

The Decisive Role Played by Leibniz in the History of Both Science and Philosophy of Knowledge

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Abstract

The present paper addresses the criticism of Kant that he ignored both the non-classical reasoning of the empiricists and Leibniz’s attempt to found mechanics anew. By taking into account this logical divergence Kant’s antinomies—actually applying Leibniz’s two labyrinths of human reason to particular subjects—represent two parallel ways of reasoning according to the two alternatives of a dichotomy regarding the kind of logic. By adding a dichotomy regarding the kind of mathematics a new conception of the foundations of the science is obtained. Leibniz’s philosophy of knowledge represents the closest approximation to these foundations in both the history of science and the history of philosophy of knowledge.

Keywords

Philosophy, Knowledge, History

1. Kant’s Antinomies as Representing Leibniz’s Two Labyrinths

In the past, superficial appraisals (in particular by Voltaire, a non-scientist and a non-philosopher!) ridiculed Leibniz’s thinking as the product of a stubborn metaphysician. Moreover, Kant made some radical criticisms of Leibniz’s philosophy. It was the first time that the fortune of a philosopher (Kant) was founded on the depreciation of the previous philosophical system. This tendency among later philosophers had a destructive effect on the historical development of philosophy as a whole. In fact, the mainstream account of the history of philosophy of knowledge diminishes Leibniz’s great achievements.

Most textbooks present Kant’s philosophy as the apex of the history of the

Western philosophy of knowledge. However, few textbooks notice that Leibniz's thinking had greatly influenced Kant¹. The latter declared that in 1768 he had been deeply impressed by reading Leibniz's *Essays* (Kant, 1798: p. 255). This reading was probably the origin in Kant of "the great light" he received in 1769. However, Kant had a partial knowledge of Leibniz's philosophy because he referred mostly to a follower of Leibniz, Wolff, who had changed Leibniz's legacy; e.g. he changed the principle of sufficient reason into a version which was compatible with classical logic; then Kant believed he had "proved" this principle of Leibniz's—actually, Wolff's principle—on the basis of the principle of non-contradiction. Hence, according to Kant there no longer existed two principles of the human mind. A consequence of this fact is that Kant did not suspect that there was anything not subjected to classical logic².

Kant is mainly credited with having reconciled the two philosophical currents which in the history of Western modern philosophy grew into mutual opposition, i.e. empiricism and the rationalism of innate ideas.

In his most celebrated book (Kant, 1781) he summarized this conflict through four dilemmas, each one between an empiricist thesis and its metaphysical antithesis. He offered a proof for each of them. He concluded that the two proofs of both thesis and antithesis of each dilemma constitute a contradiction that is insoluble by pure reason. Hence, he called these dilemmas "the antinomies of human reason". He concluded that he had established the (actually, old philosophical) idea that the thing-in-itself is unknowable. Subsequent scholars attributed great significance to Kant's treatment of antinomies; see e.g. (Williams, 2013).

However, it is rarely recalled that the content of these dilemmas repeats the content of the two labyrinths that Leibniz had seen in human reason³. Let us examine Kant's analysis in the light of Leibniz's two labyrinths.

i) Since his aim was to attack past metaphysical philosophers in the most effective way, Kant deals not with human reason, as Leibniz did, but with that about which metaphysicians usually reason, i.e. the entire World. In fact, Kant's dilemmas reduce the universal import of the two labyrinths to the import of particular instances concerning the World⁴.

ii) Kant represents each of Leibniz's labyrinths through two dilemmas. The labyrinth of infinity—either potential or actual—is represented by Kant's first two dilemmas (called "mathematical") about the finite or infinite divisibility of, in the former dilemma, space and time and, in the latter dilemma, matter. Leibniz's second labyrinth—either law or freedom—is represented by a second pair

¹On the influence of Leibniz on Kant, recently much more was claimed by (Jauernig, 2008).

²This event justifies the great step that subsequently Hegel attempted to take, i.e. to introduce non-classical logic and moreover to attribute to it the highest rank in his philosophical system.

³In his pre-critical writings, often Kant calls "labyrinth" what in the subsequent writings he calls "antinomy". See (Hinske, 1965: p. 486).

