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Two-Stage Ordering Policy under Buyer's Minimum-Commitment Quantity Contract

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Abstract

In this paper we consider a two-stage ordering problem with a buyer's minimum commitment quantity contract. Under the contract the buyer is required to give a minimum-commitment quantity. Then the manufacturer has the obligations to supply the minimum-commitment quantity and to provide a shortage compensation policy to the buyer. We formulate a dynamic optimization model to determine the manufacturer's two stage order quantities for maximizing the expected profit. The conditions for the existence of the optimal solution are defined. And we also develop a procedure to solve the problem. Numerical examples are given to illustrate the proposed solution procedure and sensitivity analyses are performed to find managerial insights.

Keywords: Two Stages Ordering, Commitment, Bayesian Information Updating

1. Introduction

In this paper we study a two-stage component ordering problem with a buyer minimum-commitment quantity contract. Under the contract, the buyer is required to commit a minimum order quantity θ_1 and for returning the buyer's commitment the manufacturer has the obligations to supply the minimum-commit quantity θ_1 and to give shortage compensation if the manufacturing supply level is under $(1+\gamma)\theta_1$ where γ is a shortage compensation range coefficient $0 \le \gamma \le 1$. Because of the presence of the long lead time of key components, the manufacturer has two opportunities to place his order to supplier before the buyer's demand realized.

The buyer's real demand *X* is uncertain following a normally distribution $N(\mu_0, \sigma_0^2)$ where μ_0 is uncertain having a normal distribution $N(\mu, \sigma_1^2)$. When the manufacturer makes his first order quantity (q_1) decision at stage 1, the unit cost of key component at stage 1 $c^{(1)}$ is known but the unit cost $C^{(2)}$ at the stage 2 is uncertain. The possible values of $C^{(2)}$ and their corresponding probabilities are known, denoted as $C^{(2)} = \left\{c_1^{(2)}, c_2^{(2)}, \cdots, c_n^{(2)}\right\}$ and $P = \left\{p_1, p_2, \cdots, p_n\right\}$ respectively. After receiving the buyer's minimum-commit quantity θ_1, the manufacturer uses θ_1 as an estimator of μ and places his first order quantity (q_1) to his supplier. At stage 2 the marketing department provides an observation θ_2 of *X*. The posterior distribution of *X* is defined by the observations of θ_1 and θ_2. Then manufacturer places his second order quantity q_2 if necessary. The time events of key component procurement process are shown in **Figure 1**.

We assume that the outputs are mainly limited by the available amounts of the key component, and the production cycle times are very short that can be neglected. After receiving the key components, the manufacturer produces the products immediately. Products are delivered to the buyer at the end of period (immediately after second stage). Due to the demand uncertainty, the manufacturer is difficult to determine two stage order quantities.

In this paper we develop a two-stage dynamic optimization model to decide the order quantity of a key component under a buyer's minimum-commitment quantity contract. The model is formulated to maximize a manufacturer's profit. The following costs are considered in the model. 1) Key component unit cost: the unit cost of key component at stage 1 is $c^{(1)}$ and the unit cost at stage 2 is $C^{(2)}$. 2) Holding cost: two kinds of inventories are considered. One is buyer responsible inventory, which only exists in the case of buyer's real demand (*x*) below the minimum guaranteed quantity θ_1. In this case, customers only take away real demand *x*, and the remaining products $(\theta_1 - x)$ are buyer responsible inventory

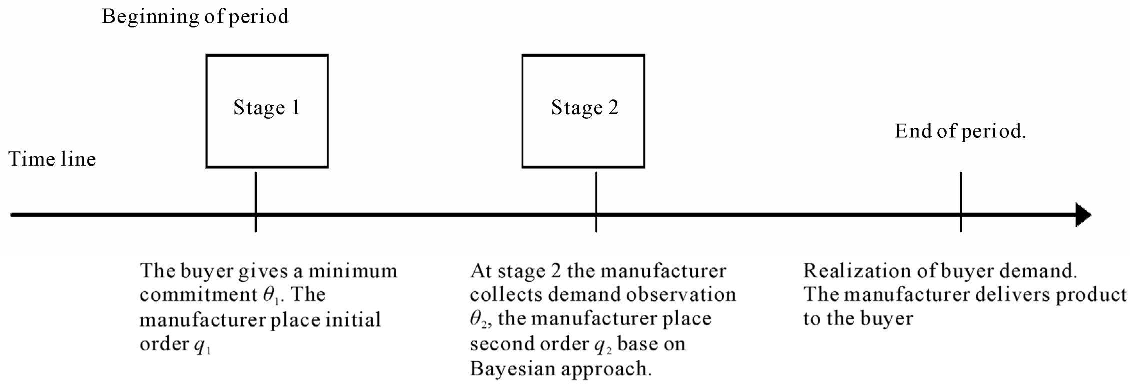

Figure 1. Time events of key component procurement process.

and will be paid in the near future. The unit holding costs of buyer responsible inventory are the interest and insurance. The other is manufacturer responsible inventory. The unit holding cost of manufacturer responsible inventory is the interest, insurance and obsolete costs. The holding costs of buyer responsible inventory and manufacturer responsible inventory are c_{h1} and c_{h2} respectively and $c_{h2} > c_{h1}$. 3) Shortage cost: two kinds of shortage cost are considered according to whether or not to pay shortage compensation. If the manufacturing output level is below $(1+\gamma)\theta_1$, then there are two shortage types may occurred. The one includes general shortage cost and the compensation cost. The other is only the general shortage cost.

The buyer's minimum commitment and demand forecast updating in this paper belong to the category of minimum purchase commitment contract [1] and inventory management with demand forecast updates respectively [2]. Durango-Cohen and Yano [3] pointed out that increasing the level of commitment and information sharing will lead to the cost down of entire supply chain. Nowadays minimum commitment quantity contracts are commonly used in electronic industry. Anupindi and Bassok [1] classified the contract of quantity commitments and flexibility as three types. The first type is the total minimum quantity commitment contract. The supply contract with total minimum quantity commitment is that a buyer gives his supplier a minimum ordering quantity commitment, and the supplier offers the buyer a discount price in return for the buyer's commitment [4]. The second type is the total minimum dollar volume commitment contract. This contract is similar to the total minimum quantity commitment contract, but a buyer commits to a minimum business on the basis of dollar volume [5], and the supplier offers discounts based on the commitment of dollar volume. The third type is the periodical commitment with flexibility contract. Under such a contract, a buyer receives discounts for committing to purchase in advance, and the buyer is allowed to update his order amount in the rolling horizon basis. The rolling horizon flexibility (RHF) contract [6-8] is one kind of the third type. The RHF contract means the buyer has a "limited" flexibility to update his advance order after he commits to purchase certain quantity.

Gallego and Ozer [9] and Sethi *et al.* [2] classified the inventory information with demand updating problems as three types. The first type is the Bayesian analysis. This approach learns about further demand from the past history [10]. Dvoretzky *et al.* [11] first analyzed Bayesian models in the inventory problem. In this type, specific classes of demand distribution were discussed, such as exponential family of distribution [12], gamma family [13,14], negative binomial distribution [15], uniform-Pareto distribution [16] and normal distribution [17,18]. The second type is time-series models used in updating demand forecast, where they assume a correlation exists in the demand realization and construct the demand as a time-series model [10,19]. The third type is concerned with forecast revisions, such as Markovian forecast revisions model [20-22], single-period, two-stage ordering problem with demand forecast updating [23-26] and multiple period ordering problem with demand forecast updating [27,28]. A more comprehensive discussion can be found in [2].

Our study differs from the previous papers because we consider a shortage compensation policy for reducing demand uncertainty. Under the buyer's minimum-commitment quantity contract and shortage compensation policy, two kinds of inventory and shortage costs are respectively formulated in the two-stage dynamic optimization model to decide the optimal order quantities.

In this paper, we define the conditions for the existence of the optimal solution and also develop a procedure to determine the two-stage optimal order quantities. The rest of this paper is organized as follows: Section 2 states the assumptions and notations for the proposed model; the two-stage dynamic optimization model and the solution procedure are proposed. Section 2 illustrates some numerical examples, and sensitivity analyses of the major parameters of the model are performed. Finally, Section 4 concludes this article.

2. Problem Formulation

2.1. Notations

θ_1 : buyer's minimum-commitment quantity. $\theta_1 > 0$
γ : shortage compensation range coefficient $0 \le \gamma \le 1$. $[\theta_1, (1+\gamma)\theta_1]$ is the shortage compensation range.
$c^{(1)}$: unit ordering cost of key component at stage 1.
$C^{(2)}$: unit ordering cost of key component at stage 2 is a random variable, $C^{(2)} = \left\{c_1^{(2)}, c_2^{(2)}, \cdots, c_n^{(2)}\right\}$ and the corresponding probability $P = \{p_1, p_2, \cdots, p_n\}$.
p : product unit selling price. $p > c^{(1)}$.
θ_2 : the demand observation at stage 2.
c_{h1} : unit holding cost of buyer responsible inventory.
c_{h2} : unit holding cost of manufacturer responsible inventory.
c_{s1} : unit shortage compensation cost; $c_{s1} > 0$
c_{s2} : unit general shortage cost; $c_{s1} \ge c_{s2} > 0$
X : buyer's real demand, realization is denoted by x.
$X^{(1)}$: $X^{(1)}$ is a random variable to forecast the buyer's demand at stage 1, $X^{(1)} = X\left|\theta_1\right.$.
$X^{(2)}$: $X^{(2)}$ is a random variable to forecast the buyer's demand at stage 2, $X^{(2)} = X\left|\theta_1, \theta_2\right.$.
$f_1(.)$: probability density function (pdf) of $X^{(1)}$, $X^{(1)} \sim N\left(\theta_1, {\sigma_0}^2 + \sigma_1^2\right)$
$f_2(.)$: probability density function (pdf) of $X^{(2)}$,

$$X^{(2)} \sim N\left(\left[\sigma_1^2\theta_2\Big/\left({\sigma_0}^2 + \sigma_1^2\right) + \sigma_0^2\theta_1\Big/\left(\sigma_0^2 + \sigma_1^2\right)\right],\right.$$
$$\left.\sigma_0^2 + \sigma_1^2\sigma_0^2\Big/\left(\sigma_0^2 + \sigma_1^2\right)\right)$$

