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Magneto Hydrodynamic Orthogonal Stagnation Point Flow of a Power-Law Fluid toward a Stretching Surface

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Abstract:

Steady two dimensional MHD stagnation point flow of a power law fluid over a stretching surface is investigated when the surface is stretched in its own plane with a velocity proportional to the distance from the stagnation point. The fluid impinges on the surface is considered orthogonally. Numerical and analytical solutions are obtained for different cases.

Keywords: Stagnation Point Flow, Galerkin's, Finite Difference Method, Stretching Surface.

1. Introduction

The stagnation point is a point on the surface of a body submerged in a fluid flow where the fluid velocity is zero. Stagnation flow, describing the fluid motion near the stagnation region, exists on all solid bodies moving in a fluid. The stagnation region encounters the highest pressure, the highest heat transfer, and the highest rates of mass deposition. The study of flow over a stretching surface has generated much interest in recent years in view of its numerous industrial applications such as extrusion of polymer sheets, continuous stretching, rolling and manufacturing plastic films and artificial fibers. The flow near a stagnation point has attracted many investigations during the past several decades because of its wide applications such as cooling of electronic devices by fans, cooling of nuclear reactors, and many hydrodynamic processes [1-5].

The two-dimensional flow of a fluid near a stagnation point was first examined by Hiemenz [6], who demonstrated that the Navier-Stokes equations governing the flow can be reduced to an ordinary differential equation of third order using similarity transformation. Later the problem of stagnation point flow was extended in numerous ways to include various physical effects. The results of these studies are of great technical importance, for example in the prediction of skin-friction as well as heat/mass transfer near stagnation regions of bodies in high speed flows and also in the design of thrust bearings and radial diffusers, drag reduction, transpiration cooling and thermal oil recovery. Axisymmetric three-dimensional stagnation point flow was studied by Homann [7]. Either in the two or three-dimensional case Navier-Stoke's equations governing the flow are reduced to an ordinary differential equation of third order using a similarity transformation. In hydromagnetics, the problem of Hiemenz flow was chosen by Na [8] to illustrate the solution of a third-order boundary value problem using the technique of finite differences. An approximate solution of the same problem has been provided by Ariel [9]. Attai [1] has made an analysis of the steady laminar flow in a porous medium of an incompressible viscous fluid impinging on a permeable stretching surface with heat generation. The steady magneto hydrodynamic (MHD) mixed convection stagnation point flow towards a vertical surface immersed in an incompressible micropolar fluid with prescribed wall heat flux was investigated by Bachok *et al.* [4]. They have transformed the governing partial differential equations into a system of ordinary differential equations, which is then solved numerically by a finite-difference method. Hayd *et al.* [2] have studied the boundary layer equations for axisymmetric point flow of power-law electrically conducting fluid through a porous medium with transverse magnetic field. McLeod and Rajagopal [10] have discussed the uniqueness of the exact analytical solution of the flow of a Newtonian fluid due to a stretching boundary. On the other hand, Rajagopal *et al.* [10,11] obtained an approximate mathematical solution of the viscoelastic boundary layer flow over a stretching plastic sheet and studied the flow

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behaviors.

Chiam [12] and Mahapatra and Gupta [13] have investigated the steady two-dimensional stagnation point flow of an incompressible viscous fluid over a flat deformable sheet when the sheet is stretched in its own plane with a velocity proportional to the distance from the stagnation point. It is shown that a boundary layer is formed near the stretching surface and that the structure of this boundary layer depends on the ratio of the velocity of the stretching surface to that of the frictionless potential flow in the neighborhood of the stagnation point. Recently, Patel *et al*. [5] have discussed the numerical solution for steady two-dimensional MHD forward stagnation point flow introducing the Crocco's independent variable with Galerkin's. In stagnation point flow, a rigid wall or a stretching surface occupies the entire horizontal *x*-axis, the fluid domain is $y > 0$ and the flow impinges on the wall either orthogonal or at an arbitrary angle of incidence.

In this paper we investigate steady two dimensional stagnation point flow of a power law fluid over a stretching surface, when the surface is stretched in its own plane with a velocity proportional to the distance from the stagnation point. The fluid impinges on the surface is considered orthogonally. Numerical and analytical solutions are obtained for different flow geometries along with graphical presentation.

2. Flow Analysis

The well-known Ostwald-de-Wale model of power-law fluid is purely phenomenological; however, it is useful in that approximately describes a great number of real non-Newtonian fluids. This model behaves properly under tensor deformation. Use of this model alone assumes that the fluid is purely viscous. Mathematically it can be represented in the form

$$\bar{\bar{\tau}} = -\left\{ m \left| \frac{1}{2} \bar{\bar{\Delta}} : \bar{\bar{\Delta}} \right|^{\frac{n-1}{2}} \right\} \bar{\bar{\Delta}} \qquad (1)$$

where m and n are called the consistency and flow behavior indices respectively. If $n < 1$, the fluid is called pseudo plastic power law fluid and if $n > 1$, it is called dilatants power law fluid since the apparent viscosity decreases or increases with the increase shear of rate according as $n < 1$ or $n > 1$, if $n = 1$ the fluid will be Newtonian.

