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Bi-Criteria Optimization Technique in Stochastic System Maintenance Allocation Problem

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ABSTRACT

In this paper, the problem of optimum allocation of repairable and replaceable components in a system is formulated as a Bi-objective stochastic non linear programming problem. The system maintenance time and cost are random variable and has gamma and normal distribution respectively. A Bi-criteria optimization technique, weighted Tchebycheff is used to obtain the optimum allocation for a system. A numerical example is also presented to illustrate the computational details.

Keywords: Selective Maintenance; Weighted Tchebycheff Technique; Multi-Criteria Optimization; Stochastic Programming; Chance Constrained; Modified E-Model; System Reliability

1. Introduction

We consider a system which requires performing a sequence of identical production runs after every given (fixed) period. A production run in the system consists of several subsystems where each subsystem can work properly if at least one of its components is operational. The following assumptions are also made:

1) all the components can be repaired if deteriorated or failed;

2) all component states are independent.

We assume that the system comprises two types of subsystem. One is the type of subsystems in which the components are very sensitive to the functioning of the whole system and, therefore, on deterioration these should be replaced by new ones. Let these subsystems range from 1 to s. The other type of subsystems is those in which the components after deterioration can be repaired and then replaced. Let such subsystems range from $s+1$ to m. In **Figure 1** the Group X consists of the s subsystems with sensitive components which on failure are replaced by new ones and Y the remaining $(m-s)$ subsystems in which the components can be repaired (see Ali *et al.* [1]).

Ideally, all the failed components in all the subsystem of Group X are replaced by new ones prior to the beginning of the next mission/ run. In a similar way, ideally all the failed components in subsystem of Group Y are repaired and then replaced prior to the beginning of the next mission/run. However, due to the constraints on the cost and time it may not be possible to repair and replace all the failed components in Groups X and Y. For this a mathematical programming frame-work is established for assisting decision-makers in determining the optimal subset of maintenance activities to perform prior to beginning of the next mission. This decision-making process

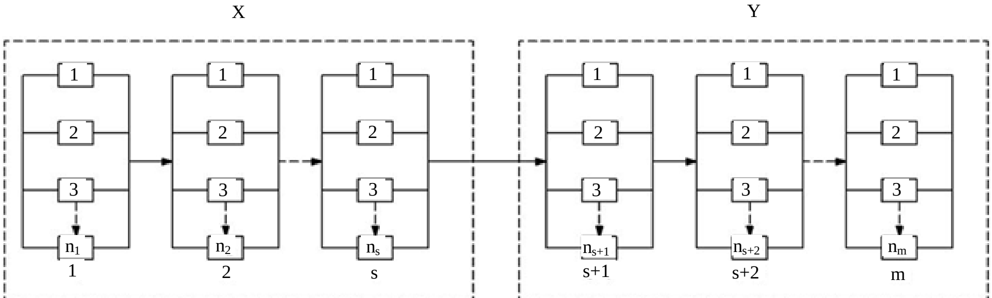

Figure 1. Parallel components in repairable and replaceable subsystem.

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is referred to as selective maintenance. The selective maintenance models presented allow the decision-maker to consider limitations on maintenance time and budget, as well as the reliability of the system. Selective maintenance is an open research area that is consistent with the modern industrial objective of performing more intelligent and efficient maintenance.

For this let us suppose a_i be the total failed components in the subsystems and d_i be the number of components in the i^{th} subsystem, which can be repaired and replaced prior to the beginning of the next mission (See Rice *et al.* [2]). Thus under the selective maintenance the number of components available for the next mission in the i^{th} subsystem will be

$$\left(n_i - k_i\right) + d_i, i = 1, 2, \cdots, m \tag{1}$$

Therefore the reliability of the subsystems range from 1 to S for a production run is given by

$$R\left(d_i\right) = \left\{\prod_{i=1}^{s}\left[1-\left(1-r_i\right)^{n_i - a_i + d_i}\right]\right\} \tag{2}$$

and the reliability of the subsystems range from $s+1$ to m for a production run is given by

$$R\left(d_i\right) = \left\{\prod_{i=S+1}^{m}\left[1-\left(1-r_i\right)^{n_i - a_i + d_i}\right]\right\} \tag{3}$$

The maintenance time constraint for the system is given as

$$\sum_{i=1}^{m} t_i d_i \le T_0 \tag{4}$$

and the maintenance cost constraint for the system is given as

$$\sum_{i=1}^{m} c_i d_i \le C_0 \tag{5}$$

However, in the event the reliability of the subsystems of Groups X and Y time are of equally serious concern. Let us consider, for instance, the following multi-objective problem (please see the Equation (6) below).

Secondly, a Bi-objective programming problem in which time and the cost spent on system maintenance is minimized simultaneously for the required reliability $R_i^*\left(d_i\right)$ (say). The mathematical model of the problem is given as Equation (7) below.

Recently many authors have discussed the allocation problem of repairable components. Among them are Rice *et al.* [2], Schneider and Cassady [3], Rajaopalan and Cassady [4], Schneider *et al.* [5], Iyoob *et al.* [6], Ali *et al.* ([1,7-10]), Faisal and Ali [11] and many others.

