

# Allocation of Repairable and Replaceable Components for a System Availability Using Selective Maintenance with Probabilistic Maintenance Time Constraints

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Received April 23, 2011; revised May 19, 2011; accepted June 8, 2011

## Abstract

In this paper, we obtain optimum allocation of replaceable and repairable components in a system design. When repair and replace time are considered as random in the constraints. We convert probabilistic constraint into an equivalent deterministic constraint by using chance constrained programming. We have used the selective maintenance policy to determine how many components to be replaced & repaired within the limited maintenance time interval and cost. A Numerical example is presented to illustrate the computational procedure and problem is solved by using LINGO Software.

**Keywords:** Selective Maintenance, Availability, Optimization and Allocation, Chance Constraints

## 1. Introduction

In many industrial environments, systems are required to perform a sequence of operations (or missions) with finite breaks between each operation. During these breaks, it may be advantageous to perform repair and replacement on some of the system components. However, it may be impossible to perform all desirable maintenance activities prior to the beginning of the next mission due to limitations on maintenance resources. In this paper, we have consider that the maintenance (*i.e.* repair and replace) time in general are unknown (are of random character). For this a mathematical programming framework is established for assisting decision-makers in determining the optimal subset of maintenance activities to perform prior to beginning of the next mission. This decision-making process is referred to as selective maintenance. The selective maintenance models presented allow the decision-maker to consider limitations on maintenance time and budget, as well as the reliability of the system. Selective maintenance is an open research area that is consistent with the modern industrial objective of performing more intelligent and efficient maintenance.

Rice *et al.* [1] define a Mathematical programming Model for solving selective maintenance problem. Cassady *et al.* [2] extend the mathematical programming

model problem by permitting subsystems to be comprised of non identical components in any structure, adding a second resource constraint (representing maintenance cost), and creating two additional selective maintenance formulations that minimize resource consumption as the objective function and include mission reliability as a constraint. Cassady *et al.* [3] extend mathematical programming model problem in two ways. First the life distributions of system components are specified to be weibull distributions. Second, the decision maker is given multiple maintenance options: minimal repair on failed components, replacement of failed components and replacement of functioning components (preventive maintenance).

Cassady *et al.* [4] formulate a set of three optimization models that capture the trade-off between improving availability performance and the investments required to achieve this improvement. Two of these models address the allocation of funds for availability improvement efforts, and the third model addresses the incorporation of availability performance considerations in the system design phase.

Now, note that the maintenance times are in general unknown (are of random character), then this is a problem of stochastic optimization. The stochastic problems can be solved by proposing an equivalent deterministic

problem; where equivalent means that the solution of the deterministic problem is a solution of the stochastic problem. The stochastic optimizations have been used in the solution of some problems in probability and statistics; see Prekopa [5]. Charnes and Cooper [6], Rao [7], Prekopa [8], Uryasev and Paradosos [9], Louveaux and Birge [10] define non linear stochastic optimization for integers.

In most of the real life problems in which the decision maker would like to optimize objective function, and the values of the parameters are uncertain to enable the decision maker to take the decision. If we consider the parameter as random variables, the resulting problem is known as stochastic programming problem. Stochastic programming is an optimization method based on the probability theory, has been developed in several ways e.g., two stage programming problem by Dantzig [11], Chance constrained programming by Charnes and Cooper [12] and a stochastic programming problem with probabilistic constraints by V. A. Bereznev [13]. Previous research on the redundancy allocation problem for series-parallel system has focused on only deterministic versions when components maintenance (*i.e.* repair and replace) are assumed to be an exact value.

In this paper we have discussed components repairable and replaceable time as a random variable in the constraint. Probabilistic constraints function is then converted into an equivalent deterministic non-linear programming form by using chance constrained programming.

