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Measuring the NBA Teams' Cross-Efficiency by DEA Game

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Abstract

In this paper, we use DEA to measure the NBA basketball teams' efficiency in seasons 2006-2007, 2007-2008, 2008-2009 and 2009-2010. In this context, each team is a DMU; we select the payroll and the average attendance to be the inputs while the wins and the average points per game to be the outputs. First, in order to obtain benchmarks, we measure the DMUs efficiency through classic DEA BCC model with an assurance region for each one of the four seasons individually and together. When we consider the four seasons together, we may analyse whether the performance of each team increases or decreases over time. Next, we evaluate the teams cross efficiency by DEA game to consider that there is no cooperation among DMUs. This approach also improves the efficiencies discrimination.

Keywords

Data Envelopment Analysis, DEA Game, Cross-Efficiency

1. Introduction

Basketball is one of the most popular sports, especially in US, moving a large amount of money. In Brazil, this sport has been achieving increasing proportions mainly with the creation of the NBB (New Brazilian Basketball) in 2009. Basketball is played by two teams with five players each. The winner is the one who gets more points at the end of the match. Draw does not happen due to an overtime whenever there is a draw in the regular time.

The main basketball league is the National Basketball Association (NBA) which has 30 teams divided into two conferences (east and west) of 15 teams, and classified according to their geographical location. Besides the

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division at conferences, there is a classification into three divisions in each conference with 5 teams in each division, which also uses geographical features to organize the teams. Each team plays 82 games in a season, 41 home matches (in their own stadium) and 41 matches out (in the opponent's stadium). These 82 games are composed by 4 games against each team of the same division, 3 games against each team that belongs to the same conference, but out of the same division group, 2 games against each team of the other conference and 6 games randomly selected with teams from the other conference. After this initial classification phase, called regular season, the eight best teams (based on number of wins) in each conference qualify to the next phase, called Playoffs, in order to define the season champion.

Due to the popularity of basketball in the US and the increasing values that this sport moves, efficient analysis is always welcome to guide the investments. In the literature, there are many papers using DEA (Data Envelopment Analysis) applied to sports [1]-[4]. Some works use DEA to evaluate the efficiency of basketball players. [5], for example, uses a procedure that selects non-zero weights in order to evaluate the efficiency of players. [6] also measures the efficiency of players, however, by the cross-efficiency method, considering some environment variables. We found one paper which applies DEA to evaluate NBA basketball teams efficiency proposed by [7]. The authors use a Network DEA approach which consists of five stages, evaluating the performance of first-team and bench-team players, the offensive and defensive systems and the ability for transforming the points made by itself and by the opponents into wins.

In this paper, we propose to evaluate through DEA, teams efficiency of NBA in the seasons 2006-2007, 2007-2008, 2008-2009 and 2009-2010. Thus, teams are the DMUs, the payroll and the average attendance of their home games are the inputs while the number of wins and the average points per game of each team are the outputs. We use the payroll to take into account the investments, average attendance because of the psychological benefit, the number of wins and the average points per game because they measure the success of a team.

First, we calculate teams' efficiency by the classic BCC output DEA model with the security region that the weights associated with the number of wins are at least two times greater than those associated with the number of points per game. Then, we calculate their cross efficiencies by *DEA game* to consider that there is no cooperation among DMUs. This approach also improves the discrimination among DMUs.

This paper is divided as follows: Section 2 describes the BCC DEA model used and its results. Section 3 describes how a cross-efficiency can be calculated by DEA game and shows final results by this approach while Section 4 shows the conclusions and the perspectives for future works.

2. DEA BCC

To calculate the efficiency of each team in the NBA basketball seasons 2006-2007, 2007-2008, 2008-2009 and 2009-2010, we use a classical output-oriented DEA BCC with a simple security region. We have, as inputs, team payroll by season and average attendance of home games by season. As outputs, we have number of wins by season and average points per game by season. The security region used is that the weight of output 1 is at least two times greater than the weight of output 2.

For each DMU k (observed DMU), we solved the model (I) to find its efficiency. The mathematical notation used in the model (I) is as follows:

- $n \rightarrow$ number of DMUs.
- $h_k \rightarrow$ continuous variable that indicates the efficiency of observed DMU k.
- $\lambda_j \rightarrow$ continuous variable that indicates the DMU j contribution for the observed DMU frontier projection (target).
- $\gamma_1 \rightarrow$ continuous variable associated with the security region constraint.
- $x_i^j \rightarrow$ input i value of DMU j.
- $y_i^j \rightarrow$ output i value of DMU j.