Consider the steady two-dimensional stagnation-point flow of power-law fluid flowing towards a flat surface coinciding with the plane $y = 0$, the flow being confined to the region $y > 0$. Two equal and opposing forces are applied on the stretching surface along the *x*-axis so that the surface is stretched keeping the origin fixed as shown in **Figure 1**. The MHD equations for steady two-dimensional stagnation-point flow in the boundary layer towards the stretching surface are, in the usual notation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial x} + \frac{\partial}{\partial y}\tau_{xy} - \frac{\sigma B_0^{\,2}}{\rho}(u - U) \qquad (3)$$

Here the magnetic Reynolds number is assumed to be very small so that the induced magnetic field is neglected. Here u and v are the velocity components along the x and y direction, respectively. Further τ_{xy} is stress tensor in the direction of Y-axis perpendicular to X-axis. $U(x)$ stands for the stagnation-point velocity in the inviscid free stream. The stress tensor is defined by Equation (1).

In the present problem we have

and $$\frac{au}{ay} > 0 \quad \text{when} \quad \frac{a}{c} > 1. \qquad (4)$$

Therefore the shear stress will convert as:

$$\tau_{xy} = k\left(\frac{\partial u}{\partial y}\right)^n \quad \text{when} \quad \frac{a}{c} > 1 \qquad (5)$$

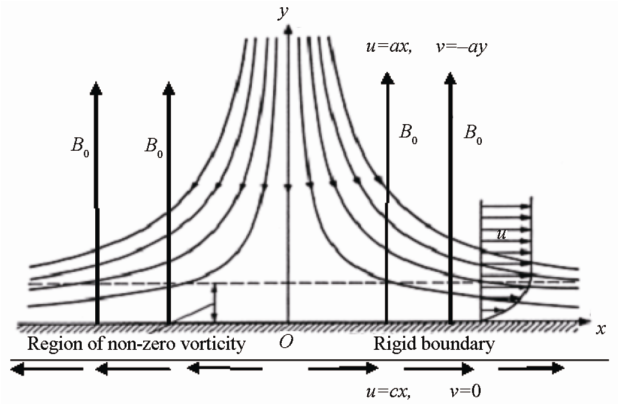

Figure 1. A sketch of the physical problem.

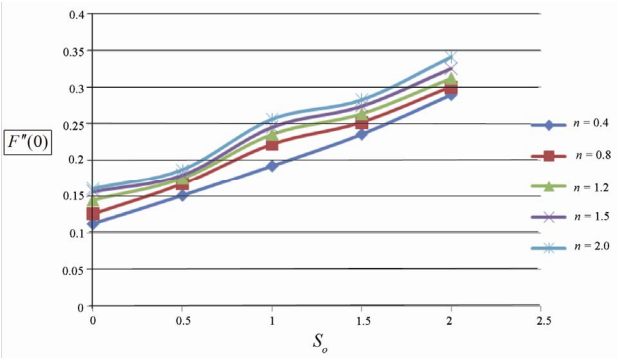

Figure 2. Variation of $F''(0)$ with S_o for $a / c = 1.1$.

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Table 1.

n	$S_o=0$	$S_o=0.5$	$S_o=1.0$	$S_o=1.5$	$S_o=2.0$
0.4	0.1123	0.1252	0.1445	0.1564	0.1601
0.8	0.1512	0.1668	0.1748	0.1799	0.1869
1.2	0.1924	0.2214	0.2356	0.2455	0.2662
1.5	0.2358	0.2514	0.2636	0.2741	0.2852
2.0	0.2897	0.2999	0.3121	0.3255	0.3412

Table 2.

n	$S_o=0$	$S_o=0.5$	$S_o=1.0$	$S_o=1.5$	$S_o=2.0$
0.4	1.2216	1.3625	1.4886	1.6289	1.7865
0.8	1.0023	1.0423	1.0858	1.1821	1.3226
1.2	0.8984	0.9672	1.0115	1.1000	1.2101
1.5	0.8864	0.9355	1.0021	1.0552	1.1001
2.0	0.8876	0.9301	0.9864	0.9945	1.0477

Table 3.

n	$S_o=0$	$S_o=0.5$	$S_o=1.0$	$S_o=1.5$	$S_o=2.0$
0.4	3.8771	4.1332	4.4755	4.8163	5.2445
0.8	2.3854	2.5446	2.5845	2.8354	2.8714
1.2	1.9328	1.9454	2.0023	2.1426	2.1735
1.5	1.7001	1.7537	1.8817	1.9583	1.9966
2.0	1.5481	1.5841	1.6120	1.6526	1.6998

Table 4. Results of finite difference method of Case-I.

