Demystification of Vedic Multiplication Algorithm

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Abstract

The Vedic multiplication algorithm is a very fast way of oral calculation. However, the basis of the algorithm is not available so far. The present paper demystifies the general Vedic algorithm for multiplication by establishment of foundation of the Vedic algorithm of product finding through end results of conventional multiplication. This novel approach, *i.e.*, finding algorithm from the end results of conventional calculations may be useful in devising algorithms similar to Vedic in cases of other calculations. Though the availability of cheap calculators made the Vedic Method obsolete, the present trend resurrected Vedic algorithms by their use in the design of computer processors for enhancing speed and performance.

Keywords

Vedic, Demystification, Multiplication, Vertically, Crosswise, Urdhvak, Tiryak, Computer, Processor

1. Introduction

Vedic mathematics [1]-[6] is undeniably a faster way of doing oral calculations. It deserves all applause. However, due to provision of cheap calculators for common people and students today, Vedic Mathematics started losing its sheen and it appeared that it would be finally rejected. Nonetheless, the speciality of Vedic Mathematical algorithms is now well recognized as per survey [7] and nowadays it is extensively used in computer processor designs [8] [9] [10] for speeding up performance. It has also been recommended for use in High-Per- formance FIR filter implementation [11], image compression [12], area and speed of wisely superior multiplier and unit multiplier [13], reduced-bit multiplication algorithm [14], multipliers as the integral components in the design of many high performance

FIR filters, image and digital signal processors [15] [16]. In spite of its so many alluring benefits and awesome speed, the Vedic Multiplication Algorithm remains mystical as the basics of these are still unknown. The natural curiosity to get the basis of Vedic algorithm has given birth to the objective of this research to remove the veil of mysticism from Vedic Multiplication Algorithm. This research institutes the very foundation and logic of Vedic Multiplication Procedure for general multiplication.

There are several Vedic algorithms for multiplication including a general one, which is universal, whereas each of other Vedic algorithms for multiplication suits a special situation.

Scope of this research is confined to finding the logic of Vedic Multiplication Algorithm for multiplication of general type as it has universal application and this finds product of a several-digit multiplicand and several-digit multiplier.

Interestingly, this research is purely based on **end results** of conventional multiplication procedure and this approach may be used in cases of other complicated mathematical procedures for finding shortcut-algorithms like the Vedic approach.

Results of this avant-garde research are very encouraging and verifiable. Besides, it may be path finder.

2. Basis of Vedic Multiplication Algorithm

It may be interesting to note that the Vedic algorithm is based on the **reorganization** of conventional multiplication method only. In Vedic jargon, the algorithmic procedure is described in terms of **urdhvak** (vertical multiplication) and **tiryak** (crosswise multiplication) operations. The purpose of this work is to handover the very basis of the Vedic formulae to the users which can be a very valuable asset to them for clear understanding.

The ordinary multiplication method is used here as a vehicle in evolving the Vedic Sutra (Algorithm) starting with multiplication of 2-digit multiplicand and 2-digit multiplier. Also, alphabetical symbols shall be used to represent digits in doing so. This will be extended later for finding products of a several-digit multiplicand by another several-digit multiplier.

3. Evolution of Vedic Algorithm

3.1. Evolution of Vedic Algorithm of Multiplication of a 2-Digit Multiplicand and 2-Digit Multiplier

Let a_1 and a_2 be digits at units and tens of multiplicand. Similarly, let b_1 and b_2 represent the digits at units and tens of multiplier. With these assumptions, the conventional multiplication is performed to evolve the algorithm for such product finding tasks. A conventional multiplication table is given below in Table 1.

Critical observation of the **result row** of Table 1 is the basis of evolution of Vedic Sutras or Vedic Formulae for multiplication shortcut. For this reason, Table 2 is made without showing the operations of multiplication.

Result at units $b_1 a_1$ prompts that this is the result of vertical multiplying the digits positioned at units for both multiplier and multiplicand (b_1 and a_1). In Vedic jargon, this is known as urdhvak. It is a Sanskrit word and when it is translated, it

means upward or rising up or vertically up. Here a_1 is vertically up or urdhvak to b_1. This is the Vedic formula to get quickly the digit of product at units. This is marked by arrows here for better understanding.

Observation of result at Tens $b_2a_1 + b_1a_2$ points out that the sum of crosswise multiplication 1) of digit at Units of multiplier with digit at Tens of multiplicand and crosswise multiplication 2) of digit at Tens of multiplier with digit at Units of multiplicand. The result at Hundreds, *i.e.*, b_2a_2 indicates that this is vertical or urdhvak multiplication of digits at the tens for both multiplier and multiplicand. This is similar to getting result at units.

