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Theory of Triple Magnetopolarons in Quantum-Dimensional Nanostructures

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ABSTRACT

A theory of triple magnetopolarons in an isolated quantum well in a strong magnetic field was developed. We study the behavior of the magnetooptical absorption peaks corresponding to the transitions of an electron at the Landau level with quantum numbers $n \geq 2$. For $n = 2$ at the point of equality of cyclotron frequency and the frequency of optical phonon (LO), there is a cross of three terms of the electron-phonon system (the electron at the Landau level $n = 2$, the electron at $n = 1$, and the optical phonon and electron at $n = 0$ and two phonons), considered as a function of the cyclotron frequency. Interaction with phonons takes off the degeneracy of the terms and leads to three disjoint branches of the electron-phonon spectrum. The theory predicts that in the resonant magnetic field, the peak of magnetooptical absorption splits into three peaks, the intensity and position of which are dependent in a complex way on the magnitude of the magnetic field and the constant of the electron-phonon coupling.

Keywords: Triple Magnetopolarons; Quantum-Dimensional Nanostucrures; Electron; Phonons; Electron-Phonon Spectrum; Magnetooptical Absorption; Resonant Magnetic Field

1. Introduction

When the following condition is valid:

$$\omega_c = \omega_0 \tag{1}$$

(where ω_0 is maxima frequency of longitudinal optical phonon (OP), $\omega_c = (eH/cm_c)$ is the cyclotron frequency, e is the electron's charge, H is the magnetic field tension, c is the velocity of light in the vacuum, m_c is the effective mass of electron), the magnetopolaron states are formed in semiconductors. This effect is called the effect of Johnson-Larsen [1,2] or magnetopolaron resonance.

When the magnetic field satisfies the condition (1), there is a resonant relation between the levels with different Landau quantum numbers n (**Figure 1**). The electronphonon interaction leads to taking off the degeneracy in points of the level's intercept which is manifested in magneto-optical phenomena. Magnetopolaron state was first discovered in a bulk InSb as the interband absorption of light [4,5].

Korovin and Pavlov showed [3] that in case of a bulk semiconductor, the magnetopolaron splitting is proportional to $\alpha_0^{2/3}\hbar\omega_0$, where α_0 is the dimensionless con-

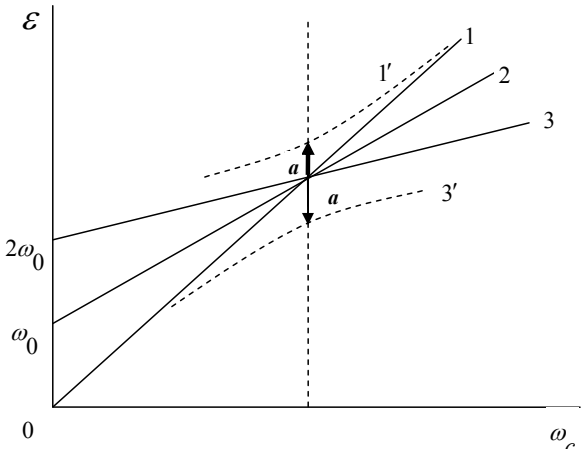

Figure 1. Schematic spectrum of electron 1—Landau level n = 2, 2—level n = 1 and optical phonon, 3—level n = 0 and two optical phonons. Dotted lines show the splitting of terms, $a = \sqrt{(A+B)\eta}$; ε —term's energy, 1' and 3'—upper and lower branches of the spectrum. Points of resonance $\omega_0 = \omega_c$ and break of the upper branch $\omega_c = \omega_0(1-a_0)$ and lower branch $\omega_c = \omega_0(1+a_0)$ of the spectrum are shown.

stant of the electron-phonon bonding, and $\alpha_0 \ll 1$.

During the last several years, a new wave of interest to the Johnson-Larsen effect [4-22] was stimulated by appearance of semiconducting objects of reduced dimension in which the effect is increased due to dimensional quantization of the electron's motion in perpendicular direction.

