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The Effect of Irradiance Related Temperature on Microalgae Growth in a Tubular Photo Bioreactor for Cleaner Energy

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Abstract

In microalgae based biofuel technology, the light is one of the important factors for the proper growth of microalgae cells as microalgae is a photosynthetic microorganism. For a large scale outdoor culture the irradiance of sunlight and associated temperature is also need to consider. In this study aims to present computational model of microalgae growth taking effect of solar irradiance and corresponding temperature in a tubular photo bioreactor for an outdoor culture system. We consider the transient behavior of temperature inside the photo bioreactor for a microalgae culture. The optimum range of temperature for outdoor cultivation of microalgae is about 22˚C - 27˚C and out of this range the microalgae cell growth inhibits. Many correlations have already been established to investigate the algal productivity based on the dynamic conditions of temperature in case of full scale outdoor cultivation. However, none of them are validated yet numerically considering the model as a function of weather conditions, operational behavior and design criteria. A tubular photobioreactor (PBR) with length 20.5 m and radius 0.05 m has taken account as a simulation model. The PBR is horizontally placed as temperature variations can be observed with greater accuracy. As the solar irradiance varies at any geographic latitude for a year and so thus temperature, equations and parameters are established relating the irradiance with the temperature to simulate the effect. We observed some significant effects of temperature on the growth of microalgae. Moreover, for the maximum growth of the cells we should control the surrounding temperature.

Keywords

Microalgae, Biofuel, Photo Bioreactor, Solar Irradiance, Temperature, Simulation

1. Introduction

With the advent of the 21st century, the world has begun to face with two major crises: one is the depletion of fossil fuel due to increasing demand and the other one is consequent dependency on the fossil fuel exporting countries. In the recent world, rapid industrialization and motorization has led the people to a steep rise for the demand of petro fuel. The burning of the fossil fuel causes the environmental hazards like: climate change, increased concentrations of GHG, depletion of ozone layer etc. So now it is a prime concern for the scientist and researchers to find out a carbon neutral droplet that would save the world from probable degradation of the environment [1] [2]. To overcome this problem plant based biofuels has become a potential resource which is carbon neutral and can replace petro based fuel. Even though biofuel production from crops still costly compared to fossil fuel as they require large amount of arable land [3]. So a new feedstock is required to mitigate the global demand and produce the biofuel commercially. Among the biofuel sources, some of the algae species have shown best potential for biofuel production. So Microalgae biofuel has received a lot of attention of researchers and engineers since the last few decades as an alternative feedstock [4].

Microalgae are unicellular photosynthetic microorganisms that utilize atmospheric carbon dioxide and sunlight to produce sugars which support biomass growth. Some species of algae yield high oil content which is 100 times faster than any terrestrial plant [5]. Technology, using and producing microalgae has been known for fifty years [4]. Extreme research efforts have been going on to develop new algal biotechnologies to get maximum productivity from microalgae. However, for full scale algae cultivation some issues like technical feasibility, environmental benefits from the culture, scaling up capability of the system still remain a big challenge [6] [7]. Though there are two culture systems are conventional: raceway pond and photo bioreactor [8] [9], none of those are yet to be proved so much fruitful for the optimization of the culture system. But photo bioreactor system is still reliable as it is free from the risk of contamination and requires less space.

Photo bioreactor technology can be used both in indoor and outdoor culture system. For full scale outdoor cultivation some environmental factors work as controlling parameters for the proper growth of microalgae. They are: fixed factors (location, geometry), variable factors (solar radiation variation, temperature, wind speed). The economics of the algal production depends largely on its occupied land area and exposure to sunlight. The amount of land area can be quantified by the sunlight reaching the ground in a definite locality and the fraction of light that is used in the photosynthesis of algae. To reduce the cost of land the factor is only limited with the sunlight. In case of full scale outdoor algae cultivation though chemical components, P^H, CO_2 injection can be easily be controlled but controlling the broth temperature still remains a challenging task as it is directly associated with solar radiation. Temperature condition is still a very crucial factor as it has straightforward implications on the growth rate of microalgae. For the optimization of the design and the efficient operation of microalgae culturing devices, temperature plays a vital role and should be taken in account. In this context, a numerical model simultaneously showing temperature fluctuation and its impact on the productivity is a challenging task. Klametson *et al.* [10] in 1985 and Losordo *et al.* [11] in 1991 first described in their literature an empirical relationship between temperature fluctuation and growth rate of microalgae in case of waste water treatment, aquaculture ponds. But this was limited for a specific species. So a universal model is required to explain the temperature phenomena on the growth rate. Bechet *et al.* [12] at first proposed a universal temperature model including all kinds of parameters location, reactor geometry, sunlight irradiance. This model can precisely predict the evolving temperature of the culture growth medium in case of outdoor cultivation. Indeed, this model includes the direct and diffuse solar radiation but did not take into account if the weather is cloudy or not. The model describes a relationship between temperature variation and growth rate which is proposed by Bernard *et al.* [13] in his literature. This relationship works out for a full kinetic range of the microalgae culture including higher and lower temperature limit which reduce growth. Before that, Bissinger *et al.* [14] has developed temperature model but the temperature effect was presented by the simple exponential Arrhenius law which can only describe the impact of temperature on the rate of chemical reactions but cannot predict the negative impact of high and low temperatures.