In this paper we assume that the system comprises two types of subsystem. One is the type of subsystems in which the components are very sensitive to the functioning of the whole system and, therefore, on deterioration these should be replaced by new ones. Let these subsystems range from 1 to  $s$ . The other types of subsystems

are those in which the components after deterioration can be repaired and then replaced. Let such subsystems range from  $s + 1$  to  $m$ . In **Figure 1** the group X consists of the  $s$  subsystems with sensitive components which on failure are replaced by new ones and Y the remaining  $(m - s)$  subsystems in which the components can be repaired. It is assumed that there is a single team for the replacing and repairing the components of group X and Y. this means that replacing and repairing of the components is various subsystems of X and Y is done in series.

### 2. Definition and Notations

Every industrial and engineering organization depends upon the effective performance of repairable and replaceable components of the system. A repairable component of a system can be defined as a component which after deterioration can be restored to an operating condition by some maintenance action. On the other hand a replaceable component is the one which after failure is replaced by a new one. We consider a system which requires to perform a sequence of identical missions after every given (fixed) period. The system consists of several subsystems where each subsystem can work properly if at least one of its components is operational. Thus we are working under the following two assumptions

Assumption 1: all the component states in a subsystem are independent

Assumption 2: the reliability, the cost and the weight of each component within a sub system are identical

Let  $a_i$  denote the probability that a component of a subsystem  $i$  survives the mission given that the component is functioning at the start of the mission, and let  $n_i$  denote the number of components in subsystem  $i$  all in the functioning state at the start of the mission. Since group X of the system is a series arrangement of the

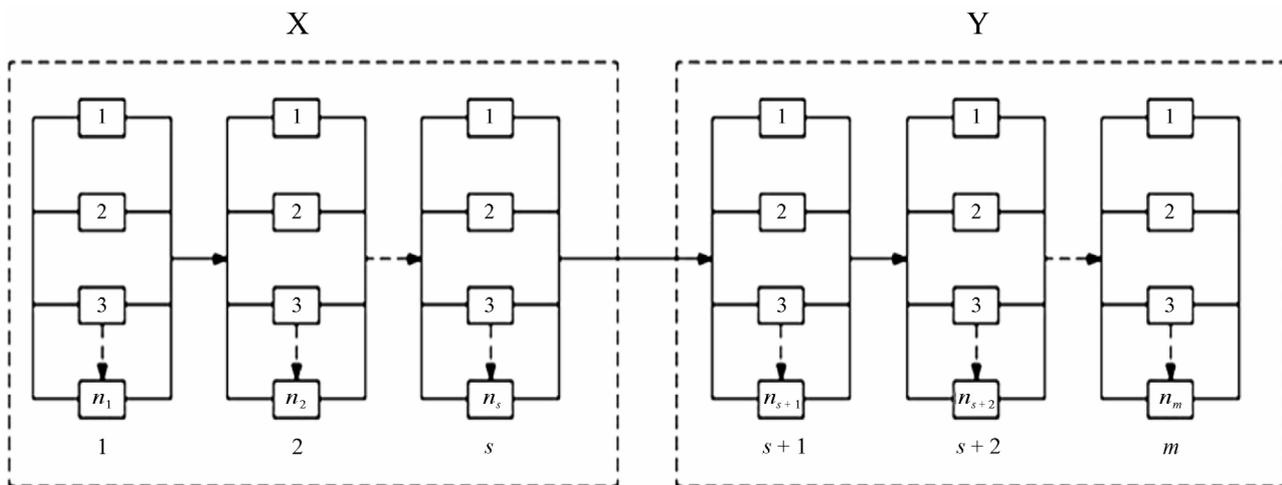


Figure 1. Parallel components in repairable and replaceable subsystems.

subsystems its availability can be defined as

$$A_1 = \prod_{i=1}^s 1 - (1 - a_i)^{n_i}, i = 1, 2, \dots, s \tag{2.1}$$

Similarly for group Y composed of  $(m - s)$  independent subsystems  $(s + 1, \dots, m)$  connected in series, the availability can be defined as

$$A_2 = \prod_{i=s+1}^m 1 - (1 - a_i)^{n_i}, i = s + 1, \dots, m \tag{2.2}$$

Since the system is a series arrangement of these two groups X and Y, the complete system availability can be defined by

$$A = \prod_{i=1}^2 A_i, i = 1, 2 \tag{2.3}$$

We will use the following notations in our formulation of the problem:

$k_i$  = Total number of failed components in the  $i^{th}$  subsystem at the end of a mission.