$$(I) \qquad \max \qquad h_k \tag{1.1}$$

$$\text{subject to} \qquad \sum_j^n x_1^j \lambda_j \le x_1^k \tag{1.2}$$

$$\sum_j^n x_2^j \lambda_j \le x_2^k \tag{1.3}$$

$$\sum_{j}^{n} y_1^j \lambda_j - \gamma_1 \geq y_1^k h_k \tag{1.4}$$

$$\sum_{j}^{n} y_2^j \lambda_j + 2\gamma_1 \geq y_2^k h_k \tag{1.5}$$

$$\sum_{j}^{n} \lambda_j = 1 \tag{1.6}$$

The previous model is known as the envelope model and for each DMU k, allows us to find a whole set of other DMUs called benchmarks set of DMU k. The DMUs belonging to this set are efficient and have similar practices as the DMU k. The set of benchmarks for each DMU is found according to its projection on the efficient frontier. When a DMU is efficient, it is the only benchmark for itself. Mathematically, a DMU j is a benchmark for the DMU k when we run the model (I) for the DMU k and the variable λ_j acquires non-null value.

Table 1, **Table 2**, **Table 3** and **Table 4** show the results for seasons 2006-2007, 2007-2008, 2008-2009 e 2009-2010, respectively. **Table 5** shows for each season, the index of the efficient teams (**Table 7** shows the correspondent team of each index). Note that there are a large number of efficient teams in each season. It happens because the BCC model is benevolent. Some teams are efficient in two or more seasons although they do

Table 1. Efficiency by model (I) for season 2006-2007.

Index	Team	Ef. BCC	Benchmarks
1	76ers	1.0000	1
2	Blazers	0.7617	1 e 24
3	Bobcats	1.0000	3
4	Bucks	0.7732	1, 23 e 24
5	Bulls	0.8943	18 e 23
6	Cavaliers	0.8294	18 e 23
7	Celtics	0.6841	1, 23 e 24
8	Clippers	0.7994	18, 23 e 24
9	Grizzlies	1.0000	9
10	Hawks	0.8476	1, 23 e 24
11	Heat	0.7543	18 e 26
12	Hornets	0.8354	18, 23 e 24
13	Jazz	0.8901	18, 23 e 24
14	Kings	0.7627	18, 23 e 24
15	Knicks	0.6588	18 e 26
16	Lakers	0.8089	18, 23 e 24
17	Magic	0.8342	18, 23 e 24
18	Mavericks	1.0000	18
19	Nets	0.8481	18, 24 e 26
20	Nuggets	0.9075	18, 24 e 26
21	Pacers	0.9258	1 e 24
22	Pistons	0.8438	18
23	Raptors	1.0000	23
24	Rockets	1.0000	24
25	Spurs	0.9685	18, 24 e 26
26	Suns	1.0000	26
27	Thunder	0.8110	1 e 24
28	Timberwolves	0.8074	1 e 24
29	Warriors	0.8524	18, 23 e 24
30	Wizards	0.8248	18, 23 e 24

Table 2. Efficiency by model (I) for season 2007-2008.

Index	Team	Ef. BCC	Benchmarks
1	76ers	0.8319	12, 13 e 14
2	Blazers	0.7988	7, 12 e 13
3	Bobcats	0.8506	12, 13 e 14
4	Bucks	0.6244	7, 12 e 13
5	Bulls	0.6883	7 e 13
6	Cavaliers	0.7683	7
7	Celtics	1.0000	7
8	Clippers	0.5720	7, 12 e 13
9	Grizzlies	1.0000	9
10	Hawks	0.7697	7, 12 e 13
11	Heat	0.4399	7
12	Hornets	1.0000	12
13	Jazz	1.0000	13
14	Kings	1.0000	14
15	Knicks	0.5404	7
16	Lakers	0.9448	7 e 13
17	Magic	0.9398	7, 12 e 13
18	Mavericks	0.8432	7
19	Nets	0.6999	7 e 12
20	Nuggets	0.8908	7 e 12
21	Pacers	1.0000	21
22	Pistons	0.9601	7 e 13
23	Raptors	0.7825	7 e 13
24	Rockets	0.8964	7 e 12
25	Spurs	0.8871	7, 12 e 13
26	Suns	0.9303	7, 12 e 13
27	Thunder	1.0000	27
28	Timberwolves	0.7194	12, 13 e 14
29	Warriors	0.8619	7 e 13
30	Wizards	0.8196	7, 12 e 13

not have a high value of outputs, such as team 9 (Grizzlies) and team 14 (Kings). Teams 18 (Mavericks) and 26 (Suns) in the season 2006-2007, 7 (Celtics) and 26 (Suns) in the season 2007-2008, 6 (Cavaliers) and 16 (Lakers) in the season 2008-2009 and 6 (Cavaliers) and 17 (Magic) in the season 2009-2010 are efficient because they have high values of output.

