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Four Steps Continuous Method for the Solution of $y''=f(x,y,y')$

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ABSTRACT

This paper proposes a continuous block method for the solution of second order ordinary differential equation. Collocation and interpolation of the power series approximate solution are adopted to derive a continuous implicit linear multistep method. Continuous block method is used to derive the independent solution which is evaluated at selected grid points to generate the discrete block method. The order, consistency, zero stability and stability region are investigated. The new method was found to compare favourably with the existing methods in term of accuracy.

Keywords: Predictor; Corrector; Collocation; Interpolation; Power Series Approximant; Continuous Block Method

1. Introduction

This paper considered the solution to the second order initial value problem of the form

$$y''=f\left(x,y(x),y'(x)\right)y(x_0)=y_0,\quad y'(x_0)=y'_0 \tag{1}$$

Predictor-corrector method for solving higher order ordinary differential equation has been proposed by many scholars. These authors are [1-5]. They individually proposed continuous implicit linear multistep method where separate reducing order of accuracy predictors was developed and Taylor series expansion provided the starting value. The major setback of this method is the cost of developing predictors. Moreover, the predictors developed are of lower order to the corrector; hence it has a great effect on the accuracy of the results.

In order to cater for the shortcoming of predictor-corrector method, scholars proposed block method. Block method gives solutions at each grid within the interval of integration without ovelapping and the burden of developing separate predictors is eradicated. The authors who proposed block method are [6-12]. These authors proposed a discrete block method, which did not enable evaluation at all points within the interval of integration.

In this paper, we propose a continuous block method which enables evaluation at all points within the interval of integration without starting the block all over. Continuous block method enables one to recover discrete block when evaluating at selected grid points.

2. Methodology

Given the general block formulae proposed by [11]

$$\boldsymbol{Y}_m=\boldsymbol{E}y_n+h^{\mu}\boldsymbol{DF}(y_n)+h^{\mu}\boldsymbol{BF}(Y_m). \tag{2}$$

where $\boldsymbol{Y}_m=\left[y_{n+1},y_{n+2},\cdots,y_{n+k}\right]^{\mathrm{T}}$, μ is the order of the differential equation, K is the steplenght, $\boldsymbol{E},\boldsymbol{D}$ and $\boldsymbol{B}$ are matrices. We then propose a prediction equation in the form

$$\boldsymbol{Y}_m^{(0)}=\boldsymbol{E}y_n+\sum_{\lambda=0}^{2}h^{\mu+\lambda}\boldsymbol{F}^{(\lambda)}(y_n). \tag{3}$$

where $\boldsymbol{F}^{(\lambda)}(y_n)=\dfrac{\partial^{\lambda}}{\partial x^{\lambda}}\left(f(x,y,y')\right)_{y_n}$. Substituting (3) into (2) gives

$$\boldsymbol{Y}_m=\boldsymbol{E}y_n+h^{\mu}\boldsymbol{DF}(y_n)+h^{\mu}B\boldsymbol{F}\left(\boldsymbol{Y}_m^{(0)}\right). \tag{4}$$

Equation (4) is non self starting since the prediction equation is not gotten directly from the block formulae as proposed by [13].

In formulating the continuous block method we now consider a continuous implicit linear multistep method given by

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$$Y_{n+k} = \sum_{k=0}^{s} \alpha_k(x) y_{n+k} + h^{\mu} \sum_{k=0}^{r} \beta_k(x) f_{n+k}. \quad (5)$$

where s and r are the numbers of interpolation and collocation points. $\alpha_k(x)$ and $\beta_k(x)$ are polynomials.

Solving (5) for the independent solution gives a continuous block formulae in the form

$$Y_{n+j} = \sum_{j=0}^{\mu-1} \frac{(jh)^m}{m!} y_n^{(m)} + h^{\mu} \sum_{j=0}^{r} \sigma_j(x) f_{n+k}. \quad (6)$$

where $\sigma_j(x)$ is a polynomial, $m = 1, 2, \cdots, \mu-1$. Evaluating (6) at selected value of j reduces to (2).