$F_1(.)$: cumulative density function (cdf) of. $X^{(1)}$
$F_2(.)$: cumulative density function (cdf) of $X^{(2)}$.
$\phi(.)$: standard normal probability density function.
$\Phi(.)$: the cumulative distribution function for standard normal distribution.
$\Phi^{-1}(.)$: inverse function of $\Phi(.)$.
$\Psi(.)$: the standard linear loss function:

$$\Psi(a) = \int_a^\infty (x-a)\mathrm{d}\Phi(x).$$

Decision variables:
q_1 : order quantity at stage 1. $q_1 \ge 0$
q_2 : order quantity at stage 2. $q_2 \ge 0$
Intermediate variables:
Ω_1: the decision space defined by $\theta_1 \le q_1 + q_2 \le (1+\gamma)\theta_1$.
Ω_2 : the decision space defined by $(1+\gamma)\theta_1 < q_1 + q_2$.
q_{11}^* : optimal order quantity in Ω_1 at stage 1
q_{21}^* : optimal order quantity in Ω_1 at stage 2
q_{12}^* : optimal order quantity in Ω_2 at stage 1
q_{22}^* : optimal order quantity in Ω_2 at stage 2
q_1^* : optimal order quantity at stage 1
q_2^* : optimal order quantity at stage 2

2.2. Problem Assumptions and Formulation

The mathematical model is formulated to determine the two stage ordering quantities of the key component for maximizing profit. The buyer's real demand *X* is uncertain to be assumed following a normal distribution with an uncertain mean μ_0 and a given variance σ_0^2, where μ_0 follows $N(\mu, \sigma_1^2)$ with an unknown μ and a given variance σ_1^2. At stage 1, after receiving the buyer's minimum commitment quantity θ_1, the manufacturer uses θ_1 as an estimator of μ. The posterior distribution of *X* after receiving θ_1 at stage 1 is denoted as $X^{(1)} = X\left|\theta_1\right.$ where

$$X^{(1)} = X\left|\theta_1\right. \sim N\left(\theta_1, \sigma_0^2 + \sigma_1^2\right) \tag{1}$$

At stage 2, the marketing department collects information θ_2 about buyer's real demand. We call it as an observation of *X*. The posterior distribution of *X* at stage 2 is denoted as $X^{(2)} = X\left|\theta_1, \theta_2\right.$ where

$$X^{(2)} = X\left|\theta_1, \theta_2\right. \sim N\left(k_2, \sigma_2^2\right) \tag{2}$$

$$k_2 = \left[\sigma_1^2\theta_2\Big/\left(\sigma_0^2 + \sigma_1^2\right) + \sigma_0^2\theta_1\Big/\left(\sigma_0^2 + \sigma_1^2\right)\right] \tag{3}$$

$$\sigma_2^2 = \sigma_0^2 + \sigma_1^2\sigma_0^2\Big/\left(\sigma_0^2 + \sigma_1^2\right) \tag{4}$$

Because the manufacturer has the obligation to provide the minimum-commitment quantity θ_1 to the buyer, the total order quantities $q_1 + q_2$ must be larger than θ_1. Now we will formulate the expected profit function and use a backward dynamic programming to determine the optimal q_1^* and q_2^*.

We formulate the problem as a dynamic programming (DP) problem. For the DP formulation, the ordering times are given as the stages, stage 1 and stage 2. Decision variable for stage *n* (*n* = 1, 2) is the ordering quantity q_n. The profit at the current stage depends upon the current decision q_n and the ordering quantity in the preceding stage q_{n-1}. We set states for each stage *n* as q_{n-1}.

With the backward solving procedure, first we should determine the optimal order quantity q_2 at stage 2 in a given state $s_2 = q_1$. We denote the expected profit function at stage 2 as $E_{(2)}\left[\Pi\left(q_2\left|q_1\right.\right)\right]$. The state at stage 1 is known as $s_1 = 0$. The expected profit function at stage 1 is denoted as $E_{(1)}\left[\Pi\left(q_1\right)\right]$, where

$$E_{(1)}\left[\Pi\left(q_1\right)\right] = E\left[-c^{(1)}q_1 + E_{(2)}\left[\Pi\left(q_2^*\left|q_1\right.\right)\right]\right] \tag{5}$$

The optimal expected profit of the manufacturer is determined as follows:

$$\begin{aligned} E_{(1)}\left[\Pi\left(q_1^*\right)\right] &= \max_{q_1 \ge 0} E_{(1)}\left[\Pi\left(q_1\right)\right] \\ &= \max_{q_1 \ge 0} E\left[-c^{(1)}q_1 + E_{(2)}\left[\Pi\left(q_2^*\middle|q_1\right)\right]\right] \end{aligned} \tag{6}$$

where

$$E_{(2)}\left[\Pi\left(q_2^*\middle|q_1\right)\right] = \max_{q_2 \ge 0} E_{(2)}\left[\Pi\left(q_2\middle|q_1\right)\right] \tag{7}$$

The items considered in $E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]$ are expected product sales, ordering costs, expected holding costs and expected shortage costs.

$$\begin{aligned} &E_{(2)}\left[\Pi\left(q_2|q_1\right)\right] = \text{Expected revenues} \\ &-\text{Expected costs} \\ &= \text{Expected product sales} \\ &-\left(\text{Ordering costs} + \text{Expected holding costs}\right. \\ &\left.+\text{Expected shortage costs}\right) \end{aligned} \tag{8}$$

The relevant items are formulated respectively as follows:

1) Expected products sales

$$= \int_X \left\{p \times \min\left\{x, q_1 + q_2\right\} f_2\left(x\right)\mathrm{d}x\right. \tag{9}$$

2) Ordering costs when $C^{(2)} = c^{(2)}$

$$c^{(2)}q_2 \tag{10}$$

3) Expected holding costs:

The unit holding cost of buyer responsible inventory is c_{h1} and holding cost of manufacturer responsible inventory is c_{h2}. The holding costs can be expressed as follows:

$$\begin{cases} c_{h1} \times \left(\theta_1 - x\right) + c_{h2} \times \left(q_1 + q_2 - \theta_1\right) & \text{for } x < \theta_1 \\ c_{h2} \times \left(q_1 + q_2 - x\right) & \text{for } \theta_1 < x < q_1 + q_2 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

The expected holding costs can be formulated as follows:

$$\int_X \left\{c_{h1} \times \max\left\{\theta_1 - x, 0\right\} + \ c_{h2} \times \left\{q_1 + q_2 - \max\left\{\theta_1 - x, 0\right\}\right\}\right\} \times f_2\left(x\right)\mathrm{d}x \tag{12}$$

4) Expected shortage costs:

The decision space of total order quantity $q_1 + q_2$ can be divided into two subspaces Ω_1 and Ω_2, Ω_1: total order quantity ($q_1 + q_2$) is less than $\left(1+\gamma\right)\theta_1$, as shown in **Figure 2(a)**; Ω_2: total order quantity ($q_1 + q_2$) is larger than $\left(1+\gamma\right)\theta_1$, as shown in **Figure 2(b)**. In Ω_1, two kinds of shortage types may occur: 1) the shortage occurred between $q_1 + q_2$ and $\left(1+\gamma\right)\theta_1$ belongs to shortage compensation range, its unit shortage costs are the shortage compensation cost and general shortage cost; 2) the others does not belong to shortage compensation range, its unit shortage cost is general shortage cost. The unit shortage costs of shortage compensation and general shortage are c_{s1} and c_{s2} respectively, $c_{s1} > c_{s2}$. In Ω_2, the shortage compensation cost does not occur. Two cases of shortage can be expressed as follows:

$$\text{expected shortage costs in } \Omega_1 = \begin{cases} c_{s1} \times \left[x - \left(q_1 + q_2\right)\right] & \text{for } q_1 + q_2 < x < \left(1+\gamma\right)\theta_1, \\ c_{s1} \times \left[\left(1+\gamma\right)\theta_1 - \left(q_1 + q_2\right)\right] + c_{s2} \times \left\{x - \left(1+\gamma\right)\theta_1\right\} & \text{for } \left(1+\gamma\right)\theta_1 < x, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

$$\text{expected shortage costs in } \Omega_2 = \begin{cases} c_{s2} \times \left[x - \left(q_1 + q_2\right)\right] & \text{for } \left(1+\gamma\right)\theta_1 < q_1 + q_2 < x, \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

Expected shortage cost can be combined as follows:

$$\int c_{s1}\left\{\max\left\{\min\left[\left(1+\gamma\right)\theta_1, x\right] - \left(q_1 + q_2\right), 0\right\} + c_{s2}\max\left\{x - \max\left\{\left(q_1 + q_2\right), \left(1+\gamma\right)\theta_1\right\}, 0\right\}\right\} \times f_2\left(x\right)\mathrm{d}x \tag{15}$$

2.3. Optimal Solution

To solve the DP problem we first provide the optimal decisions for stage 2 under a given state $s_2 = q_1$. As mentioned above the expected shortage costs in $E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]$ depend on the domain of the total order quantity $q_1 + q_2$, *i.e.*, $\Omega_1 = \left\{\left(q_1, q_2\right)\middle|\theta_1 \le q_1 + q_2 \le \left(1+\gamma\right)\theta_1\right\}$ and $\Omega_1 = \left\{\left(q_1, q_2\right)\middle|q_1 + q_2 > \left(1+\gamma\right)\theta_1\right\}$, each domain corresponds to a shortage cost equation respectively (**Figures 2(a)** and **(b)**). Therefore $E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]$ is formulated