Table 6 shows the number of times each team is a benchmark of other team in each season. Some teams are not showed in the table because they are not a benchmark for any team. Furthermore, when a team is benchmark only once in a season, it means that this team is a benchmark only for itself.

The largest number of times that a team is benchmark is 28. This result is achieved by team 17 (Magic) which is efficient in two seasons and is benchmark for 20 teams in one of them. Team 7 (Celtics) is benchmark 21 times although being efficient in just one season. Another interesting fact is the case of teams 9 (Grizzlies) and 14 (Kings), which despite of being efficient in more than a half of the seasons, the number of times they served as benchmark for other teams is relatively small (13 and 6, respectively). The Grizzlies (Kings) is benchmark just for himself in three of four seasons (two of three seasons) in which it is efficient.

Besides, we also execute the BCC model for a specific case with 120 DMUs, where each team represents a different DMU in each season. Thus, it is possible to make a temporal analysis of each DMU. The results obtained for this case are in **Table 7**. This table also shows the average efficiency of all seasons dealt in this paper and a ranking related to the average efficiency.

Table 3. Efficiency by model (I) for season 2008-2009.

Index	Team	Ef. BCC	Benchmarks
1	76ers	0.8795	17 e 21
2	Blazers	1.0000	2
3	Bobcats	0.9552	11, 20 e 21
4	Bucks	0.8435	17. 20 e 21
5	Bulls	0.7786	2 e 16
6	Cavaliers	1.0000	6
7	Celtics	0.9782	2, 16 e 17
8	Clippers	0.6595	11, 20 e 21
9	Grizzlies	1.0000	9
10	Hawks	0.9211	17, 20 e 21
11	Heat	1.0000	11
12	Hornets	0.9385	11, 20 e 21
13	Jazz	0.8861	2 e 16
14	Kings	1.0000	14
15	Knicks	0.6554	16
16	Lakers	1.0000	16
17	Magic	1.0000	17
18	Mavericks	0.8296	16
19	Nets	1.0000	19
20	Nuggets	1.0000	20
21	Pacers	1.0000	21
22	Pistons	0.7078	2 e 16
23	Raptors	0.6524	16 e 17
24	Rockets	0.9523	2, 16, 17 e 20
25	Spurs	0.9476	2, 16, 17 e 20
26	Suns	0.8506	2, 16 e 20
27	Thunder	0.6207	2, 11 e 20
28	Timberwolves	1.0000	28
29	Warriors	0.7188	2, 16 e 20
30	Wizards	0.5722	17, 20 e 21

In **Table 7**, 11 DMUs are efficient, where eight of them belong to two last seasons. Besides, the efficiency value always increases for teams 1 (76ers), 24 (Rockets), 25 (Spurs) and 28 (Timberwolves) while the efficiency always decreases for team 10 (Hawks). Team 26 (Suns) is the first of the rank, although it is not efficient in any season. It happens because he has high efficiency values for all the seasons. Despite team 9 (Grizzlies) being efficient in two of the four seasons, it is placed ninth.

We can note that the result of the BCC method leads to a poor discrimination of the efficiencies.

The Classical Cross Efficiency Method

The cross efficiency method [8] is a good way to improve the discrimination among DMUs. For the classical cross efficiency, we first obtain the weights and efficiency of each DMU *k* by (II). This model is named Multiplier and it is the dual problem of (I). The notation for the model (II) is the following:

- $n \to$ number of DMUS
- $r \to$ number of inputs
- $s \to$ number of outputs
- $x_i^j \to$ value of the input *i* for the DMU *j*.
- $y_i^j \to$ value of the output *i* for the DMU *j*.
- $u_i \to$ weight related to output *i*
- $v_i \to$ weight related to input *i*

Table 4. Efficiency by model (I) for season 2009-2010.

Index	Team	Ef. BCC	Benchmarks
1	76ers	0.7514	9 e 17
2	Blazers	0.9221	6,10 e 26
3	Bobcats	0.8902	9,10, 17 e 26
4	Bucks	0.9632	9 e 17
5	Bulls	0.7836	6 e 10
6	Cavaliers	1.0000	6
7	Celtics	0.8815	6 e 17
8	Clippers	0.6841	9, 10 e 17
9	Grizzlies	1.0000	9
10	Hawks	1.0000	10
11	Heat	0.8497	6, 10 e 17
12	Hornets	0.8488	9 e 17
13	Jazz	0.9240	6 e 17
14	Kings	1.0000	14
15	Knicks	0.6541	6, 10 e 17
16	Lakers	0.9638	6, 17 e 26
17	Magic	1.0000	17
18	Mavericks	0.9434	6, 10 e 17
19	Nets	1.0000	19
20	Nuggets	0.9566	6, 10 e 17
21	Pacers	0.8341	9 e 17
22	Pistons	1.0000	22
23	Raptors	0.8285	6 e 10
24	Rockets	0.8478	9 e 17
25	Spurs	0.8889	6, 10 e 17
26	Suns	0.9972	6, 10 e 17
27	Thunder	0.9472	6 e 10
28	Timberwolves	0.5584	9, 10 e 17
29	Warriors	0.6622	6, 10 e 17
30	Wizards	0.6464	9 e 17

Table 5. Indexes of efficient teams in each season.