Development of the Continuous Block Formula

Consider power series of a single variable as our approximate solution in the form

$$y(x) = \sum_{j=0}^{s+r-1} a_j x^j. \quad (7)$$

The second derivatives of (7) gives

$$y''(x) = \sum_{j=0}^{s+r-1} j(j-1) a_j x^{j-2}. \quad (8)$$

putting (8) into (1) gives

$$f(x, y(x), y'(x)) = \sum_{j=0}^{s+r-1} j(j-1) a_j x^{j-2}. \quad (9)$$

Let the solution to (1) be soughted on the partition $\pi_N : a = x_0 < x_1 < x_2 < \cdots < x_n < x_{n+1} < \cdots < x_N = b$ with a constant stepsize (h) given as

$$h = x_{n+i} - x_n, n = 0, 1, 2, \cdots, N.$$

Interpolating (7) at $x_{n+s}, s = 0,1$ and collocating (9) at $x_{n+r}, r = 0(1)4$ gives

$$\sum_{j=0}^{s+r-1} a_j x^j = y_{n+r}. \quad (10)$$

$$\sum_{j=2}^{s+r-1} j(j-1) a_j x^{j-2} = f_{n+r}. \quad (11)$$

solving (10) and (11) for $a_j's$ and substituting back in (7) gives a continuous linear multistep method of the form

$$y(x) = \sum_{j=0}^{1} \alpha_j(x) y_{n+j} + h^2 \sum_{j=0}^{4} \beta_j(x) f_{n+j}. \quad (12)$$

where the coefficient of y_{n+j} and f_{n+j} are given as

$$\alpha_0 = 1-t; \alpha_1 = t;$$

$$\beta_0 = \frac{1}{1440}\left(2t^6 - 30t^5 + 175t^4 - 500t^3 + 720t^2 - 376t\right);$$

$$\beta_1 = -\frac{1}{360}\left(2t^6 - 27t^5 + 130t^4 - 240t^3 - 135t\right);$$

$$\beta_2 = \frac{1}{240}\left(2t^6 - 24t^5 + 95t^4 - 120t^3 + 47t\right);$$

$$\beta_3 = -\frac{1}{360}\left(2t^6 - 21t^5 + 70t^4 - 80t^3 + 29t\right);$$

$$\beta_4 = \frac{1}{1440}\left(2t^6 - 18t^5 + 55t^4 - 60t^3 + 21t\right);$$

where

$$t = \frac{x - x_n}{h}.$$

Solving for the independent solution $y(x)$ in (13) gives a continuous block formulae (12) where the coefficient of f_{n+k} is given by

$$\sigma_0 = \frac{1}{1440}\left(2t^6 - 30t^5 + 175t^4 - 500t^3 + 720t^2\right);$$

$$\sigma_1 = -\frac{1}{360}\left(2t^6 - 27t^5 + 130t^4 - 240t^3\right);$$

$$\sigma_2 = \frac{1}{240}\left(2t^6 - 24t^5 + 95t^4 - 120t^3\right);$$

$$\sigma_3 = -\frac{1}{360}\left(2t^6 - 21t^5 + 70t^4 - 80t^3\right);$$

$$\sigma_4 = \frac{1}{1440}\left(2t^6 - 18t^5 + 55t^4 - 60t^3\right);$$

Evaluating the continuous block method at $t = 1(1)4$ gives a discrete block formulae of the form (2) where

$$\boldsymbol{E} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{D} = \begin{bmatrix} \frac{367}{1440} & \frac{53}{90} & \frac{147}{160} & \frac{56}{45} & \frac{251}{720} & \frac{29}{90} & \frac{27}{80} & \frac{14}{45} \end{bmatrix}^{\mathrm{T}},$$