In this paper, we have formulated stochastic system maintenance problem as a multi-objective programming

$$\left.\begin{array}{ll}
\text{Maximize } R\left(d_i\right) = \left\{\prod_{i=1}^{s}\left[1-\left(1-r_i\right)^{n_i - a_i + d_i}\right]\right\} & \text{(i)}\\
\text{and Maximize } R\left(d_i\right) = \left\{\prod_{i=S+1}^{m}\left[1-\left(1-r_i\right)^{n_i - a_i + d_i}\right]\right\} & \text{(ii)}\\
\text{Subject to } \sum_{i=1}^{m} t_i d_i \le T_0 & \text{(iii)}\\
\sum_{i=1}^{m} c_i d_i \le C_0 & \text{(iv)}\\
0 \le d_i \le a_i, \forall d_i \text{ are integer} & \text{(v)}\\
n_i \ge a_i, i = 1, 2, \cdots, m & \text{(vi)}
\end{array}\right\} \tag{6}$$

$$\left.\begin{array}{ll}
\text{Minimize } T = \sum_{i=1}^{m} t_i d_i & \text{(i)}\\
\text{and Minimize } C = \sum_{i=1}^{m} c_i d_i C_0 & \text{(ii)}\\
\text{Subject to } \left\{\prod_{i=1}^{s}\left[1-\left(1-r_i\right)^{n_i - a_i + d_i}\right]\right\} \ge R_1^*\left(d_i\right) & \text{(iii)}\\
\left\{\prod_{i=S+1}^{m}\left[1-\left(1-r_i\right)^{n_i - a_i + d_i}\right]\right\} \ge R_2^*\left(d_i\right) & \text{(iv)}\\
0 \le d_i \le a_i, \forall d_i \text{ are integer} & \text{(v)}\\
n_i \ge a_i, i = 1, 2, \cdots, m & \text{(vi)}
\end{array}\right\} \tag{7}$$

problem. We have discussed components repairable and replaceable time and cost as a random variable in the constraint and has Gamma and Normal distribution respectively. The Probabilistic constraints function is then converted into an equivalent deterministic non-linear programming form by using chance constrained programming.

2. The Chance Constrained Programming

In many practical situations the constraint Equations (iii) and (iv) are not fixed and taken as probabilistic. Thus the above problem (6) can be written in the following chance constrained programming form as Equation (8) below, where $p_0, 0 \le p_0 \le 1$ is a specified probability.

In the above problem (8), let us assume that t_i and c_i are independently gamma and normally distributed random variables.

Let us assume that $t_i, i = 1, \cdots, m$ are independent Gamma distributed random variables in the constraint 8 (iii), *i.e.*, $t_i \sim G(\alpha_i, \beta_i)$.

Then the,

$$\text{Mean}(t_i) = \frac{\beta_i}{\alpha_i}, \quad \text{Variance}(t_i) = \frac{\beta_i}{\alpha_i^2}.$$

Now let

$$f(d) = \sum_{i=1}^{m} t_i d_i$$

Then mean is

$$E\{f(d)\} = \sum_{i=1}^{m} d_i E(t_i) = \sum_{i=1}^{m} \frac{\beta_i}{\alpha_i} d_i$$

Further, as t_i are independently distributed, we have

$$V\{f(d)\} = \sum_{i=1}^{m} t_i^2 V(d_i) = \sum_{i=1}^{m} \frac{\beta_i}{\alpha_i^2} d_i$$

Now the constraints 8 (iii) can be written as

$$P(f(d) \le T_0) \ge p_0, \tag{9}$$

Since number of components within the system are assumed to be large we have from Liapounoff's central limit theorem

$$f(d) \sim N(E\{f(d)\}, V\{f(d)\}).$$

Thus (9) is equivalent to

$$P\left\{\frac{f(d) - E\{f(d)\}}{\sqrt{V\{f(d)\}}} \le \frac{T_0 - E\{f(d)\}}{\sqrt{V\{f(d)\}}}\right\} \ge p_0,$$

where $\left[\frac{f(d) - E\{f(d)\}}{\sqrt{V\{f(d)\}}}\right]$ is a standard normal variate with mean zero and variance one. Thus the probability of realizing $\{f(d)\}$ less than or equal to T_0 can be written as

$$P(f(d) \le T_0) = \phi\left[\frac{T_0 - E\{f(d)\}}{\sqrt{V\{f(d)\}}}\right], \tag{10}$$

where $\phi(z)$ represents the cumulative density function of the standard normal variable evaluated at *Z*. If K_α represents the value of the standard normal variable at which $\phi(K_\alpha) = p_0$, then the constraint (10) can be written as

$$\phi\left[\frac{T_0 - E\{f(d)\}}{\sqrt{V\{f(d)\}}}\right] \ge \phi(K_\alpha). \tag{11}$$

The inequality will be satisfied only if

$$\left[\frac{T_0 - E\{f(d)\}}{\sqrt{V\{f(d)\}}}\right] \ge K_\alpha,$$

$$\left.\begin{array}{ll}
\text{Maximize } R(d_i) = \left\{\prod_{i=1}^{s}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\} & \text{(i)} \\
\text{and Maximize } R(d_i) = \left\{\prod_{i=S+1}^{m}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\} & \text{(ii)} \\
\text{Subject to } P\left(\sum_{i=1}^{m} t_i d_i \le T_0\right) \ge p_0 & \text{(iii)} \\
P\left(\sum_{i=1}^{m} c_i d_i \le C_0\right) \ge p_0 & \text{(iv)} \\
0 \le d_i \le a_i, \forall d_i \text{ are int eger} & \text{(v)} \\
n_i \ge a_i, i = 1, 2, \cdots, m & \text{(vi)}
\end{array}\right\} \tag{8}$$

or equivalently,

$$\frac{T_0 - \sum_{i=1}^{m} \frac{\beta_i}{\alpha_i} d_i}{\sqrt{\sum_{i=1}^{m} \frac{\beta_i}{\alpha_i^2} d_i^2}} \ge K_\alpha,$$

Thus an equivalent deterministic constraint to the stochastic constraint is given by

$$\left(\sum_{i=1}^{m} \frac{\beta_i}{\alpha_i} d_i\right) + K_\alpha \sqrt{\sum_{i=1}^{m} \frac{\beta_i}{\alpha_i^2} d_i^2} \le T_0 \quad (12)$$

Now we consider the case when c_i are independently normally distributed random variables in the constraint 8 (iv), *i.e.* $c_i \sim N\left(\mu_i, \sigma_i^2\right)$.

