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Author(s):	Mahmoud A. E. Abdelrahman, Magdi Hakeem Armanious, Emad H. M. Zahran, Mostafa M. A. Khater*
* Corresponding author.	Email: mostafa.khater2024@yahoo.com
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The paper does not meet the standards of "American Journal of Computational Mathematics".

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The Modified Simple Equation Method and Its Applications in Mathematical Physics and Biology

Mahmoud A. E. Abdelrahman[1], Magdi Hakeem Armanious[1], Emad H. M. Zahran[2], Mostafa M. A. Khater[1*]

[1]Department of Mathematics, Faculty of Science, Mansoura University, Mansoura, Egypt
[2]Department of Mathematical and Physical Engineering, College of Engineering Shubra, Benha University, Banha, Egypt
Email: [*]mostafa.khater2024@yahoo.com

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Abstract

The modified simple equation method is employed to find the exact traveling wave solutions involving parameters for nonlinear evolution equations namely, a diffusive predator-prey system, the Bogoyavlenskii equation, the generalized Fisher equation and the Burgers-Huxley equation. When these parameters are taken special values, the solitary wave solutions are derived from the exact traveling wave solutions. It is shown that the modified simple equation method provides an effective and a more powerful mathematical tool for solving nonlinear evolution equations in mathematical physics. Comparison between our results and the well-known results will be presented.

Keywords

A Diffusive Predator-Prey System, The Bogoyavlenskii Equation, The Generalized Fisher Equation, The Burgers-Huxley Equation, The Modified Simple Equation Method, Traveling Wave Solutions, Solitary Wave Solutions

1. Introduction

The nonlinear partial differential equations of mathematical physics are major subjects in physical science [1]. Exact solutions for these equations play an important role in many phenomena in physics such as fluid mechanics, hydrodynamics, optics, plasma physics and so on. Recently many new approaches for finding these solutions have been proposed, for example, tanh-sech method [2]-[4], extended tanh-method [5]-[7], sine-cosine

[*]Corresponding author.

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method [8]-[10], homogeneous balance method [11], the $\mathrm{Exp}(-\varphi(\xi))$ expansion method [12] and [13], Jacobi elliptic function method [14]-[16], F-expansion method [17]-[19], exp-function method [20] and [21], trigonometric function series method [22], $\left(\frac{G'}{G}\right)$-expansion method [23]-[26], the modified simple equation method [27]-[32] and so on. The objective of this article is to apply the modified simple equation method for finding the exact traveling wave solution of some nonlinear partial differential equations, namely the diffusive predator-prey system [33], the Bogoyavlenskii equation [34], the generalized fisher equation [35] and the Burgers-Huxley equation [36], which play an important role in mathematical physics.

The rest of this paper is organized as follows: In Section 2, we give the description of the modified simple equation method. In Section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In Section 4, conclusions are given.

2. Description of the Modified Simple Equation Method

Consider the following nonlinear evolution equation

$$F\left(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{yy}, \cdots\right) = 0, \tag{1}$$

where F is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method [27]-[32]:

Step 1. We use the wave transformation

$$u(x, y, t) = u(\xi), \qquad \xi = x + y - ct, \tag{2}$$

where c is a nonzero constant, to reduce Equation (1) to the following ODE:

$$P\left(u, u', u'', u''', \cdots\right) = 0, \tag{3}$$

where P is a polynomial in $u(\xi)$ and its total derivatives, while $' = \frac{\mathrm{d}}{\mathrm{d}\xi}$.

Step 2. Suppose that the solution of Equation (3) has the formal solution:

$$u(\xi) = \sum_{k=0}^{N} A_k \left[\frac{\psi'(\xi)}{\psi(\xi)}\right]^k, \tag{4}$$

where A_k are arbitrary constants to be determined, such that $A_N \neq 0$, while the function $\psi(\xi)$ is an unknown function to be determined later, such that $\psi' \neq 0$.

Step 3. Determined the positive integer N in Equation (4) by considering the homogenous balance between the highest order derivatives and the nonlinear terms in Equation (3).

Step 4. Substitute Equation (4) into Equation (3), we calculate all the necessary derivative $u', u'', \cdots$ of the function $u(\xi)$ and we account the function $\psi(\xi)$. As a result of this substitution, we get a polynomial of $\psi^{-j}\ (j = 0,1,2,\cdots)$. In this polynomial, we gather all terms of the same power of $\psi^{-j}\ (j = 0,1,2,\cdots)$, and we equate with zero all coefficient of this polynomial. This operation yields a system of equations which can be solved to find A_k and $\psi(\xi)$. Consequently, we can get the exact solution of Equation (1).

3. Application

Here, we will apply the modified simple equation method described in Section 2 to find the exact traveling wave solutions and then the solitary wave solutions for the following nonlinear systems of evolution equations.

3.1. Example 1: A Diffusive Predator-Prey System

We consider a system of two coupled nonlinear partial differential equations describing the spatio-temporal dynamics of a predator-prey system [33],

$$\begin{cases} u_t = u_{xx} - \beta u + (1+\beta)u^2 - u^3 - uv, \\ v_t = v_{xx} + \kappa uv - mv - \delta v^3. \end{cases} \tag{5}$$

where κ, δ, m and β are positive parameters. The solutions of predator-prey system have been studied in various aspects [33] [37] [38]. The dynamics of the diffusive predator-prey system have assumed the following relations between the parameters, namely $m=\beta$ and $\kappa+\frac{1}{\sqrt{\delta}}=\beta+1$. Under there assumptions, Equation (5) can be rewritten in the form:

$$\begin{cases} u_t = u_{xx} - \beta u + \left(\kappa + \frac{1}{\sqrt{\delta}}\right) u^2 - u^3 - uv, \\ v_t = v_{xx} + \kappa uv - \beta v - \delta v^3. \end{cases} \tag{6}$$

We use the wave transformation $u(x,t)=u(\xi), \xi = x - ct$ to reduce Equation (6) to the following nonlinear system of ordinary differential equations:

$$\begin{cases} u'' + cu' - \beta u + \left(\kappa + \frac{1}{\sqrt{\delta}}\right) u^2 - u^3 - uv = 0, \\ v'' + cv' + \kappa uv - \beta v - \delta v^3 = 0, \end{cases} \tag{7}$$

where c is a nonzero constant.