$$\boldsymbol{B} = \begin{bmatrix} \frac{3}{8} & \frac{8}{5} & \frac{117}{40} & \frac{64}{15} & \frac{323}{360} & \frac{62}{45} & \frac{51}{40} & \frac{64}{45} \\ \frac{-47}{240} & \frac{-1}{3} & \frac{27}{80} & \frac{16}{15} & \frac{-11}{30} & \frac{4}{15} & \frac{9}{10} & \frac{8}{15} \\ \frac{29}{360} & \frac{8}{45} & \frac{3}{8} & \frac{64}{45} & \frac{53}{360} & \frac{2}{45} & \frac{21}{40} & \frac{64}{45} \\ \frac{-7}{480} & \frac{-1}{30} & \frac{-9}{160} & 0 & \frac{-19}{720} & \frac{-1}{90} & \frac{-3}{80} & \frac{14}{45} \end{bmatrix}^{\mathrm{T}},$$

$\boldsymbol{A}^{(0)} = 8 \times 8$ identity matrix.

3. Analysis of the Basic Properties of the Corrector

3.1. Order of the Method

Let the linear operator $\mathcal{L}\{y(x);h\}$ associated with the block formulae be defined as

$$\mathcal{L}\{y(x);h\} = \boldsymbol{A}^{(0)}\boldsymbol{Y}_m - \boldsymbol{E}y_n - h^{\mu}\boldsymbol{DF}(y_n) - h^{\mu}\boldsymbol{BF}(Y_m). \tag{13}$$

expanding in Taylor series and comparing the coefficient of h gives

$$\mathcal{L}\{y(x);h\} = C_0 y(x) + C_1 h y'(x) + C_2 h^2 y''(x) + \cdots + C_p h^p y^p(x) + C_{p+1} h^{p+1} y^{p+1}(x) + C_{p+2} h^{p+2} y^{p+2}(x) + \cdots \tag{14}$$

Definition 1. *The linear operator* $\mathcal{L}$ *and the associated continuous linear multistep method* (13) *are said to be of order* p *if* $C_0 = c_1 = C_2 = \cdots = C_p = C_{p+1} = 0$ *and* $C_{p+2} \neq 0$. C_{p+2} *is called the error constant and implies that the local truncation error is given by*

$$t_{n+k} = C_{p+2} h^{(p+2)} y^{(p+2)}(x_n) + O\left(h^{p+3}\right). \tag{15}$$

For our method,

$$\mathcal{L}\{y(x);h\} = \begin{bmatrix} y_{n+1} \\ y_{n+2} \\ y_{n+3} \\ y_{n+4} \\ y'_{n+1} \\ y'_{n+2} \\ y'_{n+3} \\ y'_{n+4} \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_n \\ hy'_n \end{bmatrix} - h^2 \begin{bmatrix} \frac{367}{1440} & \frac{3}{8} & \frac{-47}{240} & \frac{29}{360} & \frac{-7}{480} \\ \frac{53}{90} & \frac{8}{3} & \frac{-1}{3} & \frac{8}{45} & \frac{-1}{30} \\ \frac{147}{160} & \frac{117}{40} & \frac{27}{80} & \frac{3}{8} & \frac{-9}{160} \\ \frac{56}{720} & \frac{64}{15} & \frac{16}{15} & \frac{64}{15} & 0 \\ \frac{251}{720} & \frac{323}{360} & \frac{-11}{30} & \frac{53}{360} & \frac{-19}{720} \\ \frac{29}{90} & \frac{62}{45} & \frac{4}{15} & \frac{2}{45} & \frac{-1}{90} \\ \frac{27}{80} & \frac{51}{40} & \frac{9}{10} & \frac{21}{40} & \frac{-3}{80} \\ \frac{14}{445} & \frac{64}{45} & \frac{8}{15} & \frac{64}{45} & \frac{1}{45} \end{bmatrix} \begin{bmatrix} f_n \\ f_{n+1} \\ f_{n+2} \\ f_{n+3} \\ f_{n+4} \end{bmatrix} = 0$$