Algae species can operate its photosynthetic process at the optimal temperature though some species endure beyond this temperature range. So to control the broth temperature it is necessary to develop a temperature model that is affected by the environmental parameters.

In this study, our aim is to develop a mathematical model of horizontal loop tubular photo bioreactor to simulate the temperature distribution in the suspension with the solar heat flux variation in a definite location and probable impact on the growth rate based on thermodynamic equilibrium. For the simulation, some relevant meteorological data are gathered for Chittagong, Bangladesh such as cloudiness, daily sunshine hours. For the broth medium we have considered the strain of *Chlorella* species.

2. Mathematical Modeling

In this simulation study, a dynamic heat management model is developed to predict the temperature distribution inside the horizontal loop tubular photo bioreactor (HLTP) as well as its effect on the growth rate of the microalgae. As temperature works as a controlling parameter behind the growth rate of microalgae so for this purpose, the solar radiation that reaches the photo bioreactor directly with varying solar position and heats up the microalgae cells is considered for our simulation. The data associated with the solar radiation are considered for the geographic location Chittagong University of Engineering &

Technology (CUET), Chittagong, Bangladesh.

3. Computational Domain and Meshes

A Horizontal Loop Tubular Photo bioreactor (HLTP) with a U-loop is proposed as the domain. Each straight portion is 10 m and the U-loop is approximately 0.5 m. The radius of the photo bioreactor is 0.025 m, the surface area is about 3.136 m^2 and the volume is 0.03679 m^3 as shown in **Figure 1**. A coarse mesh design is developed with 102,822 elements for the simulation. In **Figure 2**, the mesh design for the U-loop (a) and the inlet (b) are shown respectively.

4. Governing Equation

Algae suspension is considered as Newtonian incompressible fluid. For our simulation purpose the flow dynamics is assumed to be laminar. From this point of view, the flow phenomena satisfies the continuity equation and Navier Stokes equation which are as follows

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0 \tag{1}$$

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla \rho + \nabla \cdot \mu \left(\left(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T \right) - \frac{2}{3} \mu (\nabla \cdot \boldsymbol{u}) I \right) + F \tag{2}$$

Equation (2) can be solved for the Non isothermal laminar flow *i.e.* heat transfer in flowing fluid and can be written as

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p \boldsymbol{u} \cdot \nabla T = \nabla \cdot (K \nabla T) \tag{3}$$

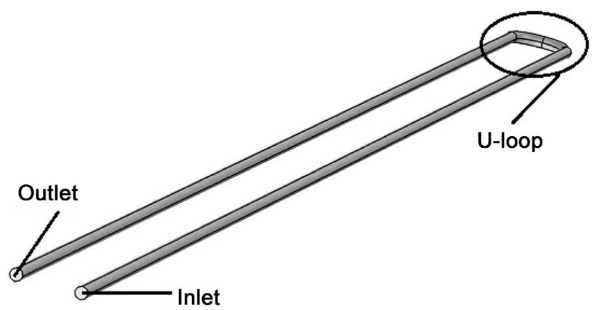

Figure 1. A computational domain of the HLTP showing inlet, outlet and U-loop.

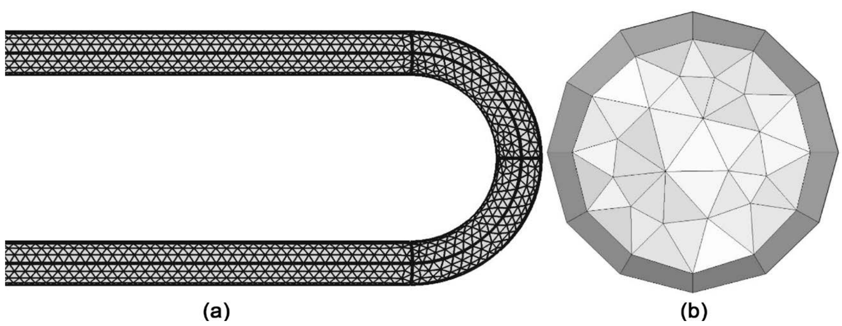

Figure 2. Coarse mesh design showing the (a) longitudinal view and (b) cross sectional view.

where, c_p is the specific heat of suspension, T is the temperature, K is the thermal conductivity.