$p_i$  = Number of failed components replaced and repaired in  $i^{th}$  subsystem prior to the next mission.

$p = (p_1, \dots, p_m)$ .

$t'_i$  = Time units required for replacing a failed component in the  $i^{th}$  subsystem of group X.

$\bar{t}'_i$  = Expected units time required for replacing a failed component in the  $i^{th}$  subsystem of group X.

$t''_i$  = Time units required for repairing and then replacing a failed component in the  $i^{th}$  subsystem of group Y.

$\bar{t}''_i$  = Expected units time required for repairing and then replacing a failed component in the  $i^{th}$  subsystem of group Y.

$\sigma_{t'_i}$  = Standard deviation of repair time for replacing a failed component in the of group  $i^{th}$  subsystem of group X.

$\sigma_{t''_i}$  = Standard deviation of replace time for repairing and then replacing a failed component in the  $i^{th}$  subsystem of group Y.

$T_1(T_2)$  = Total time required for replacing (repairing) all the failed components in the system.

$T_{01}(T_{02})$  = Total time available for replacing (repairing) the failed components in the system between two missions.

$c'_i$  = Cost units required for replacing a failed component in the  $i^{th}$  subsystem of group X.

$c''_i$  = Time units required for repairing and then replacing a failed component in the  $i^{th}$  subsystem of group Y.

$C_1(C_2)$  = Total cost required for replacing (repairing) all the failed components in the system.

$C_{01}(C_{02})$  = Total cost available for replacing (repair-

ing) the failed components in the system.

### 3. Selective Maintenance

The selective maintenance operation is an optimal decision-making activity for systems consisting of several equipments under limited maintenance duration. The main objective of the selective maintenance operation is to select the most important equipment or subsystem to maintain. It also has to determine the appropriate maintenance actions in order to minimize the sum of production losses due to machine failures and the maintenance cost during the next working time. Such kind of problems can be encountered for equipments that perform sequences of tasks and can be repaired only during intervals of tasks. Such cases occur in military equipment production lines in which maintenance actions are carried out on weekends, vehicles are maintained between two deliveries and computer systems are maintained at night, etc. We have used the selective maintenance policy in our system that comprises two types of subsystem groups X and Y. If ideally, all the failed components in all the subsystem of group X are replaced by new ones prior to the beginning of the next mission/run. In a similar way, ideally all the failed components in subsystem of group Y are repaired and then replaced prior to the beginning of the next mission/run. However, due to the constraints on the cost and time it may not be possible to repair and replace all the failed components in group X and Y. Further, the cost required for replacing the failed components by new ones in group X is

$$C_1 = \sum_{i=1}^s c'_i k_i \tag{3.1}$$

and the cost required for replacing the failed components after repairs in group Y is

$$C_2 = \sum_{i=s+1}^m c''_i k_i \tag{3.2}$$

Suppose that the total maintenance cost available for replacing of failed components between two missions is  $C_{01}$  cost units. If  $C_{01} < C_1$ , then all failed components may not be repaired prior to beginning of the next mission.

In similar way, suppose that the total maintenance cost available for repairing of failed components between two missions is  $C_{02}$  cost units. If  $C_{02} < C_2$ , then all failed components may not be replaced prior to beginning of the next mission.

Let us assume that component replace and repair time  $t'_i$  and  $t''_i$ , are independently normally distributed random variables. We write the above problem in the following chance constrained programming form. Therefore,

the time required for replacing the failed components in group X is

$$P\left(\sum_{i=1}^s (t'_i k_i) \leq T_1\right) \geq p_0 \tag{3.3}$$

and the time required for repairing and then replacing the failed components in group Y is

$$P\left(\sum_{i=s+1}^m (t''_i k_i) \leq T_2\right) \geq p_0 \tag{3.4}$$

Suppose, that the total maintenance time available for repair of failed components between two missions is  $T_{01}$  time units. If  $T_{01} < T_1$ , then also all failed components cannot be repaired prior to beginning of the next mission.