In order to solve Equation (7), let us consider the following transformation

$$v = \frac{1}{\sqrt{\delta}} u, \tag{8}$$

Substituting the transformation (8) into Equation (7), we get

$$u'' + cu' - \beta u + \kappa u^2 - u^3 = 0. \tag{9}$$

Balancing u'' with u^3 in Equation (9) yields, $N+2=3N \Rightarrow N=1$. Consequently, we get the formal solution

$$u(\xi) = A_0 + A_1\left(\frac{\psi'}{\psi}\right), \tag{10}$$

where A_0 and A_1 are constants to determined such that $A_1 \neq 0$. It is easy to see that

$$u' = A_1\left[\frac{\psi''}{\psi} - \left(\frac{\psi'}{\psi}\right)^2\right], \tag{11}$$

$$u'' = A_1\left[\frac{\psi'''}{\psi} - \frac{3\psi'\psi''}{\psi^2} + 2\left(\frac{\psi'}{\psi}\right)^3\right]. \tag{12}$$

Substituting (10)-(12) into Equation (9) and equating the coefficients of ψ^{-3}, ψ^{-2}, ψ^{-1}, ψ^{0} to zero, we respectively obtain

$$\psi^{-3} : \psi'^3 A_1\left(2 - A_1^2\right) = 0, \tag{13}$$

$$\psi^{-2} : \psi' A_1\left[-3\psi'' - \left(c - \kappa A_1 + 3A_0A_1\right)\psi'\right] = 0, \tag{14}$$

$$\psi^{-1} : A_1\left[\psi''' + c\psi'' - \left(\beta - 2\kappa A_0 + 3A_0^2\right)\psi'\right] = 0, \tag{15}$$

$$\psi^{0} : A_0\left[-\beta + \kappa A_0 - A_0^2\right] = 0. \tag{16}$$

From Equations (13) and (16), we deduce that

$$A_1 = \pm\sqrt{2},\ A_0 = 0 \text{ or } A_0 = \frac{-\kappa \pm \sqrt{\kappa^2 - 4\beta}}{-2}$$

where $\kappa^2 > 4\beta$.

Let us discuss the following cases.

Case 1. If $A_0 = 0$.

In this case, we deduce from Equations (14) and (15) that

$$\psi' = \frac{-3}{c - \kappa A_1}\psi'', \tag{17}$$

and

$$\psi''' + c\psi'' - \beta\psi' = 0. \tag{18}$$

where $c \neq \kappa A_1$.

Equations (17) and (18) yield

$$\frac{\psi'''}{\psi''} = E_0 \tag{19}$$

where $E_0 = -\left(c + \frac{3\beta}{c - kA_1}\right) \neq 0$.

Integrating (19) and using (17) we deduce that

$$\psi' = \frac{-3c_1}{c - \kappa A_1}\exp(E_0\xi), \tag{20}$$

and consequently, we get

$$\psi = \frac{-3c_1}{E_0(c - \kappa A_1)}\exp(E_0\xi) + c_2, \tag{21}$$

where c_1 and c_2 are arbitrary constants of integration.

Substituting (20) and (21) into (10) we have the exact solution:

$$u(\xi) = \pm\sqrt{2}E_0\left[\frac{\exp(E_0\xi)}{\exp(E_0\xi) + c_2}\right], \tag{22}$$

and from (8) we get

$$v(\xi) = \pm\sqrt{\frac{2}{\delta}}E_0\left[\frac{\exp(E_0\xi)}{\exp(E_0\xi) + c_2}\right]. \tag{23}$$

where $c_1 = \frac{E_0(c - \kappa A_1)}{-3}$.

If $c_2 = 1$, we have the solitary wave solutions.

$$u(\xi) = \pm\frac{E_0}{\sqrt{2}}\left[1 + \tanh\left(\frac{E_0}{2}\xi\right)\right], \tag{24}$$

and

$$v(\xi) = \pm\frac{E_0}{\sqrt{2\delta}}\left[1 + \tanh\left(\frac{E_0}{2}\xi\right)\right], \tag{25}$$

while, if $c_2 = -1$, we get

$$u(\xi) = \pm\frac{E_0}{\sqrt{2}}\left[1 + \coth\left(\frac{E_0}{2}\xi\right)\right], \tag{26}$$

and

$$v(\xi) = \pm\frac{E_0}{\sqrt{2\delta}}\left[1 + \coth\left(\frac{E_0}{2}\xi\right)\right]. \tag{27}$$

Case 2. If $A_0 \neq 0$.

In this case, we deduce from Equations (14) and (15) that

$$\psi' = \left(\frac{-3}{c - \kappa A_1 + 3A_0 A_1}\right)\psi'' \tag{28}$$

and

$$\psi''' + c\psi'' - \left(\beta - 2\kappa A_0 + 3A_0^2\right)\psi' = 0. \tag{29}$$

Substituting (28) into (29), we get

$$\frac{\psi'''}{\psi''} = E_1, \tag{30}$$

where $E_1 = -\left[c + \frac{3\left(\beta - 2\kappa A_0 + 3A_0^2\right)}{c - \kappa A_1 + 3A_0 A_1}\right] \neq 0$.

Integrating (30) and using (28), we deduce that

$$\psi' = \frac{-3c_3}{c - \kappa A_1 + 3A_0 A_1}\exp\left(E_1\xi\right), \tag{31}$$

and consequently, we get

$$\psi = \frac{-3c_3}{E_1\left(c - \kappa A_1 + 3A_0 A_1\right)}\exp\left(E_1\xi\right) + c_4, \tag{32}$$

where c_3 and c_4 are arbitrary constants of integration.