As we know, radiation heat flux is directly negative proportional to $\nabla \cdot (K \nabla T)$, so a heat balance equation for the total direct solar radiation that reaches the photo bioreactor and heated up the microalgae cells can be expressed as

$$\rho c_p \frac{\partial T}{\partial t} = Q_{\text{radiation(total)}} \tag{4}$$

where, Q_{radt} is the total solar heat flux from sun depending on the latitude of any geographical position in W/m^2.

The total heat flux comprises of the vertically and horizontally incident heat flux [15] shown in Equation (5)

$$Q_{\text{radiation(total)}} = \left(Q_{\text{top}} + Q_{\text{lateral}}\right) f(t) \tag{5}$$

where, $f(t)$ is considered as shading function. The value of $f(t)$ is set to 1 when there is available sunlight and set to 0 at night during outdoor cultivation of microalgae. The direct solar radiation that reaches the PBR vertically can be expressed by Equation (6)

$$Q_{\text{top}} = \varepsilon_{\text{reactor}} \tau H_d \pi R_r^2 \tag{6}$$

where, $\varepsilon_{\text{reactor}}$ is the emissivity of the reactor, τ is the transmissivity of the reactor, H_d is the intensity of the solar radiation reaching to the ground vertically.

The solar radiation received by the PBR laterally is given by

$$Q_{\text{lateral}} = \varepsilon_R \tau H_d \tan(\theta) \pi R_r^2 \tag{7}$$

Generally, solar radiation reaches the ground surface in lateral position varies with the angular position (θ) of the incident sunlight. This angular position is a function of five parameters, they are: declination (δ), solar hour (*sh*), geographic latitude (φ), surface slope (β), surface azimuth angle (τ) and the hour angle. The relation can be expressed by the Equation (8).

$$\begin{aligned} \cos\theta &= \sin\delta \sin\psi \cos\beta - \sin\delta \cos\psi \sin\beta \cos\tau \\ &\quad + \cos\delta \cos\psi \sin\beta \cos\tau \cos\omega + \cos\delta \sin\beta \sin\tau \sin\omega \end{aligned} \tag{8}$$

Grima *et al.* [16] showed in his study that The PBR placed in horizontal position has an exposure to larger illumination area than any other position with respect to the variation in solar hour. For this reason, the surface slope (β) is set to zero degree that provides the Equation (8) in its simplest form

$$\cos\theta = \sin\delta \sin\psi + \cos\delta \cos\psi \cos\omega \tag{9}$$

Duffie and Beckman [17] stated the solar declination angle (δ) as a function of number of days and the equation is as follows

$$\delta = 23.45 \sin\left[\frac{360}{365}(284 + N)\right] \tag{10}$$

The hour angle (ω) can be calculated from the following equation

$$\omega = 15(sh - 12) \tag{11}$$

The hour angle (ω) varies from negative in the morning to positive in the afternoon with 15 degrees angular displacement per hour for the earth rotation from the east to the west.

In most of the cases, the direct solar radiation (H_d) is a function of total solar radiation (H) which comprises of diffuse plus direct solar radiation

$$H_d = (1 - K_d)H \tag{12}$$

where, K_d is the fraction of the diffused reaction reaching the ground surface. Typically its value ranges between 0.33 and 0.5 from low altitude areas to high altitude areas [18].

Almorox *et al.* established a relationship between the total solar radiation (H) and the global solar radiation (H_0) which is as follows

$$\frac{H}{H_0} = a \cdot \left(\frac{s}{s_0}\right)^b \tag{13}$$

In the above equation, a and b are regression coefficients that depend on the specific geographical location. As CUET is located in Chittagong, Bangladesh thus the values are taken for Chittagong, collected from the data from Sarkar [19].