In similar way, suppose, the total maintenance time available for replacement of failed components between two missions is  $T_{02}$  time units. If  $T_{02} < T_2$ , then also all failed components can not be replaced prior to beginning of the next mission.

In such a case, a method is needed to decide which failed components should be repaired and replaced prior to the next mission and which components should be left in a failed condition. This process is referred as Selective Maintenance.

For this let us suppose  $p_i$  be the number of components in the  $i^{th}$  subsystem, which can be repaired and replaced prior to the beginning of the next mission (See Rice *et al.* [1]).

Thus under the selective maintenance the number of components available for the next mission in the  $i^{th}$  subsystem will be

$$(n_i - k_i) + p_i \quad i = 1, 2, \dots, m \tag{3.5}$$

### 4. Formulation of the Problem

Now we discuss the mathematical programming model, with stochastic maintenance time constraint. The problem at hand addresses the issue of maximize the total system availability within the limited available repair and replacement (maintenance) budget and maintenance time between two missions, where repair & replace time of the component are random variable. From (2.3) and (3.5), the availability of the system to be maximized is given by

$$A = \left[ \prod_{i=1}^s \left\{ 1 - (1 - a_i)^{(n_i - k_i) + p_i} \right\} \right] \times \left[ \prod_{i=s+1}^m \left\{ 1 - (1 - a_i)^{(n_i - k_i) + p_i} \right\} \right] \tag{4.1}$$

Since the total cost of replacing the failed components should not exceed  $C_{01}$ , we have

$$\sum_{i=1}^s c'_i p_i \leq C_{01} \tag{4.2}$$

And the total cost of replacing the failed components after repairs should not exceed  $C_{02}$ , we have

$$\sum_{i=s+1}^m c''_i p_i \leq C_{02} \tag{4.3}$$

The maximum tolerable time between two missions (Spent in the maintenance of the components in various subsystems of group X and Y simultaneously by single server/team) is given by  $T_{01}$  and  $T_{02}$ . Thus we have the constraints.

$$P\left(\sum_{i=1}^s (t'_i p_i) \leq T_{01}\right) \geq p_0 \tag{4.4}$$

$$P\left(\sum_{i=s+1}^m (t''_i p_i) \leq T_{02}\right) \geq p_0 \tag{4.5}$$

We have  $p_i$  cannot be exceed  $k_i$

$$0 \leq p_i \leq k_i \text{ and integer } i = 1, 2, \dots, m \tag{4.6}$$

The probabilistic constraints (4.4) and (4.5) converted into an equivalent deterministic non-linear programming form by using chance constrained programming. The mathematical formulation of the problem is to maximize (4.1) under the constraints (4.2) to (4.6).

## 5. Solution Using Chance Constrained Programming

### 5.1. When Replace Time Treated to be Random Variable

The replace time  $t'_i$ ,  $i = 1, \dots, s$  in the constraint function are assumed to be independently and normally distributed random variables. Let  $\underline{t}'_i = (t_1, \dots, t_s)$  and  $\underline{p}_i = (p_1, \dots, p_s)$ . Then the constraint function  $(t'_i p_i)$ , will also be normally distributed random variables with mean  $E(t'_i p_i)$  and variance  $V(t'_i p_i)$ . If  $t'_i \sim N(\mu_i, \sigma_i^2)$ , then the joint distribution of  $(t_1, \dots, t_s)$  will be given by

$$f(\underline{t}'_i) = \frac{1}{(2\pi)^{s/2} \prod_{i=1}^s \sigma_i} \exp\left(-\frac{1}{2} \sum_{i=1}^s \frac{(t'_i - \mu_i)^2}{\sigma_i^2}\right) \quad i = 1, \dots, s.$$