⁴In my opinion this is the origin of Kant's "transcendental fallacy"—i.e. the collapse of epistemology (i.e. how we know the World) into ontology (i.e. what there is in the World)—, which was suggested by (Ferraris, 2013). Also the neo-kantian Cassirer (1954: pp. 269-270) rejects this application of Kant.

of Kant's dilemmas (called "dialectical"); the former dilemma repeats the same content of Leibniz's second labyrinth and the latter dilemma translates this labyrinth into the terms of the existence or not of an absolute cause of the World.

iii) At Kant's time the distinction between potential infinity and actual infinity was manifest to the philosophers owing to an animated debate among Leibniz, Euler, Berkeley, D'Alembert and Hobbes on the foundations of the most important advance in mathematics, i.e. infinitesimal analysis. Despite living a century after the invention of this mathematical theory, Kant knew very little of it. In fact, in the dilemmas concerning infinity he drastically reduces the import of the distinction between potential infinity and actual infinity to that conceived by the ancient Greeks, i.e. the distinction between the "finite" and an undefined "infinity"⁵.

iv) In the former two dilemmas Kant links the problem of the kind of infinity with that of the organisation of a totality, the World; which actually is the subject of the other labyrinth; hence, in the former two dilemmas he mixes together the subjects of Leibniz's two labyrinths. In sum, Kant reduces the problem of Leibniz's two labyrinths to the problems of their applications to some un-systematic and inaccurate instances.

2. Kant's Inadequate Conclusion on Each Pair of Proofs in Each Dilemma

v) Kant offers a proof for each thesis and its antithesis. By omitting Kant's arguments⁶, I take into account only their common logical form, which is that of an *ad absurdum* proof. Why did Kant choose this kind of proof? In classical logic it is always possible to change these proofs into direct proofs (Gardiès, 1991). Neither Kant—who believed that it was impossible to depart from Aristotelian logic—, nor subsequent philosophers or logicians changed these proofs into direct proofs. This fact constitutes the first evidence that non-classical logic—in which *ad absurdum* proofs are pertinent—plays a role in Kant's proofs.

vi) Kant believes that classical logic was the only logic⁷. All past commentators of Kant's antinomies assume the same belief. Recently, one particular non-classical logic, intuitionist logic was recognised to be on a par with classical logic. Their laws are mutually incompatible; in particular, the law of double negation fails in intuitionist logic; this failure constitutes the borderline of the two kinds

⁵Moreover, in the past several scholars (e.g. Couturat, 1905: p. 301, fn. 5) have remarked that Kant's conception of the notion of infinity is inconsistent; sometimes he means actual infinity, sometimes potential infinity.

⁶Some scholars have severely criticised Kant's analysis of antinomies. A partial review of these criticisms is given by Loparic (1990) who suggests a tentative rebuttal of them.

⁷Actually even the knowledge of this subject was not good (Kneale and Kneale, 1962, V, 4). About Kant's logic let us recall the drastic appraisal: "terrifyingly narrow-minded and mathematically trivial". (Hanna, 2013) Loparic (1990) interprets Kant's antinomies as caused by the failure of the excluded middle law, more precisely by a divergence between classical logic and a logic whose negation is defined by him as a limitative predicate. However, in such a way Kant plays the improbable role of a (partial and incomplete) forerunner of a strange non-classical logic.

of logic; e.g. a Court's verdict of "not guilty"⁸ does not mean innocent, for lack of evidence of his being extraneous to the crime⁹.

It is easy to recognise—although no empiricist philosopher has noticed this point—that empiricism essentially relies on non-classical logic. For instance, a major instance of empiricist philosophy, Hume's main work (Hume, 1759), includes a large number of doubly negated propositions, which are not equivalent to the corresponding affirmative propositions because of the lack of evidence for the latter (DNPs). Hume often obscured the two negations of a DNPs by means of words of a conversational style. In particular, he often makes use of the word "only" which means "nothing other than, ...", i.e. a DNP. Moreover, he often uses the word "(im)possible" or some other modal words; which are all equivalent to DNPs *via* the S4 model of modal logic (Chellas, 1980: p. 78ff).

In Hume's book the relevance of the DNPs is manifest in the crucial Sect. VII ("On the idea of necessary connection"). Let us consider e.g. the following DNP concerning the relation between cause and effect:

But when many uniform instances appear and the same object is always followed by the same event; we then begin to entertain the notion of cause and connection. We then feel a new sentiment or impression, ... (no. 61)

His word "feel" is equivalent to "it is not true that it is not, ..."; it alludes to a new hypothesis to be tested by both experiment and deduction.

Let us remark also the following DNP:

It seems a proposition which will not admit of much dispute, that all our ideas are nothing but copies of our impressions, or, in other words, that it is impossible for us to think of any thing which we have not antecedently felt, either by our external or internal senses (No. 49).

In this quotation the reasoning leads to state as "impossible" what works as an absurdity. Hence, Hume makes use of *ad absurdum* arguments (although informally presented), which represent the most accurate evidence of reasoning in non-classical logic.

