 1 {19, 7, 18}, in cell 2 {35, 23, 29,}, in cell 4 {8, 6, 16}, in cell 5 {17, 3, 5, 9, 14, 24, 34, 33, 15, 30}, in cell 6 {32, 4, 25, 11, 20}, in cell 7 {13, 1, 2, 10, 12, 22, 21}, and in cell 8 {28, 26, 27, 31} gives the optimum results and the total production time is 2553 hours.

5. Conclusions

In this paper, we have proposed a new method based on linear programming for a parallel machine scheduling problem involving sequence dependent setup times. The criterion is to minimize the total production time. As shown in the real world problem section, total production time reduced from 2819 to 2553 hour. For this problem, average saving based on total production time is 9.4%, and due to the fact that demands are changing every month, the saving will change every month.

Figure 1 displays the variation of production times that are constructed by summation of processing and setup times for each type. It can be inspected from the figure that significant reduction in production times can be observed for the types where the demand is low (e.g. Type 11). Contrarily for the types where demands are distinctively higher (e.g. Type 20), it is hard to observe any significant change in the production times. According to this graphic, it can be concluded that the firm has found a proper assignment for high demand product types. As displayed in **Figure 2**, variations in setup times show significant reduction for the future situation versus the current one. For large majority of the product types, setup times decrease explicitly where the commentary for this situation should be the appropriate transition of product types according to the objective of this scheduling problem.

	Type Sequence y_{kml}					
k	m	1	- Demand	Production Time (s)		
13	1	7	850	19,150		
1	2	7	400	9800		
17	3	5	300	8700		
32	4	6	1300	26,500		
3	5	5	240	7560		
8	6	4	2370	45,630		
19	7	1	6220	112,940		
16	8	4	650	13,250		
5	9	5	1020	20,340		
2	10	7	1710	35,490		
25	11	6	3280	71,320		
10	12	7	1280	22,360		
21	13	7	600	12,300		
9	14	5	640	15,120		
33	15	5	5158	88,286		
6	16	4	44,600	758,800		
30	17	5	4950	85,050		
7	18	1	26,000	497,000		
18	19	1	26,880	516,720		
11	20	6	46,000	884,200		
22	21	7	43,680	787,140		
12	22	7	26,000	442,900		
35	23	2	41,000	703,000		
14	24	5	5376	102,768		
4	25	6	4032	79,776		
28	26	8	41,522	796,118		
26	27	8	29,441	561,179		
31	28	8	1344	27,336		
23	29	2	25,600	493,600		
15	30	5	20,160	383,940		
27	31	8	6048	122,112		
20	32	6	25,920	449,640		
34	33	5	1920	38,640		
24	34	5	39,360	715,680		
29	35	2	12,096	237,024		
	Total Produ	action Time (h)		2553		

Table 7. Results of linear mathematical programming.

Another prominent gain is the usage of 8 production cells instead of 11 observed in the current situation. This is achieved by our linear programming approach with optimal product type assignments into production cells.

Topic of a future research is to apply heuristic methods; such as tabu search or simulated annealing algorithm for more product types. Because it is difficult to solve sequence-dependent scheduling problem for more product types.



Figure 1. Comparison of production times [h] (Current vs. Proposed).



Figure 2. Comparison of setup times [h] (Current vs. Proposed).

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