Formation of polaron states takes place both in three-dimensional (3D) [1-3,22] and quasi-two-dimensional (2D) systems [4-22]. A difference between the systems is in electron's spectra in the presence of quantizing magnetic field: in 3D-system these are one-dimensional Landau zones, and in 2D system these are discrete Landau levels. This difference leads to different magnitudes of pushing aside the levels of electron-phonon system [3,8,9].

In 3D and 2D systems, the magnetopolaron states play an important role in formation of frequency dependence of magnetooptical phenomena, such as the intarband absorption of light [1,2,6], the cyclotron resonance [4,5, 7,10] and Raman scattering of light [12-14].

In 2D-systems, the effect is enhanced, and the distance between the components of the splitting of the peak becomes proportional to $\alpha_0^{1/2}\hbar\omega_0$ [8-15].

When the Condition (1) is implemented, and when Landau levels $n = 1$ and $n = 0$ plus one LO-phonon are connected, the double polarons are formed [22]. If in this condition, three Landau levels $n = 2$, $n = 1$ plus one LO-phonon and $n = 0$ plus two LO-phonons are connected then triple polarons are formed [22]. The triple polarons in a bulk semiconductor were discussed in [23] and in the quantum wells they were considered in works [12-14,16].

The energy spectrum of magnetopoarons—both usual (classical) and combined [18]—was defined in two ways, which give the same results. One of them was first used in [3] and consisted in determining the poles of the single-particle Green's function of an electron. It was also used in [8,17,18]. Another method was described in work [14] devoted to double magnetopolaron of *A* type. Wave functions of the polaron are represented as the superposetion of wave functions of unperturbed states (in the case of the *A*-polaron, this state ($n = 1$ and $n = 0$ plus one LO-phonon) is that with yet unknown coefficients). Schrodinger equation reduces to a system of two equations for two coefficients. Equating the determinant to zero, we obtain a quadratic equation for the energies of polaron states $p = a$ and $p = b$. The advantage, as compared with the first method, is that simultaneously with the calculation of the energies we find the wave functions of magnetopolarons, and these functions are necessary to describe a lot of magneto-optical effects. In [22], the results of [14] for the *A*-polaron were generalized to the case of all double polarons, including regular combined and special polaron states.

In this paper, we present the results of theoretical investigation of the energy spectrum of a triple mangotopolaron, and the influence of the triple magnotopolaron spectrum on the frequency dependence of magneto-optical interband peaks in semiconductor quantum wells.

2. Problem Statement and Analysis of the Mass Operator in Case of a Low Temperature

We consider a semiconductor in which in a surface layer there is a single-dimensional potential well that quantizes the motion of electrons in a direction normal to the interface (we mean InSb in MDS system and GaAs heterojunction). In the magnetic field, which is normal to the interface, the energy levels in the well become discrete ones (with infinitely fold degeneracy), and their classification depends on the ratio of the energy in the well and the cyclotron energy. Below we assume that the energy of quantization in the well is high as compared with the cyclotron one, and only the lowest level with adjacent Landau levels is taken into account

We assume that the valence (V) and conductance (C) bands are located in the center of Brillouin zone, and the direct dipole transition between them is allowed. Interaction with LO-phonons, which determines in this case the splitting of peaks, we assume to be a weak one. In many semi-conductors the following condition is valid: $(m_c/m_v) \ll 1$ (where m_v is the effective mass of the hole). We consider in this case the interband optical transitions, which creates an electron in a potential well of the conduction band and a hole near the peak of the Landau band or in a potential well in the valence band in the case of GaAs heterojunction (possibility of formation of exciton states is neglected). If the temperature is low, and the magnetic field is close to the resonance (see (1)), then the hole states will be stationary ones in the chosen mechanism of interaction, because the hole can not actually emit the LO-phonon due to a lack of energy. The hole cannot also absorb phonons because of their absence. An electron in the conductance band can, in this case, actually emit the LO-phonon and move to the level with quantum number $n - 1$.