Global solar radiation (H_0) can be expressed as follows [17]

$$H_0 = \frac{(24 \times 3600 \times G_{sc})}{\pi}\left(1 + 0.033\cos\frac{360N}{365}\right)(\cos\phi\cos\delta\sin\omega) + \frac{(\pi\omega)}{180}\sin\varphi\sin\delta \tag{14}$$

The day length (S_0) can be obtained from the following equation according to Duffie and Beckman [17].

$$S_0 = \frac{2}{15}\omega \tag{15}$$

Rangarajan *et al.* [20] established a correlation between the total sunshine hours (S) and the cloud fraction (c) *i.e.* the monthly average daytime the sky is concealed with cloud

$$c = 1 - \frac{s}{s_0} \tag{16}$$

The term c indicates the clearness index and the average value is taken for Chittagong from the data set in the paper of Sarkar [19].

The transmittance property of the photo bioreactor keeps a vital effect on the growth of microalgae cells as how much radiation is transmitted through the tube and reaches the microalgae cells to heat them up to maintain the broth temperature within the optimum range. Transmitted radiation can be determined from the product of transmittance of the reactor and the transmittance of the microalgae.

$$\tau = \tau_T * \tau_A \tag{17}$$

where, τ_T is the reactor transmittance and can be evaluated from the following equation stated by Duffie and Beckman [17]

$$\tau_T = 0.5 * \left(\frac{1 - R_{\text{parallel}}}{1 + R_{\text{parallel}}} + \frac{1 - R_{\text{perpendicular}}}{1 + R_{\text{perpendicular}}} \right), \tag{18}$$

where R_{parallel} and $R_{\text{perpendicular}}$ are the parallel and perpendicular reflection from the tube and can be evaluated by the following equations

$$R_{\text{parallel}} = \left(\tan\left(\theta_2 - \theta\right) * \pi/180\right)^2 / \left(\tan\left(\theta + \theta_2\right) * \pi/180\right)^2, \tag{19}$$

$$R_{\text{perpendicular}} = \left(\sin\left(\theta_2 - \theta\right) * \pi/180\right)^2 / \left(\sin\left(\theta + \theta_2\right) * \pi/180\right)^2, \tag{20}$$

where θ_2 is the angle after refraction from the transparent tube surface. It is a function of the angle of incidence of sunlight and the function of refraction index of air and the reactor.

$$\sin\theta_2 = \left(\frac{IR_{\text{air}}}{IR_{\text{reactor}}} \right) * \sin\theta. \tag{21}$$

Due to some associated losses the effective reflection of the tube is calculated as half of the perpendicular and parallel reflection of the tube

$$R_{\text{effective}} = 0.5 * \left(R_{\text{perpendicular}} + R_{\text{parallel}} \right). \tag{22}$$

The transmittance of the algae cells can be calculated using Bouger's law stated in Duffie and Beckman [17] which is given below

$$\tau_a = \exp\left(-K_a * \frac{PL}{\cos\theta} \right) \tag{23}$$

where, K_a is the proportionally constant which the extinction coefficient of the microalgae cells. The value of K_a is taken for the species *Chlorella Vulgaris* which varies with the variation of species. The total path length *PL* is assumed to be the 60% of the total tube diameter [21].

To quantitatively account for the temperature distribution in the culture medium, a suitable model should be developed to predict the impact of the temperature fluctuation on the growth of microalgae culture.

Bernard *et al.* [13] expressed their model of the growth rate as a function of the maximum specific growth rate, maximum temperature, minimum temperature and optimum temperature. The model has named after cardinal temperature model with inflexion (CTMI).

$$\mu_m = \mu_{\text{opt}} \frac{\left(T - T_{\max}\right)\left(T - T_{\min}\right)^2}{\left(T_{\text{opt}} - T_{\min}\right)\left[\left(T_{\text{opt}} - T_{\min}\right)\left(T - T_{\text{opt}}\right) - \left(T_{\text{opt}} - T_{\max}\right)\left(T_{\text{opt}} + T_{\min} - 2T\right)\right]} \tag{24}$$

In the above equation, T is the temperature in Kelvin or degrees Celsius (˚C), μ_m is the growth rate in minute^{-1}, μ_{opt} is the maximum specific growth rate at the temperature T_{opt}, $T_{\max}$ and $T_{\min}$ is the hypothetical maximum and minimum temperature limit. Growth rate is zero except the temperatures between $T_{\max}$ and $T_{\min}$. The maximum, minimum and optimum temperatures are called cardinal temperatures.

5. Boundary and Initial Conditions

In our simulation, the microalgae suspension flow is considered as a uniform flow and initially the velocity at the inlet is zero, *i.e.* $\boldsymbol{u}=0$; no slip condition at the wall of the reactor and zero normal stress at the outlet of the domain which can be written as

$$\left[-PI+\eta(t)\left(\nabla\boldsymbol{u}+(\nabla\boldsymbol{u})^{\mathrm{T}}\right)\right]\boldsymbol{n}=0 \quad (25)$$

where, P is the pressure and I is the identity matrix.