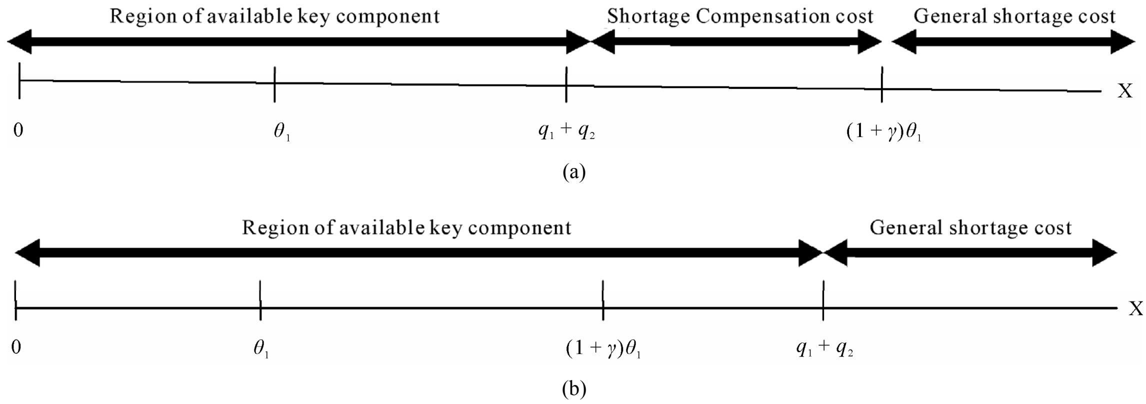

Figure 2. (a) in Ω_1: total order quantity $q_1 + q_2$ is less than $(1+\gamma)\theta_1$; (b) in Ω_2: total order quantity $q_1 + q_2$ is larger than $(1+\gamma)\theta_1$.

for each domain as follows:

$$\begin{aligned} E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_1} &= \int_{-\infty}^{\theta_1} p\theta_1 f_2(x)\mathrm{d}x + \int_{\theta_1}^{q_1+q_2} pxf_2(x)\mathrm{d}x + \int_{q_1+q_2}^{\infty} p(q_1+q_2) f_2(x)\mathrm{d}x - c_{h1}\int_{-\infty}^{\theta_1}(\theta_1 - x) f_2(x)\mathrm{d}x \\ &\quad - c_{h2}\int_{-\infty}^{\theta_1}(q_1+q_2-\theta_1) f_2(x)\mathrm{d}x - c_{h2}\int_{\theta_1}^{q_1+q_2}(q_1+q_2-x) f_2(x)\mathrm{d}x \\ &\quad - c_{s1}\int_{q_1+q_2}^{(1+\gamma)\theta_1}(x-q_1-q_2) f_2(x)\mathrm{d}x - c_{s2}\int_{(1+\gamma)\theta_1}^{\infty}\left(x-(1+\gamma)\theta_1\right) f_2(x)\mathrm{d}x - c^{(2)}q_2 \end{aligned} \quad (16)$$

and

$$\begin{aligned} E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_2} &= \int_{-\infty}^{\theta_1} p\theta_1 f_2(x)\mathrm{d}x + \int_{\theta_1}^{q_1+q_2} pxf_2(x)\mathrm{d}x + \int_{q_1+q_2}^{\infty} p(q_1+q_2) f_2(x)\mathrm{d}x - c_{h1}\int_{-\infty}^{\theta_1}(\theta_1 - x) f_2(x)\mathrm{d}x \\ &\quad - c_{h2}\int_{-\infty}^{\theta_1}(q_1+q_2-\theta_1) f_2(x)\mathrm{d}x - c_{h2}\int_{\theta_1}^{q_1+q_2}(q_1+q_2-x) f_2(x)\mathrm{d}x \\ &\quad - c_{s2}\int_{q_1+q_2}^{\infty}(x-q_1-q_2) f_2(x)\mathrm{d}x - c^{(2)}q_2 \end{aligned} \quad (17)$$

We will show $E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_1}$ and $E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_2}$ are both concave functions. Then we can determine the optimal q_2 in the interval $[0,\infty)$ for the two cases respectively.

Proposition 1. If $p+c_{s1}+c_{h2}>0$ and $p+c_{s2}+c_{h2}>0$ hold for $q_2\in[0,\infty)$, then $E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_1}$ and $E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_2}$ are concave functions, *i.e.*, $\frac{d^2E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_1}}{dq_2^2}\le 0$ and $\frac{d^2E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_2}}{dq_2^2}\le 0$ for $q_2\in[0,\infty)$.

Proof. See appendix A.

Proposition 2. Maximizing $E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_1}$ and $E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_2}$ with respect to q_2, we can get the optimal order quantity q_2 denoted as q_{21}^* and q_{22}^* for the two domains respectively as follows:

$$q_{21}^* = \begin{cases} \max\left\{0, k_2+\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}(t_1)-q_1\right\} \\ \quad \text{if } \theta_1 \le k_2+\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}(t_1)\le(1+\gamma)\theta_1 \\ \max\left\{0,(1+\gamma)\theta_1-q_1\right\} \\ \quad \text{if } k_2+\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}(t_1)>(1+\gamma)\theta_1 \\ \max\left\{0,\theta_1-q_1\right\} \\ \quad \text{if } k_2+\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}(t_1)<\theta_1 \end{cases},$$

where

$$t_1 = \left(p+c_{s1}F_2\left((1+\gamma)\theta_1\right)-c^{(2)}\right)\Big/\left(p+c_{h2}+c_{s1}\right) \quad (18)$$

$$q_{22}^{*}=\begin{cases}\max\left\{0,(1+\gamma)\theta_1-q_1\right\}\\ \quad \text{if } \theta_1\le k_2+\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}(t_2)\le(1+\gamma)\theta_1\\ \max\left\{0,k_2+\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}(t_2)-q_1\right\}\\ \quad \text{if } k_2+\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}(t_2)>(1+\gamma)\theta_1\end{cases},$$

where

$$t_2=\left(p+c_{s2}-c^{(2)}\right)\Big/\left(p+c_{h2}+c_{s2}\right) \tag{19}$$

Proof: See appendix B.

Then we provide the optimal solution for stage 1. At stage 1, the profit functions

$$\Pi(q_1)_{(q_1,q_2)\in\Omega_1}=-c^{(1)}q_1+E_{(2)}\left[\Pi\left(q_{21}^{*}\middle|q_1\right)\right]_{(q_1,q_2)\in\Omega_1}$$

and

$$\Pi(q_1)_{(q_1,q_2)\in\Omega_2}=-c^{(1)}q_1+E_{(2)}\left[\Pi\left(q_{22}^{*}\middle|q_1\right)\right]_{(q_1,q_2)\in\Omega_2}$$

correspond to Ω_1 and Ω_2 respectively. Due to k_2 and $C^{(2)}$ being uncertain, the sample space of $C^{(2)}$ is $C^{(2)}=\left\{c_1^{(2)},c_2^{(2)},\cdots,c_n^{(2)}\right\}$ with respect to the probability $P=\{p_1,p_2,\cdots,p_n\}$, and the distribution of k_2 is $N\left(\theta_1,\sigma_1^4\big/\left(\sigma_0^2+\sigma_1^2\right)\right)$. Let

$$E_{(1)}\left[\Pi(q_1)\right]_{(q_1,q_2)\in\Omega_1}=E\left[-c^{(1)}q_1+E_{(2)}\left[\Pi\left(q_{21}^{*}\middle|q_1\right)\right]_{(q_1,q_2)\in\Omega_1}\right]$$

and

$$E_{(1)}\left[\Pi(q_1)\right]_{(q_1,q_2)\in\Omega_2}=E\left[-c^{(1)}q_1+E_{(2)}\left[\Pi\left(q_{22}^{*}\middle|q_1\right)\right]_{(q_1,q_2)\in\Omega_2}\right]$$

be the expectation of $\Pi(q_1)_{(q_1,q_2)\in\Omega_1}$ and $\Pi(q_1)_{(q_1,q_2)\in\Omega_2}$ respectively, that are formulated as follows:

$$\begin{aligned}E_{(1)}\left[\Pi(q_1)\right]_{(q_1,q_2)\in\Omega_1}=\sum_{i=1}^{n}p_i\Bigg(&\int_{-\infty}^{q_1-\sqrt{d_1+d_3}\Phi^{-1}(t_1)}E_{(2)}\left[\Pi(q_2=0,q_1)_{(q_1,q_2)\in\Omega_1}\right]f_{K_2}(k_2)\mathrm{d}k_2\\&+\int_{q_1-\sqrt{d_1+d_3}\Phi^{-1}(t_1)}^{\infty}E_{(2)}\left[\Pi(q_2>0,q_1)_{(q_1,q_2)\in\Omega_1}\right]f_{K_2}(k_2)\mathrm{d}k_2-c^{(1)}q_1\Bigg)\end{aligned} \tag{20}$$

$$\begin{aligned}E_{(1)}\left[\Pi_2(q_1)\right]_{(q_1,q_2)\in\Omega_2}=\sum_{i=1}^{n}p_i\Bigg(&\int_{-\infty}^{q_1-\sqrt{d_1+d_3}\Phi^{-1}(t_2)}E_{(2)}\left[\Pi(q_2=0,q_1)_{(q_1,q_2)\in\Omega_2}\right]f_{K_2}(k_2)\mathrm{d}k_2\\&+\int_{q_1-\sqrt{d_1+d_3}\Phi^{-1}(t_2)}^{\infty}E_{(2)}\left[\Pi(q_2>0,q_1)_{(q_1,q_2)\in\Omega_2}\right]f_{K_2}(k_2)\mathrm{d}k_2-c^{(1)}q_1\Bigg)\end{aligned} \tag{21}$$