The constraint $P\left(f(c) \le C_0\right) \ge p_0$, is equivalent to

$$P\left\{\frac{f(c) - E\{f(c)\}}{\sqrt{V\{f(c)\}}} \le \frac{C_0 - E\{f(c)\}}{\sqrt{V\{f(c)\}}}\right\} \ge p_0, \quad (13)$$

Now in this case

$$E\{f(c)\} = \sum_{i=1}^{m} d_i E(c_i) = \sum_{i=1}^{m} \mu_i d_i$$

and

$$V\{f(c)\} = \sum_{i=1}^{m} d_i^2 V(c_h) = \sum_{i=1}^{m} \sigma_i^2 d_i^2$$

Therefore, the deterministic equivalent of 8 (iv) in this case is

$$\left(\sum_{i=1}^{m} \mu_i d_i\right) + K_\alpha \sqrt{\sum_{i=1}^{m} \sigma_i^2 d_i^2} \le C_0 \quad (14)$$

The equivalent deterministic non-linear programming problem (8) to the stochastic programming problem is given by

$$\left.\begin{array}{lr}
\text{Maximize } R(d_i) = \left\{\prod_{i=1}^{s}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\} & \text{(i)} \\
\text{and Maximize } R(d_i) = \left\{\prod_{i=S+1}^{m}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\} & \text{(ii)} \\
\text{Subject to } \left(\sum_{i=1}^{m} \frac{\beta_i}{\alpha_i} d_i\right) + K_\alpha \sqrt{\sum_{i=1}^{m} \frac{\beta_i}{\alpha_i^2} d_i^2} \le T_0 & \text{(iii)} \\
\left(\sum_{i=1}^{m} \mu_i d_i\right) + K_\alpha \sqrt{\sum_{i=1}^{m} \sigma_i^2 d_i^2} \le C_0 & \text{(iv)} \\
0 \le d_i \le a_i, \forall d_i \text{ are int eger} & \text{(v)} \\
n_i \ge a_i, i = 1, 2, \cdots, m & \text{(vi)}
\end{array}\right\} \quad (15)$$

3. Modified E-Model

Consider the situations in which the time taken and cost spent on maintenance are not fixed and taken as probabilistic in the objective function in Equations (i) and (ii). Thus the above problem (7) can be written in the following probabilistic objective function form as:

$$\left.\begin{array}{lr}
\text{Minimize } T = p\left(\sum_{i=1}^{m} t_i d_i\right) & \text{(i)} \\
\text{and Minimize } C = p\left(\sum_{i=1}^{m} c_i d_i\right) & \text{(ii)} \\
\text{Subject to } \left\{\prod_{i=1}^{s}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\} \times \left\{\prod_{i=S+1}^{m}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\} \ge R^*(d_i) & \text{(iii)} \\
0 \le d_i \le a_i, \forall d_i \text{ areint eger} & \text{(v)} \\
n_i \ge a_i, i = 1, 2, \cdots, m & \text{(vi)}
\end{array}\right\} \quad (16)$$

Using Modified E-model technique, the problem (16) is formulated as

$$\left.\begin{array}{ll}\text{Minimize } T = k_1\left(\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i}d_i\right)+k_2\sqrt{\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i^2}d_i^2} & (\text{i})\\ \text{and Minimize } C = k_1\left(\sum_{i=1}^{m}\mu_i d_i\right)+k_2\sqrt{\sum_{i=1}^{m}\sigma_i^2 d_i^2} & (\text{ii})\\ \text{Subject to}\left\{\prod_{i=1}^{s}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\}\times\left\{\prod_{i=S+1}^{m}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\}\ge R^*(d_i) & (\text{iii})\\ 0\le d_i\le a_i, \forall d_i \text{ are int eger} & (\text{iv})\\ n_i\ge a_i, i=1,2,\cdots,m & (\text{v})\end{array}\right\} \quad (17)$$

where k_1 and k_2 are non-negative constants, and their values show the relative importance of the expectation and the variance. Some authors suggest that $k_1 + k_2 = 1$, see Rao [12].

The two others Bi-objective programming models in different prospects for the decision-makers are

Model 1:

$$\left.\begin{array}{ll}\text{Minimize } T = k_1\left(\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i}d_i\right)+k_2\sqrt{\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i^2}d_i^2} & (\text{i})\\ \text{and Maximize } R_2(d_i)=\left\{\prod_{i=S+1}^{m}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\} & (\text{ii})\\ \text{Subject to}\left(\sum_{i=1}^{m}\mu_i d_i\right)+K_\alpha\sqrt{\sum_{i=1}^{m}\sigma_i^2 d_i^2}\le C_0 & (\text{iii})\\ \left\{\prod_{i=1}^{s}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\}\times\left\{\prod_{i=S+1}^{m}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\}\ge R^*(d_i) & (\text{iv})\\ 0\le d_i\le a_i, \forall d_i \text{ are int eger} & (\text{v})\\ n_i\ge a_i, i=1,2,\cdots,m & (\text{vi})\end{array}\right\} \quad (18)$$

Model 2:

$$\left.\begin{array}{ll}\text{Minimize } C = k_1\left(\sum_{i=1}^{m}\mu_i d_i\right)+k_2\sqrt{\sum_{i=1}^{m}\sigma_i^2 d_i^2} & (\text{i})\\ \text{and Maximize } R_1(d_i)=\left\{\prod_{i=1}^{s}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\} & (\text{ii})\\ \text{Subject to}\left(\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i}d_i\right)+K_\alpha\sqrt{\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i^2}d_i^2}\le T_0 & (\text{iii})\\ \left\{\prod_{i=1}^{s}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\}\times\left\{\prod_{i=S+1}^{m}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\}\ge R^*(d_i) & (\text{iv})\\ 0\le d_i\le a_i, \forall d_i \text{ are int eger} & (\text{v})\\ n_i\ge a_i, i=1,2,\cdots,m & (\text{vi})\end{array}\right\} \quad (19)$$

4. A Multi-Criteria Weighted Tchebycheff Optimization Technique

Let us consider a multi-objective programming problem

$$\left.\begin{array}{l}\text{Min } f(x) = f_1(x), f_2(x), \cdots, f_k(x)\\ \text{Subject to } x\in s\end{array}\right\} \quad (20)$$

assumed to have $k\left(k \geq 2\right)$ competing objective functions $\left(f_i : \Re^n \rightarrow \Re\right)$ that are to be minimized simultaneously. The following definitions illustrate the concepts of *efficient and weakly efficient* decision vectors.