Substituting (31) and (32) into (10), we have the exact solution:

$$u(\xi) = \frac{-\kappa \pm \sqrt{\kappa^2 - 4\beta}}{-2} \pm \sqrt{2}E_1\left[\frac{\exp\left(E_1\xi\right)}{\exp\left(E_1\xi\right) + c_4}\right], \tag{33}$$

and from (8) we get

$$v(\xi) = \frac{-\kappa \pm \sqrt{\kappa^2 - 4\beta}}{-2\sqrt{\delta}} \pm \sqrt{\frac{2}{\delta}}E_1\left[\frac{\exp\left(E_1\xi\right)}{\exp\left(E_1\xi\right) + c_4}\right], \tag{34}$$

where $c_3 = \frac{E_1\left(c - kA_1 + 3A_0 A_1\right)}{-3}$.

If $c_4 = 1$, we get the solitary solutions.

$$u(\xi) = \frac{-\kappa \pm \sqrt{\kappa^2 - 4\beta}}{-2} \pm \frac{E_1}{\sqrt{2}}\left[1 + \tanh\left(\frac{E_1}{2}\xi\right)\right], \tag{35}$$

and

$$v(\xi) = \frac{-\kappa \pm \sqrt{\kappa^2 - 4\beta}}{-2\sqrt{\delta}} \pm \sqrt{\frac{2}{\delta}}\frac{E_1}{2}\left[1 + \tanh\left(\frac{E_1}{2}\xi\right)\right], \tag{36}$$

while, if $c_4 = -1$, we get

$$u(\xi) = \frac{-\kappa \pm \sqrt{\kappa^2 - 4\beta}}{-2} \pm \frac{E_1}{\sqrt{2}}\left[1 + \coth\left(\frac{E_1}{2}\xi\right)\right], \tag{37}$$

and

$$v(\xi) = \frac{-\kappa \pm \sqrt{\kappa^2 - 4\beta}}{-2\sqrt{\delta}} \pm \sqrt{\frac{2}{\delta}}\frac{E_1}{2}\left[1 + \coth\left(\frac{E_1}{2}\xi\right)\right]. \tag{38}$$

3.2. Example 2: The Bogoyavlenskii Equation

We consider the Bogoyavlenskii equation [34] in the form

$$\begin{cases} 4u_t + u_{xxy} - 4u^2 u_y - 4u_x v = 0, \\ u u_y = v_x. \end{cases} \tag{39}$$

Equation (39) was derived by Kudryashov and Pickering [39] as a member of $a(2 + 1)$ Schwarzian breaking soliton hierarchy. The above equation also appeared in [40] as one of the equations associated to nonisospectral scattering problems. Estevez *et al*. [41] showed that Equation (39) possesses the Painleve property. Equation (39) is the modified version of a breaking soliton equation, $4u_{xt} + 8u_x u_{xy} + 4u_y u_{xx} + u_{xxxy} = 0$, which describes the (2 + 1)-dimensional interaction of a Riemann wave propagation along the y-axis with a long wave the x-axis. To a certain extent, a similar interaction is observed in waves on the surface of the sea. It is well-known that the solution and its dynamics of the equation can make researchers.

In this subsection, we determine the exact solutions and the solitary wave solutions of Equation (39). To this end, we use the wave transformation (2) to reduce Equation (39) to the following nonlinear system of ordinary differential equations.

$$\begin{cases} -4cu' + u''' - 4u^2 u' - 4u'v = 0, \\ \dfrac{u^2}{2} = v. \end{cases} \tag{40}$$

Substituting the second equation of (40) into the first one, and integrating the resultant equation

$$u'' - 2u^3 - 4cu = 0, \tag{41}$$

with zero constant of integration.

Balancing u'' with u^3 in Equation (41) yields, $N + 2 = 3N \Rightarrow N = 1$. Consequently, we get the same formal solution (10).

Substituting (10)-(12) into Equation (41) and equating the coefficients of ψ^{-3}, ψ^{-2}, ψ^{-1}, ψ^{0} to zero, we obtain

$$\psi^{-3} : 2A_1 \psi'^3 \left[1 - A_1^2\right] = 0, \tag{42}$$

$$\psi^{-2} : -3A_1 \psi' \left[\psi'' + 2A_0 A_1 \psi'\right] = 0, \tag{43}$$

$$\psi^{-1} : A_1 \left[\psi''' - \left(6A_0^2 + 4c\right)\psi'\right] = 0, \tag{44}$$

$$\psi^{0} : -2A_0 \left[A_0^2 + 2c\right] = 0. \tag{45}$$

From Equations (42) and (45), we deduce that

$$A_1 = \pm 1,\ A_0 = 0 \text{ or } A_0 = \pm\sqrt{-2c}, \text{ where } c < 0.$$

Case 1. If $A_0 \neq 0$.

In this case, we deduce from Equations (43) and (44) that

$$\psi' = \frac{-1}{2A_0 A_1} \psi'', \tag{46}$$

and

$$\psi''' - \left(6A_0^2 + 4c\right)\psi' = 0, \tag{47}$$

Equations (46) and (47) yield

$$\frac{\psi'''}{\psi''} = E_2, \tag{48}$$

where $E_2 = \frac{-6A_0^2 - 4c}{2A_0A_1} \neq 0$.

Integrating (48) and using (46), we deduce that

$$\psi' = \frac{-c_5}{2A_0A_1}\exp(E_2\xi), \tag{49}$$

$$\psi = \frac{-c_5}{2E_2A_0A_1}\exp(E_2\xi) + c_6, \tag{50}$$

where c_5 and c_6 are arbitrary constants of integration.

Substituting (49) and (50) into (10), we have the exact solution:

$$u(\xi) = \pm\sqrt{-2c} \pm E_0\left[\frac{\exp(E_2\xi)}{c_6 + \exp(E_2\xi)}\right], \tag{51}$$

and

$$v(\xi) = \frac{1}{2}\left[\pm\sqrt{-2c} \pm E_2\left[\frac{\exp(E_2\xi)}{c_6 + \exp(E_2\xi)}\right]\right]^2. \tag{52}$$

where $c_5 = -2E_0A_0A_1$.

