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The Effect of Self-Driving Car on Urban Traffic

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Abstract

Based on the idea of infinitesimal analysis, we establish the basic model of relation between speed and flow. Since putting a certain amount of self-driving car will affect the average speed of mixed traffic flow, we choose the proportion of self-driving car to be a variable, denoted by k. Based on the least square method, we find two critical values of k that are 38.63% and 68.26%. When k < 38.63%, the self-driving cars have a negative influence to the traffic. When 38.63% < k < 68.26%, they have a positive influence to the traffic. When k > 68.26%, they have significant improvement to the traffic capacity of the road.

Keywords

Self-Driving Car, Least Square Method, Mixing Speed, Traffic Flow

1. Introduction

In order to indicate the origin of self-driving, cooperating cars, the following background is worth mentioning.

1.1. Background

As we all known that the traffic capacity is limited in many areas of the United States owing to the number of lanes of roads. However, self-driving, cooperating cars have been provided a solution to increase the capacity of highways without increasing the number of lanes of roads. The self-driving, cooperating cars are popular with its advantages, for example, faster speed, faster acceleration, and shorter baking distance.

1.2. Literature Review

Lu [1] analyzed the influence of mixed traffic flow car's speed under different traffic flow on the highway. He studied the relationship between traffic flow and traffic speed change and built the model of speed-flow. At the same time, he also analyzed the impact of medium-sized car under different ratios on highway mixed traffic flow speed, researched the relation between the ratio of large and medium-sized cars with the changes of the rate of traffic and built relationship model between speed with the speed-large and the proportion of medium-sized cars. Daganzo and Knoop [2] derived the analytic form of the capacity formula inspired by analytic upper and lower bounds derived with variational theory for a version of the problem where cars are treated as a fluid. Daganzo and Lehe [3] considered a signalized street of uniform width and blocks of various lengths. It has shown that the macroscopic fundamental diagram of an arterial street controlled by traffic signals was defined by the solution of a linear program. Sorigueraa *et al.* [4] examined both the macroscopic and microscopic effects of different speed limits on a traffic stream. Jamshidnejad *et al.* [5] proposed a framework to interface and integrate macroscopic flow models and microscopic emission models. Yang *et al.* [6] addressed the planning and optimization of intermodal hub-and-spoke (IH & S) network considering mixed uncertainties in both transportation cost and travel time. Celikoglu and Silgu [7] evaluated the performance of a dynamic approach to classify flow patterns reconstructed by a switching-mode macroscopic flow model considering a multivariate clustering method. Yang *et al.* [8] proposed a bi-objective hub-and-spoke (H & S) network design problem with type-2 (T2) fuzzy transportation cost and travel time described by parametric secondary possibility distributions. Those give us some inspiration about research the mixed ratio of self-driving cars and the mixed speed of flows. We will consider mainly the delay time to self-driving cars deal with signal which results to the changes of the traffic flows.

1.3. Our Approach

We use the infinitesimal analysis method to establish the model of relationship between flow and velocity, and combine with the least square method to calculate driving the delivery ratio as a function of the average speed of mixed traffic flow. Using least square method can easily find the function relationship between the variables, and make the error between the predicted value and the actual value of sum of squares to a minimum. Therefore, it is reasonable for us to find the best function relation between the variables with the least squares approximation.

The rest of this paper is organized as follows. In Section 2, we describe the problem in detail. In Section 3, we establish the model and derive the solution based on the least square method. Finally, some conclusions are made in Section 4.

2. Problem Description

Traffic capacity is limited the designed capacity of the road networks. Once over the capacity of the road networks, drivers would experience long delays during peak traffic hours. Self-driving, cooperating cars have been proposed as a solution to increase capacity of highway without increasing number of lanes. Our model is required to solve the influence of traffic capacity about traffic lane number, peak or average, self-drive and percentage of self-driving. Further, to achieve a certain number, the number of self-drive capacity has a significant effect for roads, whether should set lanes for self-drive and corresponding policy change. By analysis, we proposed to decompose the problem into three sub-problems:

1) Sure the driving cars with the self-drive cars in different sections of mixed traffic flow speed.