Then, the mean is obtained as follows

$$E(t'_i p_i) = E\left(\sum_{i=1}^s t'_i p_i\right) = \sum_{i=1}^s p_i E(t'_i) = \sum_{i=1}^s p_i \mu_i \tag{5.1a}$$

and the variance as follows

$$V(t'_i p_i) = V\left(\sum_{i=1}^s t'_i p_i\right) = \sum_{i=1}^s p_i^2 V(t'_i) = \sum_{i=1}^s p_i^2 \sigma_{t'_i}^2 \quad (5.2a)$$

Now, we consider the probabilistic constraint (4.4) can be expressed as

$$P(f(t'_i) \leq T_{01}) \geq p_0 \quad (5.3a)$$

where  $f(t'_i) = \sum_{i=1}^s t'_i p_i$

It can be simplified as

$$P\left\{\frac{f(t'_i) - E\{f(t'_i)\}}{\sqrt{V\{f(t'_i)\}}} \leq \frac{T_{01} - E\{f(t'_i)\}}{\sqrt{V\{f(t'_i)\}}}\right\} \geq p_0 \quad (5.4a)$$

where  $\left[\frac{f(t'_i) - E\{f(t'_i)\}}{\sqrt{V\{f(t'_i)\}}}\right]$  is a standard normal variate

with mean zero and variance one. thus the probability of realizing  $V\{f(t'_i)\}$  less than or equal to one can be written as

$$P(f(t'_i) \leq T_{01}) = \phi\left[\frac{T_{01} - E\{f(t'_i)\}}{\sqrt{V\{f(t'_i)\}}}\right] \quad (5.5a)$$

where  $\phi(z)$  represent the cumulative density function of the standard normal variate evaluated at  $z$ . if  $K_\alpha$  represents the value of the standard normal variate at which  $\phi(K_\alpha) = p_0$ , then the constraint (5.5a) can be stated as

$$\phi\left[\frac{T_{01} - E\{f(t'_i)\}}{\sqrt{V\{f(t'_i)\}}}\right] \geq \phi(K_\alpha) \quad (5.6a)$$

the inequality will be satisfied only if

$$\left[\frac{T_{01} - E\{f(t'_i)\}}{\sqrt{V\{f(t'_i)\}}}\right] \geq K_\alpha$$

or equivalently,

$$\begin{aligned} E\{f(t'_i)\} + K_\alpha \sqrt{V\{f(t'_i)\}} &\leq T_{01} \\ E\left(\sum_{i=1}^s t'_i p_i\right) + K_\alpha \sqrt{V\left(\sum_{i=1}^s t'_i p_i\right)} &\leq T_{01} \end{aligned} \quad (5.7a)$$

substituting Equations (5.1a) and (5.2a) in Equation (5.7a), we get

$$\left(\sum_{i=1}^s \mu_i p_i\right) + K_\alpha \sqrt{\sum_{i=1}^s p_i^2 \sigma_{t'_i}^2} \leq T_{01} \quad (5.8a)$$

Here, functions in the constraint (5.8a) are given in

terms of the population expected values and variance of  $(t'_i p_i)$ , which are unknown (by hypothesis), then we will use estimators of mean  $E(t'_i p_i)$  and variance  $V(t'_i p_i)$ .

The estimator of  $E(t'_i p_i)$  is

$$\hat{E}(t'_i p_i) = \sum_{i=1}^s p_i E(t'_i) = \sum_{i=1}^s p_i \bar{t}'_i, \text{ say.} \quad (5.9a)$$

and the estimator of  $V(t'_i p_i)$

$$\hat{V}(t'_i p_i) = \sum_{i=1}^s p_i^2 E(t_i^2) = \sum_{i=1}^s p_i^2 \bar{\sigma}_{t'_i}^2, \text{ say.} \quad (5.10a)$$

where  $\bar{t}'_i$  and  $\bar{\sigma}_{t'_i}^2$  are the estimated mean and variance from the sample. Thus, an equivalent deterministic constraint to the stochastic constraint is given by

$$\left(\sum_{i=1}^s \bar{t}'_i p_i\right) + K_\alpha \sqrt{\sum_{i=1}^s p_i^2 \bar{\sigma}_{t'_i}^2} \leq T_{01} \quad (5.11a)$$

### 5.2. When Repair Time Treated to be Random Variable

The repair time  $t''_i, i = s+1, \dots, m$  in the constraint function are assumed to be independently and normally distributed random variables. Let  $t''_i = (t_{s+1}, \dots, t_m)$  and  $p_i = (p_{s+1}, \dots, p_m)$ . Then the constraint function  $(t''_i p_i)$ , will also be normally distributed random variables with mean  $E(t''_i p_i)$  and variance  $V(t''_i p_i)$ .