No. of iteration	$S_o=0$	$S_o=0.5$	$S_o=1.0$	$S_o=1.5$	$S_o=2.0$
1	1.02326	1.2326	1.2326	1.2326	1.2326
2	1.1688	1.3593	1.5393	1.7882	1.8385
3	1.1743	1.3637	1.5407	1.7890	1.8339
4	1.1715	1.3618	1.5392	1.7662	1.8328
5	1.1717	1.3621	1.5394	1.7914	1.8330
6	1.1717	1.3620	1.5394	1.7914	1.8330

Now the momentum Equation (3) become, in non-dimensional form, when $\frac{a}{c}>1$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\partial U}{\partial x}+\frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n-\frac{\sigma B_0^{\,2}}{\rho}(u-U) \quad (6)$$

The boundary conditions are:

$$u=cx,\ v=0\ at\ \mathrm{y}=0 \quad (7)$$

$$u=U(x)=ax,\ v=-ay,\ at\ \mathrm{y}\to\infty \quad (8)$$

where a and c are positive constants.

Introducing stream function $\psi(x,\ Y)$, where

$$u=\frac{\partial\psi}{\partial y}\ \text{ and }\ v=-\frac{\partial\psi}{\partial x} \quad (9)$$

Following Labropulu [3], we assume that

$$\psi=cxF(y) \quad (10)$$

Using Equations (9) and (10) into the Equation (6), we obtained

$$n\left[F''(y)\right]^{n-1}F'''(y)+\left(\frac{2n}{n+1}\right)F(y)F''(y)$$

$$-F'^2(y)-S_oF'(y)+S_o\frac{a}{c}+\frac{a^2}{c^2}=0 \quad (11)$$

with the boundary conditions:

$$F(0)=0,\ \ F'(0)=1,\ \ F'(\infty)=\frac{a}{c} \quad (12)$$

where $S_o=\frac{\sigma B_0^{\,2}}{\rho c}$ is the magnetic parameter.

Case-I: Newtonian fluid: consider $n=1$ and $a/c=1$ Equation (11) is converted in the following equation

$$F'''(y)+F(y)F''(y)-F'^2(y)-S_oF'(y)+S_o+1=0 \quad (13)$$

with the boundary conditions:

$$F(0)=0,\ \ F'(0)=1,\ \ F'(\infty)=\frac{a}{c} \quad (14)$$

Case-II: if we consider case for $n=1$, and let $U=0$ (*i.e.* $a=0$) then the Equation (11) is converted in

$$F'''(y)+F(y)F''(y)-F'^2(y)-S_oF'(y)=0 \quad (15)$$

Subject to the boundary conditions;

$$F(0)=0,\ F'(0)=1,\ F'(\infty)=0 \quad (16)$$

It is interesting to note that the above BVP Equation (15) has a simple analytic solution of the form

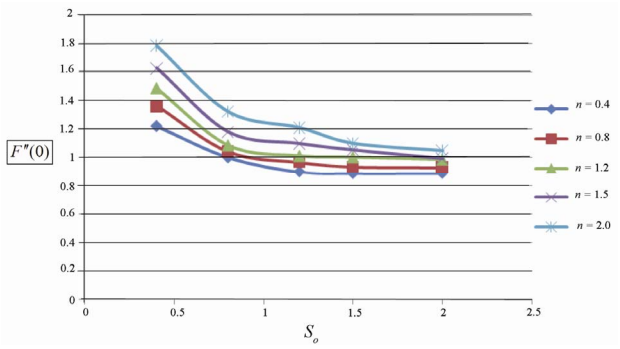

Figure 3. Variation of $F''(0)$ with S_o for $a/c=1.5$.

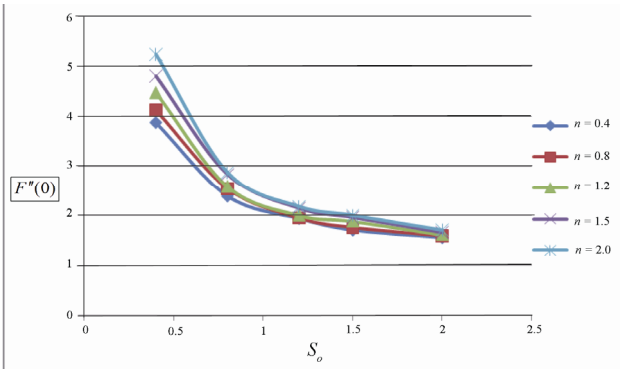

Figure 4. Variation of $F''(0)$ with S_o for $a/c=2.0$.