where

$$\begin{aligned}E_{(2)}\left[\Pi(q_2>0,q_1)_{(q_1,q_2)\in\Omega_1}\right]=&\left(p-c_{h1}+c_{h2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\,\Psi\left(\left(\theta_1-k_2\right)\Big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\\&-\left(p+c_{h2}+c_{s1}\right)\sqrt{\sigma_0^2+\sigma_2^2}\,\Psi\left(\Phi^{-1}(t_1)\right)\\&+\left(c_{s1}-c_{s2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\,\Psi\left(\left((1+\gamma)\theta_1-k_2\right)\Big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\\&+\left\{p-c_{h1}+c_{h2}+c_{s1}\left[1-\Phi\left(\left((1+\gamma)\theta_1-k_2\right)\Big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\right]\right\}\theta_1\\&+\left\{c_{h1}-c_{h2}-c^{(2)}-c_{s1}\left[1-\Phi\left(\left((1+\gamma)\theta_1-k_2\right)\Big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\right]\right\}k_2\\&-\left\{c^{(2)}+c_{h2}+c_{s1}\left[1-\Phi\left(\left((1+\gamma)\theta_1-k_2\right)\Big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\right]\right\}\sqrt{\sigma_0^2+\sigma_2^2}\,\Psi\left(\Phi^{-1}(t_1)\right)\\&+c_{s1}\gamma\theta_1\left[1-\Phi\left(\left((1+\gamma)\theta_1-k_2\right)\Big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\right]+c^{(2)}q_1,\end{aligned} \tag{22}$$

$$t_1=\left[p+c_{s_1}F_2\left((1+\gamma)\theta_1\right)-c^{(2)}\right]\Big/\left(p+c_{h2}+c_{s1}\right)$$

$$
\begin{aligned}
E_{(2)}\left[\Pi\left(q_2=0,q_1\right)_{(q_1,q_2)\in\Omega_1}\right] = &-\left(p+c_{h2}+c_{s1}\right)\sqrt{\sigma_0^2+\sigma_2^2}\,\Psi\left(\left(q_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\\
&+\left(p-c_{h1}+c_{h2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\,\Psi\left(\left(\theta_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\\
&+\left(c_{s1}-c_{s2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\,\Psi\left(\left((1+\gamma)\theta_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\\
&+\left\{p-c_{h1}+c_{h2}+c_{s1}\left[1-\Phi\left(\left((1+\gamma)\theta_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\right]\right\}\theta_1\\
&-\left\{c_{h2}+c_{s1}\left[1-\Phi\left(\left((1+\gamma)\theta_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\right]\right\}q_1\\
&+c_{s1}\left[1-\Phi\left(\left((1+\gamma)\theta_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\right]\gamma\,\theta_1+c_{h1}k_2,
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
E_{(2)}\left[\Pi\left(q_2>0,q_1\right)_{(q_1,q_2)\in\Omega_2}\right] = &\left(p-c_{h1}+c_{h2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\,\Psi\left(\left(\theta_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\\
&-\left(p+c_{h2}+c_{s2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\,\Psi\left(\Phi^{-1}\left(t_2\right)\right)+\left(p-c_{h1}+c_{h2}\right)\theta_1\\
&+\left(c_{h1}-c_{h2}-c^{(2)}\right)k_2-\left(c^{(2)}+c_{h2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\,\Psi\left(\Phi^{-1}\left(t_2\right)\right)+c^{(2)}q_1,
\end{aligned}
\tag{24}
$$

$$t_2=\left(p+c_{s2}-c^{(2)}\right)\big/\left(p+c_{h2}+c_{s2}\right)$$

$$
\begin{aligned}
E_{(2)}\left[\Pi\left(q_2=0,q_1\right)_{(q_1,q_2)\in\Omega_2}\right] = &-\left(p+c_{h2}+c_{s2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\,\Psi\left(\left(q_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\\
&+\left(p-c_{h1}+c_{h2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\,\Psi\left(\left(\theta_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\\
&+\left(p-c_{h1}+c_{h2}\right)\theta_1+c_{h1}k_2-c_{h2}q_1.
\end{aligned}
\tag{25}
$$

Proof: See appendix C.

We will show $E_{(1)}\left[\Pi\left(q_1\right)\right]_{(q_1,q_2)\in\Omega_1}$ and $E_{(1)}\left[\Pi\left(q_1\right)\right]_{(q_1,q_2)\in\Omega_2}$ are both concave functions. Then we can search for optimal ordering quantity for each domain, expressed as (q_{11}^*, q_{21}^*) and (q_{12}^*, q_{22}^*) respectively.

Proposition 3. If $p+c_{s1}+c_{h2}>0$ and $p+c_{s2}+c_{h2}>0$ hold for $q_1\in[0,\infty)$, then $E_{(1)}\left[\Pi\left(q_1\right)\right]_{(q_1,q_2)\in\Omega_1}$ and $E_{(1)}\left[\Pi\left(q_1\right)\right]_{(q_1,q_2)\in\Omega_2}$ are concave functions of q_1. where

$$\frac{\partial^2 E_{(1)}\left[\Pi\left(q_1\right)_{(q_1,q_2)\in\Omega_1}\right]}{\partial q_1^2}=\sum_{i=1}^{n}p_i\left(-\left(p+c_{s1}+c_{h2}\right)B_4\Phi\left(\frac{q_1-\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}\left(t_1\right)-B_2}{\sqrt{B_1}}\right)\right) \tag{26}$$

$$\frac{\partial^2 E_{(1)}\left[\Pi\left(q_1\right)_{(q_1,q_2)\in\Omega_2}\right]}{\partial q_1^2}=\sum_{i=1}^{n}p_i\left(-\left(p+c_{s2}+c_{h2}\right)B_4\Phi\left(\frac{q_1-\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}\left(t_2\right)-B_2}{\sqrt{B_1}}\right)\right) \tag{27}$$

$$B_1=\frac{\sigma_1^4\left(\sigma_2^2+\sigma_0^2\right)}{\sigma_1^4+\left(\sigma_1^4+\sigma_0^2\right)\left(\sigma_2^2+\sigma_0^2\right)},$$

$$B_2=\frac{\sigma_1^4 q_1+\theta_1\left(\sigma_1^2+\sigma_0^2\right)\left(\sigma_2^2+\sigma_0^2\right)}{\sigma_1^4+\left(\sigma_1^2+\sigma_0^2\right)\left(\sigma_1^2+\sigma_0^2\right)},$$

$$B_3=\frac{\sigma_1^4 q_1^2+\theta_1^2\left(\sigma_1^2+\sigma_0^2\right)\left(\sigma_2^2+\sigma_0^2\right)}{\sigma_1^4+\left(\sigma_1^2+\sigma_0^2\right)\left(\sigma_2^2+\sigma_0^2\right)},$$

$$B_4=\frac{\sqrt{\sigma_1^2+\sigma_0^2}\sqrt{B_1}}{\sqrt{2\pi}\sigma_1^2\left(\sigma_2^2+\sigma_0^2\right)}e^{\left(B_2^2-B_3\right)/2B_1}.$$

Proposition 4. If $\overline{c}^{(2)}-c^{(1)}<0$ where $\overline{c}^{(2)}=\sum_{i=1}^{n}p_i c_i^{(2)},\ i=1,\cdots,n$, then at stage 1 the optimal order quantity $q_{11}^*=0$ and $q_{12}^*=0$.

Proof: See appendix D.

Proposition 5. There exists an optimal ordering quantity for each domain respectively, and the optimal ordering quantity (q_{11}^*, q_{21}^*) and (q_{12}^*, q_{22}^*) can be determined by the following procedure:

Step 1: At stage 1, we find q_{11}^*, q_{12}^* such that

$$\partial E_{(1)}\left[\Pi\left(q_1\right)_{(q_1,q_2)\in\Omega_1}\right]\Big/\partial q_1\Big|_{q'_{11}}=0$$

and

$$\partial E_{(1)}\left[\Pi\left(q_1\right)_{(q_1,q_2)\in\Omega_2}\right]\Big/\partial q_1\Big|_{q'_{12}}=0,$$

where

$$\partial E_{(1)}\left[\Pi\left(q_1\right)_{(q_1,q_2)\in\Omega_1}\right]\Big/\partial q_1=\sum_{i=1}^{n}p_i\Bigg(\left(p+c_{s1}-c^{(2)}\right)\Phi\left(\left[q_1-\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}\left(t_1\right)-\theta_1\right]\Big/\left[\sqrt{\sigma_1^4\big/\left(\sigma_0^2+\sigma_1^2\right)}\right]\right)$$
$$+\left(c^{(2)}-c^{(1)}\right)-\left(p+c_{s1}+c_{h2}\right)\times\int_{-\infty}^{q_1-\sqrt{d_1+d_3}\Phi^{-1}(t_1)}\Phi\left(\left(q_1-k_2\right)\Big/\sqrt{\sigma_0^2+\sigma_2^2}\right)f_{K_2}\left(k_2\right)\mathrm{d}k_2\Bigg) \quad (28)$$

$$t_1=\left[p+c_{s1}F_{x_2}\left(\left(1+\gamma\right)\theta_1\right)-c^{(2)}\right]\Big/\left(p+c_{h2}+c_{s1}\right),$$

$$\partial E_{(1)}\left[\Pi\left(q_1\right)_{(q_1,q_2)\in\Omega_2}\right]\Big/\partial q_1=\sum_{i=1}^{n}p_i\Bigg(\left(p+c_{s2}-c^{(2)}\right)\Phi\left(\left[q_1-\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}\left(t_2\right)-\theta_1\right]\Big/\left[\sqrt{\sigma_1^4\big/\left(\sigma_0^2+\sigma_1^2\right)}\right]\right)$$
$$+\left(c^{(2)}-c^{(1)}\right)-\left(p+c_{s2}+c_{h2}\right)\times\int_{-\infty}^{q_1-\sqrt{d_1+d_3}\Phi^{-1}(t_2)}\Phi\left(\left(q_1-k_2\right)\Big/\sqrt{\sigma_0^2+\sigma_2^2}\right)f_{K_2}\left(k_2\right)\mathrm{d}k_2\Bigg) \quad (29)$$

$$t_2=\left(p+c_{s2}-c^{(2)}\right)\Big/\left(p+c_{h2}+c_{s2}\right)$$

then q_{11}^* and q_{12}^* can be derived as follows:

$$q_{11}^*=\min\left[q'_{11},\left(1+\gamma\right)\theta_1\right] \quad (30)$$

and

$$q_{12}^*=q'_{12} \quad (31)$$

Step 2: At stage 2, q_{21}^* and q_{22}^* can be derived by Proposition 2 as follows:

$$q_{21}^*=\begin{cases}\max\left\{0,k_2+\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}\left(t_1\right)-q_{11}^*\right\} \\ \quad \text{if } \theta_1\le k_2+\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}\left(t_1\right)\le\left(1+\gamma\right)\theta_1 \\ \max\left\{0,\left\{\left(1+\gamma\right)\theta_1-q_{11}^*\right\}\right\} \\ \quad \text{if } k_2+\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}\left(t_1\right)>\left(1+\gamma\right)\theta_1 \\ \max\left\{0,\left\{\theta_1-q_{11}^*\right\}\right\} \\ \quad \text{if } k_2+\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}\left(t_1\right)<\theta_1\end{cases},$$

where

$$t_1=\left(p+c_{s1}F_2\left(\left(1+\gamma\right)\theta_1\right)-c^{(2)}\right)\Big/\left(p+c_{h2}+c_{s1}\right). \quad (32)$$

$$q_{22}^*=\begin{cases}\max\left\{0,\left\{\left(1+\gamma\right)\theta_1-q_{12}^*\right\}\right\} \\ \quad \text{if } \theta_1\le k_2+\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}\left(t_2\right)\le\left(1+\gamma\right)\theta_1 \\ \max\left\{0,\left\{k_2+\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}\left(t_2\right)-q_{12}^*\right\}\right\} \\ \quad \text{if } k_2+\sqrt{\sigma_0^2+\sigma_2^2}\,\Phi^{-1}\left(t_2\right)>\left(1+\gamma\right)\theta_1\end{cases},$$

where

$$t_2=\left(p+c_{s2}-c^{(2)}\right)\Big/\left(p+c_{h2}+c_{s2}\right). \quad (33)$$

Step 3:

After determining optimal ordering quantity (q_{11}^*, q_{21}^*) and (q_{12}^*, q_{22}^*) for each domain, the optimal order quantity q_1^* and q_2^* can be derived as follows:

$$q_1^*=Arg_{(q_1)}\max\left\{E_{(1)}\left[\Pi\left(q_1\right)_{(q_1)\in\Omega_1}\middle|q_1=q_{11}^*\right],\ E_{(1)}\left[\Pi\left(q_1\right)_{(q_1)\in\Omega_2}\middle|q_1=q_{12}^*\right]\right\} \quad (34)$$

$$q_2^*=\begin{cases}q_{21}^*,\ \text{if } q_1^*=q_{11}^* \\ q_{22}^*,\ \text{if } q_1^*=q_{12}^*\end{cases} \quad (35)$$

In next section we will demonstrate the proposed procedure with some given numerical examples and the sensitivity analysis.

3. Computational Study

3.1. Numerical Examples

Three examples are presented to demonstrate the proposed solution procedure. The relevant parameters are shown in **Table 1**. The optimum q_1^*, q_2^* and the corresponding expected profit for each example are also shown in **Table 1**.

Example 1. Suppose the buyer demand follows a normal distribution with the standard deviation terms $\sigma_0=3$, $\sigma_1=5$, and buyer's minimum-commitment quantity (θ_1) is 30 and γ is 0.1, that is, shortage compensation range is (30, 33). The demand observation at the second stage (θ_2) is 33, and the other relevant parameters are given as follows: product unit selling price *p* is 100, the unit cost of key component at stage 1, $c^{(1)}$= \$30, there is a 70%

Table 1. Examples.

	Example 1	Example 2	Example 3
σ_0	3	3	3
σ_1	5	5	5
p	100	100	100
c_1	30	30	30
$c_1^{(2)}(p_1 = 0.7)$	40	40	40
$c_2^{(2)}(p_2 = 0.3)$	20	20	20
θ_1	30	30	30
γ	0.1	0.4	0.1
θ_2	33	33	38
c_{h1}	10	10	10
c_{h2}	15	15	15
c_{s1}	15	15	15
c_{s2}	10	10	10
Fitted domain	Ω_2	Ω_1	Ω_2
Optimal Solution q_1^*, q_2^*	$q_1^* = 27.1216$ $q_2^* = 5.8784$ $(c^{(2)} = 40)$ $q_2^* = 7.3876$ $(c^{(2)} = 20)$	$q_1^* = 27.2491$ $q_2^* = 5.7159$ $(c^{(2)} = 40)$ $q_2^* = 7.3787$ $(c^{(2)} = 20)$	$q_1^* = 27.1216$ $q_2^* = 9.3574$ $(c^{(2)} = 40)$ $q_2^* = 11.0641$ $(c^{(2)} = 20)$
Optimal Profit	2084.91.	2079.22	2220.19

chance that the ordering cost at stage 2 will 40 and a 30% that the ordering cost at stage 2 is 20 ($c_1^{(2)} = 40$, $p_1 = 0.7$, $c_2^{(2)} = 20$, $p_2 = 0.3$), per unit holding cost of buyer responsible inventory (c_{h1}) = \$10, per unit holding cost of manufacturer responsible inventory (c_{h2}) = \$15, per unit shortage compensation cost (c_{s1}) = \$15 and per unit general shortage cost (c_{s2}) = \$10.

With the proposed solution procedure, in step 1 we find that $q'_{11} = 27.3127$ and $q'_{12} = 27.1216$, then

$$q_{11}^* = \min\left[27.3127, (1+\gamma)\theta_1 = 33\right] = 27.3127$$

and $q_{12}^* = 27.1216$. In step 2, if $c_1^{(2)} = 40$ at stage 2,

$$(\theta_1 = 30) < \left(k_2 + \sqrt{\sigma_0^2 + \sigma_2^2}\,\Phi^{-1}(t_1) = 32.4876\right) < \left((1+\gamma)\theta_1 = 33\right)$$

and

$$(\theta = 30) < \left(k_2 + \sqrt{\sigma_0^2 + \sigma_2^2}\,\Phi^{-1}(t_2) = 32.8025\right) < \left((1+\gamma)\theta_1 = 33\right),$$

then

$$q_{21}^* = \max\left\{0, (32.4876 - 27.3127)\right\} = 5.1749$$

and

$$q_{22}^* = \max\left\{0, \{33 - 27.1216\}\right\} = 5.8784.$$

If $c_2^{(2)} = 20$ at stage 2,

$$\left((1+\gamma)\theta_1 = 33\right) < \left(k_2 + \sqrt{\sigma_0^2 + \sigma_2^2}\,\Phi^{-1}(t_1) = 34.0793\right)$$

and

$$\left((1+\gamma)\theta_1 = 33\right) < \left(k_2 + \sqrt{\sigma_0^2 + \sigma_2^2}\,\Phi^{-1}(t_2) = 34.5092\right),$$

then

$$q_{21}^* = \max\left\{0, \{33 - 27.3127\}\right\} = 5.6873$$

and

$$q_{22}^* = \max\left\{0, \{34.5092 - 27.1216\}\right\} = 7.3876.$$

In step 3, we get

$$q_1^* = Arg_{(q_1)} \max\left\{E_{(1)}\left[\Pi(q_1)_{(q_1)\in\Omega_1} \middle| q_1 = q_{11}^*\right], E_{(1)}\left[\Pi(q_1)_{(q_1)\in\Omega_2} \middle| q_1 = q_{12}^*\right]\right\} = Arg_{(q_1)} \max\{2081.85, 2084.91\} = 27.1216,$$

$q_2^* = 5.8784$ (if $c_1^{(2)} = 40$) or $q_2^* = 7.3876$ (if $c_2^{(2)} = 20$) and optimal expected profit is 2084.91.

Example 2. In example 1 the value of γ is changed from 0.1 to 0.4 while other parameters remain unchanged. With the proposed solution procedure, in step 1 we find that $q'_{11} = 27.2491$ and $q'_{12} = 27.1216$, then

$$q_{11}^* = \min\left[27.2491, (1+\gamma)\theta_1 = 42\right] = 27.2491$$

and $q'_{12} = 27.1216$. In step 2, if $c_1^{(2)} = 40$ at stage 2,

$$(\theta_1 = 30) < \left(k_2 + \sqrt{\sigma_0^2 + \sigma_2^2}\,\Phi^{-1}(t_1) = 32.965\right) < \left((1+\gamma)\theta_1 = 42\right)$$

and

$$(\theta = 30) < \left(k_2 + \sqrt{\sigma_0^2 + \sigma_2^2}\,\Phi^{-1}(t_2) = 32.8025\right) < \left((1+\gamma)\theta_1 = 42\right),$$

then

$$q_{21}^* = \max\left\{0, (32.965 - 27.2491)\right\} = 5.7159$$

and

$$q_{22}^* = \max\left\{0, \{42 - 27.1216\}\right\} = 14.8784.$$

If $c_2^{(2)} = 20$ at stage 2,

$$\left(\theta_1 = 30\right) < \left(k_2 + \sqrt{\sigma_0^2 + \sigma_2^2}\Phi^{-1}(t_1) = 34.6278\right) < \left(\left(1+\gamma\right)\theta_1 = 42\right)$$

and

$$\left(k_2 + \sqrt{\sigma_0^2 + \sigma_2^2}\Phi^{-1}\left(t_2\right) = 34.5092\right) < \left(\left(1+\gamma\right)\theta_1 = 42\right),$$

then

$$q_{21}^* = \max\left\{0, \left\{34.6278 - 27.2491\right\}\right\} = 7.3787$$

and

$$q_{22}^* = \max\left\{0, \left\{42 - 27.1216\right\}\right\} = 14.8784 .$$

In step 3, we get

$$q_1^* = Arg_{(q_1)} \max\left\{E_{(1)}\left[\Pi\left(q_1\right)_{(q_1)\in\Omega_1}\middle| q_1 = q_{11}^*\right], E_{(1)}\left[\Pi\left(q_1\right)_{(q_1)\in\Omega_2}\middle| q_1 = q_{12}^*\right]\right\} = Arg_{(q_1)} \max\left\{2079.22, 1846.81\right\} = 27.2491,$$

$q_2^* = 5.7159$ (if $c_1^{(2)} = 40$) or $q_2^* = 7.3787$ (if $c_2^{(2)} = 20$) and optimal expected profit is 2079.22.