Wave functions of electron in this case can be written as

$$\Psi_\alpha(x,y,z) = \exp(ikx)\varphi_n(y)\chi_l(z) \qquad (2)$$

where $\varphi_n(y)$ is a wave function of a harmonic oscillator in the magnetic field, α represents *l, n* and *k* quantum numbers that characterize the state of electron in the potential well in the magnetic field (*l* is a number of the quantization level along *z* axis, *i.e.* along the normal direction to the interface, that describes the levels of dimensional quantization in the well; *n* is the number of

Landau level; k is the continuous quantum number, which describes the degeneration of discrete levels). $\chi_l(z)$ describes the quantum states in the well. In case when only the lowest level $l = 0$ is occupied, the function $\chi_0(z)$ can be well approximated by the following expression:

$$\chi_0(z) = \sqrt{b^3/2}\, z \cdot \exp(-bz/2) \tag{3}$$

where

$$b^3 = \left(48\pi m_0 e^2/\varepsilon_p \hbar^2\right)\left[N_d + (11/32) N_S\right], \tag{4}$$

Here, m_o is the mass of a free electron, N_d is the concentration of ionized impurities in the depleted layer, N_S is the concentration of electrons (cm^{-2}) on $l = 0$ level, ε_p is the static dielectric constant of a semiconductor. The absorption will be characterized by a fraction of the absorbed energy [24]

$$W = W_0 \omega_c \sum_{\alpha} \mathrm{Re}\, iG_r(\alpha, \omega - \omega_{v\alpha}) \tag{5}$$

where

$$G_r(\alpha,\varepsilon) = \left[\varepsilon - \omega_{c\alpha} - \sum(\alpha,\varepsilon) + i\delta\right]^{-1}; \quad \delta \to +0 \tag{6}$$

is the single-particle retarded Green's function of the electron

$$\omega_{c\alpha} = \omega_c\left(n_\alpha + \frac{1}{2}\right) + \left(\varepsilon_{c_0}/\hbar\right) \tag{7}$$

$$\omega_{v\alpha} = \left(E_g/\hbar\right) + \omega_v\left(n_\alpha + \frac{1}{2}\right) + \left(\varepsilon_{v_0}/\hbar\right) \tag{8}$$

$$W_0 = \frac{8\sqrt{\varepsilon_p}}{\left(\sqrt{\varepsilon_p} + \sqrt{\varepsilon_l}\right)^2}\left(\frac{e^2}{c\hbar}\right)\frac{\left|P_{cv}^y\right|^2}{m_0 E_g}\left(\frac{m_c}{m_0}\right), \ \omega_v = \left(eH/m_v c\right) \tag{9}$$

Here, P_{cv}^y is the interband momentum matrix element, evaluated at the Bloch modulating factors, E_g is the band gap, ε_l is the statistical permittivity of the dielectric, $\varepsilon_{c(v)0}$ is the energy of l = 0 level in the potential well.

Below we consider in detail the transition of electron under the influence of light on Landau level with n = 2 in the conductance band. In this case, three terms of electron-phonon system (electron at n = 2 level, electron at n = 1 level and one LO-phonon, and electron at n = 0 level and two LO-phonons), which are considered as functions of ω_c, intersect in the point $\omega_c = \omega_0$ (**Figure 1**). The electron-phonon interaction leads to break of degeneracy in the point $\omega_0 = \omega_c$ and, consequently, to appearance of three non-intersecting branches of the spectrum.

The problem of calculation of the spectrum is complicated by the fact that emission of phonons is connected with transitions of electrons between Landau levels, where the density of states is high. Formally, it means the impropriety of usual series expansion of the mass operator with respect to coupling constant, and necessity of selective summation of the series.