Expanding in Taylor series gives

$$\begin{bmatrix} \sum_{j=0}^{\infty}\frac{(h)^j}{j!}y_n^j - y_n - hy'_n - \frac{367}{1440}h^2 y_n^{(2)} - \sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{j+2}\left\{\frac{3}{8}(1)^j + \frac{47}{240}(2)^j + \frac{29}{360}(3)^j - \frac{7}{480}(4)^j\right\} \\ \sum_{j=0}^{\infty}\frac{(2h)^j}{j!}y_n^j - y_n - 2hy'_n - \frac{53}{90}h^2 y_n^{(2)} - \sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{j+2}\left\{\frac{8}{5}(1)^j - \frac{1}{3}(2)^j + \frac{8}{45}(3)^j - \frac{1}{30}(4)^j\right\} \\ \sum_{j=0}^{\infty}\frac{(3h)^j}{j!}y_n^j - y_n - 3hy'_n - \frac{147}{160}h^2 y_n^{(2)} - \sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{j+2}\left\{\frac{117}{40}(1)^j + \frac{27}{80}(2)^j + \frac{3}{8}(3)^j - \frac{9}{160}(4)^j\right\} \\ \sum_{j=0}^{\infty}\frac{(4h)^j}{j!}y_n^j - y_n - 4hy'_n - \frac{56}{45}h^2 y_n^{(2)} - \sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{j+2}\left\{\frac{64}{15}(1)^j + \frac{16}{15}(2)^j + \frac{64}{45}(3)^j\right\} \\ \sum_{j=0}^{\infty}\frac{(h)^j}{j!}y_n^{j+1} - hy'_n - \frac{251}{720}hy_n^{(2)} - \sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{j+2}\left\{\frac{323}{360}(1)^j - \frac{11}{30}(2)^j + \frac{53}{360}(3)^j - \frac{19}{720}(4)^j\right\} \\ \sum_{j=0}^{\infty}\frac{(2h)^j}{j!}y_n^{j+1} - 2hy'_n - \frac{29}{90}hy_n^{(2)} - \sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{j+2}\left\{\frac{62}{45}(1)^j - \frac{4}{15}(2)^j + \frac{2}{45}(3)^j - \frac{1}{90}(4)^j\right\} \\ \sum_{j=0}^{\infty}\frac{(3h)^j}{j!}y_n^{j+1} - 3hy'_n - \frac{27}{80}hy_n^{(2)} - \sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{j+2}\left\{\frac{51}{40}(1)^j + \frac{9}{10}(2)^j + \frac{21}{40}(3)^j - \frac{3}{80}(4)^j\right\} \\ \sum_{j=0}^{\infty}\frac{(4h)^j}{j!}y_n^{j+1} - 4hy'_n - \frac{14}{45}hy_n^{(2)} - \sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{j+2}\left\{\frac{64}{45}(1)^j + \frac{8}{45}(2)^j + \frac{64}{45}(3)^j + \frac{14}{45}(4)^j\right\} \end{bmatrix} = 0$$

hence,

$$C_0 = C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C_7 = 0$$

$$C_8 = \begin{bmatrix} \frac{107}{10080} & \frac{8}{315} & \frac{9}{225} & \frac{16}{315} & \frac{3}{160} & \frac{1}{90} & \frac{3}{160} & \frac{-8}{945} \end{bmatrix}$$

4. Zero Stability

Definition 2. *The block* (2) *is said to be zero stable, if the roots* $z_s, s = 1, 2, \cdots, N$ *of the first characteristic polynomial* $\rho(z)$ *defined by* $\rho(z) = \det\left(zA^{(0)} - E\right)$ *satisfies* $|z_s| \le 1$ *have multiplicity not exceeding the order of the differential equation. Moreover as* $h \to 0$, $\rho(z) = z^{r-\mu}(z-1)^{\mu}$, *where* μ *is the order of the differential equation, r is the order of the matrix* $\boldsymbol{A}^{(0)}$ *and* $\boldsymbol{E}$ (*see* [11] *for details*).