Season	Indexes of efficient teams
06-07	1, 3, 9, 18, 23, 24 e 26
07-08	7, 9, 12, 13, 14, 21 e 27
08-09	2, 6, 9, 14, 16, 17, 19, 20, 21 e 28
09-10	6, 9, 10, 14, 17, 19 e 22

$$(II) \qquad \min \quad \sum_{i=1}^{r} v_i x_i^k + v_k^* \tag{1.7}$$

$$\text{Subject to} \quad \sum_{i=1}^{s} u_i y_i^k = 1 \tag{1.8}$$

$$\sum_{i=1}^{r} v_i x_i^j - \sum_{i=1}^{s} u_i y_i^j + v_k^* \geq 0, \forall j \in \{1, \cdots, n\} \tag{1.9}$$

$$u_1 \geq 2u_2 \tag{1.10}$$

Table 6. Number of times that a team is benchmark for each season.

Original Index	Team	Number of times that a team is benchmark 06-07	07-08	08-09	09-10	Total
17	Magic	-	-	8	20	28
7	Celtics	-	21	-	-	21
24	Rockets	19	-	-	-	19
13	Jazz	-	17	-	-	17
18	Mavericks	17	-	-	-	17
6	Cavaliers	-	-	1	15	16
10	Hawks	-	-	-	15	15
12	Hornets	-	15	-	-	15
9	Grizzlies	1	1	1	10	13
23	Raptors	13	-	-	-	13
16	Lakers	-	-	11	-	11
20	Nuggets	-	-	11	-	11
2	Blazers	-	-	10	-	10
21	Pacers	-	1	8	-	9
1	76ers	8	-	-	-	8
14	Kings	-	4	1	1	6
26	Suns	5	-	-	-	5
19	Nets	-	-	1	1	2
3	Bobcats	1	-	-	-	1
22	Pistons	-	-	-	1	1
27	Thunder	-	1	-	-	1
28	Timberwolves	-	-	1	-	1

"-": The team is not efficient.

For each pair of DMUs k and d, we calculate the DMU k efficiency using the weights from DMU $d\left(E_{dk}\right)$. Thus, E_{dk} is calculated as follows:

$$E_{dk} = \frac{\sum_{i=1}^{r} v_i^d x_i^k + v_d^*}{\sum_{i=1}^{s} u_i^d y_i^k} \tag{1.11}$$

The average cross efficiency of DMU k is calculated by the following way:

$$E_k = \frac{\sum_{d=1}^{n} E_{dk}}{n}, \tag{1.12}$$

To avoid $E_{dk} < 0$ [9] [10], we add the following group of constraints to model (II):

$$\sum_{i=1}^{r} v_i x_i^j + v_k^* \geq 0, \forall j \in \{1, \cdots, n\} \tag{1.13}$$

It should be noted that with constraint (1.13) the model is no longer BCC, being less benevolent. The efficient frontier may be changed.

Some papers propose secondary objectives in cross efficiency because different weight sets can generate the same efficiency value [8] [11].

In this paper we apply a cross efficiency method named Dea Game that is not affected by the previous situation. In this method, the weight set of each DMU is obtained iteratively with the constraint that the best efficiency for the others DMUs does not decrease. We describe the method in Section 3.

Table 7. Teams efficiency by BCC model for all seasons at the same time.