Example 3. In example 1 the value of θ_2 is changed from 33 to 38 while other parameters remain unchanged. With the proposed solution procedure, in step 1 we find that $q_{11}' = 27.4702$ and $q_{12}' = 27.1216$, then

$$q_{11}^* = \min\left[27.4702, \left(1+\gamma\right)\theta_1 = 42\right] = 27.4702$$

and $q_{12}' = 27.1216$. In step 2, if $c_1^{(2)} = 40$ at stage 2,

$$\left(\left(1+\gamma\right)\theta_1 = 33\right) < \left(k_2 + \sqrt{\sigma_0^2 + \sigma_2^2}\Phi^{-1}\left(t_1\right) = 35.7675\right)$$

and

$$\left(\left(1+\gamma\right)\theta_1 = 33\right) < \left(k_2 + \sqrt{\sigma_0^2 + \sigma_2^2}\Phi^{-1}\left(t_2\right) = 36.479\right),$$

then

$$q_{21}^* = \max\left\{0, \left(33 - 27.4702\right)\right\} = 5.5298$$

and

$$q_{22}^* = \max\left\{0, \left\{36.479 - 27.1216\right\}\right\} = 9.3574 .$$

If $c_2^{(2)} = 20$ at stage 2,

$$\left(\left(1+\gamma\right)\theta_1 = 33\right) < \left(k_2 + \sqrt{\sigma_0^2 + \sigma_2^2}\Phi^{-1}\left(t_1\right) = 37.3228\right)$$

and

$$\left(\left(1+\gamma\right)\theta_1 = 33\right) < \left(k_2 + \sqrt{\sigma_0^2 + \sigma_2^2}\Phi^{-1}\left(t_2\right) = 38.1857\right),$$

then

$$q_{21}^* = \max\left\{0, \left\{33 - 27.4702\right\}\right\} = 5.5298$$

and

$$q_{22}^* = \max\left\{0, \left\{38.1857 - 27.1216\right\}\right\} = 11.0641 .$$

In step 3, we get

$$q_1^* = Arg_{(q_1)} \max\left\{E_{(1)}\left[\Pi\left(q_1\right)_{(q_1)\in\Omega_1}\middle| q_1 = q_{11}^*\right], E_{(1)}\left[\Pi\left(q_1\right)_{(q_1)\in\Omega_2}\middle| q_1 = q_{12}^*\right]\right\} = Arg_{(q_1)} \max\left\{2131.75, 2220.19\right\} = 27.1216,$$

$q_2^* = 9.3574$ (if $c_1^{(2)} = 40$) or 11.0641 (if $c_2^{(2)} = 20$) and optimal expected profit is 2220.19.

3.2. Sensitivity Analysis

θ_2 is an observation of $N\left(\theta_1, \sigma_0^2 + \sigma_1^2\right)$ at stage 2. At stage 1 we don't know what value of θ_2 will be observed. With Monte Carlo method we randomly generate 100 values of θ_2 from $N\left(\theta_1, \sigma_0^2 + \sigma_1^2\right)$, denoted as $\theta_2^{(i)}$, $i = 1, \cdots, 100$. With the proposed solution procedure, we can find $q_1^{*(i)}$ with respect to $\theta_2^{(i)}$. Then we evaluate the average expected profit value for using $q_1^{*(i)}$ in 100 values of $\theta_2^{(i)}$. The steps of Monte Carlo method are stated as follows:

Step 1: Randomly generate $\theta_2^{(i)}$, $i = 1, \cdots, 100$ from $N\left(\theta_1, \sigma_0^2 + \sigma_1^2\right)$.

Step 2: With each $\theta_2^{(i)}$ we can find $q_1^{*(i)}$ by Proposition 5, $i = 1, \cdots, 100$ respectively.

Step 3: Compute the expected profit values for using $q_1^{*(i)}$ in each $\theta_2^{(i)}$ $i = 1, \cdots, 100$, denoted as $\Pi^{(1)}\left(q_1^{*(i)}\right)$, $\cdots$, $\Pi^{(100)}\left(q_1^{*(i)}\right)$ (excluding θ_2, parameters are fixed in $\Pi^{(i)}\left(q_1^{*(i)}\right)$, $i = 1, \cdots, 100$). Let

$$\Pi_{(i)}^* = \left[\Pi^{(1)}\left(q_1^{*(i)}\right) + \Pi^{(2)}\left(q_1^{*(i)}\right) + \cdots + \Pi^{(100)}\left(q_1^{*(i)}\right)\right] \big/ 100, \quad i = 1, \cdots, 100.$$

Step 4: optimal profit and optimal order quantity q_1^* can be derived as follows:

$$q_1^* = Arg_{\left(q_1^{*(i)}\right)} \max\left\{\Pi_{(1)}^*, \Pi_{(2)}^*, \cdots, \Pi_{(100)}^*\right\}.$$

The relationships among optimal expected profit, shortage compensation range coefficient (γ), and buyer's minimum-commitment quantity (θ_1) are shown in **Table 1** with the related parameters given in the illustrating example. And the relationships among optimal expected profit, shortage compensation range coefficient (γ) and shortage compensation cost c_{s1}, are shown in **Table 2** and **Table 3**.

In **Table 2** we find that: 1) given θ_1, we define the optimal expected profit turning point of decreasing as

Table 2. Relationships among optimal expected profit, shortage compensation coefficient and buyer's minimum-commitment quantity.

γ \ θ_1	20	25	30	35	40
0	1836.02	2074.59	2345.81	2653.83	2990.32
0.1	1836.02	2074.59	2345.81	2653.83	2990.32
0.2	1836.02	2074.59	2345.81	2653.83	2990.32
0.3	1836.02	2074.59	2345.81	2653.83	2990.32
0.4	1836.02	2074.59	2345.81	2648.21	2985.97
0.5	1836.02	2067.19	2337.45	2645.42	2982.73
0.6	1832.17	2064.98	2335.56	2643.19	2980.85
0.7	1829.75	2064.86	2335.53	2643.11	2980.44
0.8	1829.53	2064.67	2335.47	2643.1	2980.41
0.9	1829.49	2064.65	2335.47	2643.1	2980.39
1	1829.48	2064.65	2335.45	2643.1	2980.39

Table 3. Relationships among optimal expected profit, shortage compensation coefficient and unit shortage compensation cost (c_{s1}) (given θ_1 = 30).

γ \ c_{s1}	15	30	45	60	75
0	2345.81	2345.81	2345.81	2345.81	2345.81
0.1	2345.81	2345.81	2345.81	2345.81	2345.81
0.2	2345.81	2345.81	2345.81	2345.81	2345.81
0.3	2345.81	2345.81	2345.81	2345.81	2345.81
0.4	2345.81	2345.81	2345.81	2345.81	2345.81
0.5	2337.45	2321.75	2312.85	2299.03	2270.26
0.6	2335.56	2315.03	2301.06	2283.1	2264.03
0.7	2335.53	2303.37	2290.98	2270.67	2243.73
0.8	2335.47	2302.57	2287.84	2266.5	2229.04
0.9	2335.47	2302.48	2286.71	2260.49	2228.85
1	2335.45	2302.47	2286.7	2260.49	2228.84

γ_{θ_1}, that is, when $\gamma < \gamma_{\theta_1}$, the optimal expected profit is kept unchanged, but when $\gamma > \gamma_{\theta_1}$, the optimal expected profit decreases accordingly, *i.e.*, γ_{20} = 0.5, γ_{25} = γ_{30} = 0.4, γ_{35} = γ_{40} = 0.3. The larger θ_1 is, the smaller γ_{θ_1} is. 2) The marginal optimal expected profit is increased as θ_1 increases. In **Table 3**, **Table 4** and **Figure 3** with a given θ_1 we find that when $\gamma < \gamma_{\theta_1}$ (γ_{30} = 0.4, γ_{40} = 0.3), the optimal total order quantity $q_1^* + q_2^*$ is over $(1+\gamma)\theta_1$ which causes the shortage compensation cost can not be occurred. Hence, if $\gamma < \gamma_{40}$ then the manufacturer can give the buyer a larger shortage compensation value of c_{s1} and take buyer's increasing the value of θ_1. For example, if the original shortage compensation coefficient γ = 0.2 and θ_1 = 30, then we find γ_{30} = 0.4, the manufacturer can give γ = 0.3 and an arbitrarily large value of compensation cost c_{s1} to the buyer and take the buyer increasing the value of θ_1 from 30 to 40, then the optimal expected profit in the case of γ = 0.3 and θ_1 = 40 will be larger than it in the case of γ = 0.2 and θ_1 = 30.

Base on the above description, the management insights observed from **Table 2** to **Table 4** can be concluded as follows:

1) The more the buyer's minimum-commitment quantity θ_1 is, the more the expected profit of manufacturer is.

2) There are some ways to induce the buyer to increase the value of θ_1:

a) From **Table 2**, we know the upper bound of γ (γ_{θ_1}) which the manufacturer can give to the buyer to attract the buyer increasing the minimum commitment value of θ_1 without losing the expected profit.

b) If the optimal total order quantity $q_1^* + q_2^*$ is larger than $(1+\gamma)\theta_1$, then the manufacturer can give attractive values of shortage compensation cost c_{s1} (**Tables 3** and **4**) and shortage compensation coefficient (γ) (**Table 2** to **Table 4** to take buyer increasing the value of θ_1.

The manufacturer can provide alternatives for the buyer as **Table 5** to induce the buyer to increase the value of θ_1.

Table 4. Relationships among optimal expected profit, shortage compensation coefficient and unit shortage compensation cost (c_{s1}) (given θ_1 = 40).