For our method

$$\rho(z) = \left[z\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right] = 0$$

$\rho(z) = z^6(z-1)^2$; hence our method is zero stable.

Region of Absolute Stability

Definition 3. *The method* (2) *is said be absolutely stable if for a given value oh h, all the roots* z_s *of the characteristic polynomial* $\pi(z,\bar{h}) = \rho(z) + \bar{h}\sigma(z) = 0$, *satisfies* $|z_s| < 1, s = 1, 2, \cdots, n$ *where* $\bar{h} = \lambda^2 h^2$ *and* $\lambda = \frac{\partial f}{\partial y}$.

We adopted the boundary locus method for the region of absolute stability if the block method. Substituting the test equation $y'' = -\lambda^2 y$ into the block formula gives

$$\boldsymbol{A}^{(0)}\boldsymbol{Y}_m(r) = \boldsymbol{E}y_n(r) - h^2\lambda^2\boldsymbol{D}y_n(r) - h^2\lambda^2\boldsymbol{B}\boldsymbol{Y}_m(r).$$

$$\bar{h}(r,h) = -\left(\frac{\boldsymbol{A}^{(0)}\boldsymbol{Y}_m(r) - \boldsymbol{E}y_n(r)}{\boldsymbol{D}y_n(r) + \boldsymbol{B}\boldsymbol{Y}_m(r)}\right)$$

writing in trigonometric ratios gives

$$\bar{h}(r,\theta) = -\left(\frac{\boldsymbol{A}^{(0)}\boldsymbol{Y}_m(\theta) - \boldsymbol{E}\boldsymbol{y}_n(\theta)}{\boldsymbol{D}y_n(\theta) + \boldsymbol{B}\boldsymbol{Y}_m(\theta)}\right). \quad (16)$$

where $r = e^{i\theta}$. Equation (16) is our characteristics polynomial (see [14] for details). Applying to our method

$$\bar{h}(r,\theta) = -\left(\frac{6975\cos 4\theta - 6975}{168\cos\theta - 793}\right)$$

which gives the stability region to be [−14.35, 0] after evaluation $\bar{h}(\theta,h)$ at interval of 30° within [0, 180] The stability is shown in **Figure 1**.

5. Numerical Experiments

5.1. Test Problems

We test our scheme on second order initial value problems.

Problem 1: Consider the non linear initial value problem (I.V.P) which was solved by [11] using block method and [2] for step size $h = 0.003125$ which is of order 8

$$y'' - x(y') = 0, y(0) = 1, y'(0) = \frac{1}{2}$$

Exact solution: $y(x) = 1 + \frac{1}{2} In\left(\frac{2+x}{2-x}\right)$

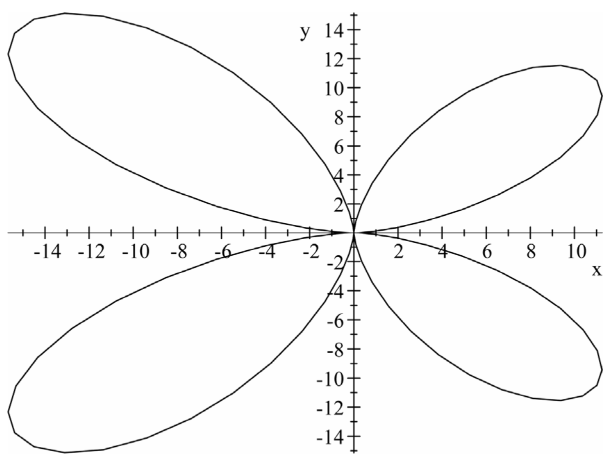

Figure 1. Showing region of absolute stability of the block method.