If $c_2 = 1$, we have the solitary wave solutions.

$$u(\xi) = \pm\sqrt{-2c} \pm \frac{E_2}{2}\left[1 + \tanh\left(\frac{E_2}{2}\xi\right)\right], \tag{53}$$

and

$$v(\xi) = \frac{1}{2}\left[\pm\sqrt{-2c} \pm \frac{E_2}{2}\left[1 + \tanh\left(\frac{E_2}{2}\xi\right)\right]\right]^2, \tag{54}$$

while, if $c_2 = -1$, we get

$$u(\xi) = \pm\sqrt{-2c} \pm \frac{E_2}{2}\left[1 + \coth\left(\frac{E_2}{2}\xi\right)\right], \tag{55}$$

and

$$v(\xi) = \frac{1}{2}\left[\pm\sqrt{-2c} \pm \frac{E_2}{2}\left[1 + \coth\left(\frac{E_2}{2}\xi\right)\right]\right]^2. \tag{56}$$

Case 2. If $A_0 \neq 0$.

In this case, we deduce from Equations (40) and (41) that $\psi' = 0$. This case is rejected.

3.3. Example 3. The Genaralized Fisher Equation with Nonlinearity

We consider a nonlinear partial differential equation describing the generalized Fisher equation [35]

$$u_t + cu_x = Du_{xx} + \alpha u - \beta u^2 - \gamma u^3, \tag{57}$$

where D is the diffusion coefficient, u is the concentration or density, c represent the convective velocity, and α, β, γ are the constants in different contexts. Substituting the wave transformation $u(x,t) = u(\xi), \xi = x - kt$ into Equation (57), we get

$$Du'' + \alpha u - \beta u^2 - \gamma u^3 + (k - c)u' = 0, \text{ where } k \neq c, \tag{58}$$

where k is arbitrary constant.

Balancing u'' with u^3 in Equation (58) yields, $N + 2 = 3N \Rightarrow N = 1$. Consequently, we get the same formal solution (10).

Substituting (10)-(12) into Equation (58) and equating the coefficients of ψ^{-3}, ψ^{-2}, ψ^{-1}, ψ^{0} to zero, we respectively obtain

$$\psi^{-3}: A_1\psi'^3\left[2D-\gamma A_1^2\right]=0, \tag{59}$$

$$\psi^{-2}: A_1\psi'\left[-3D\psi''-\left(\beta A_1+3\gamma A_0A_1+k-c\right)\psi'\right]=0, \tag{60}$$

$$\psi^{-1}: A_1\left[D\psi'''+\left(\alpha-2\beta A_0-3\gamma A_0^2\right)\psi'+\left(k-c\right)\psi''\right]=0, \tag{61}$$

$$\psi^{0}: A_0\left[\alpha-\beta A_0-\gamma A_0^2\right]=0. \tag{62}$$

From Equations (59) and (62), we deduce that

$$A_1=\pm\sqrt{\frac{2D}{\gamma}} \text{ and } A_0=0 \text{ or } A_0=\frac{\beta\pm\sqrt{\beta^2+4\gamma\alpha}}{-2\gamma}.$$

where D, γ are nonzero real constants.

Let us now discuss the following cases.

Case 1. If $A_0=0$.

In this case, we deduce from Equations (60) and (61) that

$$\psi'=\frac{-3D}{\beta A_1+k-c}\psi'', \tag{63}$$

and

$$D\psi'''+\alpha\psi'+\left[k-c\right]\psi''=0, \tag{64}$$

Equations (63) and (64) yield

$$\frac{\psi'''}{\psi''}=\frac{-E_3}{D}, \tag{65}$$

where $E_3=\left[\frac{-3\alpha D}{\beta A_1+k-c}+k-c\right]\neq 0.$

Integrating (65) and using (63), we deduce that

$$\psi'=\frac{-3Dc_7}{\beta A_1+k-c}\exp\left(\frac{-E_3}{D}\xi\right), \tag{66}$$

and consequently, we get

$$\psi=\frac{3D^2c_7}{E_3\left(\beta A_1+k-c\right)}\exp\left(\frac{-E_3}{D}\xi\right)+c_8, \tag{67}$$

where c_7 and c_8 are arbitrary constants of integration.

Substituting (66) and (67) into (10), we have the exact solution:

$$u(\xi)=\mp E_3\sqrt{\frac{2}{D\gamma}}\left[\frac{\exp\left(\frac{-E_3}{D}\xi\right)}{c_8+\exp\left(\frac{-E_3}{D}\xi\right)}\right], \tag{68}$$

where $c_7=\frac{E_0\left(\beta A_1+c_1-c\right)}{3D^2}$ and $\frac{2}{D\gamma}>0$.

If $c_8=\pm 1$ and $\frac{E_3}{D}>0$, we have the solitary wave solution.

$$u(\xi) = \mp \frac{E_3}{\sqrt{2D\gamma}}\left[1 - \tanh\left(\frac{E_3}{2D}\xi\right)\right], \tag{69}$$

$$u(\xi) = \mp \frac{E_3}{\sqrt{2D\gamma}}\left[1 - \coth\left(\frac{E_3}{2D}\xi\right)\right], \tag{70}$$

while, if $c_8 = \pm 1$ and $\frac{E_3}{D} < 0$, we get

$$u(\xi) = \mp \frac{E_3}{\sqrt{2D\gamma}}\left[1 + \tanh\left(\frac{E_3}{2D}\xi\right)\right], \tag{71}$$

$$u(\xi) = \mp \frac{E_3}{\sqrt{2D\gamma}}\left[1 + \coth\left(\frac{E_3}{2D}\xi\right)\right]. \tag{72}$$

Case 2. If $A_0 \neq 0$.