We start the analysis of the self-energy part of $\sum(\alpha,\varepsilon)$ with consideration of the graph in **Figure 2**, *a*, which is

$$\sum\nolimits_1(\alpha,\varepsilon) = \sum_{\boldsymbol{q}}\sum_{\alpha_1} \hbar^{-2}\left|C_q\right|^2 \left|J_{\alpha\alpha_1}(\boldsymbol{q})\right|^2 \left(\varepsilon - \omega_0 - \omega_{c\alpha_1} + i\delta\right)^{-1} \tag{10}$$

where

$$\left.\begin{aligned} \left|C_q\right|^2 &= \left(\hbar\omega_0\right)^2\left(4\pi\alpha_0 l_0/Vq^2\right), \quad l_0^2 = \left(\hbar/2m_c\omega_0\right), \\ \alpha_0 &= \left(e^2/2\hbar\omega_0 l_0\right)\left(\varepsilon_\infty^{-1} - \varepsilon_p^{-1}\right) \end{aligned}\right\} \tag{11}$$

and $J_{\alpha\alpha'}(\boldsymbol{q})$ is the matrix element of $\exp(i\boldsymbol{qr})$ operator, evaluated at the wave functions (3)

$$J_{\alpha\alpha'}(\boldsymbol{q}) = M_0(q_z)\delta(k_{\alpha'} - k_\alpha + q_x)\, e^{-u/2} u^{\frac{|n_\alpha - n_{\alpha'}|}{2}} L_v^{|n_\alpha - n_{\alpha'}|}(u) \times \exp\left\{i\left(n_{\alpha'} - n_\alpha\right)\varphi + iq_y\left(R^2/2\right)\left(k_\alpha + k_{\alpha'}\right)\right\} \tag{12}$$

$$M_0(q_z) = \int_0^\infty \mathrm{d}z \left|\chi_0(z)\right|^2 \exp(iq_z z), \quad \mathrm{tg}\varphi = \left(q_y/q_x\right), \\ u = \left(q_x^2 + q_y^2\right)\left(R^2/2\right) = \left(q_{11}^2 R^2/2\right), \quad R^2 = \left(c\hbar/eH\right) \tag{13}$$

Here $\boldsymbol{q} = (q_x, q_y, q_z)$—is the wave-vector of phonon, ε_∞—is the high-frequency dielectric constant, V—is the normalization volume, $L_v^{|n_\alpha - n_{\alpha'}|}(u)$ is Laguerre polynomial [25], $v = \min\left(n_\alpha, n_{\alpha^1}\right)$.

In case then the electron is thrown by light to Landau level n = 2 in the sum by n_{α_1} the term with $n_{\alpha_1} = 1$ will be a resonant one since it corresponds to a real resonant transition between the neighboring Landau levels. Assuming $n_\alpha = 2$, $n_{\alpha_1} = 1$ for the resonant term we will have

$$\sum\nolimits_1^{(p)}(2,\varepsilon) = \eta A\omega_0\left(\gamma + \lambda + i\delta\right)^{-1} \tag{14}$$

where

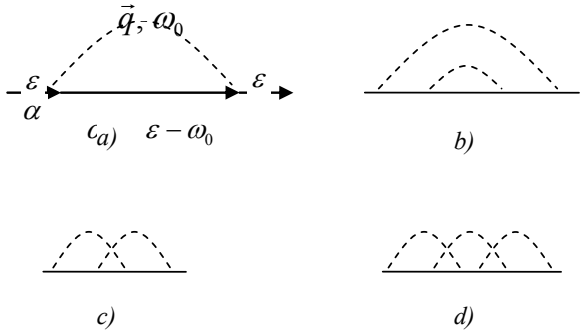

Figure 2. Examples of graphs, which are essential for calculation of the self-energy part.