6. Simulation Parameters

For the simulation, some of the simulation parameters are chosen specifically for the definite region *i.e.* Chittagong, Bangladesh. So the model does not apply for universal condition.

The simulation parameters are given in Table 1.

Table 1. Parameter used for simulation.

Name	Value	Description
R	0.025 m	Radius of the reactor
V	0.03679 m^3	Volume of the reactor
A	3.136 m^2	Area of the reactor
IR_{air}	1	Refraction index of air
IR_{tube}	1.49	Refraction index of acrylic tube
ϕ	22.46°	Latitude of CUET
N	75	Day of the year
ε	0.94	Emissivity of the reactor
K_a	36.9 m^{-1}	Extinction coefficient of *Chlorella Vulgaris*
μ_{max}	0.0631 h^{-1}	Maximum growth rate of microalgae
a	1	Constant value
b	200	Constant value
C_0	0.55	Initial concentration of microalgae
N_w	0.001 Pa * S	Water viscosity
U_{in}	0.5 ms^{-1}	Inlet initial velocity
G	1367 w/m^2	Solar constant
PL	0.03 m	Path length of the radiation
S	8.1 h	Bright sunshine hour in the month of march
$T_{\max}$	318 [K]	Maximum temperature above which growth rate becomes zero
$T_{\min}$	278 [K]	Minimum temperature below which growth rate becomes zero
$T_{\rm opt}$	300.5 [K]	Optimum temperature at which the growth rate becomes maximum

7. Numerical Simulation

The aim of this study is to observe the temperature fluctuation in the photo bioreactor from dawn to dusk and consequently its effect on the growth of microalgae for specific geometry, specific species and specific location. All the parameters are considered here for the outdoor culture condition to observe the phenomena whether the temperature distribution is in the cardinal temperatures range or not. The COMSOL MULTIPHYSICS version 4.2a software is used to simulate the problem. The simulation is carried out for the seventh day of the microalgae culture and the photo bioreactor is assumed to be illuminated with varying solar radiation from morning to evening. The initial solution is kept at $\boldsymbol{u} = 0$ for the whole domain except at the inlet. The simulation is carried out for three definite time ranges with definite time interval. The time ranges are: In the morning (6.00 am to 6.15 am), the noon (11.45 am to 12.00 pm) and the afternoon (3.00 pm to 3.15 pm). For all cases, the time interval was 100 s. For the geographical location, we have considered the latitude of CUET, Bangladesh. As in the Bangladesh the bright sunshine hour is in the month of March in the summer season so we have chosen the 16th March for our simulation [22].

8. Results and Discussions

The simulation results are analyzed to observe the temperature distribution in three different times and temperature effects on a microalgae cell in the photo bioreactor. Also the most important factor growth curve against the temperatures collected from the simulation data is produced to gain knowledge on the productivity level of microalgae in this region. In **Figure 3**, surface temperatures are shown for three different times whereas in **Figure 3(c)** surface temperature reaches 325 K at 3.15 pm which is beyond the upper cardinal temperature. In this situation the microalgae cell growth inhibits. In this case, water spraying system or shading system can be arranged to control the broth temperature.

In **Figure 4**, temperature slices of the suspension flow are presented in the morning, the noon and the afternoon. The slices are taken at the length of 5 m, at the middle of the U-loop and at the length of 15.5 m. In case of **Figures 4(a)-(h)** the temperature slice of the suspension shows the congenial environment for the growth of microalgae but in **Figure 4(i)**, the temparature slice crosses the optimum range which is near the outlet of the domain at the afternoon. By observing **Figure 4(b)** & **Figure 4(e)** & **Figure 4(h)** and **Figure 4(c)** & **Figure 4(f)** & **Figure 4(i)** we have found that there is abrupt change of temperature from the middle portion of the u-loop to the outlet. This is due to the rapid mixing of the inner microalgae cells with the cells of the outer periphery while crossing the U-loop and thus receive more heat from the surface. Consequently, in **Figure 4(c)** & **Figure 4(f)** & **Figure 4(i)** the outer pheriphery shows higher temperature than the inner portion of the suspension.