In this case, we deduce from Equations (60) and (61) that

$$\psi' = \frac{-3D}{\beta A_1 + 3\gamma A_0 A_1 + k - c}\psi'', \tag{73}$$

and

$$D\psi''' + (k-c)\psi'' + \left(\alpha - 2\beta A_0 - 3\gamma A_0^2\right)\psi' = 0. \tag{74}$$

Equations (73) and (74) yield

$$\frac{\psi'''}{\psi''} = \frac{-E_4}{D}, \tag{75}$$

where $E_4 = \left[\frac{-3\alpha D\left(\alpha - 2\beta A_0 - 3\gamma A_0^2\right)}{\beta A_1 + +3\gamma A_0 A_1 + k - c} + k - c\right] \neq 0$.

Integrating (75) and using (72), we deduce that

$$\psi' = \frac{-3Dc_9}{\beta A_1 + 3\gamma A_0 A_1 + k - c}\exp\left(\frac{-E_4}{D}\xi\right), \tag{76}$$

and consequently, we get

$$\psi = \frac{3D^2 c_9}{E_4\left(\beta A_1 + 3\gamma A_0 A_1 + k - c\right)}\exp\left(\frac{-E_4}{D}\xi\right) + c_{10}, \tag{77}$$

where c_9 and c_{10} are arbitrary constants of integration.

Substituting (76) and (77) into (10), we have the exact solution:

$$u(\xi) = \frac{\beta \pm \sqrt{\beta^2 + 4\alpha\gamma}}{-2\gamma} \mp E_4\sqrt{\frac{2}{D\gamma}}\left[\frac{\exp\left(\frac{-E_4}{D}\xi\right)}{c_{10} + \exp\left(\frac{-E_4}{D}\xi\right)}\right], \tag{78}$$

where $c_9 = \frac{E_4\left(\beta A_1 + 3\gamma A_0 A_1 + k - c\right)}{3D^2}$ and $\frac{2}{D\gamma} > 0$.

If $c_{10} = \pm 1$ and $\frac{E_4}{D} > 0$, we have the solitary wave solution.

$$u(\xi) = \frac{\beta \pm \sqrt{\beta^2 + 4\alpha\gamma}}{-2\gamma} \mp \frac{E_4}{\sqrt{2D\gamma}}\left[1 - \tanh\left(\frac{E_4}{2D}\xi\right)\right], \tag{79}$$

$$u(\xi)=\frac{\beta\pm\sqrt{\beta^2+4\alpha\gamma}}{-2\gamma}\mp\frac{E_4}{\sqrt{2D\gamma}}\left[1-\coth\left(\frac{E_4}{2D}\xi\right)\right], \tag{80}$$

while, if $c_{10}=\pm 1$ and $\frac{E_4}{D}<0$, we have

$$u(\xi)=\frac{\beta\pm\sqrt{\beta^2+4\alpha\gamma}}{-2\gamma}\mp\frac{E_4}{\sqrt{2D\gamma}}\left[1+\tanh\left(\frac{E_4}{2D}\xi\right)\right], \tag{81}$$

$$u(\xi)=\frac{\beta\pm\sqrt{\beta^2+4\alpha\gamma}}{-2\gamma}\mp\frac{E_4}{\sqrt{2D\gamma}}\left[1+\coth\left(\frac{E_4}{2D}\xi\right)\right]. \tag{82}$$

3.4. Example 4. The Burgers-Huxley Equation

We consider a nonlinear partial differentail equation describing the burgers-Huxley equation [36]

$$u_t+\alpha uu_x-vu_{xx}=\beta u(1-u)(u-\gamma), \tag{83}$$

where u is the concentration or density, c represents the convective velocity, and α, β, γ are real constants in different contexts. In which v plays the role of diffusion-like coefficient. Note that Equation (83) reduces to Hodgkin-Huxley equation for $\alpha=0$ and to Burgers equation for $\beta=0$. Subsituting the wave transformation $u(x,t)=u(\xi),\xi=x-kt$ into Equation (83) we get

$$\beta u(1-u)(u-\gamma)+ku'-\alpha uu'+vu''=0, \tag{84}$$

where k is arbitrary constant.

Balancing between u'' and u^3 in Equation (84) yields, $N+2=3N\Rightarrow N=1$. Consequently, we get the same formal solution (10).

Substituting Equations (10)-(12) into Equation (84) and equating the coefficients of ψ^{-3}, ψ^{-2}, ψ^{-1}, ψ^{0} to zero, we respectively obtain.

$$\psi^{-3}: A_1\psi'^3\left[-\beta A_1^2+\alpha A_1+2v\right]=0, \tag{85}$$

$$\psi^{-2}: A_1\psi'\left[\left(-3\beta A_0A_1+\beta A_1+\beta A_1\gamma-k+\alpha A_0\right)\psi'-\left(\alpha A_1+3v\right)\psi''\right]=0, \tag{86}$$

$$\psi^{-1}: A_1\left[\left(2\beta A_0-3\beta A_0^2+2\beta A_0\gamma-\beta\gamma\right)\psi'+\left(k-\alpha A_0\right)\psi''+v\psi'''\right], \tag{87}$$

$$\psi^{0}: \beta A_0\left[\gamma-A_0^2+A_0\gamma\right]=0. \tag{88}$$

From Equations (85) and (89), we deduce that

$$A_1=\frac{-\alpha\pm\sqrt{\alpha^2+8\beta v}}{-2\beta},\ A_0=0 \text{ or } A_0=\frac{\gamma}{2}\left[1\pm\sqrt{1+\frac{4}{\gamma}}\right]$$

where α, β, γ are nonzero real constants.

Let us now discuss the following cases.

Case 1. If $A_0=0$.

In this case, we deduce from Equations (86) and (87) that

$$\psi'=\frac{\alpha A_1+3v}{\beta A_1+\beta A_1\gamma-k}\psi''', \tag{89}$$

and

$$v\psi'''+k\psi''-\beta\gamma\psi'=0, \tag{90}$$

Equations (89) and (90) yield

$$\frac{\psi'''}{\psi''}=\frac{-E_5}{v}, \tag{91}$$

where $E_5 = \left[\frac{(\alpha A_1 + 3\nu)(-\beta\gamma)}{\beta A_1 + \beta A_1\gamma - k} + k\right] \neq 0$.