$$\eta=\frac{\alpha_0}{2}\left(\frac{\omega_c}{\omega_0}\right)^{\frac{1}{2}},\quad A=\frac{1}{2}\int_0^\infty \mathrm{d}u\sqrt{u}\,(2-u)^2\,F(u,a)\,\mathrm{e}^{-u}$$

$$F(u,a)=\left(1+\sqrt{u/a}\right)^{-3}\left(1+\frac{9}{8}\sqrt{u/a}+\frac{3}{8}\frac{u}{a}\right) \tag{15}$$

$$\gamma=\left(\varepsilon-\omega_{c2}\right)\big/\omega_0,\ \lambda=\left(\omega_c-\omega_0\right)\big/\omega_0,\ a=\left(R^2b^2/2\right)$$

Graphs with two phonon lines, which are presented in **Figures 2(b)** and **(c)** in the area of resonance when $\lambda=0$ are, correspondingly, equal to

$$\sum\nolimits_{21}^{(p)}(2,\varepsilon)=\omega_0\left(\eta^2/\gamma^3\right)A\cdot B \tag{16}$$

$$\sum\nolimits_{22}^{(p)}(2,\varepsilon)=\omega_0\left(\eta^2/\gamma^3\right)D \tag{17}$$

where

$$B=\int_0^\infty \mathrm{d}u\sqrt{u}F(u,a)\mathrm{e}^{-u},$$

$$D=\int_0^\infty \mathrm{d}u\sqrt{u}\,(2-u)\,F(u,a)\mathrm{e}^{-u} \tag{18}$$

$$\int_0^\infty \mathrm{d}u'\sqrt{u'}\,(2-u')F(u',a)\mathrm{e}^{-u'}J_0\left(2\sqrt{uu'}\right)$$

The general selection rule of the diagrams for the considered case of low temperatures and $\omega_c=\omega_0$ is as follows. For the Landau level with quantum number *n* the essential graphs will be those ones in which *n*-multiple resonant transition with emission of LO-phonons and serial transitions of the electron between the neighboring Landau levels is allowed. Graphs in which number of emitted phonons is greater than the number of possible resonant transitions will have smallness with respect to bonding constant, which is greater, the more "extra" phonon lines in the graph. That is why, for the level *n* = 2, besides graphs in **Figures 2(b)** and **(c)**, another graphs will be essential, for example in **Figure 2(d)**. Graphs presented in **Figure 3** are small with respect to the bonding constant since they contain non-resonant transitions and, correspondingly, non-resonant denominators.

3. Evaluation of the Spectrum of Electron-Phonon System

In order to obtain a qualitative picture, we neglect first of all graphs, which contain apical parts. In this case it is enough to take into account a series of graphs presented in **Figure 4**. Non-resonant terms in all graphs are neglected due to their smallness with respect to the resonant ones. In this case, the summation of the series in **Figure 4** leads to the following equation

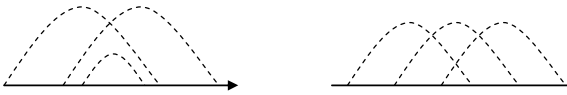

Figure 3. Graphs, which are small as compared with those presented in Figure 2.

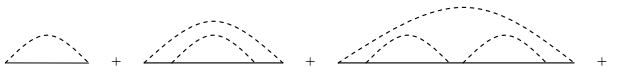

Figure 4. A series of graphs that leads to Equation (19).

$$\gamma-\frac{A\eta}{\gamma+\lambda-\dfrac{B\eta}{\gamma+2\lambda}}=0 \tag{19}$$