In **Figures 5(a)-(c)** the temperature distributions against three different time ranges (6.00 am to 6.15 am), (11.45 am to 12.00 pm) and (3.00 pm to 3.15 pm)

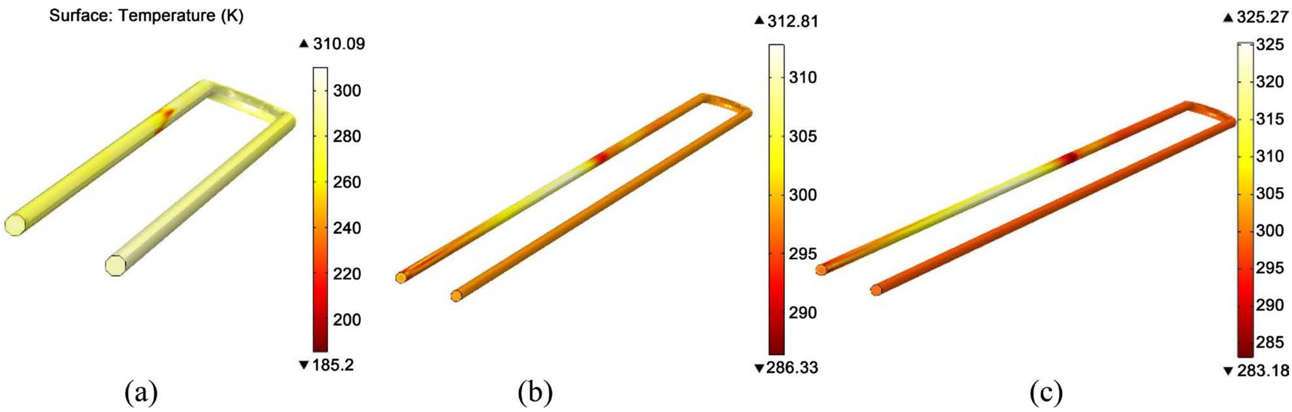

Figure 3. Surface temperature (a) at 6.15 am; (b) at 12.00 pm; (c) at 3.15 pm.