⁸Here and in the following the two negative words of a DNP will be underlined in order to facilitate the recognition of the nature of this kind of proposition. The occurrences of essential DNPs in a text constitutes the clearest evidence for recognizing non-classical logic in a text illustrating a theory.

⁹(Prawitz & Malmnaess, 1968; Grize, 1970; Prawitz, 1976; Dummett, 1977). In the past this point was misinterpreted owing to a widespread opinion that, as a common slogan says, two negations affirm. But this slogan wordy expresses the classical law of double negation, whereas several kinds of non-classical logic are possible when this law fails. Moreover, it is a traditional "dogma" of the Anglo-Saxon linguists that a double negation is a Latinate which is characterized by ambiguity; so that it has to be suppressed in all cases (Horn, 2002: p. 82ff). Moreover, it is a widespread prejudice that non-classical logic cannot be applied to reality. Yet, several kinds of non-classical logic are at present applied to for instance computer science. Remarkably, in analysing a text it is easy to recognize a failure of the double negation law, whereas a failure of the law of excluded middle is not apparent except for specific author's declaration. This new characterization of the borderline between non-classical logic and classical logic by means of the double negation law allow a new readings of old texts and also new interpretation of them. Notice that in the following I will disregard the fact that doubly negated propositions may be of various kinds, because I assume that philosophers of the past used this linguistic figure intuitively, i.e. by referring more to the intended semantic than to formal rules.

The conclusion is a universal DNP:

...where we cannot find some preceding impression, we may be certain that there is no idea. (No. 61)

From the initial guess, “It seems...” of the previous quotation, Hume obtains to “be certain”.

I conclude that the reasoning in the main book of the most illustrious representative of empiricism belongs to intuitionist logic.

Let us come back to Kant’s proofs. In the light of non-classical logic, we see that Kant has wrongly equated the empiricists’ theses, which actually are DNPs, with the negations of the metaphysicians’ theses, or *vice versa*.

Surely, Kant shares a common linguistic mistake. Quine qualified Popper’s philosophy of science as a “Negative epistemology”, (Quine, 1974); Popper’s basic propositions are however not negations, but DNPs (e.g. “Science is fallible [owing to negative experiments]”) (Drago and Venezia, 2007). Even in Mathematical logic the logical translation from classical logic to intuitionist logic obtained by adding two negations to each predicate of classical logic is commonly called “Negative translation”! (Troelstra & van Dalen, 1988: p. 50).

Let us now correctly formalize each dilemma concerning an empiricist thesis and its metaphysical antithesis according to a pluralist logical viewpoint, allowing both kinds of logic: respectively the theses P and \bar{P} . Let us now examine Kant’s comparison of these proofs. First, the *ad absurdum* proofs are justified only for the empiricists’ theses, not for the anti-theses which by belonging to classical logic may be translated into direct proofs. Second, the above two predicates are separated by the failure of the double negation law, which characteristically distinguishes their respective kinds of logic. Hence, the two conclusions also pertain to two different kinds of logic; hence, they do not constitute a contradiction within a single logical theory, but are merely predicates of two mutually incompatible kinds of logic¹⁰. Hence, Kant interpreting the proofs as being mutually contradictory, ignores that they actually represent two different ways of reasoning, each of which is legitimate. In conclusion, Kant was wrong in interpreting his two-track proofs of a dilemma as a contradiction; this situation represents a *formal dichotomic branch* concerning two kinds of logic rather than an antinomy between two basic propositions pertaining to a same logical theory.

3. The Paradoxical Current Account of the History of the Philosophy of Knowledge

In addition to the antinomies, Kant suggested some categories of our knowledge

¹⁰Kant’s “proofs” of at the same time a thesis-expressing an empirical view-and the corresponding antithesis represents a modern instance of what in ancient times the Sophists were able to do. Later Greek philosophers wanted to overcome these apparent antinomies of human reason by re-establishing its correct power: Socrates showed its efficiency of reasoning within even a slave’s mind; Plato assured knowledge by deriving it from absolute Ideas; Aristotle built theories on all aspects of reality, including the theory of the reasoning and even a theory of the theory. Yet after Kant’s “proofs” of the antinomies, later philosophers continued to be fascinated by his presentation of these contradictions, to the extent that they presented them as a milestone of the historical development of the Western philosophy. In particular, Hegel introduced a “logic of contradiction” (which actually is a mere reiteration of Cusanus’ first, inadequate attempt to renew logic; Drago, 2010).