Lemma: In all product finding cases, the result of first place, *i.e.*, units and last place (depending on number of digits) are vertical (urdhvak) products.

It may be noted that like conventional multiplication, carry, if any, is to be carried forward for addition to next higher place value.

Example 1: find product 34 and 67.

The above problem is tabulated in Table 3.

Table 1. Table of conventional multiplication of a 2-digit variable number with another 2-digit variable number.

Tags for position of digits for place values → Description, operations and result	Hundreds	Tens	Units
Multiplicand		a_2	a_1
Multiplier		b_2	b_1
First operation		b_1a_2	b_1a_1
Second operation	b_2a_2	b_2a_1	X
Result of product	b_2a_2	$b_1a_2 + b_2a_1$	b_1a_1

Table 2. Table 1 after deletion of operations.

Tags for position of digits for place values → Description, operations and result	Hundreds	Tens	Units
Multiplicand		a_2	a_1
Multiplier		b_2	b_1
Result of product	b_2a_2	$b_2a_1 + b_1a_2$	b_1a_1

Table 3. Tabular form of the above multiplication problem.

Tags for position of digits for place values → Description, operations and result	Hundreds	Tens	Units
Multiplicand		3	4
Multiplier		6	7
Vedic operations	6 * 3 = 18 Add carry 4 to it: 18 + 4 = 22	7 * 3 + 6 * 4 = 45 Add carry 2, it is 47	7 * 4 = 28
Resulting Digit at place	22	7	8
Carry		4	2

So, 34 * 67 = 2.278.

Example 2: Find 78 * 59.

Orally 9 * 8 = 72. 2 is kept as result at units whereas 7 is carry. 9 * 7 + 5 * 8 = 63 + 40 = 103. Adding carry 7 to it, it is 110. So, 0 is retained as answer digit at tens and 11 is carry. Lastly 7 * 5 = 35. Adding carry 11 to it, it is now 46. This goes to hundreds. Therefore the final answer is 4602.

3.2. Evolution of Vedic Algorithm of Multiplication of a 3-Digit Multiplicand with Another 3-Digit Multiplier

Let a_1, a_2 and a_3 be digits at units, tens and hundreds respectively of multiplicand. Similarly, let b_1, b_2 and b_3 be digits at units, tens and hundreds respectively of multiplier. With these assumptions, the conventional multiplication is performed to evolve the Vedic algorithm for product finding tasks. Conventional multiplication table is given in Table 4.

From the result row in Table 4, it can be visualized that the Vedic formulae for finding product's digits at units and ten-thousands would be same as in case of multiplication of 2-digit variable number by another 2-digit variable number. Here ten-thousandth place is the last position and hence the last digits of multiplicand and multiplier will be multiplied vertically (urdhvak). As in previous case, Table 5 is made after deleting the rows for operations and keeping result at only hundreds.

Observations of results at hundreds:

1) $b_1 a_3$ is found after multiplying multiplier's digit at tens with multiplicand's digit at hundred. This is shown by slant arrow *i.e.* crosswise or **tiryak** of Vedic

Table 4. Table of conventional multiplication of a 3-digit variable number with another 3-digit variable number.

Tags for position of digits for place values → Description, operations and result ↓	Ten-thousands	Thousands	Hundreds	Tens	Units
Multiplicand			a_3	a_2	a_1
Multiplier			b_3	b_2	b_1
First operation			$b_1 a_3$	$b_1 a_2$	$b_1 a_1$
Second operation		$b_2 a_3$	$b_2 a_2$	$b_2 a_1$	X
Third operation	$b_3 a_2$	$b_3 a_2$	$b_3 a_1$	X	
Result of product	$b_3 a_2$	$b_2 a_3 + b_3 a_2$	$b_1 a_3 + b_2 a_2 + b_3 a_1$	$b_1 a_2 + b_2 a_1$	$b_1 a_1$

Table 5. Table 4 after deletion of operations.

Tags for position of digits for place values → Description, operations and result ↓	Ten-thousands	Thousands	Hundreds	Tens	Units
Multiplicand			a_3	a_2	a_1
Multiplier			b_3	b_2	b_1
Result of product			$b_1 a_3 + b_2 a_2 + b_3 a_1$		

multiplication.

2) b_3a_1 is also obtained by multiplying multiplier's digit at hundreds by multiplicand's digit at units crosswise or tiryak way.