Definition: A decision vector $x^* \in X$ is efficient (Pareto optimal) for multi-objective programming problem if there does not exist a $x \in X, x \neq x^*$ such that $f_i\left(x\right) \leq f_i\left(x^*\right)$ for $i = 1, 2, \cdots, k$ with strict inequality holding for at least one index i. ($x^* \in X$ is efficient, $f\left(x^*\right)$ is non-dominated).

Definition: A decision vector $x^* \in X$ is weakly efficient (weakly Pareto optimal) for multi-objective programming problem if there does not exist a $x \in X$, $x \neq x^*$ such that $f_i\left(x\right) < f_i\left(x^*\right)$ for $i = 1, 2, \cdots, k$. ($x^* \in X$ is weakly efficient, $f\left(x^*\right)$ is weakly non-dominated).

There are several metrics that are found in the literature related to multi-objective programming problem. If ξ_i is the reference point and the ideal point,

$$\xi_i = \text{Min}_{x \in X} f_i\left(x\right),$$

is used as the reference point, the general weighted L_p-metric$\left(1 \leq p \leq \infty\right)$ is defined as

$$\left.\begin{aligned} &\text{Min}\left(\sum_{i=1}^{k} w_i \left| f_i\left(x\right) - \xi_i \right|^p\right)^{1/p} \\ &\text{Subject to } x \in X \end{aligned}\right\} \quad (21)$$

We assume that $w_i \geq 0, \forall i = 1, 2, \cdots, k$ and $\sum_{i=1}^{k} w_i = 1$, where the w_i 's are weighting coefficients provided by the decision maker reflecting the relative importance. If $p = \infty$, problem (21) reduces to a "weighted Tchebycheff Technique" (see Bowman [13]).

$$\left.\begin{aligned} &\text{Min} \underset{i=1,2,\cdots,k}{\text{Max}}\left[w_i \left| f_i\left(x\right) - \xi_i \right|\right] \\ &\text{Subject to } x \in X \end{aligned}\right\} \quad (22)$$

If the reference point is the global optimal solution of $f_i\left(x\right)$, then the absolute value signs in problem (22) can be removed (see Miettinen [14]) yielding

$$\left.\begin{aligned} &\text{Min} \underset{i=1,2,\cdots,k}{\text{Max}}\left[w_i \left(f_i\left(x\right) - \xi_i \right)\right] \\ &\text{Subject to } x \in X \end{aligned}\right\} \quad (23)$$

Miettinen [14] also showed that if the objectives and constraints are differentiable form of problem (23) can be defined as

$$\left.\begin{aligned} &\text{Min } \delta \\ &\text{Subject to } \delta \geq \left\{w_i \left(f_i\left(x\right) - \xi_i \right)\right\}, \forall i, x \in X \end{aligned}\right\} \quad (24)$$

The solution of problem (24) is guaranteed weekly non-dominated for positive weights and at least one non-dominated solution is also guaranteed. If the solution is unique, then it is non-dominated, however if it is not unique, then it might be weakly non-dominated (see Wierzbicki [15].

The two objective functions in the Equation (15) are to be maximizing the total reliability of replaceable components of Group X and the reliability of repairable components of Group Y. We have convert the following maximizing problem into minimization problem using the property $\text{Max}Z = \text{Min}\left(-Z\right)$. Therefore the problem defined in Equation (15) is converted into a two criterion minimization problem: $\text{Min} - \left\{Z_1, Z_2\right\}$ subject to the constraints (see Khasawneh *et al.* [16]). Now the efficient solution is obtained by using the weighted Tchebycheff technique

$$\left.\begin{aligned}
&\text{Minimize } \delta && (\text{i}) \\
&\text{Subject to } \delta \geq w_1 \left\{\left\{-\prod_{i=1}^{s}\left[1-\left(1-r_i\right)^{n_i - a_i + d_i}\right]\right\} - \xi_1\right\} && (\text{ii}) \\
&\delta \geq w_2 \left\{\left\{\prod_{i=S+1}^{m}\left[1-\left(1-r_i\right)^{n_i - a_i + d_i}\right]\right\} - \xi_2\right\} && (\text{iii}) \\
&\left(\sum_{i=1}^{m} \frac{\beta_i}{\alpha_i} t_i\right) + K_\alpha \sqrt{\sum_{i=1}^{m} \frac{\beta_i}{\alpha_i^2} d_i^2} \leq T_0 && (\text{iv}) \\
&\left(\sum_{i=1}^{m} \mu_i d_i\right) + K_\alpha \sqrt{\sum_{i=1}^{m} \sigma_i^2 d_i^2} \leq C_0 && (\text{v}) \\
&w_1 + w_2 = 1, w_1, w_2 \geq 0 && (\text{vi}) \\
&0 \leq d_i \leq a_i, \forall d_i \text{ are integer} && (\text{vii}) \\
&n_i \geq a_i, i = 1, 2, \cdots, m && (\text{viii})
\end{aligned}\right\} \quad (25)$$