Index	Team	BCC's Efficiency					
		06-07	07-08	08-09	09-10	average	Ranking
1	76ers	0.6527	0.7814	0.7997	1.0000	0.8085	15
2	Blazers	0.8753	0.9179	0.7570	0.6649	0.8038	17
3	Bobcats	0.8208	0.7251	0.7758	1.0000	0.8304	13
4	Bucks	0.8526	0.7174	0.6203	0.6663	0.7142	24
5	Bulls	0.7245	0.7390	0.6504	0.8703	0.7461	21
6	Cavaliers	0.9440	0.9911	0.7618	0.8253	0.8806	7
7	Celtics	0.8379	0.9590	1.0000	0.6030	0.8500	11
8	Clippers	0.6565	0.5304	0.5617	0.7499	0.6246	28
9	Grizzlies	1.0000	0.7221	1.0000	0.7688	0.8727	9
10	Hawks	1.0000	0.8312	0.7502	0.7208	0.8256	14
11	Heat	0.8066	0.8304	0.4382	0.7460	0.7053	25
12	Hornets	0.7591	0.8443	1.0000	0.7618	0.8413	12
13	Jazz	0.8682	0.8191	0.9503	0.8662	0.8760	8
14	Kings	0.7562	0.6690	0.9603	0.6962	0.7704	20
15	Knicks	0.6162	0.6554	0.5371	0.6447	0.6134	30
16	Lakers	0.9016	1.0000	0.9314	0.7768	0.9025	2
17	Magic	0.9596	0.9635	0.9121	0.7558	0.8978	3
18	Mavericks	0.8822	0.8296	0.8359	1.0000	0.8869	5
19	Nets	0.5845	0.7201	0.7010	0.7698	0.6939	26
20	Nuggets	0.9019	0.9222	0.8912	0.8325	0.8870	4
21	Pacers	0.7216	0.7833	1.0000	0.7209	0.8065	16
22	Pistons	0.6780	0.6934	0.9098	0.8430	0.7811	19
23	Raptors	0.8034	0.6498	0.7465	1.0000	0.7999	18
24	Rockets	0.7949	0.8849	0.8960	0.8962	0.8680	10
25	Spurs	0.8465	0.8800	0.8836	0.9267	0.8842	6
26	Suns	0.9477	0.8238	0.9242	0.9821	0.9195	1
27	Thunder	0.9223	0.5591	0.7969	0.6951	0.7434	22
28	Timberwolves	0.5218	0.6139	0.6671	0.6740	0.6192	29
29	Warriors	0.6378	0.6535	0.8405	0.7984	0.7326	23
30	Wizards	0.6048	0.5223	0.7891	0.7781	0.6736	27

3. DEA Game

In DEA game, proposed by [12], each DMU is seen as a competitor in an uncooperative environment. In this context, to calculate the cross-efficiency of DMU k related to DMU d, a set of weights is found in order to maximize the efficiency of DMU k with the additional constraint that d efficiency does not decrease. In this context, to calculate the efficiency of each DMU, it is necessary to know the efficiencies of the others, and vice-versa. This problem is solved through an iterative process, where the DMUs efficiencies are found, and these values represent a Nash equilibrium.

Model (III) calculates the cross-efficiency of DMU k related to d with current cross-efficiency α_d using DEA game.

$$(III) \qquad \min \qquad E'_{k,d} = \sum_{i=1}^{r} v_i x_i^k \tag{1.14}$$

$$\text{s.t.} \qquad \sum_{i=1}^{s} u_i y_i^k = 1 \tag{1.15}$$

$$\sum_{i=1}^{r} v_i x_i^j - \sum_{i=1}^{s} u_i y_i^j \geq 0, \forall j \in \{1, \cdots, n\} \tag{1.16}$$

$$u_1 \geq 2u_2 \tag{1.17}$$

$$\sum_{i=1}^{r} v_i x_i^d - \alpha_d \sum_{i=1}^{s} u_i y_i^d \geq 0 \tag{1.18}$$

The constraint (1.18) ensures that the efficiency of the DMU *d* is greater than α_d.

Iterative **Algorithm 1** describes the steps to find the efficiency of DMUs that represent a Nash equilibrium solution. In this algorithm, α_j^t represents the efficiency of DMU *j* at iteration *t*.

Liang *et al*. [12] proved that the algorithm converges and that the final solution represents a Nash equilibrium.

We apply DEA game for seasons 2006-2007, 2007-2008, 2008-2009 and 2009-2010. The results for each of these seasons are shown in **Table 8**, **Table 9**, **Table 10** and **Table 11**, respectively. Those tables show for each season, the BCC efficiencies, the average classical cross-efficiency, DEA game efficiency described in **Algorithm 1**, rankings associated with the average cross-efficiencies and the DEA game efficiencies along with the difference between these rankings.

Results show a significant difference among the rankings generated by each season of at least 10 units. It is expected since the average cross-efficiency is affected by the multiplicity of optimal weights.

The DEA game is more indicated for noncooperative environment. Besides, it helps differentiating the DMUs, creating a ranking that is not affected by the optimal weights multiplicity. Results concerning this method show that six teams were in three of the four seasons among the top 10 (teams 16, 17, 20, 24, 25 and 26), two teams were among the last ten of all four seasons (teams 8 and 15) and three teams were among the last ten in three of four seasons (teams 5, 14 and 22).

4. Conclusions

In this paper, we show through DEA methodology, classical and cross-efficiency measures of basketball teams belonging to the NBA for 2006-2007, 2007-2008, 2008-2009 and 2009-2010 seasons. In this context, the 30 teams are the DMUs, the two inputs are annual payroll and average attendance at season and outputs 1 and 2 are the number of wins and the average number of points per game, where output 1 has a weight two times higher compared to output 2. For the classical efficiency, we use the BCC method while we use the DEA Game for the cross efficiency.