3) b_2a_2 is the result of finding product of digits at tens for both multiplicand and multiplier vertically or in urdhvak way.

Table 5 is now made after deleting all results save result at thousands to find Vedic algorithm to get product's digit at this place. Also, forget the digits at units now and onwards for the sake of calculations. Therefore, the column for units has been deleted.

Observations of result at thousands are in Table 4.

1) b_2a_3 is crosswise or tiryak product of multiplier's digit at tens and multiplicand's digit at hundred. This is shown by arrow in Table 6.

2) Similarly, b_3a_2 is also crosswise or tiryak product of multiplier's digit at hundreds and multiplicand's digit at tens. This is shown by arrow in Table 6.

Table 3 is reproduced below as Table 7 to observe result at ten-thousands. Digit at ten-thousands is b_3a_2, which is nothing but vertical multiplication of digits at hundreds for both multiplier and multiplicand. It is noteworthy to keep in mind that results at extreme places (here units and ten-thousands) are always vertical multiplications. This rule is universally applicable to multiplication of any-digit number by other any-digit number.

Example 1: Find product of 467 and 235.

The above problem is tabulated below in Table 8.

Therefore, 467 * 235 = 109,745.

Proceeding this way, one can find formulae for getting the result of multiplication of an n-digit multiplicand by n-digit multiplier. As an example, an 8-digit

Table 6. Table showing crosswise or Tiryak multiplications to get result at the thousands position.

Tags for position of digits for place values → Description, operations and result	Ten-thousands	Thousands	Hundreds	Tens
Multiplicand			a_3	a_2
Multiplier			b_3	b_2
Result of product		$b_2a_3 + b_3a_2$		

Table 7. Reproduced Table 3 for result at ten-thousand.

Tags for position of digits for place values → Description, operations and result	Ten-thousands	Thousands	Hundreds	Tens	Units
Multiplicand			a_3	a_2	a_1
Multiplier			b_3	b_2	b_1
First operation			b_1a_3	b_1a_2	b_1a_1
Second operation		b_2a_3	b_2a_2	b_2a_1	X
Third operation	b_3a_2	b_3a_2	b_3a_1	X	
Result of product	b_3a_2	$b_2a_3 + b_3a_2$	$b_1a_3 + b_2a_2 + b_3a_1$	$b_1a_2 + b_2a_1$	b_1a_1

multiplicand is multiplied by 8-digit multiplier in order to show the efficacy of the findings of this research. The Vedic formulae are worked out on the basis of conventional multiplication output. Table 9 shows the conventional multiplication of an 8-digit multiplicand by 8-digit multiplier. From this conventional multiplication result, the product of any two 8-digit numbers can be orally found. The end results of conventional product finding in Table 9 is used to get the Vedic formulae for such product finding. The next table, Table 10, uses this algorithm to find the product of 8-digit multiplicand and 8-digit multiplier orally.

Example: Find the product of 10,231,021 and 21,021,103.

Table 8. Table for Vedic multiplication of above problem.

Tags for position of digits for place values / Description, operations and result	Ten-thousands	Thousands	Hundreds	Tens	Units
Multiplicand			4	6	7
Multiplier			2	3	5
Operations	4 * 2 = 8 Adding 2 carry it is 10	4 * 3 + 2 * 6 = 24 Adding 5 carry it is 29	5 * 4 + 2 * 7 + 6 * 3 = 52. Adding 5 carry it is 57	5 * 6 + 3 * 7 = 51 Adding 3 carry it is 54	7 * 5 = 35
Resulting Digit	10	9	7	4	5
Carry		2	5	5	3

Table 9. Table of conventional multiplication of an 8-digit number by another 8-digit number.