In practice, ξ_i can be defined as the minimum individual values of the following problems:

$$\left.\begin{array}{ll}\xi_1 = \text{Min}\left(-R\left(d_i\right)\right) = -\left\{\prod_{i=1}^{s}\left[1-\left(1-r_i\right)^{n_i-a_i+d_i}\right]\right\} & \text{(i)}\\ \text{Subject to}\left(\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i}t_i\right)+K_\alpha\sqrt{\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i^2}d_i^2}\le T_0 & \text{(ii)}\\ \left(\sum_{i=1}^{m}\mu_i d_i\right)+K_\alpha\sqrt{\sum_{i=1}^{m}\sigma_i^2 d_i^2}\le C_0 & \text{(iii)}\\ 0\le d_i\le a_i, \forall d_i \text{ are integer} & \text{(iv)}\\ n_i\ge a_i, i=1,2,\cdots,m & \text{(v)}\end{array}\right\} \tag{26}$$

and similarly

$$\left.\begin{array}{ll}\xi_2 = \text{Min}\left(-R\left(d_i\right)\right) = -\left\{\prod_{i=s+1}^{m}\left[1-\left(1-r_i\right)^{n_i-a_i+d_i}\right]\right\} & \text{(i)}\\ \text{Subject to}\left(\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i}t_i\right)+K_\alpha\sqrt{\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i^2}d_i^2}\le T_0 & \text{(ii)}\\ \left(\sum_{i=1}^{m}\mu_i d_i\right)+K_\alpha\sqrt{\sum_{i=1}^{m}\sigma_i^2 d_i^2}\le C_0 & \text{(iii)}\\ 0\le d_i\le a_i, \forall d_i \text{ are integer} & \text{(iv)}\\ n_i\ge a_i, i=1,2,\cdots,m & \text{(v)}\end{array}\right\} \tag{27}$$

In similar way, the problem defined in Equation (17) is also a two criterion minimization problem: $\text{Min}\{T, C\}$ subject to the constraints. Now the efficient solution is obtained by using the weighted Tchebycheff technique

$$\left.\begin{array}{ll}\text{Minimize } \delta & \text{(i)}\\ \text{Subject to } \delta\ge w_1\left\{\left\{k_1\left(\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i}t_i\right)+k_2\sqrt{\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i^2}d_i^2}\right\}-\xi_1\right\} & \text{(ii)}\\ \delta\ge w_2\left\{k_1\left(\sum_{i=1}^{m}\mu_i d_i\right)+k_2\sqrt{\sum_{i=1}^{m}\sigma_i^2 d_i^2}-\xi_2\right\} & \text{(iii)}\\ \left\{\prod_{i=1}^{s}\left[1-\left(1-r_i\right)^{n_i-a_i+d_i}\right]\right\}\times\left\{\prod_{i=S+1}^{m}\left[1-\left(1-r_i\right)^{n_i-a_i+d_i}\right]\right\}\ge R^*\left(d_i\right) & \text{(iv)}\\ w_1+w_2=1, w_1, w_2\ge 0 & \text{(v)}\\ 0\le d_i\le a_i, \forall d_i \text{ are integer} & \text{(vi)}\\ n_i\ge a_i, i=1,2,\cdots,m & \text{(vii)}\end{array}\right\} \tag{28}$$

The values of ξ_i can be defined as the minimum individual values of the following problems:

$$\left.\begin{aligned}
&\xi_1 = \text{Min}T = k_1\left(\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i}d_i\right) + k_2\sqrt{\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i^2}d_i^2} && \text{(i)}\\
&\text{Subject to}\left\{\prod_{i=1}^{s}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\}\times\left\{\prod_{i=S+1}^{m}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\} \ge R^*(d_i) && \text{(ii)}\\
&0 \le d_i \le a_i, \forall d_i \text{ are int eger} && \text{(iii)}\\
&n_i \ge a_i, i = 1,2,\cdots,m && \text{(iv)}
\end{aligned}\right\} \quad (29)$$

and similarly

$$\left.\begin{aligned}
&\xi_2 = \text{Min}C = k_1\left(\sum_{i=1}^{m}\mu_i d_i\right) + k_2\sqrt{\sum_{i=1}^{m}\sigma_i^2 d_i^2} && \text{(i)}\\
&\text{Subject to}\left\{\prod_{i=1}^{s}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\}\times\left\{\prod_{i=S+1}^{m}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\} \ge R^*(d_i) && \text{(ii)}\\
&0 \le d_i \le a_i, \forall d_i \text{ are int eger} && \text{(iii)}\\
&n_i \ge a_i, i = 1,2,\cdots,m && \text{(iv)}
\end{aligned}\right\} \quad (30)$$

Now the problem defined in Model 1; Equation (18) is also a two criterion minimization problem: $\text{Min}\{T - R_2(d_i)\}$ subject to the constraints. Now the efficient solution is obtained by using the weighted Tchebycheff technique

$$\left.\begin{aligned}
&\text{Minimize } \delta && \text{(i)}\\
&\text{Subject to } \delta \ge w_1\left\{\left\{k_1\left(\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i}t_i\right) + k_2\sqrt{\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i^2}d_i^2}\right\} - \xi_1\right\} && \text{(ii)}\\
&\delta \ge w_2\left\{\left\{\prod_{i=S+1}^{m}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\} - \xi_2\right\} && \text{(iii)}\\
&\left\{\prod_{i=1}^{s}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\}\times\left\{\prod_{i=S+1}^{m}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\} \ge R^*(d_i) && \text{(iv)}\\
&\left(\sum_{i=1}^{m}\mu_i d_i\right) + K_\alpha\sqrt{\sum_{i=1}^{m}\sigma_i^2 d_i^2} \le C_0 && \text{(v)}\\
&w_1 + w_2 = 1, w_1, w_2 \ge 0 && \text{(vi)}\\
&0 \le d_i \le a_i, \forall d_i \text{ are int eger} && \text{(vii)}\\
&n_i \ge a_i, i = 1,2,\cdots,m && \text{(viii)}
\end{aligned}\right\} \quad (31)$$

The values of ξ_1 and ξ_2 is the minimum individual values obtained as

$$\xi_1 = \text{Minimize } T = k_1\left(\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i}d_i\right) + k_2\sqrt{\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i^2}d_i^2}$$

subject to the (iii) to (vi) of Equation (18)

$$\xi_2 = \text{Min}R_2(d_i) = -\left\{\prod_{i=S+1}^{m}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\}$$

subject to the (iii) to (vi) of Equation (18).