Regarding teams analysis we can observe Suns is the top ranked because this team has a very high points average and reached a great number of wins. In the last two seasons Cavaliers reached the highest number of wins, but could not achieve the efficiency because of the high investment in payroll and a very high average attendance. The most improved team in our analysis is Charlotte Bobcats which increased a lot the number of wins between seasons 2008-2009 (35 wins) and 2009-2010 (44 wins). Another issue we observed is the benevolence of BCC model which considered Atlanta Hawks and Vancouver Grizzlies as efficient because these teams have the lowest payroll or the lowest average attendance. BCC results showed the efficient teams and benchmarks for each season while DEA Game considered the noncooperative environment, improved the discrimination between the DMUs and gave us a ranking.

In future papers, we intend to deal with the fact that each team belongs to a specific conference and division.

Algorithm 1. DEA game.

Require: ϵ

Step 1: Set $t = 1$. For each DMU *k*, calculate the classical average cross-efficiency E_k and set $\alpha_k^t = E_k, \forall k \in \{1, \cdots, n\}$.

Step 2: For each pair of DMUs *k* and *d*, solve model (III) and obtain $E'_{k,d}$.

Step 3: Set $\alpha_k^{t+1} = \frac{\sum_{d=1}^{n} E'_{dk}}{n}$.

Step 4: If for some *k*, $\left|\alpha_k^{t+1} - \alpha_k^t\right| > \epsilon$, then return to step 2. Otherwise, the algorithm ends and α_k^{t+1} is the optimum cross-efficiency of DMU *k*.

Table 8. Results summary for the 2006-2007 season.

Index	Team	BCC Eff.	Average Cross-Eff.	DEA game Eff.	Cross-Eff. Rank	Dea Game Rank	Diff.
1	76ers	1.0000	0.0443	0.9834	23	3	20
2	Blazers	0.7617	0.0421	0.7485	27	27	0
3	Bobcats	1.0000	0.0374	0.9398	29	7	22
4	Bucks	0.7732	0.0449	0.7676	21	25	−4
5	Bulls	0.8943	0.0557	0.8039	11	20	−9
6	Cavaliers	0.8294	0.0559	0.8156	10	18	−8
7	Celtics	0.6841	0.0409	0.6781	28	29	−1
8	Clippers	0.7994	0.0534	0.7928	15	22	−7
9	Grizzlies	1.0000	0.0151	0.8253	30	16	14
10	Hawks	0.8476	0.0450	0.8363	20	13	7
11	Heat	0.7543	0.0503	0.7302	18	28	−10
12	Hornets	0.8354	0.0526	0.8263	17	15	2
13	Jazz	0.8901	0.0633	0.8822	4	9	−5
14	Kings	0.7627	0.0493	0.7597	19	26	−7
15	Knicks	0.6588	0.0433	0.6435	24	30	−6
16	Lakers	0.8089	0.0567	0.8037	8	21	−13
17	Magic	0.8342	0.0532	0.8311	16	14	2
18	Mavericks	1.0000	0.0645	0.9926	3	2	1
19	Nets	0.8481	0.0538	0.8459	14	12	2
20	Nuggets	0.9075	0.0587	0.9038	5	8	−3
21	Pacers	0.9258	0.0445	0.8658	22	10	12
22	Pistons	0.8438	0.0544	0.7710	12	24	−12
23	Raptors	1.0000	0.0544	0.9825	13	5	8
24	Rockets	1.0000	0.0586	0.9991	6	1	5
25	Spurs	0.9685	0.0670	0.9624	1	6	−5
26	Suns	1.0000	0.0653	0.9832	2	4	−2
27	Thunder	0.8110	0.0424	0.8051	26	19	7
28	Timberwolves	0.8074	0.0425	0.7722	25	23	2
29	Warriors	0.8524	0.0575	0.8483	7	11	−4
30	Wizards	0.8248	0.0562	0.8202	9	17	−8

Table 9. Results summary for the 2007-2008 season.