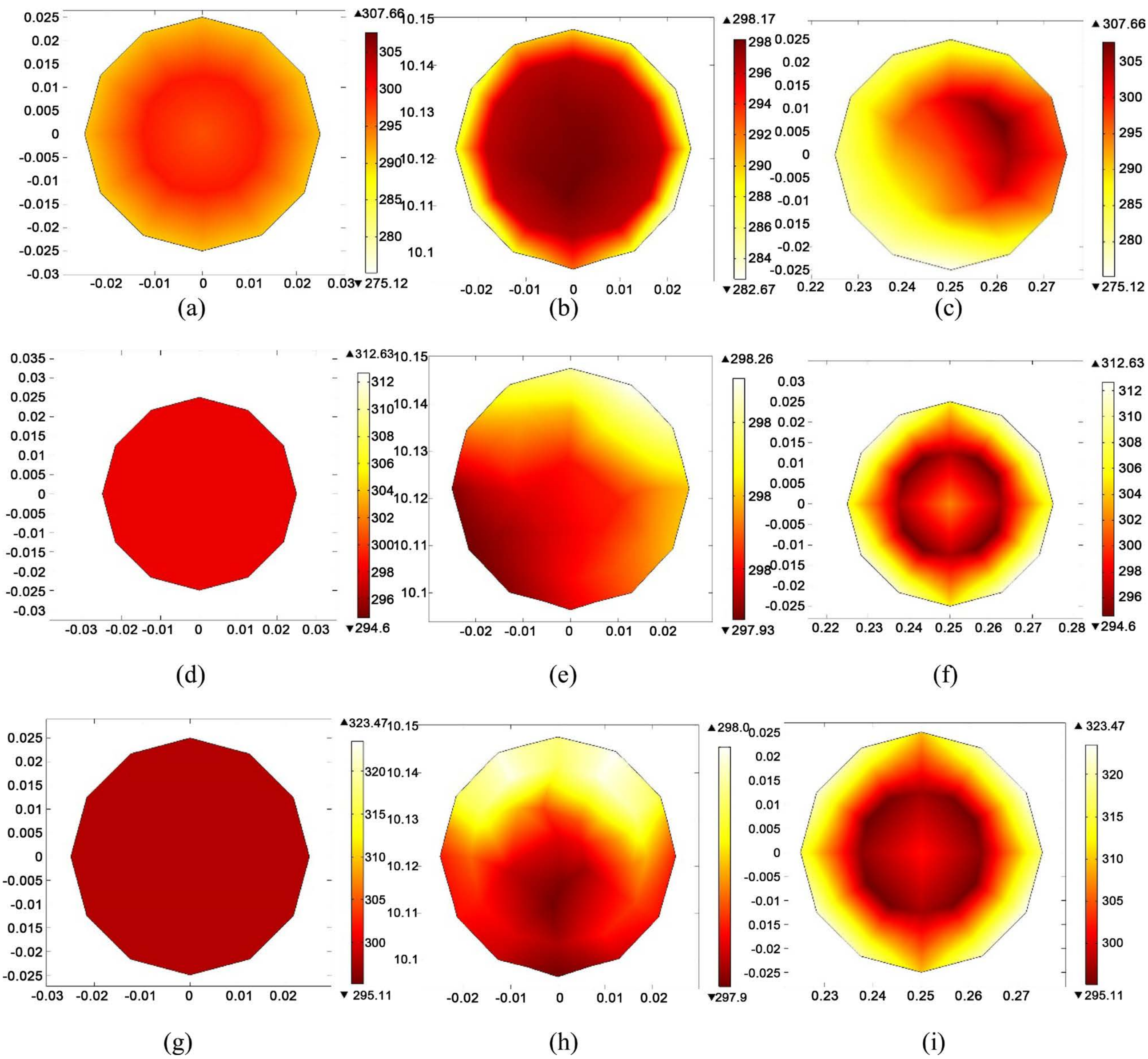

Figure 4. Temperature slice at 6.15 am: arclength (a) 5 m, (b) 10.25 m, (c) 15.5 m; at 12.00 pm: arclength (d) 5 m (e) 10.25 m (f) 15.5 m; at 3.15 pm: arclength (g) 5 m, (h) 10.25 m, (i) 15.5 m.

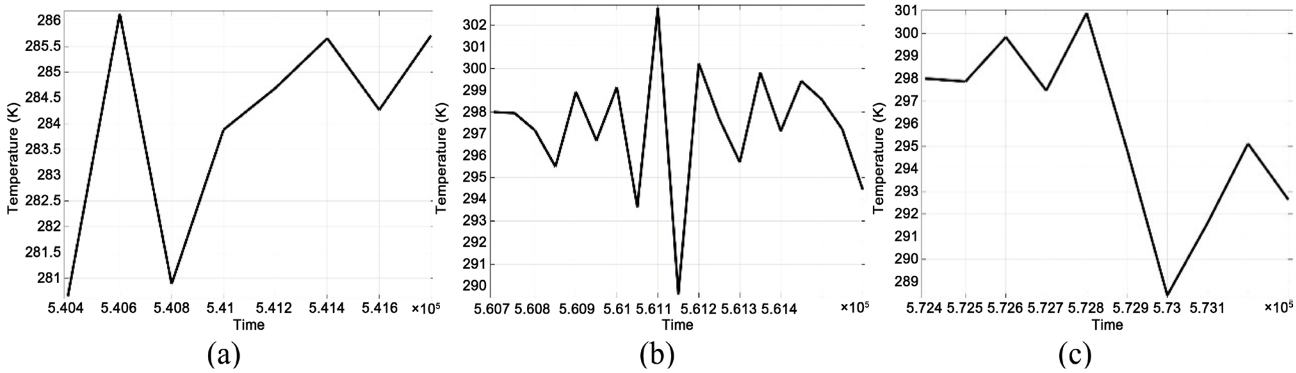

Figure 5. Temperature vs time for a mciroalgae cell at the outlet of the Photobioreactor (a) at 6 am - 6.15 am; (b) at 11.45 am to 12.00 pm; (c) at 3.00 pm to 3.15 pm.