In Model 2, Equation (19) is also a two criterion minimization problem: $\text{Min}\{C - R_1(d_i)\}$ subject to the constraints. Now the efficient solution is obtained by using the weighted Tchebycheff technique

$$\left.\begin{array}{ll} \text{Minimize } \delta & \text{(i)} \\ \text{Subject to } \delta \ge w_1\left\{k_1\left(\sum_{i=1}^{m}\mu_i d_i\right)+k_2\sqrt{\sum_{i=1}^{m}\sigma_i^2 d_i^2}-\xi_1\right\} & \text{(ii)} \\ \delta \ge w_2\left\{\left\{\prod_{i=1}^{s}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\}-\xi_2\right\} & \text{(iii)} \\ \left\{\prod_{i=1}^{s}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\}\times\left\{\prod_{i=S+1}^{m}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\}\ge R^*(d_i) & \text{(iv)} \\ \left(\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i}t_i\right)+K_\alpha\sqrt{\sum_{i=1}^{m}\frac{\beta_i}{\alpha_i^2}d_i^2}\le T_0 & \text{(v)} \\ w_1+w_2=1, w_1, w_2\ge 0 & \text{(vi)} \\ 0\le d_i\le a_i, \forall d_i \text{ are int eger} & \text{(vii)} \\ n_i\ge a_i, i=1,2,\cdots,m & \text{(viii)} \end{array}\right\} \quad (32)$$

where the values of ξ_1 and ξ_2 is the minimum individual values obtained as

$$\xi_1 = \text{Minimize } C = k_1\left(\sum_{i=1}^{m}\mu_i d_i\right)+k_2\sqrt{\sum_{i=1}^{m}\sigma_i^2 d_i^2}$$

subject to the (iii) to (vi) of Equation (19)

$$\xi_2 = \text{Min } R_1(d_i) = -\left\{\prod_{i=1}^{s}\left[1-(1-r_i)^{n_i-a_i+d_i}\right]\right\}$$

subject to the (iii) to (vi) of Equation (19).

5. Numerical Illustrations

Consider a system having the Group X consisting of 3 subsystems and also the Group Y consisting of 4 subsystems. The available time between two missions for repairing and replacing is 150 time units. The available cost of maintenance for repairing and replacing for the next mission is 860 units. For simplicity we have considered in the above numerical illustration: the reliability of each component in a subsystem is same, cost spent and time taken on replacing and repairing each component within a subsystem are same. The remaining parameters for the various subsystems are given in **Table 1**.

5.1. Solution of Chance Constrained Programming by Using Weighted Tchebycheff Technique

Before applying the Weighted Tchebycheff Technique firstly we find the individual optimum values $\xi_i = (\xi_1, \xi_2)$. For the values given in **Table 1**, the SNLPP (26) for the first optimum value is

$$\left.\begin{array}{l} \xi_1 = \text{Min}\left(-R_1(d_i)\right) = -\left\{\left[1-(1-0.8)^{(3+d_1)}\right]\times\left[1-(1-0.75)^{(2+d_2)}\right]\times\left[1-(1-0.8)^{(4+d_3)}\right]\right\}, \\ \text{Subject to}\left[2d_1+3d_2+d_3+20d_4+28d_5+22d_6+22d_7\right] \\ +2.99\sqrt{0.33d_1^2+0.60d_2^2+0.10d_3^2+2.86d_4^2+3.11d_5^2+1.83d_6^2+2.2d_7^2}\le 150 \\ \left[120d_1+110d_2+120d_3+40d_4+30d_5+45d_6+65d_7\right] \\ +2.99\sqrt{13d_1^2+10d_2^2+15d_3^2+4d_4^2+3d_5^2+5d_6^2+6d_7^2}\le 860 \\ 0\le d_i\le a_i, \forall d_i \text{ are int eger } n_i\ge a_i, i=1,2,\cdots,7. \end{array}\right\} \quad (33)$$

The optimal solution of (33) provided by LINGO is

$$d_1^* = 2, d_2^* = 3, d_3^* = 2, d_4^* = 0, d_5^* = 0, d_6^* = 0, d_7^* = 0$$

with the value of objective function as

$$R_1(d_{ij}) = 0.9986398.$$

Similarly using (27) the SNLPP for the second optimum values

Table 1. The number of failed components and the respective cost and time etc. in the various subsystems.

	Group X (Replaced)			Group Y (Repair)			
Subsystem	**1**	**2**	**3**	**4**	**5**	**6**	**7**
n_i	6	5	10	7	9	12	10
r_i	0.8	0.75	0.8	0.8	0.75	0.8	0.7
β_i	12	15	10	140	252	264	220
α_i	6	5	10	7	9	12	10
$E(t_i)=\frac{\beta_i}{\alpha_i}$	2	3	1	20	28	22	22
$V(t_i)=\frac{\beta_i}{\alpha_i^2}$	0.33	0.60	0.10	2.86	3.11	1.83	2.2
$\bar{c}_i$	120	110	120	40	30	45	65
$\sigma_{c_i}^2$	13	10	15	4	3	5	6
a_i	3	3	6	5	7	9	7

$$\left.\begin{array}{l}\xi_2=\text{Min}\left(-R_2\left(d_i\right)\right)=-\left\{\left[1-(1-0.8)^{(2+d_1)}\right]\times\left[1-(1-0.75)^{(2+d_2)}\right]\times\left[1-(1-0.8)^{(3+d_3)}\right]\times\left[1-(1-0.70)^{(3+d_2)}\right]\right\},\\ \text{Subject to}\left[2d_1+3d_2+d_3+20d_4+28d_5+22d_6+22d_7\right]\\ +2.99\sqrt{0.33d_1^2+0.60d_2^2+0.10d_3^2+2.86d_4^2+3.11d_5^2+1.83d_6^2+2.2d_7^2}\le 150\\ \left[120d_1+110d_2+120d_3+40d_4+30d_5+45d_6+65d_7\right]+2.99\sqrt{13d_1^2+10d_2^2+15d_3^2+4d_4^2+3d_5^2+5d_6^2+6d_7^2}\le 860\\ 0\le d_i\le a_i,\forall d_i \text{ are int eger } n_i\ge a_i, i=1,2,\cdots,7.\end{array}\right\}\quad(34)$$