Index	Team	BCC Eff.	Average Cross-Eff.	DEA game Eff.	Cross-Eff. Rank	Dea Game Rank	Diff.
1	76ers	0.8319	0.0443	0.8164	18	14	4
2	Blazers	0.7988	0.0495	0.7911	16	16	0
3	Bobcats	0.8506	0.0392	0.7837	21	17	4
4	Bucks	0.6244	0.0392	0.6207	22	27	−5
5	Bulls	0.6883	0.0438	0.6703	19	24	−5
6	Cavaliers	0.7683	0.0496	0.7132	15	21	−6
7	Celtics	1.0000	0.0645	0.9756	1	3	−2
8	Clippers	0.5720	0.0363	0.5662	24	28	−4
9	Grizzlies	1.0000	0.0217	0.7064	29	22	7
10	Hawks	0.7697	0.0473	0.7671	17	20	−3
11	Heat	0.4399	0.0284	0.4260	28	30	−2
12	Hornets	1.0000	0.0614	1.0000	5	1	4
13	Jazz	1.0000	0.0605	0.9924	6	2	4
14	Kings	1.0000	0.0343	0.9043	26	8	18
15	Knicks	0.5404	0.0349	0.4931	25	29	−4
16	Lakers	0.9448	0.0631	0.9229	2	6	−4
17	Magic	0.9398	0.0593	0.9307	8	4	4
18	Mavericks	0.8432	0.0544	0.7728	12	18	−6
19	Nets	0.6999	0.0433	0.6917	20	23	−3
20	Nuggets	0.8908	0.0575	0.8389	10	12	−2
21	Pacers	1.0000	0.0369	0.8340	23	13	10
22	Pistons	0.9601	0.0628	0.9284	3	5	−2
23	Raptors	0.7825	0.0501	0.7695	14	19	−5
24	Rockets	0.8964	0.0580	0.8550	9	10	−1
25	Spurs	0.8871	0.0602	0.8667	7	9	−2
26	Suns	0.9303	0.0626	0.9102	4	7	−3
27	Thunder	1.0000	0.0025	0.6677	30	25	5
28	Timberwolves	0.7194	0.0325	0.6570	27	26	1
29	Warriors	0.8619	0.0573	0.8407	11	11	0
30	Wizards	0.8196	0.0517	0.8109	13	15	−2

Table 10. Results summary for the 2008-2009 season.

Index	Team	BCC Eff.	Average Cross-Eff.	DEA game Eff.	Cross-Eff. Rank	Dea Game Rank	Diff.
1	76ers	0.8795	0.0492	0.8537	15	15	0
2	Blazers	1.0000	0.0586	0.9580	8	5	3
3	Bobcats	0.9552	0.0373	0.8850	23	14	9
4	Bucks	0.8435	0.0459	0.8170	17	18	−1
5	Bulls	0.7786	0.0501	0.7202	14	23	−9
6	Cavaliers	1.0000	0.0645	0.9179	4	11	−7
7	Celtics	0.9782	0.0690	0.9528	1	6	−5
8	Clippers	0.6595	0.0333	0.6298	27	26	1
9	Grizzlies	1.0000	0.0330	0.8157	28	19	9
10	Hawks	0.9211	0.0549	0.9116	11	12	−1
11	Heat	1.0000	0.0414	0.9789	21	3	18
12	Hornets	0.9385	0.0523	0.9266	13	10	3
13	Jazz	0.8861	0.0553	0.8487	10	16	−6
14	Kings	1.0000	0.0164	0.7832	29	21	8
15	Knicks	0.6554	0.0423	0.6086	20	28	−8
16	Lakers	1.0000	0.0645	0.9765	3	4	−1
17	Magic	1.0000	0.0651	0.9998	2	1	1
18	Mavericks	0.8296	0.0535	0.7614	12	22	−10
19	Nets	1.0000	0.0343	0.9104	26	13	13
20	Nuggets	1.0000	0.0602	0.9985	6	2	4
21	Pacers	1.0000	0.0374	0.9481	22	7	15
22	Pistons	0.7078	0.0470	0.6439	16	25	−9
23	Raptors	0.6524	0.0446	0.6136	19	27	−8
24	Rockets	0.9523	0.0599	0.9456	7	8	−1
25	Spurs	0.9476	0.0606	0.9281	5	9	−4
26	Suns	0.8506	0.0573	0.8336	9	17	−8
27	Thunder	0.6207	0.0373	0.6067	24	29	−5
28	Timberwolves	1.0000	0.0163	0.8069	30	20	10
29	Warriors	0.7188	0.0448	0.6991	18	24	−6
30	Wizards	0.5722	0.0344	0.5655	25	30	−5

Table 11. Results summary for the 2009-2010 season.