We solved this problem using our method for stepsize h = 0.01. The result is shown in **Table 1**. Our method performed better than the methods compared with. This problem method was also solved [10] using block of lower order for h = 1/32. Though the details are not shown but our result performed better in term of accuracy.

Problem 2: Consider a linear second order initial value problem

$$y'' + \left(\frac{6}{x}\right)y' + \left(\frac{4}{x^2}\right)y = 0,\ y(1) = 1,\ y'(1) = 1$$

Exact solution: $y(x) = \dfrac{5x^3 - 2}{3x^3}$.

This problem was also solved by [10]. The result in **Table 2** shows clearly that our method gives better approximation than the existing methods.

5.2. Numerical Results

The following notations are used in the table

ESBK: Error in [11];

EAK: Error in [2];

EBK: Error in [10];

ERR = $\left|\text{Exact result} - \text{computed result}\right|$.

6. Conclusion

We have proposed a non self starting continuous block method in this paper. It has been shown clearly that the non self starting method gives a better approximation than

Table 1. Showing results of problem 1.

X	Exact result	Computed result	ERR	ESBK	EAK
0.1	1.05004172927849	1.05004172927848	9.992 (−15)	6.442 (−11)	6.550 (−11)
0.2	1.10033534773107	1.10033534773099	8.149 (−14)	5.456 (−10)	5.480 (−10)
0.3	1.15114043593646	1.15114043593599	4.700 (−13)	1.921 (−09)	1.925 (−09)
0.4	1.20273255405408	1.20273255405244	1.637 (−12)	4.797 (−09)	4.802 (−09)
0.5	1.25541281188299	1.25541281187833	4.664 (−12)	9.998 (−09)	1.000 (−08)
0.6	1.30951960420311	1.30951960419194	1.116 (−11)	1.871 (−08)	1.872 (−08)
0.7	1.36544375427139	1.36544375424638	2.501 (−11)	3.272 (−08)	3.274 (−08)
0.8	1.43264893019360	1.42364893014144	5.215 (−11)	5.479 (−08)	5.396 (−08)
0.9	1.48470027859405	1.48470027848636	1.076 (−11)	8.929 (−08)	8.800 (−08)
1.0	1.54930614433405	1.54930614411698	2.170 (−10)	1.434 (−07)	1.435 (−07)

Table 2. Showing results of problem 2.

X	Exact result	Computed result	ERR	ESBK	EBK
1.0031	1.00307652585769	1.0030765258576	1.332 (−15)	3.381 (−10)	3.835 (−05)
1.0062	1.00605750308351	1.0060575030835	8.215 (−15)	1.666 (−09)	7.500 (−05)
1.0094	1.00894499508837	1.0089449950888	1.909 (−14)	4.023 (−09)	1.059 (−04)
1.0125	1.01174101816798	1.0117410181680	3.508 (−14)	7.282 (−09)	1.355 (−04)
1.0156	1.01444754268641	1.0144475426864	5.484 (−14)	1.147 (−08)	1.556 (−04)
1.0187	1.01706649423567	1.0170664942357	7.904 (−14)	1.649 (−08)	1.864 (−04)
1.0219	1.01959975475628	1.019599547563	1.063 (−13)	2.237 (−08)	1.961 (−04)
1.0250	1.02204916362943	1.0220491636295	1.374 (−13)	2.898 (−08)	2.210 (−04)
1.0281	1.02441651873840	1.0244165187385	1.723 (−13)	3.638 (−08)	2.056 (−04)
1.0312	1.02670357750080	1.0267035775010	2.100 (−13)	4.446 (−08)	2.779 (−04)

the self starting method. The continuous parameter introduced in the block method allows evaluation at all points within the interval of integration; hence, it enables researchers to understand the behaviour of the system under investigation. We therefore recommend this new method when seeking for the solution of initial value problems.

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