The optimal solution of (34) provided by LINGO is

$$d_1^*=0,d_2^*=0,d_3^*=0,d_4^*=2,d_5^*=1,d_6^*=1,d_7^*=2$$

with the value of objective function as

$$R_2\left(d_{ij}\right)=0.9788431.$$

From the Equations (33) and (34) the optimum values $\xi=(-0.9986398,-0.9788431)$. For simplicity we assumed that the reliability of both the Groups X and Y subsystems are equally important, that is $w_1=w_2=0.5$. For the values given in **Table 1**, the SNLPP (25) efficient solution is obtained by using the weighted Tchebycheff Technique

$$\left.\begin{array}{l}\text{Minimize }\delta\\ \text{Subject to }\delta\ge w_1\left\{-\left\{\left[1-(1-0.8)^{(3+d_1)}\right]\times\left[1-(1-0.75)^{(2+d_2)}\right]\times\left[1-(1-0.8)^{(4+d_3)}\right]\right\}-(-0.9983842)\right\}\\ \delta\ge w_2\left\{-\left\{\left[1-(1-0.8)^{(2+d_4)}\right]\times\left[1-(1-0.75)^{(2+d_5)}\right]\times\left[1-(1-0.8)^{(3+d_6)}\right]\times\left[1-(1-0.7)^{(3+d_7)}\right]\right\}-(-0.9994604)\right\}\\ \left[2d_1+3d_2+d_3+20d_4+28d_5+22d_6+22d_7\right]\\ +2.99\sqrt{0.15d_1^2+0.18d_2^2+0.10d_3^2+0.35d_4^2+0.40d_5^2+0.50d_6^2+0.60d_7^2}\le 150\\ \left[120d_1+110d_2+120d_3+40d_4+30d_5+45d_6+65d_7\right]+2.99\sqrt{10d_1^2+8d_2^2+15d_3^2+8d_4^2+5d_5^2+7d_6^2+9d_7^2}\le 860\\ 0\le d_i\le a_i,\forall d_i \text{ are int eger,} w_1+w_2=1,w_1,w_2\ge 0,n_i\ge a_i,i=1,2,\cdots,7.\end{array}\right\}\quad(35)$$

The optimum allocation under the Weighted Tchebycheff Technique

$$d^*_{\text{Tcheb}} = \left(\underline{d}^*_1, d^*_2, \underline{d}^*_3, \underline{d}^*_4, \underline{d}^*_5, \underline{d}^*_6, \underline{d}^*_7\right)$$

is obtained as

$$d^*_1 = 2, d^*_2 = 3, d^*_3 = 0, d^*_4 = 2, d^*_5 = 2, d^*_6 = 0, d^*_7 = 1.$$

The corresponding value of objective function is 0.00076.

5.2. Solution of Modified E-Model by Using Weighted Tchebycheff Technique

The individual optimum values $\xi_i = (\xi_1, \xi_2)$. For the values given in **Table 1** and for simplicity take $k_1 = k_2 = 0.5$ then the SNLPP (29) for the first optimum value is

$$\left.\begin{aligned}
&\xi_1 = \text{Min}T = k_1\left[2d_1 + 3d_2 + d_3 + 20d_4 + 28d_5 + 22d_6 + 22d_7\right] \\
&\quad + k_2\sqrt{0.33d_1^2 + 0.60d_2^2 + 0.10d_3^2 + 2.86d_4^2 + 3.11d_5^2 + 1.83d_6^2 + 2.2d_7^2} \\
&\text{Subject to} \left\{\left[1-(1-0.8)^{(3+d_1)}\right]\times\left[1-(1-0.75)^{(2+d_2)}\right]\times\left[1-(1-0.8)^{(4+d_3)}\right]\right\} \\
&\qquad \times\left\{\left[1-(1-0.8)^{(2+d_1)}\right]\times\left[1-(1-0.75)^{(2+d_2)}\right]\times\left[1-(1-0.8)^{(3+d_3)}\right]\times\left[1-(1-0.70)^{(3+d_2)}\right]\right\} \ge 0.99 \\
&0 \le d_i \le a_i, \forall d_i \text{ are integer}, n_i \ge a_i, i = 1, 2, \cdots, 7.
\end{aligned}\right\} \quad (36)$$

And the SNLPP (30) for the second optimum value

$$\left.\begin{aligned}
&\xi_2 = \text{MinC} = k_1\left[120d_1 + 110d_2 + 120d_3 + 40d_4 + 30d_5 + 45d_6 + 65d_7\right] \\
&\quad + k_2\sqrt{10d_1^2 + 8d_2^2 + 15d_3^2 + 8d_4^2 + 5d_5^2 + 7d_6^2 + 9d_7^2} \\
&\text{Subject to} \left\{\left[1-(1-0.8)^{(3+d_1)}\right]\times\left[1-(1-0.75)^{(2+d_2)}\right]\times\left[1-(1-0.8)^{(4+d_3)}\right]\right\} \\
&\qquad \times\left\{\left[1-(1-0.8)^{(2+d_1)}\right]\times\left[1-(1-0.75)^{(2+d_2)}\right]\times\left[1-(1-0.8)^{(3+d_3)}\right]\times\left[1-(1-0.70)^{(3+d_2)}\right]\right\} \ge 0.99 \\
&0 \le d_i \le a_i, \forall d_i \text{ are integer}, n_i \ge a_i, i = 1, 2, \cdots, 7.
\end{aligned}\right\} \quad (37)$$

From the Equations (36) and (37) the optimum values $\xi = (101.73, 418.40)$.