If we make a substitution $\gamma=x-\lambda$, then (19) can be presented as follows

$$x^3-\left[\lambda^2+(A+B)\eta\right]x+(B-A)\eta=0 \tag{20}$$

The roots of (20) are

$$x_1=-\frac{2}{\sqrt{3}}\sqrt{\left[\lambda^2+(A+B)\eta\right]}\cos\left(\frac{\alpha}{3}-\frac{\pi}{3}\right),$$

$$x_2=-\frac{2}{\sqrt{3}}\sqrt{\left[\lambda^2+(A+B)\eta\right]}\cos\left(\frac{\alpha}{3}+\frac{\pi}{3}\right), \tag{21}$$

$$x_3=-\frac{2}{\sqrt{3}}\sqrt{\left[\lambda^2+(A+B)\eta\right]}\cos\frac{\alpha}{3}.$$

where

$$\cos\alpha=-\frac{\sqrt{27}\,(B-A)\eta\lambda}{2\sqrt{\left[\lambda^2+(A+B)\eta\right]^3}}. \tag{22}$$

Taking into account the made substitution we have for the spectrum of the electron-phonon system

$$\gamma_1=-\lambda+x_1,\ \gamma_2=-\lambda+x_2,\ \gamma_3=-\lambda+x_3 \tag{23}$$

and when $\lambda=0$ we have

$$\gamma_1=-\sqrt{(A+B)\eta},\ \gamma_2=0,\ \gamma_3=\sqrt{(A+B)\eta}. \tag{24}$$

When $\lambda>0$ and $\lambda^2\gg(A+B)\eta$ from (23) we have

$$\gamma_1=-2\lambda-\frac{(A+B)\eta}{2\lambda^2},\ \gamma_2=-\lambda,\ \gamma_3=\frac{(A+B)\eta}{2\lambda^2}\to 0. \tag{25}$$

i.e., the lower branch of the spectrum disappears, at high magnetic fields two branches remain (see **Figure 1**).

If $\lambda<0$ and $|\lambda|\gg\sqrt{(A+B)\eta}$, then we have a reverse picture

$$\gamma_1=-\frac{(A+B)\eta}{2\lambda^2}\to 0,\ \gamma_2=|\lambda|,\ \gamma_3=2|\lambda|+\frac{(A+B)\eta}{2\lambda^2}. \tag{26}$$

i.e., upper branch of the spectrum disappear, and also two branches remain (see **Figure 1**).

Under disappearance of the upper branch of the spectrum γ_1 we should understand disappearance of the

bond between states of electron with $n = 1$ and $n = 0$. Actually, when the magnetic field decreases the cyclotron frequency becomes so much lesser than ω_0 that the transition of electron thrown by light to level $n = 2$ trough level $n = 1$ becomes impossible. Since the energy levels are discrete ones the energy conservation law is not valid. With rising the magnetic field the cyclotron frequency becomes so much higher than ω_0 that the transition of electron from level $n = 2$ to level $n = 1$ and then to level $n = 0$ becomes impossible, and in this case the bond between the levels $n = 2$ and $n = 1$ disappears and two levels $n = 1$ and $n = 0$ remain, and, correspondingly, two branches γ_1 and γ_2 of the spectrum.

Now we account in Equation (19) the apical points. For Landau level $n = 2$ it is necessary to take into account the main graphs of those diverging in the resonance point, which are presented by the series presented in **Figure 5(a)**, where the bold line presents the series in **Figure 5(b)**.

The general term of the series is equal to

$$\sum_{m2} = \frac{\eta^m C_m \omega_0}{\left(\gamma + \lambda - \frac{B\eta}{\gamma + 2\lambda}\right)^m (\gamma + 2\lambda)^{m-1}} \quad (27)$$

where $m = 2, 3, \ldots$

$$\begin{aligned} C_m = & \int_0^\infty \mathrm{d}u_1 \sqrt{u_1} F(u_1, a)(2 - u_1)\mathrm{e}^{-u_1} \\ & \times \int_0^\infty \mathrm{d}u_2 \sqrt{u_2} F(u_2, a)(2 - u_2)\mathrm{e}^{-u_2} \times \cdots \\ & \times \int_0^\infty \mathrm{d}u_{m-1} \sqrt{u_{m-1}} F(u_{m-1}, a)(2 - u_{m-1})\mathrm{e}^{-u_{m-1}} \\ & \times \int_0^\infty \mathrm{d}u_m \sqrt{u_m} F(u_m, a)(2 - u_m)\mathrm{e}^{-u_m} \\ & \times I_0\left(2\sqrt{u_1 u_2}\right) \times I_0\left(2\sqrt{u_2 u_3}\right) \times \cdots \times I_0\left(2\sqrt{u_{m-1} u_m}\right). \end{aligned} \quad (28)$$