which most scholars have claimed to be the last word on the philosophy of science. Newton elevated the Euclidean notion of space to absolute space and the common notion of time to absolute time (in opposition to him, Leibniz advocated the notions of relative space and relative time). Kant promoted these notions to the first two a priori transcendental categories of human knowledge. Yet, shortly after his time these categories proved to be inadequate in several non-Newtonian theories (above all classical chemistry, thermodynamics), whose basic notions are very different from Newtonian space and time; in particular, the non-Euclidean geometries, born some decades after Kant's death, changed the notion of space so radically that later no one could reconcile it with his philosophy of knowledge. The subsequent developments in logic, geometry, physics and chemistry all proved to be at variance with Kant's philosophy of knowledge. Even those scientists (e.g. Comte, Mach, Enriques, ...) who built "spontaneous" philosophies of knowledge did not confirm Kantianism.

We have to conclude that Kant's philosophy of knowledge was far from having recognized the true foundations of science. Nevertheless, Kantianism enjoys an unjustified dominant position even in the present philosophy of science:

Despite a fundamental lack of clarity on the epistemic status of its "critical philosophy" itself and the penetrating absolute idealist, realist and naturalist attacks on it, Kantianism influences almost all contemporary non-positivist philosophies of science, as well as formalist and intuitionist philosophies of mathematics (Bhaskar, 1981: p. 223).

Few scholars challenged this mainstream philosophy of science.

Kant's criticism, ... represents one of the most serious stoppages we [logicians] received from Kant (Scholz, 1931: p. 55).

4. The Foundations of Science as Constituted by Two Basic Dichotomies

The crucial problem of Western philosophy of knowledge was to recognize its foundations. By comparing past mathematical and physical theories I have obtained a new conception of the foundations of science. In the above a dichotomy regarding the two main kinds of logic was exemplified in a crucial case-study of the history of philosophy. In addition, in Mathematics some decades ago a long and obscure work achieved a new formalization of calculus and more in general of the whole of Mathematics; this constructive mathematics is new to the extent that it avoids (almost all) idealistic notions (Markov, 1962; Bishop, 1967). At present, although its premises are incompatible with those of classical mathematics, it is considered by mathematicians to be on a par with classical mathematics. I conclude that the foundations of science include two basic dichotomies, one in mathematics and the other in logic (Drago, 1987; Drago, 1996).

In particular, I showed that the basic notions and the mathematical techniques of some scientific theories—e.g. L. Carnot's three scientific theories, S. Carnot's thermodynamics, Classical chemistry, Einstein's first theory of quanta, etc.—do

not appeal to idealistic notions representing actual infinity (**AI**), but only notions representing at most potential infinity (**PI**); formally, the mathematics of all these theories belongs to constructive mathematics. Moreover, the original texts of almost all the above theories present a different organization from the Aristotelian ideal—in which the truths are all derived from few axioms—(**AO**); an author of these theories looks for a new scientific method aimed at solving a problem which was unsolvable by common means (**PO**). It is easy to recognize that, owing to its deductive nature, an AO theory is governed by classical logic, whereas a PO theory, owing to its investigative nature, by intuitionist logic (Drago, 2012a). In sum, on the one hand, the two dichotomies may be represented by philosophical notions, respectively infinity and the organization of a theory; on the other hand, they are formally defined, respectively in mathematics and in mathematical logic.

Let us consider how the historical development of science has revealed the basic choices concerning these dichotomies. In ancient times science was dominated by Euclidean geometry, whose choices are AO (being it characterised by the application of the deductive method to five principles) and PI (it makes use of only ruler and compass). With the birth of modern science, Galileo introduced, as the basis of a scientific theory, the experimental evidence obtained by operative tools (which in mathematics are represented by constructive techniques, hence, PI). Shortly after, Descartes deliberately founded the geometrical optics as an AO theory deduced from two principles and making use of the mathematics of ruler and compass, hence PI, i.e. the same pair of choices as Euclid's geometry. After some decades the invention of infinitesimal analysis occurred, which introduced AI into Mathematics, because it dismissed the ancient tradition of making use of PI only. These two notions of infinity gave rise to a conflictual co-existence within modern science. The next physical theory, Newton's mechanics, was deduced from the three celebrated principles; hence, it was again an AO theory as were those of Euclid and Descartes. Rather Newton changed the kind of mathematics, by making use of infinitesimals, i.e. AI¹¹. Newton did not comment on this pair of choices (rather he emphatically commented on the absolute nature he attributed to the corresponding two basic notions, space and time). This pair of choices proved to be so powerful in cultural terms that it established this theory as a paradigm of theoretical physics for the next two centuries.