γ \ c_{s1}	15	30	45	60	75
0	2990.32	2990.32	2990.32	2990.32	2990.32
0.1	2990.32	2990.32	2990.32	2990.32	2990.32
0.2	2990.32	2990.32	2990.32	2990.32	2990.32
0.3	2990.32	2990.32	2990.32	2990.32	2990.32
0.4	2985.97	2963.11	2940.85	2934.03	2930.33
0.5	2982.73	2956.39	2932.06	2909.27	2887.73
0.6	2980.85	2952.73	2926.98	2903.1	2880.74
0.7	2980.44	2951.93	2925.84	2901.67	2879.04
0.8	2980.41	2951.84	2925.71	2901.5	2878.85
0.9	2980.39	2951.83	2925.7	2901.49	2878.84
1	2980.39	2951.83	2925.7	2901.49	2878.84

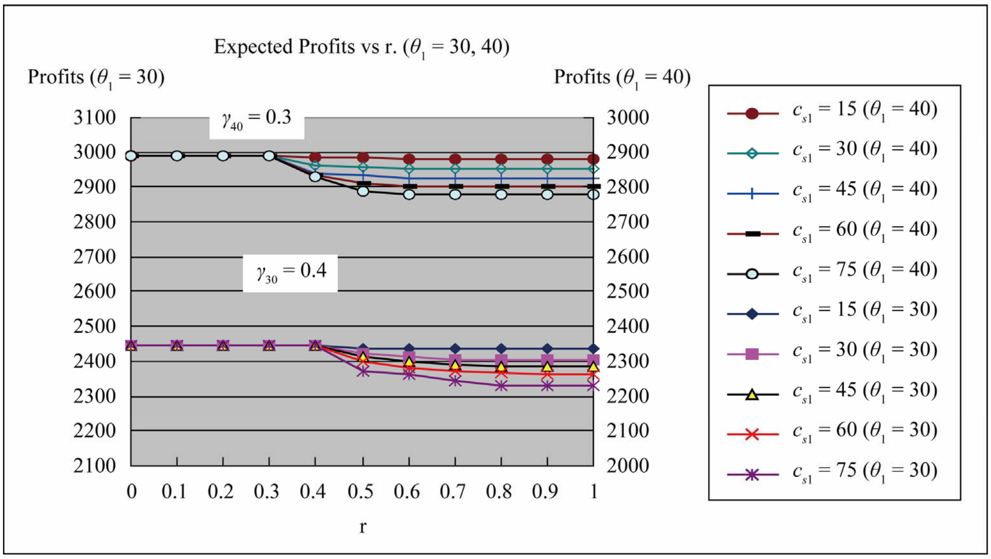

Figure 3. Relationships among the expected profit, shortage compensation cost (c_{s1}), shortage compensation coefficient (γ) and unit shortage compensation cost (c_{s1}).

Table 5. Alternatives for the buyer.

Alternative	Buyer's minimum-commitment quantity θ_1	Shortage compensation coefficient γ	Shortage compensation cost c_{s1}
1	$\theta_1 = 20$	$\gamma = 0.1$	$c_{s1} = 15$
2	$\theta_1 = 30$	$\gamma = 0.2$	$c_{s1} = 30$
3	$\theta_1 = 40$	$\gamma = 0.3$	$c_{s1} = 45$

4. Conclusions

This paper proposes a two-stage dynamic optimization model for an ODM CMOS camera module manufacturer to determine its optimal order quantities to maximize optimal expected profit based on buyer's minimum-commitment quantity contract and shortage compensation policy. The manufacturer can update the distribution of buyer's demand by collecting the market information, and this situation is common and realistic for entrepreneurs in industry. In this paper, two kinds of inventories and shortage costs that are taken into consideration, the conditions for the two-stage optimal order quantities are derived, and the solution procedure is proposed. Numerical examples are to be illustrated and some management insights are provided. The upper bound of γ (γ_{θ_1}) which the manufacturer can give to the buyer to attract the buyer increasing the minimum commitment value of θ_1 without losing the expected profit, the manufacturer can use the upper bound of γ (γ_{θ_1}) to redefine an attractive values of shortage compensation coefficient (γ) and shortage compensation cost (c_{s1}) to induce the buyer to increase the value of θ_1 that can improve expected profit of the manufacturer. It would be of interest to extend the model to allow for the manufacturer having third or above order opportunity before the buyer's real demand occurred.

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Appendix A

Proof of Proposition 1:

$$\mathrm{d}E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_1}\Big/\mathrm{d}q_2$$

$$=\int_{q_1+q_2}^{\infty} p f_2(x)\mathrm{d}x - c_{h2}\int_{-\infty}^{\theta_1} f_2(x)\mathrm{d}x$$

$$-c_{h2}\int_{\theta_1}^{q_1+q_2} f_2(x)\mathrm{d}x - c_{s1}\int_{q_1+q_2}^{(1+\gamma)\theta_1} f_2(x)\mathrm{d}x - c^{(2)}$$

After rearranging the above equation, we have

$$\mathrm{d}E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_1}\Big/\mathrm{d}q_2$$

$$=-\left(p+c_{h2}+c_{s1}\right)F_2\left(q_1+q_2\right)+p+c_{s1}F_2\left((1+\gamma)\theta_1\right)-c^{(2)}$$

and $\mathrm{d}^2E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_1}\Big/\mathrm{d}q_2^2$

$$=-\left(p+c_{h2}+c_{s1}\right)f_2\left(q_1+q_2\right).$$

If $p+c_{s1}+c_{h2}>0$ hold for $q_2\in[0,\infty)$, then

$$\mathrm{d}^2E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_1}\Big/\mathrm{d}q_2^2\le 0\cdot$$

Therefore $E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_1}$ is concave for $q_2\in[0,\infty)$. The proof of $E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega 2}$ is the same as $E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega_1}$.

Appendix B

Proof of Proposition 2: For Ω_1, let

$$\partial E_{(2)}\left[\Pi\left(q_2|q_1\right)\right]_{(q_1,q_2)\in\Omega 2}\Big/\partial q_2$$

$$=-\left(p+c_{h2}+c_{s1}\right)F_2\left(q_1+q_2\right)+p$$

$$+c_{s1}F_2\left((1+\gamma)\theta_1\right)-c^{(2)}=0$$

If

$$\theta_1\le k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_1\right)\le(1+\gamma)\theta_1,$$

$$q_{21}^*=\max\left\{0,k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_1\right)-q_1\right\},$$

where $t_1=\left(p+c_{s1}F_2\left((1+\gamma)\theta_1\right)-c^{(2)}\right)\Big/\left(p+c_{h2}+c_{s1}\right)$.

If $k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_1\right)>(1+\gamma)\theta_1$, we take the upper bound of $\Omega_1\left((1+\gamma)\theta_1\right)$ instead of $\left(k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_1\right)\right)$, $q_{21}^*=\max\left\{0,(1+\gamma)\theta_1-q_1\right\}$ If $k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_1\right)<\theta_1$, we take the lower bound of $\Omega_1\left(\theta_1\right)$ instead of $\left(k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_1\right)\right)$, $q_{21}^*=\max\left\{0,\theta_1-q_1\right\}$. The proof of Ω_2 is the same as Ω_1.

Appendix C

Proof of Equations (20)-(25): From (18), when $k_2>q_1-\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_1\right)$, we can know $q_{21}>0$; when $k_2>q_1-\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_1\right)$, $q_{21}=0$. For given $q_1,c^{(2)}$ and k_2, we can discuss $E_{(1)}\left[\Pi\left(q_1\right)\right]_{(q_1,q_2)\in\Omega_1}$ with two conditions: $q_2>0$, $q_2=0$ and expected profit at stage 2 can be expressed as follows:

when $q_2>0,k_2>q_1-\sqrt{\sigma_o^2+\sigma_2^2}\Phi^{-1}\left(t_1\right)$

$$E_{(2)}\left[\Pi\left(q_2>0,q_1\right)_{(q_1,q_2)\in\Omega_1}\right]=\int_{-\infty}^{\theta_1}p\theta_1 f_2(x)\mathrm{d}x+\int_{\theta_1}^{k_2+\sqrt{\sigma_o^2+\sigma_2^2}\Phi^{-1}(t_1)}px\,f_2(x)\mathrm{d}x$$

$$+\int_{k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}(t_1)}^{\infty}P\left(k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_1\right)\right)f_2(x)\mathrm{d}x-c_{h1}\int_{-\infty}^{\theta_1}\left(\theta_1-x\right)f_2(x)\mathrm{d}x-c_{h2}\int_{-\infty}^{\theta_1}\left(k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_1\right)-\theta_1\right)f_2(x)\mathrm{d}x$$

$$-c_{h2}\int_{\theta_1}^{k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}(t_1)}\left(k_2+\sqrt{\sigma_0^2+\sigma_1^2}\Phi^{-1}\left(t_1\right)-x\right)f_2(x)\mathrm{d}x-c_{s1}\int_{k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}(t_1)}^{(1+\gamma)\theta_1}\left(x-k_2-\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_1\right)\right)f_2(x)\mathrm{d}x$$

$$-c_{s2}\int_{(1+\gamma)\theta_1}^{\infty}\left(x-(1+\gamma)\theta_1\right)f_2(x)\mathrm{d}x-c^{(2)}\left(k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_1\right)-q_1\right)=\left(p-c_{h1}+c_{h2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\Psi\left(\left(\theta_1-k_2\right)\Big/\sqrt{\sigma_0^2+\sigma_2^2}\right)$$

$$-\left(p+c_{h2}+c_{s1}\right)\sqrt{\sigma_0^2+\sigma_2^2}\Psi\left(\Phi^{-1}\left(t_1\right)\right)+\left(c_{s1}-c_{s2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\Psi\left(\left((1+\gamma)\theta_1-k_2\right)\Big/\sqrt{\sigma_0^2+\sigma_2^2}\right)$$

$$+\left\{p-c_{h1}+c_{h2}+c_{s1}\left[1-\Phi\left(\left((1+\gamma)\theta_1-k_2\right)\Big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\right]\right\}\theta_1$$

$$+\left\{c_{h1}-c_{h2}-c^{(2)}-c_{s1}\left[1-\Phi\left(\left((1+\gamma)\theta_1-k_2\right)\Big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\right]\right\}k_2$$