Integrating (91) and using (89), we have

$$\psi' = F_1 c_{11} \exp\left(\frac{-E_5}{\nu}\xi\right), \tag{92}$$

where, $F_1 = \frac{\alpha A_1 + 3\nu}{\beta A_1 + \beta A_1\gamma - k}$,

$$\psi = \frac{-F_1 \nu c_{11}}{E_5} \exp\left(\frac{-E_5}{\nu}\xi\right) + c_{12}, \tag{93}$$

where c_{11} and c_{12} are arbitrary constants of integration.

Substituting (92) and (93) into (10), we have the exact solution:

$$u(\xi) = \frac{-E_5\left(-\alpha \pm \sqrt{\alpha^2 + 8\beta\nu}\right)}{-2\beta\nu}\left[\frac{\exp\left(\frac{-E_5}{\nu}\xi\right)}{c_{12} + \exp\left(\frac{-E_5}{\nu}\xi\right)}\right], \tag{94}$$

where $c_{11} = \frac{-E_2}{F_1 \nu}$.

If $c_{12} = \pm 1$ and $\frac{E_5}{\nu} > 0$, we have the solitary wave solution

$$u(\xi) = \frac{-E_5\left(-\alpha \pm \sqrt{\alpha^2 + 8\beta\nu}\right)}{-4\beta\nu}\left[1 - \tanh\left(\frac{E_5}{2\nu}\xi\right)\right], \tag{95}$$

$$u(\xi) = \frac{-E_5\left(-\alpha \pm \sqrt{\alpha^2 + 8\beta\nu}\right)}{-4\beta\nu}\left[1 - \coth\left(\frac{E_5}{2\nu}\xi\right)\right], \tag{96}$$

while, if $c_{12} = \pm 1$ and $\frac{E_5}{\nu} < 0$, we get

$$u(\xi) = \frac{-E_5\left(-\alpha \pm \sqrt{\alpha^2 + 8\beta\nu}\right)}{-4\beta\nu}\left[1 + \tanh\left(\frac{E_5}{2\nu}\xi\right)\right], \tag{97}$$

$$u(\xi) = \frac{-E_5\left(-\alpha \pm \sqrt{\alpha^2 + 8\beta\nu}\right)}{-4\beta\nu}\left[1 + \coth\left(\frac{E_5}{2\nu}\xi\right)\right]. \tag{98}$$

Case 2. If $A_0 \neq 0$.

In this case, we deduce from Equations (86) and (87) that

$$\psi' = F_2 \psi'', \tag{99}$$

where $F_2 = \frac{\alpha A_1 + 3\nu}{-3\beta A_0 A_1 + \beta A_1 + \beta A_1\gamma - k + \alpha A_0}$.

and

$$\nu\psi''' + (k - \alpha A_0)\psi'' + \left(2\beta A_0 - 3\beta A_0^2 + 2\beta A_0\gamma - \beta\gamma\right)\psi' = 0. \tag{100}$$

Equations (99) and (100) yield

$$\frac{\psi'''}{\psi''}=\frac{-E_6}{\nu}, \tag{101}$$

where $E_6=\left[\left(k-\alpha A_0\right)+F_4\left(2\beta A_0-3\beta A_0^2+2\beta A_0\gamma-\beta\gamma\right)\right]\neq 0$.

Integrating (101) and using (99), we deduce that

$$\psi'=F_2c_{13}\exp\left(\frac{-E_6}{\nu}\xi\right), \tag{102}$$

and consequently, we get

$$\psi=\frac{-\nu F_2c_{13}}{E_6}\exp\left(\frac{-E_6}{\nu}\xi\right)+c_{14}, \tag{103}$$

where c_{13} and c_{14} are arbitrary constants of integration.

Substituting (102) and (103) into (10), we have the exact solution:

$$u(\xi)=\frac{\gamma}{2}\left[1\pm\sqrt{1+\frac{4}{\gamma}}\right]-\frac{E_6\left(-\alpha\pm\sqrt{\alpha^2+4\beta\gamma}\right)}{-2\beta\nu}\left[\frac{\exp\left(\frac{-E_6}{\nu}\xi\right)}{c_{14}+\exp\left(\frac{-E_6}{\nu}\xi\right)}\right], \tag{104}$$

where $c_{13}=\frac{-E_6}{\nu F_2}$. If $c_{14}=\pm 1$ and $\frac{E_6}{\nu}>0$, we have the solitary wave solution.

$$u(\xi)=\frac{\gamma}{2}\left[1\pm\sqrt{1+\frac{4}{\gamma}}\right]-\frac{E_6\left(-\alpha\pm\sqrt{\alpha^2+4\beta\gamma}\right)}{-4\beta\nu}\left[1-\tanh\left(\frac{E_6}{2\nu}\xi\right)\right], \tag{105}$$

$$u(\xi)=\frac{\gamma}{2}\left[1\pm\sqrt{1+\frac{4}{\gamma}}\right]-\frac{E_6\left(-\alpha\pm\sqrt{\alpha^2+4\beta\gamma}\right)}{-4\beta\nu}\left[1-\coth\left(\frac{E_6}{2\nu}\xi\right)\right], \tag{106}$$

while, if $c_{14}=\pm 1$ and $\frac{E_6}{\nu}<0$, we get

$$u(\xi)=\frac{\gamma}{2}\left[1\pm\sqrt{1+\frac{4}{\gamma}}\right]-\frac{E_6\left(-\alpha\pm\sqrt{\alpha^2+4\beta\gamma}\right)}{-4\beta\nu}\left[1+\tanh\left(\frac{E_6}{2\nu}\xi\right)\right], \tag{107}$$

$$u(\xi)=\frac{\gamma}{2}\left[1\pm\sqrt{1+\frac{4}{\gamma}}\right]-\frac{E_6\left(-\alpha\pm\sqrt{\alpha^2+4\beta\gamma}\right)}{-4\beta\nu}\left[1+\coth\left(\frac{E_6}{2\nu}\xi\right)\right]. \tag{108}$$