Index	Team	BCC Eff.	Average Cross-Eff.	DEA game Eff.	Cross-Eff. Rank	Dea Game Rank	Diff.
1	76ers	0.7514	0.0405	0.7054	25	23	2
2	Blazers	0.9221	0.0578	0.8779	11	13	−2
3	Bobcats	0.8902	0.0531	0.8691	14	15	−1
4	Bucks	0.9632	0.0515	0.9175	17	9	8
5	Bulls	0.7836	0.0509	0.7511	19	21	−2
6	Cavaliers	1.0000	0.0645	0.9680	2	4	−2
7	Celtics	0.8815	0.0572	0.8721	12	14	−2
8	Clippers	0.6841	0.0419	0.6766	24	24	0
9	Grizzlies	1.0000	0.0512	0.9260	18	8	10
10	Hawks	1.0000	0.0599	1.0000	8	1	7
11	Heat	0.8497	0.0558	0.8459	13	16	−3
12	Hornets	0.8488	0.0485	0.8070	20	19	1
13	Jazz	0.9240	0.0605	0.9012	7	11	−4
14	Kings	1.0000	0.0106	0.7324	28	22	6
15	Knicks	0.6541	0.0433	0.6387	22	27	−5
16	Lakers	0.9638	0.0629	0.9399	5	6	−1
17	Magic	1.0000	0.0668	0.9999	1	2	−1
18	Mavericks	0.9434	0.0630	0.9171	4	10	−6
19	Nets	1.0000	0.0054	0.5383	29	29	0
20	Nuggets	0.9566	0.0624	0.9483	6	5	1
21	Pacers	0.8341	0.0449	0.7812	21	20	1
22	Pistons	1.0000	0.0031	0.6478	30	26	4
23	Raptors	0.8285	0.0515	0.8168	16	18	−2
24	Rockets	0.8478	0.0527	0.8332	15	17	−2
25	Spurs	0.8889	0.0590	0.8828	9	12	−3
26	Suns	0.9972	0.0639	0.9901	3	3	0
27	Thunder	0.9472	0.0585	0.9307	10	7	3
28	Timberwolves	0.5584	0.0318	0.5379	27	30	−3
29	Warriors	0.6622	0.0424	0.6544	23	25	−2
30	Wizards	0.6464	0.0394	0.6288	26	28	−2

References

[1] Lozano, S., Villa, G., Guerrero, F. and Cortes, P. (2002) Measuring the Performance of Nations at the Summer Olympics Using Data Envelopment Analysis. *Journal of the Operational Research Society*, **53**, 501-511.

[2] Haas, D.J. (2003) Productive Efficiency of English Football Teams—A Data Envelopment Analysis Approach. *Managerial and Decision Economics*, **24**, 403-410. http://dx.doi.org/10.1002/mde.1105

[3] Lins, M.P.E., Gomes, E.G., Soares de Mello, J.C.C.B. and Soares de Mello, A.J.R. (2003) Olympic Ranking Based on a Zero Sum Gains DEA Model. *European Journal of Operational Research*, **148**, 312-322. http://dx.doi.org/10.1016/S0377-2217(02)00687-2

[4] Barros, C.P. and Leach, S. (2006) Performance Evaluation of the English Premier Football League with Data Envelopment Analysis. *Applied Economics*, **38**, 1449-1458. http://dx.doi.org/10.1080/00036840500396574

[5] Cooper, W.W. and Ruiz, J.L. (2009) Selecting Non-Zero Weights to Evaluate Effectiveness of Basketball Players with DEA. *European Journal of Operational Research*, **195**, 563-574. http://dx.doi.org/10.1016/j.ejor.2008.02.012

[6] Bai, F. (2009) Testing the Effects of Environmental Variables on Efficiency and Generating Multiple Weight Sets for Cross-Evaluation with DEA: An Application to the National Basketball Association. City University of Hong Kong, Hong Kong.

[7] Moreno, P. and Lozano, S. (2012) A Network DEA Assessment of Team Efficiency in the NBA. *Annals of Operations Research*, **214**, 99-124.

[8] Doyle, J. and Green, R. (1994) Efficiency and Cross-Efficiency in DEA: Derivations, Meanings and Uses. *Journal of the Operational Research Society*, **45**, 567-578. http://dx.doi.org/10.1057/jors.1994.84

[9] Soares de Mello, J.C.C.B., Soares de Mello, A.J.R., Lins, M.P.E. and Gomes, E.G. (2002) Construction of a Smoothed DEA Frontier. *Pesquisa Operacional*, **22**, 183-201. http://dx.doi.org/10.1590/S0101-74382002000200006

[10] Wu, J., Liang, L. and Chen, Y. (2009) DEA Game Cross-Efficiency Approach to Olympic Rankings. *Omega*, **37**, 909-918. http://dx.doi.org/10.1016/j.omega.2008.07.001

[11] Liang, L., Wu, J., Cook, W.D. and Zhu, J. (2008) Alternative Secondary Goals in DEA Cross-Efficiency Evaluation. *International Journal of Production Economics*, **113**, 1025-1030. http://dx.doi.org/10.1016/j.ijpe.2007.12.006

[12] Liang, L., Wu, J., Cook, W.D. and Zhu, J. (2008) The DEA Game Cross-Efficiency Model and Its Nash Equilibrium. *Operations Research*, **56**, 1278-1288. http://dx.doi.org/10.1287/opre.1070.0487