Tags for position of digits for place values / Description, operations and result	Hundred-quadrillion	Ten-quadrillion	Quadrillion	Hundred-trillions	Ten-trillions	Trillions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units
Multiplicand								a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1
Multiplier								b_8	b_7	b_6	b_5	b_4	b_3	b_2	b_1
								b_1a_8	b_1a_7	b_1a_6	b_1a_5	b_1a_4	b_1a_3	b_1a_2	b_1a_1
							b_2a_8	b_2a_7	b_2a_6	b_2a_5	b_2a_4	b_2a_3	b_2a_2	b_2a_1	x
						b_3a_8	b_3a_7	b_3a_6	b_3a_5	b_3a_4	b_3a_3	b_3a_2	b_3a_1	x	
					b_4a_8	b_4a_7	b_4a_6	b_4a_5	b_4a_4	b_4a_3	b_4a_2	b_4a_1	x		
Operational calculations				b_5a_8	b_5a_7	b_5a_6	b_5a_5	b_5a_4	b_5a_3	b_5a_2	b_5a_1	x			
			b_6a_8	b_6a_7	b_6a_6	b_6a_5	b_6a_4	b_6a_3	b_6a_2	b_6a_1	x				
		b_7a_8	b_7a_7	b_7a_6	b_7a_5	b_7a_4	b_7a_3	b_7a_2	b_7a_1	x					
	b_8a_8	b_8a_7	b_8a_6	b_8a_5	b_8a_4	b_8a_3	b_8a_2	b_8a_1	x						
									b_1a_8+						
							b_2a_8+	b_2a_7+	b_1a_7+						
						b_3a_8+	b_3a_7+	b_3a_6+	b_2a_6+	b_1a_6+					
					b_4a_8+	b_4a_7+	b_4a_6+	b_4a_5+	b_3a_5+	b_2a_5+	b_1a_5+				
Result				b_5a_8+	b_5a_7+	b_5a_6+	b_5a_5+	b_5a_4+	b_4a_4+	b_3a_4+	b_2a_4+	b_1a_4+			
			b_6a_8+	b_6a_7+	b_6a_6+	b_6a_5+	b_6a_4+	b_6a_3+	b_5a_3+	b_4a_3+	b_3a_3+	b_2a_3+	b_1a_3+		
		b_7a_8+	b_7a_7+	b_7a_6+	b_7a_5+	b_7a_4+	b_7a_3+	b_7a_2+	b_6a_2+	b_5a_2+	b_4a_2+	b_3a_2+	b_2a_2+	b_1a_2+	
	b_8a_8	b_8a_7	b_8a_6	b_8a_5	b_8a_4	b_8a_3	b_8a_2	b_8a_1	b_7a_1	b_6a_1	b_5a_1+	b_4a_1	b_3a_1	b_2a_1	b_1a_1

Table 10. Table of Vedic multiplication for above problem.

Description, operations and result	Hundred-quadrillion	Ten-quadrillion	Quadrillion	Hundred-trillions	Ten-trillions	Trillions	Hundred-millions	Ten-millions	Millions	Hundred-Thousands	Ten-thousands	Thousands	Hundreds	Tens	Units
Multiplicand								1	0	2	3	1	0	2	1
Multiplier								2	1	0	2	1	1	0	3
Operations (see block below)															
Result	2	1	5	0	6	7	3	4	6	2	3	6	1	6	3
Carry			1			1	1			1	1				

Operations:

```
2 * 1  1 * 1+ 0 * 1  2 * 1+ 1 * 1+ 1 * 1+ 0 * 1+ 3 * 1+ 3 * 0+ 3 * 2+ 3 * 3+ 3 * 1+ 3 * 0+ 3 * 2+ 3 * 1
       2 * 0  2 * 2+ 2 * 3+ 2 * 1+ 2 * 0+ 2 * 2+ 2 * 1+ 1 * 1+ 0 * 1+ 2 * 1+ 1 * 1+ 1 * 1+ 0 * 1
              0 * 1  0 * 0+ 0 * 2+ 0 * 1+ 0 * 1+ 0 * 0+ 2 * 0+ 0 * 3+ 1 * 0+ 0 * 0+ 2 * 0
                     2 * 1  1 * 3+ 1 * 1+ 1 * 0+ 1 * 2+ 0 * 2+ 2 * 2+ 1 * 2+ 1 * 2
                            2 * 0+ 2 * 2+ 1 * 2+ 2 * 1+ 3 * 1+ 1 * 1+ 1 * 0
                                   0 * 3  0 * 1+ 0 * 0+ 2 * 0+ 1 * 0
                                          3 * 2  3 * 1+ 1 * 1
                                                 2 * 1+
```

This example problem is tabulated in Table 10.

Therefore, 10,231,021 * 21,021,103 = 21,506,734,623.

4. Conclusions

1) The basis of the Vedic general multiplication algorithm can be found from the end results of ordinary multiplication. This demystifies the Vedic algorithm for general multiplication.

2) The approach of this paper may be used to find algorithm shortcuts for other types of arithmetical calculations.

3) A Vedic algorithm for the multiplication of a n-digit multiplicand by n-digit multiplier can be easily obtained by using alphabetic variables in lieu of numerals in the general conventional multiplication.

4) Vedic mathematics will continue in spite of the availability of electronic calculators or other calculating gadgets.

Acknowledgements

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