2) Built a model to deal with the impact of traffic lane number, peak or average model analysis, the driving and percentage of self-drive influence on traffic capacity.

3) Reaching a certain number, the number of self-drive consideration for self-drive lanes and pipe policy tendency, and find out under different number of lanes, the number of lanes and self-drive best delivery ratio.

3. The Model

3.1. Definitions and Symbols

In order to model the above mentioned problem, the following notations are employed (Table 1).

Since they drive on the proportion of directly affects the road traffic capacity, this article defined equilibrium and tipping point as follows.

Table 1. The list of notations.

Symbol	Definition	Units
$\bar{v}$	The average velocity of mixed traffic flow	mile/hour
s_{sum}	The vehicle by the total distance of a road	mile
t_{sum}	Some sections of the vehicle by the total time	hour
l_1	The safe distance from driving	mile
l_2	The length of self-driving, cooperating cars	mile
k	The proportion of self-driving, cooperating cars	mile
s	Through a section of distance	mile
T_1	Through a road the total time of driving cars	hour
T_2	Through a road the total time of self-driving cars	hour
T_3	Under the influence of the driving cars, the self-drive delay time	hour
k	The proportion of self-driving	
Q	A road traffic flow	hour
ρ	Some sections of the vehicle density	
k_j	A road block coefficient	
v_1	The average speed of driving cars	mile/hour
v_2	The average speed of self-driving cars	mile/hour
v_0	The average speed of zero flow	mile/hour

Tipping point: The proportion of the self-driving, cooperating cars when self-driving cars on road traffic capacity has the largest inhibition effect (Chen *et al.* [9]).

Equilibrium: The proportion of the self-driving, cooperating cars when self-driving cars and driving cars and driving cars go to the Nash equilibrium point (Chen *et al.* [10]).

3.2. Assumptions

In order to establish the model, we make the following assumptions, which have been used extensively in Gupta and Dhiman [11], Qu *et al.* [12], and Tang *et al* [13].

Assumption 1. Self-driving cars evenly distributed in the driving cars.

Assumption 2. All the driving and the self-drive conductor are equal.

Assumption 3. Ignore to slope impact on the traffic of flow.

3.3. The Foundation of Model

We divide the establishing process of the model into the following two steps.

Step 1 The mixed speed of self-driving cars and driving cars

Based on our assumption that all the self-drive uniform distribution in the self-drive, as shown in Figure 1. In order to calculate the average speed of mixed traffic flow, we since the driving ratio of 10% (Chen *et al.* [14]).

$$\overline{v} = \frac{s_{\text{sum}}}{t_{\text{sum}}} \tag{1}$$

$$s_{\text{sum}} = \sum_{i=0}^{9}\left(\int_{t_{i-1}}^{t_i}\frac{\mathrm{d}s_i}{\mathrm{d}t} + i\left(l_1 + l_2\right)\right) \tag{2}$$

$$t_{\text{sum}} = T_1 + T_2 + k * T_3 \tag{3}$$

$$T_1 = \frac{\sum_{i=0}^{8}\left(\int_{t_{i-1}}^{t_i}\frac{\mathrm{d}s_i}{\mathrm{d}t} + i\left(l_1 + l_2\right)\right)}{v_1} \tag{4}$$

$$T_2 = \frac{s + 9\left(l_1 + l_2\right)}{v_2} \tag{5}$$

According to (1)-(5), we can get the average speed of mixed traffic flow as

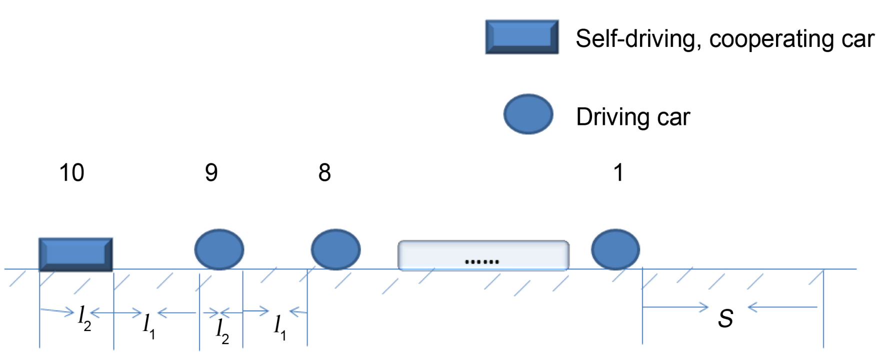

Figure 1. Hybrid vehicle distribution.