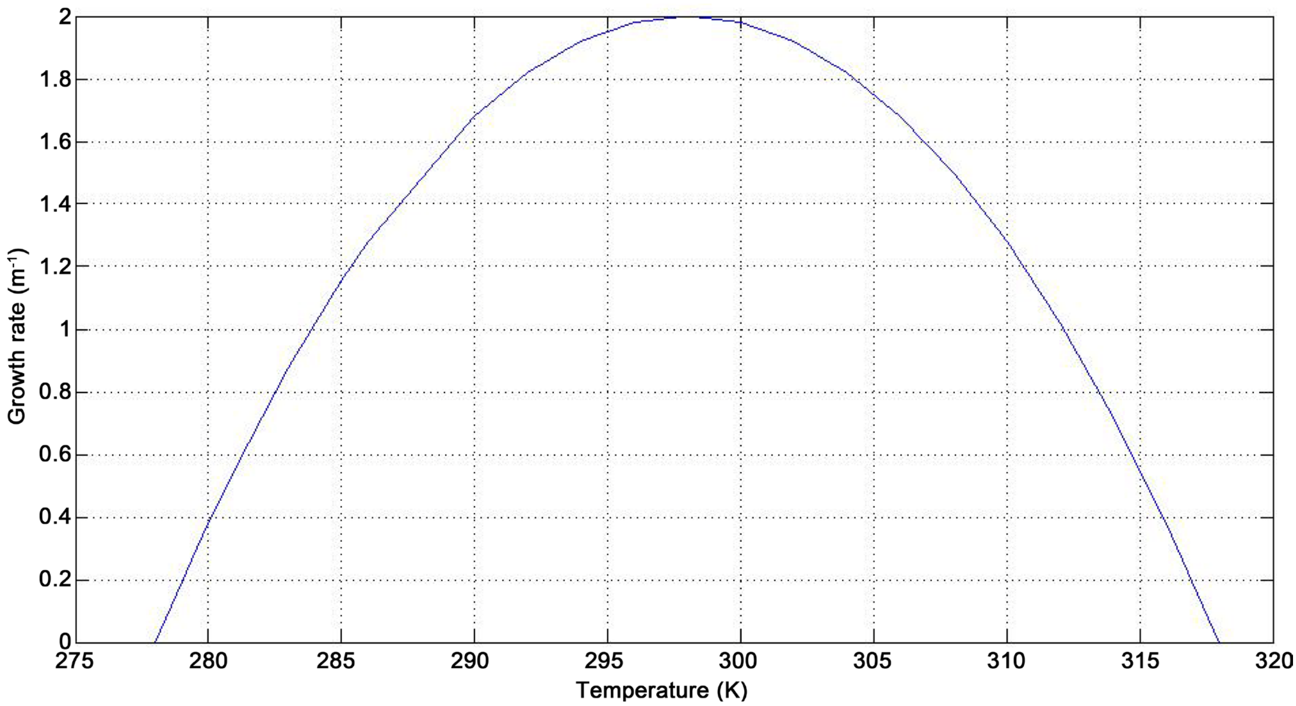

Figure 6. Temperatre vs growth rate (m^{-1}) (Temperatures collected from the simulation results).

are shown for a microalgae cell at the outlet. The cell is chosen from the lower position *i.e.* far from the upper surface to observe whether it receives the adequate temperature for its growth. In each case, we can see the temperature fluctuation lies between the cardinal temperatures.

As temperature fluctuation and its effect on the growth of the microalge is very crucial factor a growth curve against the temperatures collected from the simulation data is shown in the **Figure 6**. By using equation 24 the growth rate per minute is found. The graph shows a parabolic curve of growth rate vs temperature. From the **Figure 6**, it is shown that at the temperature 298 K the growth rate is maximum. At 278 K and at 318 K the growth rate becomes zero. It satisfies the cardinal temperatures of *Chlorella Vulgaris* as the *Chlorella* species can sustain between 5˚C to 45˚C. Out of this range the cells start to demise.

9. Conclusions

A CFD based study is performed to focus on the temperature variation on the growth rate of microalgae for a definite locality whether the place is suitable or not for the culture of microalgae. From the numerical results of the suspension flow it is conspicuous that the temperature always lies in the cardinal temperature range *i.e.*, 5˚C to 45˚C most of the time through the entire domain except in the afternoon. At 3.00 pm to 3.15 pm the microalgae suspension temperature exceeds the threshold value for the optimum growth. So in this case temperature control of the culture broth should be maintained by applying proper engineering methods. Also it is an important fact to maintain the suspension the microalgae suspension temperature always close to the optimum temperature so that maximum growth can be ensured. Indeed, it is visualized by a symmetrical growth curve versus temperature. Beyond this optimum temperature, the decrease of the growth rate becomes linear and depending on the species reaches to the lethal temperature. The increasing rate of mortality with the temperature exceeds optimum value is a real fact but how much time these changes are experienced is an important issue to diagnose the extent of the mortality rate. The CFD model shows that temperature control techniques are necessary for the large scale production of microalge biofuel in this region to ensure the maximum productivity. The Combining of models on heat flux variation coupled with the effect of the temperature on the cell growth may lead to the temperature control strategies to achieve a tradeoff between the cooling cost and productivity. However, better outcome will be found if the simulation can be run for the daylong.

This paper is an extended version of our very recent paper [23] by putting more results and upgrade the growth model. In this paper the tempearature variation is observed in the broth culture of algae only for a single day perhaps the longest day hour. However, it is necessary to observe a full set of numerical data of temperature for the whole year to scale up and optimize the productivity. Recently Khanam *et. al.* [3] has given an average data of solar irradiance for 12 months of a year. So in the future, we would extend our work by applying the average irradiance for a month in a year as controlling parameter to observe the growth of microalgae cells for a large scale outdoor culture.

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