If  $t''_i \sim N(\mu_i, \sigma_i^2)$ , then the joint distribution of  $(t_{s+1}, \dots, t_m)$  will be given by

$$f(t''_i) = \frac{1}{(2\pi)^{m/2} \prod_{i=s+1}^m \sigma_{t''_i}} \exp\left(-\frac{1}{2} \sum_{i=s+1}^m \frac{(t''_i - \mu_i)^2}{\sigma_{t''_i}^2}\right),$$

$$i = s+1, \dots, m.$$

Then, the mean is obtained as follows

$$E(t''_i p_i) = E\left(\sum_{i=s+1}^m t_i p_i\right) = \sum_{i=s+1}^m p_i E(t''_i) = \sum_{i=s+1}^m p_i \mu_i \quad (5.1b)$$

and the variance as follows

$$\begin{aligned} V(t''_i p_i) &= V(t''_i p_i) = V\left(\sum_{i=s+1}^m t_i p_i\right) \\ &= \sum_{i=s+1}^m p_i^2 V(t''_i) = \sum_{i=s+1}^m p_i^2 \sigma_{t''_i}^2 \end{aligned} \quad (5.2b)$$

now, we consider the probabilistic constraint (4.5) can be expressed as

$$P(f(t''_i) \leq T_{02}) \geq p_0 \quad (5.3b)$$

where  $f(t''_i) = \sum_{i=s+1}^m t''_i p_i$

It can be simplified as

$$P \left\{ \frac{f(t_i^n) - E\{f(t_i^n)\}}{\sqrt{V\{f(t_i^n)\}}} \leq \frac{T_{02} - E\{f(t_i^n)\}}{\sqrt{V\{f(t_i^n)\}}} \right\} \geq p_0 \quad (5.4b)$$

where  $\left[ \frac{f(t_i^n) - E\{f(t_i^n)\}}{\sqrt{V\{f(t_i^n)\}}} \right]$  is a standard normal variate

with mean zero and variance one. Thus the probability of realizing  $V\{f(t_i^n)\}$  less than or equal to one can be written as

$$P(f(t_i^n) \leq T_{02}) = \phi \left[ \frac{T_{02} - E\{f(t_i^n)\}}{\sqrt{V\{f(t_i^n)\}}} \right] \quad (5.5b)$$

where  $\phi(z)$  represent the cumulative density function of the standard normal variate evaluated at  $z$ . if  $K_\alpha$  represents the value of the standard normal variate at which  $\phi(K_\alpha) = p_0$ , then the constraint (5.5b) can be stated as

$$\phi \left[ \frac{T_{02} - E\{f(t_i^n)\}}{\sqrt{V\{f(t_i^n)\}}} \right] \geq \phi(K_\alpha) \quad (5.6b)$$

the inequality will be satisfied only if

$$\left[ \frac{T_{02} - E\{f(t_i^n)\}}{\sqrt{V\{f(t_i^n)\}}} \right] \geq K_\alpha$$

or equivalently,

$$E\{f(t_i^n)\} + K_\alpha \sqrt{V\{f(t_i^n)\}} \leq T_{02}$$

$$E \left( \sum_{i=s+1}^m t_i^n p_i \right) + K_\alpha \sqrt{V \left( \sum_{i=s+1}^m t_i^n p_i \right)} \leq T_{02} \quad (5.7b)$$

Substituting Equations (5.1b) and (5.2b) in Equation (5.7b), we get

$$\left( \sum_{i=s+1}^m \mu_i p_i \right) + K_\alpha \sqrt{\sum_{i=s+1}^m p_i^2 \sigma_i^2} \leq T_{02} \quad (5.8b)$$

Here, functions in the constraint (5.8b) are given in terms of the population expected values and variance of  $(t_i^n p_i)$ , which are unknown (by hypothesis), then we will use estimators of mean  $E(t_i^n p_i)$  and variance  $V(f(t_i^n))$ .