($I_0(x)$ is the first order Bessel function [25]). When $m \gg 1$, $C_m \to 0$. For InSb ($\hbar\omega_c = \hbar\omega_0 = 24.4$ meV, $m_c = 0.013 m_0$) $a = 1.8$ and $C_2 = 0.175$, $C_3 = 0.013$ etc.

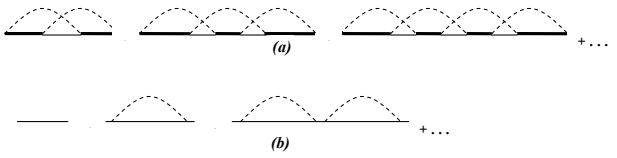

Figure 5. Essential apical parts.

Taking into account the term $\sum_{22}$ from (17) in the dispersion Equation (19) leads to addition of term (17) to the left side of the equation. Solution of (19) by taking into account this addition gives respectively small change in roots, which can be neglected.

4. Splitting of Magnetooptical Absorption and Results Discussion

Three terms of the electron-phonon system, which in the absence of interaction, are interceptions in point $\omega_c = \omega_0$, after accounts of electron-phonon bonding are splitting, according to results of Section 2, into three branches of electron-phonon spectrum. The peak of magnetooptical absorption, which corresponds to excitation of electron by light to Landau level with $n = 2$, will also split at $\omega_c \cong \omega_0$ into three δ-like peaks. The absorption will be determined by three branches of electron-phonon spectrum in the conduction band since when the condition $(m_c/m_v) \ll 1$ is valid, the holes do not contribute into non-stationarity of the levels.

Now we calculate the absorption as a function of light frequency in the region of the studied peak by neglecting the small contribution of apical parts.

By taking into account Expressions (5), (6) and (19), as well as inequality of effective masses we have

$$W = W_0 \pi \left(\frac{\omega_c}{\omega_0}\right) \delta\left(\Gamma - \frac{A\eta}{\Gamma + \lambda - \frac{B\eta}{\Gamma + 2\lambda}}\right), \quad (29)$$

where

$$\Gamma = \left(\omega - \omega_g - \omega_{0c} - \frac{5}{2}\omega_c\right) \Big/ \omega_0, \quad \omega_{0c} = E_{0c}/\hbar. \quad (30)$$

Using the properties of δ-function [26], from (29) we have

$$\begin{aligned} W \neq W_0 \pi \left(\frac{\omega_c}{\omega_0}\right) = & \left(\frac{1}{2} F_-(\lambda) \delta\left[\Gamma + \lambda + \sqrt{\lambda^2 + (A + B)\eta}\right] + B\eta\delta(\Gamma + \lambda)\right. \\ & \left. + \frac{1}{2} F_+(\lambda) \delta\left[\Gamma + \lambda - \sqrt{\lambda^2 + (A + B)\eta}\right]\right) \times \left[\lambda^2 + (A + B)\eta\right]^{-1}, \end{aligned} \quad (31)$$

where

$$F_\pm(\lambda) = \lambda^2 \pm \lambda\sqrt{\lambda^2 + (A + B)\eta} + A\eta. \quad (32)$$