4. Conclusion

The modified simple equation method has been successfully used to find the exact traveling wave solutions of some nonlinear evolution equations. As an application, the traveling wave solutions for Bogoyavlenskii equation and a diffusive predator-prey system which have been constructed using the modified simple equation method. Let us compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of a diffusive predator-prey system and Bogoyavlenskii equation are new and different from those obtained in [42]-[44] and also our results of the generalized Fisher equation and Burgers-Huxley equation are new and different from those obtained in [45]. It can be concluded that this method is reliable and propose a variety of exact solutions NPDEs. The performance of this method is effective and can be applied to many other nonlinear evolution equations. **Figures 1-3** represent the solitary traveling wave solution for a di usive predator-prey system and Bogoyavlenskii equation and the generalized Fisher equation and Burgers-Huxley equation.

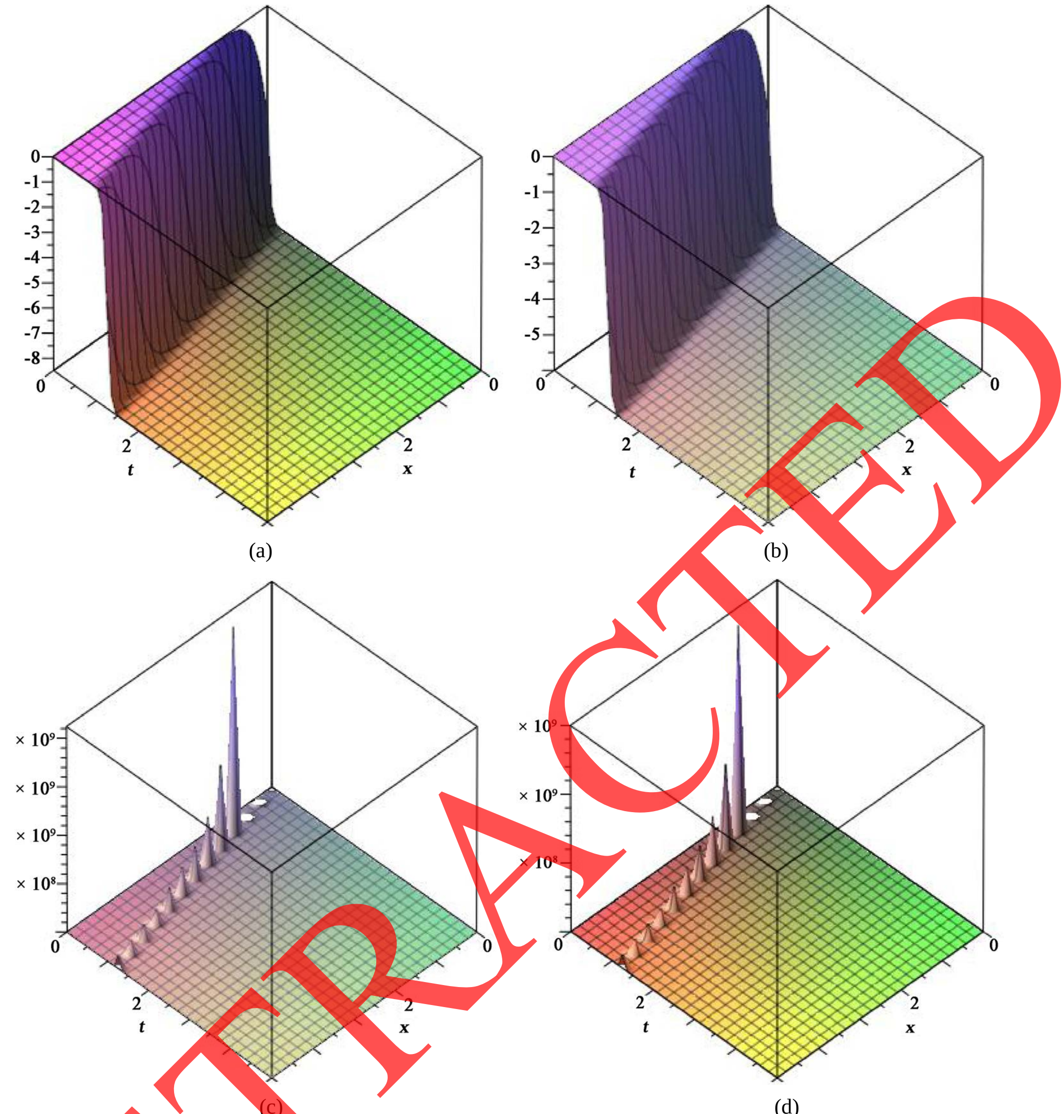
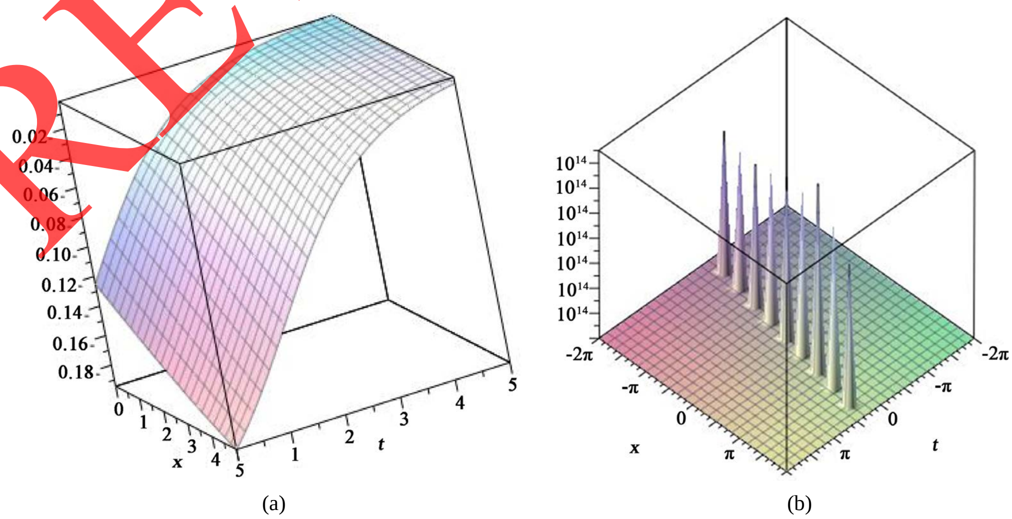
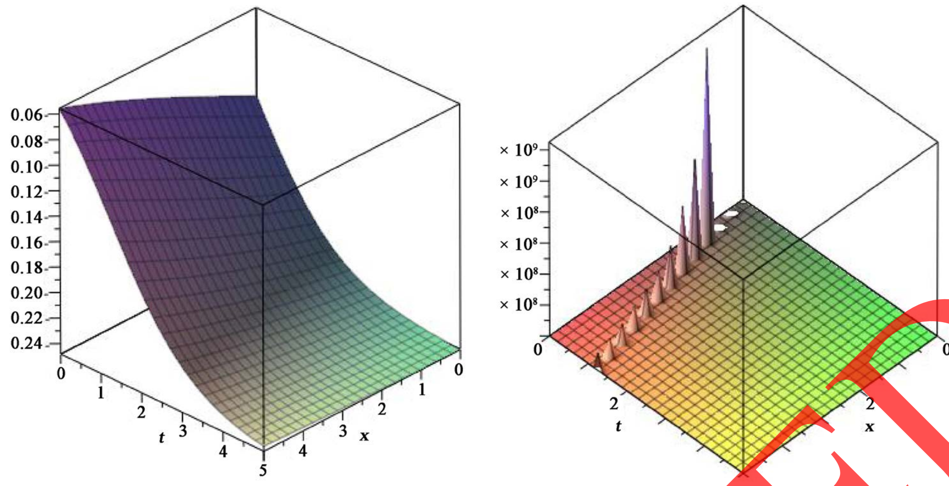