The estimator of  $E(t_i^n p_i)$

$$\hat{E}(t_i^n p_i) = \sum_{i=s+1}^m p_i E(t_i^n) = \sum_{i=s+1}^m p_i \bar{t}_i^n, \text{ say} \quad (5.9b)$$

and the estimator of  $V(f(t_i^n))$

$$\hat{V}(t_i^n p_i) = \sum_{i=s+1}^m p_i^2 E(t_i^{n2}) = \sum_{i=s+1}^m p_i^2 \bar{\sigma}_{t_i^n}^2, \text{ say} \quad (5.10b)$$

where  $\bar{t}_i^n$  and  $\bar{\sigma}_{t_i^n}^2$  are the estimated mean and variance from the sample. Thus, an equivalent deterministic constraint to the stochastic constraint is given by

$$\left( \sum_{i=s+1}^m \bar{t}_i^n p_i \right) + K_\alpha \sqrt{\sum_{i=s+1}^m p_i^2 \bar{\sigma}_{t_i^n}^2} \leq T_{02} \quad (5.11b)$$

These are the deterministic non linear constraints equivalent to the original probabilistic constraints. Thus, the solution of the probabilistic programming problem can be obtained by solving the deterministic non-linear programming problem. The resulting mathematical programming formulation is given as

$$\left. \begin{aligned} \text{MaxA}(p_i) &= \left[ \prod_{i=1}^s \left\{ 1 - (1 - a_i)^{(n_i - k_i) + p_i} \right\} \right] \times \left[ \prod_{i=s+1}^m \left\{ 1 - (1 - a_i)^{(n_i - k_i) + p_i} \right\} \right] \quad (i) \\ \text{subject to} & \\ & E \left( \sum_{i=1}^s t_i' p_i \right) + K_\alpha \sqrt{V \left( \sum_{i=1}^s t_i' p_i \right)} \leq T_{01} \quad (ii) \\ & E \left( \sum_{i=s+1}^m t_i^n p_i \right) + K_\alpha \sqrt{V \left( \sum_{i=s+1}^m t_i^n p_i \right)} \leq T_{02} \quad (iii) \\ & \sum_{i=1}^s c_i' p_i \leq C_{01} \quad (iv) \\ & \sum_{i=s+1}^m c_i^n p_i \leq C_{02} \quad (v) \\ & 0 \leq p_i \leq k_i \text{ and integer, } i = 1, \dots, m \quad (vi) \end{aligned} \right\} \quad (A)$$

Or equivalently

$$\begin{aligned}
 \text{MaxA}(p_i) &= \left[ \prod_{i=1}^s \left\{ 1 - (1 - a_i)^{(n_i - k_i) + p_i} \right\} \right] \times \left[ \prod_{i=s+1}^m \left\{ 1 - (1 - a_i)^{(n_i - k_i) + p_i} \right\} \right] \quad (i) \\
 \text{subject to} & \\
 & \left( \sum_{i=1}^s \bar{t}_i' p_i \right) + K_\alpha \sqrt{\left( \sum_{i=1}^s p_i^2 \bar{\sigma}_i'^2 \right)} \leq T_{01} \quad (ii) \\
 & \left( \sum_{i=s+1}^m \bar{t}_i'' p_i \right) + K_\alpha \sqrt{\left( \sum_{i=s+1}^m p_i^2 \bar{\sigma}_i''^2 \right)} \leq T_{02} \quad (iii) \\
 & \sum_{i=1}^s c_i' p_i \leq C_{01} \quad (iv) \\
 & \sum_{i=s+1}^m c_i'' p_i \leq C_{02} \quad (v) \\
 & 0 \leq p_i \leq k_i \text{ and integer, } i = 1, \dots, m \quad (vi)
 \end{aligned} \tag{B}$$

### 6. Numerical Illustration

Consider a system having the group X consisting of 3 subsystems and also the group Y consisting of 3 subsystems. The available time between two missions for repairing and replacing is 80 and 8 time units. The available cost of maintenance for repairing and replacing is for the next mission is 480 and 200 units. The remaining parameters for the various subsystems are given in **Table 1**.