As one can see from (32) when $\lambda = 0$ the integral intensities of left and right peaks are equal to each other $I_л = I_п = A/(A+B)$, since the central peak has the integral intensity equal to $I_ц = B/(A+B)$ and it is (A/B) times lesser than the intensity of lateral peaks. The peaks are located in points $\Gamma_л = -\sqrt{(A+B)\eta}$ (left peak), $\Gamma_ц = 0$ (central peak) and $\Gamma_п = +\sqrt{(A+B)\eta}$ (right peak). The distance between peaks in this case is the smallest one. If $\lambda > 0$ and $\lambda^2 \gg (A+B)\eta$, then

$$I_л = \frac{(3A+B)\eta}{4\lambda^2}, \quad I_ц = \frac{B\eta}{\lambda^2}, \quad I_п = \left[1 - \frac{(A+B)\eta}{4\lambda^2}\right], \quad i.e.$$

the right peak becomes dominant, $\Gamma_л = -2\lambda - \frac{(A+B)\eta}{2\lambda}$,

$\Gamma_ц = -\lambda$, $\Gamma_п = \frac{(A+B)\eta}{2\lambda}$. If $\lambda < 0$ $\left(|\lambda| \gg \sqrt{(A+B)\eta}\right)$, then the intensity of the left and the right peaks shuffle, and the intensity of the central peak remains unchanged:

$$I_л = \left[1 - \frac{(A+B)\eta}{4\lambda^2}\right], \quad I_ц = \frac{B\eta}{\lambda^2}, \quad I_п = \frac{(3A+B)\eta}{4\lambda^2}, \text{ and}$$

$$\Gamma_л = -\frac{(A+B)\eta}{2\lambda}, \quad \Gamma_ц = |\lambda|, \quad \Gamma_п = 2|\lambda| + \frac{(A+B)\eta}{2\lambda}.$$

Above we assumed that the resonant transition with emission of optical phonon occurs between Landau levels, which correspond to dimension-quantized level with zero number $l = 0$. This limitation is not principled, the difference is just in the magnitude of the matrix element $J_{\alpha\alpha'}$, which determines the magnitude of the levels' splitting. On the other hand, the real depths of quantum-dimensional potential wells in quantum-dimensional nanostructures are quite enough for formation at least two or three Landau levels, which correspond to one dimensional-quantized sub-level with number $l = 0$ and space from each other on the magnitude $\hbar\omega_0$ [27].

If we choose $\hbar\omega_c = \hbar\omega_0 = 24.4$ meV, $m_c = 0.013m_0$, $\eta = 0.01$ (the chosen parameters correspond to InSb), then $R^2 = 2.4\times10^{-12}$ cm, $b^2 = 1.5\times10^{12}$ cm^{-1} (variation parameter b coincides by order of magnitude with inverse thickness of the quantum-dimensional potential well), $a = (R^2b^2/2) = 1.8$, $A = 0.353$, $B = 0.336$. The magnitude of splitting is $\hbar\omega_0\sqrt{(A+B)\eta} = 0.002$ эв, which is possible to be measured.

Since dissipative mechanisms of scattering (for example, by acoustic phonons or impurities) are neglected, the levels of the electronic sub-system are discrete ones, and the lines of absorption appear to be δ-like ones. Naturally, in a real situation the lines are broadened and the picture of splitting becomes less pronounced. However, the considered mechanism of scattering due to LO-phonons is sharply non-monotonous one (in contrast with dissipative mechanisms, which depend monotonically on magnetic field and external frequency) and can be extracted on their background.

Strictly speaking, the developed theory is valid if the energy of ionization of exciton in the magnetic field is small as compared with the energy of electron-phonon bonding, and the magnetic field is restricted by condition $\omega_c = \omega_0$. For example, the ionization energy of exciton in the field $H = 3.5 \times 10^4$ Oe for GaAs is 1.5×10^{-5} eV [28], and this corresponds to the level $n = 0$. With the rising number of Landau levels the ionization energy sharply decreases such, for $n = 2$ it is significantly lesser than above magnitude. On the other hand, splitting effects, as shown above, are 0.002 эВ, since the neglect of exciton states is possible.

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