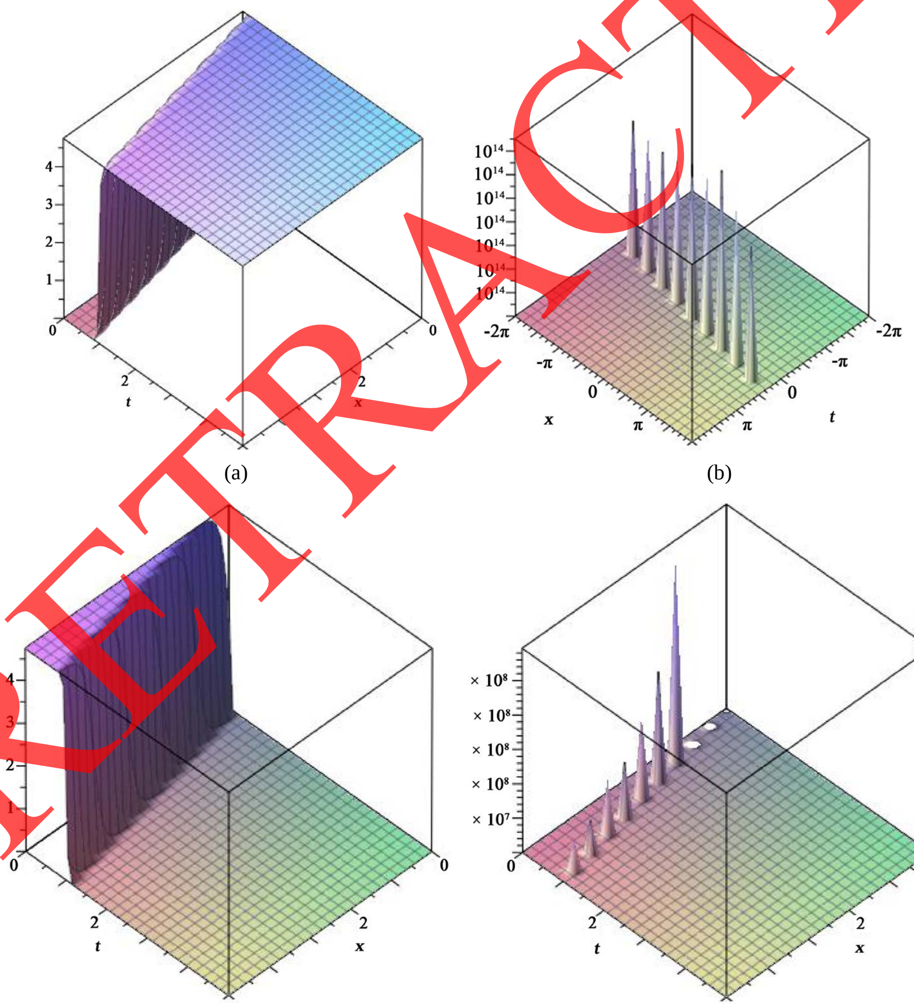
Figure 1. Solution of Equations (24)-(27). (a) Equation (24); (b) Equation (25); (c) Equation (26); (d) Equation (27).





(c) (d)

Figure 2. Solution of Equations (69)-(72). (a) Equation (69); (b) Equation (70); (c) Equation (71); (d) Equation (72).



(a) (b)

(c) (d)

Figure 3. Solution of Equations (95)-(98). (a) Equation (95); (b) Equation (96); (c) Equation (97); (d) Equation (98).

Finally, the physical meaning of our new results in this article can be summarized as follows: the solutions (24), (25), (35), (36) (53), (54), (68), (69), (79), (81), (95), (97), (105), (107) represent the kink shaped solitary wave while the solutions (26), (27), (37), (38), (55), (56), (70), (71), (3.76), (80), (82), (96), (106), (108) represent the singular kink solitary wave.

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