$$-\left\{c^{(2)}+c_{h2}+c_{s1}\left[1-\Phi\left(\left((1+\gamma)\theta_1-k_2\right)\Big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\right]\right\}\sqrt{\sigma_0^2+\sigma_2^2}\Psi\left(\Phi^{-1}\left(t_1\right)\right)$$

$$+c_{s1}\gamma\theta_1\left[1-\Phi\left(\left((1+\gamma)\theta_1-k_2\right)\Big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\right]+c^{(2)}q_1$$

when $q_{21}=0$,

$$E_{(2)}\left[\Pi\left(q_2=0,q_1\right)_{(q_1,q_2)\in\Omega_1}\right]=\int_{-\infty}^{\theta_1} p\theta_1 f_2\left(x\right)\mathrm{d}x+\int_{\theta_1}^{q_1} pxf_2\left(x\right)\mathrm{d}x+\int_{q_1}^{\infty} pq_1 f_2\left(x\right)\mathrm{d}x-c_{h2}\int_{-\infty}^{\theta_1}\left(\theta_1-x\right)f_2\left(x\right)\mathrm{d}x$$

$$-c_{h2}\int_{-\infty}^{\theta_1}\left(q_1-\theta_1\right)f_2\left(x\right)\mathrm{d}x-c_{h2}\int_{\theta_1}^{q_1}\left(q_1-x\right)f_2\left(x\right)\mathrm{d}x-c_{s1}\int_{q_1}^{(1+\gamma)\theta_1}\left(x-q_1\right)f_2\left(x\right)\mathrm{d}x$$

$$-c_{s2}\int_{(1+\gamma)\theta_1}^{\infty}\left(x-\left(1+\gamma\right)\theta_1\right)f_2\left(x\right)\mathrm{d}x=-\left(p+c_{h2}+c_{s1}\right)\sqrt{\sigma_0^2+\sigma_2^2}\Psi\left(\left(q_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)$$

$$+\left(p-c_{h1}+c_{h2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\Psi\left(\left(q_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)+\left(c_{s1}-c_{s2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\Psi\left(\left(\left(1+\gamma\right)\theta_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)$$

$$+\left\{p-c_{h1}+c_{h2}+c_{s1}\left[1-\Phi\left(\left(\left(1+\gamma\right)\theta_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\right]\right\}\theta_1-\left\{c_{h2}+c_{s1}\left[1-\Phi\left(\left(\left(1+\gamma\right)\theta_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\right]\right\}q_1$$

$$+c_{s1}\left[1-\Phi\left(\left(\left(1+\gamma\right)\theta_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)\right]\gamma\theta_1+c_{h1}k_2$$

Therefore

$$E_{(1)}\left[\Pi\left(q_1\right)\right]_{(q_1,q_2)\in\Omega_1}=\sum_{i=1}^{n}p_i\left(\int_{-\infty}^{q_1-\sqrt{d_1+d_3}\Phi^{-1}(t_1)}E_{(2)}\left[\Pi\left(q_2=0,q_1\right)_{(q_1,q_2)\in\Omega_1}\right]f_{K_2}\left(k_2\right)\mathrm{d}k_2\right.$$
$$\left.+\int_{q_1-\sqrt{d_1+d_3}\Phi^{-1}(t_1)}^{\infty}E_{(2)}\left[\Pi\left(q_2>0,q_1\right)_{(q_1,q_2)\in\Omega_1}\right]f_{K_2}\left(k_2\right)\mathrm{d}k_2-c^{(1)}q_1\right)$$

By the same discussions, $E_{(1)}\left[\Pi\left(q_1\right)\right]_{(q_1,q_2)\in\Omega_2}$ can be expressed as follows:

when $q_2>0,k_2>q_1-\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_2\right)$

$$E_{(2)}\left[\Pi\left(q_2>0,q_1\right)_{(q_1,q_2)\in\Omega_1}\right]=\int_{-\infty}^{\theta_1} p\theta_1 f_{X_2}\left(x_2\right)\mathrm{d}x_2+\int_{\theta_1}^{k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}(t_2)} pxf_2\left(x\right)\mathrm{d}x$$

$$+\int_{k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}(t_2)}^{\infty}p\left(k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_2\right)\right)f_2\left(x\right)\mathrm{d}x-c_{h1}\int_{-\infty}^{\theta_1}\left(\theta_1-x\right)f_2\left(x\right)\mathrm{d}x$$

$$-c_{h2}\int_{-\infty}^{\theta_1}\left(k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_2\right)-\theta_1\right)f_2\left(x\right)\mathrm{d}x-c_{h2}\int_{\theta_1}^{k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}(t_2)}\left(k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_2\right)-x\right)f_2\left(x\right)\mathrm{d}x$$

$$-c_{s2}\int_{k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}(t_2)}^{\infty}\left(x_2-k_2-\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_2\right)\right)f_2\left(x\right)\mathrm{d}x-c^{(2)}\left(k_2+\sqrt{\sigma_0^2+\sigma_2^2}\Phi^{-1}\left(t_2\right)-q_1\right)$$

$$=\left(p-c_{h1}+c_{h2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\Psi\left(\left(\theta_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)-\left(p+c_{h2}+c_{s2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\Psi\left(\Phi^{-1}\left(t_2\right)\right)$$

$$+\left(p-c_{h1}+c_{h2}\right)\theta_1+\left(c_{h1}-c_{h2}-c_2\right)k_2-\left(c^{(2)}+c_{h2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\Psi\left(\Phi^{-1}\left(t_2\right)\right)+c^{(2)}q_1$$

when $q_{22}=0$,

$$E_{(2)}\left[\Pi\left(q_2=0,q_1\right)_{(q_1,q_2)\in\Omega_2}\right]=\int_{-\infty}^{\theta_1} p\theta_1 f_2\left(x\right)\mathrm{d}x+\int_{\theta_1}^{q_1} pxf_2\left(x\right)\mathrm{d}x+\int_{q_1}^{\infty} pq_1 f_2\left(x\right)\mathrm{d}x$$

$$-c_{h1}\int_{-\infty}^{\theta_1}\left(\theta_1-x\right)f_2\left(x\right)\mathrm{d}x-c_{h2}\int_{-\infty}^{\theta_1}\left(q_1-\theta_1\right)f_2\left(x\right)\mathrm{d}x-c_{h2}\int_{\theta_1}^{q_1}\left(q_1-x\right)f_2\left(x\right)\mathrm{d}x-c_{s2}\int_{q_1}^{\infty}\left(x-q_1\right)f_2\left(x\right)\mathrm{d}x$$

$$=-\left(p+c_{h2}+c_{s2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\Psi\left(\left(q_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)+\left(p-c_{h1}+c_{h2}\right)\sqrt{\sigma_0^2+\sigma_2^2}\Psi\left(\left(\theta_1-k_2\right)\big/\sqrt{\sigma_0^2+\sigma_2^2}\right)$$

$$+\left(p-c_{h1}+c_{h2}\right)\theta_1+c_{h1}k_2-c_{h2}q_1$$

Therefore

$$E_{(1)}\left[\Pi_2\left(q_1\right)\right]_{(q_1,q_2)\in\Omega_2}=\sum_{i=1}^{n}p_i\left(\int_{-\infty}^{q_1-\sqrt{d_1+d_3}\Phi^{-1}(t_2)}E_{(2)}\left[\Pi\left(q_2=0,q_1\right)_{(q_1,q_2)\in\Omega_2}\right]f_{K_2}\left(k_2\right)\mathrm{d}k_2+\int_{q_1-\sqrt{d_1+d_3}\Phi^{-1}(t_2)}^{\infty}E_{(2)}\left[\Pi\left(q_2>0,q_1\right)_{(q_1,q_2)\in\Omega_2}\right]f_{K_2}\left(k_2\right)\mathrm{d}k_2-c^{(1)}q_1\right)$$

Appendix D

Proof of Proposition 4:

In Ω_1, we redefine $\partial E_{(1)}\left[\Pi\left(q_1\right)\right]_{(q_1,q_2)\in\Omega_1}\big/\partial q_1=G\left(q_1\right)$. If $\overline{c}^{(2)}-c^{(1)}\geq 0$, $G\left(q_1\right)=\overline{c}^{(2)}-c^{(1)}\geq 0$ as $q_1\to-\infty$ and $G\left(q_1\right)=-c_{h1}-c^{(1)}<0$ as $q_1\to\infty$, by the intermediate value theorem, there exists a $q_{11}\in[-\infty,\infty]$ such that $\partial E_{(1)}\left[\Pi\left(q_1\right)\right]_{(q_1,q_2)\in\Omega_1}\big/\partial q_1=0$. If $\overline{c}^{(2)}-c^{(1)}<0$, $G\left(q_1\right)<0$ as $q_1\to-\infty$ and $G\left(q_1\right)<0$ as $q_1\to\infty$, hence $q_1=0$ when $\overline{c}^{(2)}-c^{(1)}<0$.

In Ω_2, we redefine $\partial E_{(1)}\left[\Pi\left(q_1\right)\right]_{(q_1,q_2)\in\Omega_2}\big/\partial q_1=G\left(q_1\right)$, if $\overline{c}^{(2)}-c^{(1)}\geq 0$, $G\left(q_1\right)=\overline{c}^{(2)}-c^{(1)}\geq 0$ as $q_1\to-\infty$ and $G\left(q_1\right)=-c_{h2}-c^{(1)}<0$ as $q_1\to\infty$, by the intermediate value theorem, there exists a $q_{12}\in[-\infty,\infty]$ such that $\partial E_{(1)}\left[\Pi\left(q_1\right)\right]_{(q_1,q_2)\in\Omega_2}\big/\partial q_1=0$. If $\overline{c}^{(2)}-c^{(1)}<0$, $G\left(q_1\right)<0$ as $q_1\to-\infty$ and $G\left(q_1\right)<0$ as $q_1\to\infty$, hence $q_1=0$ when $\overline{c}^{(2)}-c^{(1)}<0$ and this completes the proof.