follows.

$$\overline{v}=\frac{\sum_{i=0}^{9}\left(\int_{t_{i-1}}^{t_i}\frac{\mathrm{d}s_i}{\mathrm{d}t}+i\left(l_1+l_2\right)\right)}{\frac{9s+36\left(l_1+l_2\right)}{v_1}+\frac{s+9\left(l_1+l_2\right)}{v_2}+k*T_3} \tag{6}$$

Step 2 Speed-flow model

In the continuous traffic flow, average speed, the density of flow and between the following relations:

$$Q=\rho*\overline{v} \tag{7}$$

We based on the hypothesis that density has a linear relation with the flow, can derive the average speed and flow rate of the quadratic parabola relationship model:

$$Q=k_j\left(\overline{v}-\frac{\overline{v}^2}{v_0}\right) \tag{8}$$

Among from them, the for block coefficient, the average of speed at the time of zero flow. In theory, when the traffic flow to maximum, the average speed of traffic flow half of vitamin to zero flow speed, while the maximum flow is traffic capacity.

3.4. Solution and Result

Step 1 Establish automatic driving vehicle delay time

Because of self-drive obstacle information processing, with the percentage of self-drive on different, delay would happen. According to the driving on the proportion of 50% had the greatest influence of traffic capacity [5] [15] and Arizona from driving the test data [4] [16] [17], on the proportion of self-drive delay time to simulate, you can get different delivery delay time under the proportion, as shown in Table 2. The computational experiments are carried out on a personal computer (Dell with Intel (R) Core (TM)i5-2450M CPU 2.50 GHZ and RAM 2.50 GB), using the Microsoft Windows 7 operating system (Cheng *et al.* [18]).

Step 2 The driving with the self-drive mixed average speed

A safe distance from $l_1=0.06$ miles, conductor $l_2=0.0031$ miles, the distance by vitamin $s=200$ miles, the self-drive miles per hour, the speed of the driving miles per hour, the speed of the delay time of self-drive, in (6) available in the average speed of mixed traffic flow under different delivery ratio, the results are shown in Table 3.

According to the data in Table 3, in order to establish the average velocity $\overline{v}$ and put in the function $\overline{v}(k)$.

Table 2. The delay time under different delivery ratios.

Proportion	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
$t_3(s)$	0.3	0.8	1.2	1.6	1.85	1.6	1.2	0.9	0.6	0.1

Table 3. Average velocity under different delivery ratios.

Proportion	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
$\overline{v}$	39.62	38.47	37.75	37.05	36.88	38.47	40.83	43.02	45.46	49.38

Using the MATLAB software, first of all, the relationship between the scatterplot, found that increasing its change trend to decline after the first, in line with the basic facts.

Next, by adopting the idea of least square method, average speed can be on different on the proportion of mixed traffic flow with the quadratic curve fitting function of $\overline{v}(k) = ak^2 + bk + c$, the average speed of mixed traffic flow and the function relation between driving the proportion is the function:

$$\overline{v} = 0.3259k^2 - 2.5180k + 41.9942 \tag{9}$$

When the driving on the tipping point $k_1 = 38.63\%$, at this point the drive to prevent traffic capacity of the largest. When driving on the proportion of equilibrium $k_2 = 68.26\%$, the point where the driving influence on traffic capacity. When the delivery ratio $k < k_1$, self-drive for road traffic capacity has obvious inhibiting effect. When put in the proportion of $k_1 < k < k_2$, self-drive for road traffic capacity and some improvement. When dropping ratio $k_1 > k_2$, consider setting up self-drive lanes.

Step 3 Model validation

We will block coefficient k_j, average speed v_0 at the time of zero flow and mixed flow into the average speed $\overline{v}$ of these parameters (8), about 5, 90, 405, and 520 road traffic simulation calculation, obtained under different on the proportion of road traffic. The results are as shown in Table 4.