Let the chance constraint (4.4) and (4.5) be required to be satisfied with 99% probability. Then  $k_\alpha$  is such that  $\phi(k_\alpha) = 0.99$ . The value of standard normal variate  $K_\alpha$  corresponding to 99% confidence limits is 2.33 (by linear interpolation). Thus, the (non-linear programming) problem (B) is obtained as:

$$\begin{aligned}
 \text{MaxA}(t) &= \left[ 1 - (1 - 0.8)^{(1+p_1)} \right] \times \left[ 1 - (1 - 0.75)^{(2+p_2)} \right] \\
 & \times \left[ 1 - (1 - 0.8)^{(2+p_3)} \right] \times \left[ 1 - (1 - 0.8)^{(1+p_4)} \right] \\
 & \times \left[ 1 - (1 - 0.75)^{(1+p_5)} \right] \times \left[ 1 - (1 - 0.8)^{(2+p_6)} \right]
 \end{aligned}$$

Subject to

$$\begin{aligned}
 (2p_1 + 3p_2 + p_3) + 2.33 \sqrt{(0.15p_1^2 + 0.18p_2^2 + 0.10p_3^2)} &\leq 8 \\
 (20p_4 + 28p_5 + 12p_6) + 2.33 \sqrt{(0.35p_4^2 + 0.40p_5^2 + 0.50p_6^2)} &\leq 80 \\
 120p_1 + 110p_2 + 120p_3 &\leq 480 \\
 50p_4 + 40p_5 + 45p_6 &\leq 200 \\
 0 \leq p_1 \leq 2, \quad 0 \leq p_2 \leq 1, \quad 0 \leq p_3 \leq 2 \quad \text{and} \quad 0 \leq p_4 \leq 3, \\
 0 \leq p_5 \leq 2, \quad 0 \leq p_6 \leq 3 \\
 \text{and } p_i \geq 0, i = 1, 2, \dots, 6 \text{ integer}
 \end{aligned}$$

The above nonlinear programming problem represented by, is solved by using LINGO computer program. The optimal Solution obtained after 347 iterations as follows.

$$p_1 = 1, p_2 = 1, p_3 = 1 \text{ and } p_4 = 1, p_5 = 1, p_6 = 2 \text{ with } \text{MaxA}(t) = 0.9188.$$

**Table 1. The number of failed components and the respective cost and time etc. in the various subsystems.**

Subsystem	1	2	3	Subsystem	4	5	6
$n_i$	4	4	4	$n_i$	4	4	4
$r_i$	0.8	0.75	0.8	$r_i$	0.8	0.75	0.8
$\bar{t}_i'$	2	3	1	$\bar{t}_i''$	20	28	12
$\sigma_i'^2$	0.15	0.18	0.10	$\sigma_i''^2$	0.35	0.40	0.50
$c_i'$	120	110	120	$c_i''$	50	40	45
$k_i$	2	1	2	$k_i$	3	2	3

So in subsystem 1, 2 and 3 we replace 1, 1 and 1 components respectively while in the subsystems 4, 5 and 6 we repair and then replace 1, 1 and 2 components respectively.

## 7. Conclusions

This paper has provided a profound study of the selective maintenance problem for an optimum allocation of repairable and replaceable components in system availability as a problem of non-linear stochastic programming in which we maximize the system availability under the upper bounds on maintenance (*i.e.*, repair and replace) time and cost. The maintenance time considered as a random variable and has normal distribution. An equivalent deterministic form of the stochastic non-linear programming problem (SNLPP) is established by using the chance constrained programming problem. Many authors have solved the allocation of repairable component problem. But to solve the above problem with probabilistic maintenance time will be much more helpful to demonstrate the practically complicated situations related to system maintenance problem.

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