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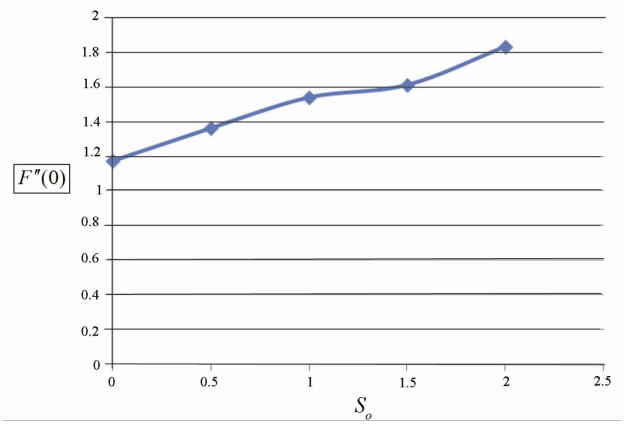

Figure 5. Variation of $F''(0)$ with S_o for $a/c=1$, $n=1$.

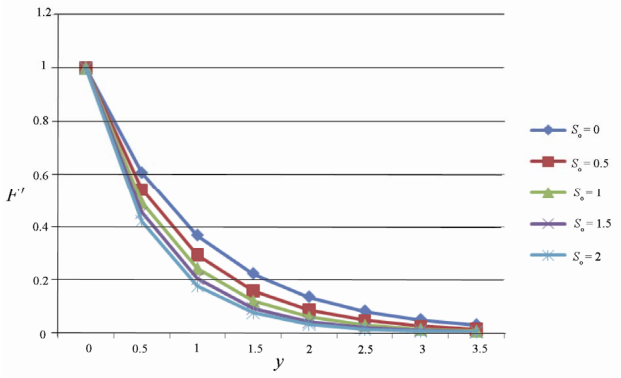

Figure 6. Variation of $F'(y)$ with y.

$$F(y)=\frac{1}{p}[1-\exp(-py)] \qquad (17)$$

where $p=(1+S_0)^{1/2}$

Method of solution: The transformed momentum Equation (11) is solved using Galerkin's method after introducing Crocco's variables [5,14]. The equation for Case-I (Equation (13)) is solved by T. Y. Na [8] using Finite difference method the graphical presentation of those tabular values (**Table 3**) is shown in **Figure 2**. And the equation of Case-II (Equation (15)) is solved analytically.

Results and discussion: The computed variation of $F''(0)$ with S_o and n is summarized in **Tables 1** and **2** for $a/c=1.1$, 1.5 and 2.0, respectively. It can be conclude (from the above **Tables 1-3**, **Figures 2-4**) that for a fixed value of S_o, $F''(0)$ increases with increase in n in a small neighbourhood of $a/c=1$ then it decreases for other values of a/c. From **Tables 1-3**, it is clear that for a fixed value of n and a/c, the value of $F''(0)$ decreases with increase in S_o. This behaviour may perhaps be attributed to the change in the character of the flow as a/c changes its values.

It is interesting to note that when the velocity of the stretching surface is equal to the velocity of the inviscid stream $(a=c)$, Equation (16) admit to the exact analytic solution $F(y)=-y$. From this we can infer that when $a=c$, the velocity distribution near the stretching surface is the same as athat of a flow away from the surface so that no boundary layer is formed near the surface. It should be mentioned here that when , the flow is not frictionless in a strict sense. In fact in this case the friction is uniformly distributed and does not, therefore, affect the motion.

If we consider $n=1$ then the entire flow geometry is reduced in Newtonian fluid, which is discussed as a Case-I with $a=c$ (*i.e.* $a/c=1$, **Table 4**, **Figure 5**). For that case $F''(0)$ increases with increase in the value of S_o for a fixed value of $n=1$. If we consider stream velocity is zero $(n=1)$ then the flow is treated as a uniform stagnation point flow, in this case $F'(y)$ decreases with increase in y for a fixed value of S_o. The graphical representation is shown in **Figure 6**.

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Nomenclature:

u, v – velocity components in X, Y directions respectively
U – main stream velocity in X direction
a, c – positive constants
τ, τ_{ij} – usual shear stress tensor
Δ, e_{ij} – usual rate of deformation tensor/ Strain rate component
τ_{yx} – stress tensor in the direction of X-axis perpendicular to Y-axis.
K – kinematic Viscosity
m – Physical constant
n – flow behavior indices
B_0 – Imposed magnetic field
MHD – Magneto hydro dynamics
ρ – field density
σ – Electrical conductivity
C_p – Specific heat
ψ – Stream function
F – Similarity function
S_o – Magnetic parameter – $\dfrac{\sigma B_0^2}{\rho}$

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