Table 4 shows that when the driving on the proportion between 10% - 70%, the stretch of road traffic is only the self-drive road traffic to reduce, and when the self-drive delivery ratio above 70%, the stretch of road traffic significantly higher than that of only the self-drive road traffic. Results are basically consistent with the above theoretical analysis.

3.5. Analysis of the Result

In order to further validate our conclusion, the rationality of the velocity-flow model is established and calculated under different on the proportion of road traffic, as shown in Table 4, since the driving input to a certain extent, can actually increase the road capacity. When driving on the ratio between 10% - 70%, the stretch of road traffic is lower than only the self-drive road traffic, and when the self-drive delivery ratio above 70%, the stretch of road traffic significantly higher than that of only the self-drive road traffic. It is also consistent with our vehicles in this paper the relationship between average speed ratio coefficient and mixing road.

4. Conclusion

Through the above analysis, we concluded that when the driving on the tipping

Table 4. In each part of the road traffic simulation calculation results.

Rou	Star	End	0	10%	20%	30%	40%	50%	60%	70%	80%	90%
5	100.93	101.87	6500	64,441	63,592	60,492	59,967	60, 198	61,016	65,392	69,439	74,435
5	101.87	103.17	85,000	82,213	80,689	78,555	79,139	77,876	79,768	87,036	92,954	94,632
5	103.17	103.42	108,000	107,718	104,230	102,251	100,918	98, 426	102,491	109,671	115,172	121,168
5	103.42	104.81	101,000	101,814	96,983	93,928	94,268	94,291	97,284	101,195	110,261	113,215
5	104.81	105.63	144,000	144,111	139,951	134,188	132,234	131,349	138,550	146,979	156,448	163,125
5	105.63	106.23	123,000	120,271	118,952	116,400	112,257	113,363	117,849	126,851	131,878	138,749
5	106.23	107.09	143,000	141,044	136,960	134,101	131,554	129,897	136,787	144,993	154,294	161,905
90	1.94	2.04	13,000	13,624	13,622	11,064	10,362	12,540	12,572	11,373	14,318	16,487
90	2.04	2.4	44,000	42,141	44,142	43,456	40,198	40,108	41,304	43,530	47,197	51,600
90	2.4	2.54	23,000	22,827	23,802	22,545	22,620	22,269	20,898	24,812	23,085	27,543
90	2.54	2.79	23,000	23,665	21,662	20,407	20,162	21,818	20,486	22,255	26,051	25,167
405	0	0.09	75,000	76,154	73,434	71,480	69,242	70,702	70,275	78,301	81,162	84,328
405	0.09	0.37	110,000	107,338	107,047	104,788	100,101	99,640	104,950	110,705	117,876	123,697
405	0.37	0.54	169,000	167,564	163,762	160,859	154,890	154,211	160,862	174,094	179,787	192,726
405	0.54	0.75	176,000	173,921	170,119	166,163	164,215	162,871	170,889	179,841	189,466	200,169
520	0	0.36	48,000	47,413	46,140	44,157	43,533	44,203	46,132	48,699	51,041	52,886
520	0.36	0.85	79,000	78,255	75,589	73,549	74,231	72,697	77,210	81,160	83,491	88,078
520	0.85	1.43	45,000	46,211	45,263	41,374	42,904	40,015	42,589	47,264	47,029	52,221
520	1.43	1.63	61,000	59,239	57,717	58,379	54,919	57,788	58,873	61,342	63,852	70,598
520	1.63	4.4	68,000	66,701	66,067	65,190	62,865	63,394	64,961	69,009	73,941	78,245

point $k_1 = 38.63\%$, it prevents traffic capacity of the largest. It influences the driving traffic capacity. When the delivery ratio $k < k_1$, self-drive for road traffic capacity has obvious inhibiting effect. When put in the proportion $k_1 < k < k_2$, they have a positive influence to the traffic. When dropping ratio $k_1 > k_2$, they have the significant improvement to the traffic capacity of the road. In this paper, we only consider the ordinary road traffic capacity. For certain capacity, we need to continue to look for a function of flow rate, density, time between.

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