For simplicity we assumed that the maintenance time taken and cost spent for both the Groups X and Y subsystems are equally important, that is $w_1 = w_2 = 0.5$. For the values given in **Table 1**, the SNLPP (28) efficient solution is obtained by using the weighted Tchebycheff Technique

$$\left.\begin{aligned}
&\text{Minimize } \delta \\
&\text{Subject to } \delta \ge w_1\left\{\left\{0.5\left[2d_1 + 3d_2 + d_3 + 20d_4 + 28d_5 + 22d_6 + 22d_7\right]\right.\right. \\
&\qquad \left.\left. + 0.5\sqrt{0.33d_1^2 + 0.60d_2^2 + 0.10d_3^2 + 2.86d_4^2 + 3.11d_5^2 + 1.83d_6^2 + 2.2d_7^2}\right\} - 101.73\right\} \\
&\qquad \delta \ge w_2\left\{-\left\{0.5\left[120d_1 + 110d_2 + 120d_3 + 40d_4 + 30d_5 + 45d_6 + 65d_7\right]\right.\right. \\
&\qquad \left.\left. + 0.5\sqrt{10d_1^2 + 8d_2^2 + 15d_3^2 + 8d_4^2 + 5d_5^2 + 7d_6^2 + 9d_7^2}\right\} - 418.40\right\} \\
&\text{Subject to} \left\{\left[1-(1-0.8)^{(3+d_1)}\right]\times\left[1-(1-0.75)^{(2+d_2)}\right]\times\left[1-(1-0.8)^{(4+d_3)}\right]\right\} \\
&\qquad \times\left\{\left[1-(1-0.8)^{(2+d_1)}\right]\times\left[1-(1-0.75)^{(2+d_2)}\right]\times\left[1-(1-0.8)^{(3+d_3)}\right]\times\left[1-(1-0.70)^{(3+d_2)}\right]\right\} \ge 0.99 \\
&0 \le d_i \le a_i, \forall d_i \text{ are integer}, n_i \ge a_i, i = 1, 2, \cdots, 7.
\end{aligned}\right\} \quad (38)$$

The optimum allocation under the Weighted Tchebycheff Technique

$$d^*_{\text{Tcheb}} = \left(\underline{d}^*_1, d^*_2, \underline{d}^*_3, \underline{d}^*_4, \underline{d}^*_5, \underline{d}^*_6, \underline{d}^*_7\right)$$

is obtained as

$$d_1^* = 1, d_2^* = 3, d_3^* = 0, d_4^* = 3, d_5^* = 3, d_6^* = 1, d_7^* = 2.$$

The corresponding value of objective function is 6.48.

The individual optimum values of Model 1 are $\xi = (64.47, -0.9994)$. Therefore, the optimum allocation under the Weighted Tchebycheff Technique of SNLPP (31) is obtained as

$$d_1^* = 1, d_2^* = 3, d_3^* = 1, d_4^* = 2, d_5^* = 1, d_6^* = 1, d_7^* = 1.$$

The corresponding value of objective function is 0.01306.

In the same manner, we obtained the individual optimum values for Model 2 are $\xi = (277.57, -0.9989)$. Therefore, the optimum allocation under the Weighted Tchebycheff Technique of SNLPP (32) is obtained as

$$d_1^* = 1, d_2^* = 2, d_3^* = 0, d_4^* = 2, d_5^* = 2, d_6^* = 0, d_7^* = 1.$$

The corresponding value of objective function is 0.00299.

The optimum allocations obtained corresponding to the various Bi-criteria models are summarized as given below in **Table 2**.

6. Conclusions

The multi-objective problem of allocation of repairable and replaceable components becomes complicated because an allocation that is optimal for one objective is usually far from optimal for other objectives. In such situations, we need a compromise criterion that gives an allocation which is optimum for all objectives in some sense. This paper is an attempt to utilize weighted Tchebycheff approach to the solution of optimum compromise allocation of repairable and replaceable components in a system.

The allocation problem of repairable and replaceable components for a parallel-series system considered as a Bi-objective stochastic optimization problem and discussed the four different situations. In the first situation, the reliabilities of Groups X and Y are considered as two different objectives. While in the next three situations, the maintenance cost and time, maintenance cost and Group X subsystem reliabilities, maintenance time and Group Y subsystem reliabilities respectively are considered as two different objectives. Selective maintenance policy is used to select the repairable and replaceable components.

An equivalent deterministic model of these Bi-objective stochastic optimization problems is established by using Chance Constrained programming method. The following four different stochastic problems are then solved by using the Bi-criteria optimization technique, weighted Tchebycheff. The weighted Tchebycheff technique provides compromise allocations of repairable and replaceable components which are optimum for both the objectives function (see **Table 2**). Since a compromise criteria differ from method to method, so comparison can not be made.

Table 2. Optimum allocation of replaceable and repairable components under various Bi-criteria models.

S. No.	Bi-criteria models	d_1	d_2	d_3	d_4	d_5	d_6	d_7
1	$-\text{Min}\{R_1, R_2\}$	2	3	0	2	1	0	2
2	$\text{Min}\{T, C\}$	2	1	0	2	1	0	3
3	$\text{Min}\{C, -R_1\}$	1	2	0	2	2	0	1
4	$\text{Min}\{T, -R_2\}$	1	3	1	2	1	1	1

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