However, before him Galileo had performed a remarkable reflection on the foundations of science. He recognized all the basic choices: PI and AI in the first journey of the *Discourse*, AO and PO through the structure of the previous book and also the antecedent book, *Dialogue*; in each of them he alternated deductive parts, written in Latin, clearly in an AO-style, with inductive, investigative parts, written in Italian, clearly referring to a PO-style. Yet, in the latter book he confessed that he was unable to decide between PI and AI (and implicitly also between AO and PO) (Coppola and Drago, 1984; Drago, 2017a). In sum, among

¹¹Koyré's celebrated interpretation of the birth of modern science is based, as the same title of his main book declares (Koyré, 1957) on the introduction of the notion of infinity in scientific thinking.

the founders of modern science, only Galileo recognised the four choices; yet he was unable to characterize their roles in theoretical physics¹². Newton's two subsequent choices enjoyed undisputed authority for two centuries thanks to the marvellous results of his theory.

5. The Emergence of the Basic Choices in Philosophy

Let us quickly re-visit the past history of modern philosophy of knowledge in the light of these two dichotomies. *We will investigate to what extent the basic choices regarding them were recognized by the leading philosophers of the past.* Of course, the resulting review of their philosophies will be very sketchy; it deals very little with the issues that are usually recalled by textbooks, and even less of the past debates among philosophers.

A very quick analysis of ancient philosophies shows that they have implicitly suggested something of the basic choices concerning the two dichotomies. Socrates' dialogical method is related to choices IP (adherence to hard facts) and OP (the various problems of his dialogues); then Plato derived human knowledge from Ideas of an AI nature; whereas Aristotle based knowledge on the choices OA (apodictic science) and syncategorematic infinity, i.e. PI. Notice that he considered his choices separately.

Cassirer (1927) presented Nicholas Cusanus (1401-1464) as "the first modern philosopher of knowledge". Indeed, Cusanus made infinity knowable through formal means; as well as representing PI through an approximating series of mathematical objects, e.g. polygons towards a circle (a series already suggested by Archimedes), he introduced—through the wordy definition of an infinitesimal—the AI in geometry. Moreover, he stressed that the mind works through two faculties; one is the discursive *ratio*, proceeding by means of the principle of non-contradiction; the other one is the *intellectus*, producing, independently from the previous principle, *coniecturae* (roughly: conjectures) which are governed by what Cassirer (1927: pp. 15, 31) has called "a new logic" (Drago, 2017b). Indeed, I have recognized in Cusanus' writings the use of, as opposed to the dominant choice AO, the PO and moreover the use of double negations of intuitionist logic. As an instance of this use, it is enough to recall that he named God as *Non Aliud* and moreover he stressed that these two negative words are

¹²Two more ways to obtain the same two dichotomies are represented by the interpretations of the crises that occurred in Mathematics and in Physics around the beginnings of the 20th century. Against the dominant paradigm of Newtonian mechanics, relying on both the choices AI and AO, on one hand the discovery of quanta introduced the discrete (PI), and on the other hand, moreover, special relativity made use not of a differential equation (AI), but the Lorenz' transformations group (PI); moreover, the latter theory was organized as a "principle theory" (in Einstein's words), i.e. a PO theory. In Mathematics, against both the paradigmatic choice AI of rigorous calculus and the choice AO of Euclidean geometry, the intuitionist Brouwer respectively introduced a kind of constructive mathematics (PI) and he distrusted the axioms of classical logic (and hence AO); also the adversary mathematicians, the Formalists, had to accept that the AO is destabilized by Goedel's theorems. The first PO mathematical theory was Lobachevsky's theory of parallels (1840) and the first PO logical theory was Kolmogorov's minimal logic in 1925. As a further verification of the two above dichotomies I have characterized through them the interpretative categories of the main historians of science (Koyré, Kuhn, Mach, etc.) (Drago, 2001a).

not equivalent to the corresponding affirmative name, *Idem*; hence, here he recognized the failure of the double negation law, i.e. occurrence of the characteristic phenomenon of intuitionist logic¹³.

In sum, against the paradigmatic AO Cusanus made use of the PO and moreover its corresponding non-classical logic; in addition he made use of the two kinds of infinity. Hence, he was aware of all four basic choices concerning the two dichotomies. Unfortunately, his suggestions were subsequently largely ignored.

As a summary of previous analysis, the following **Table 1** compares the two dichotomies with both Leibniz's two labyrinths and Kant's four dilemmas.

6. The Decisive Role Played by Leibniz in the History of Modern Science

Assuredly, Leibniz invented infinitesimal analysis, the most fruitful theory in the history of modern mathematics. Of it he recognized two possible formulations, founded respectively on either infinitesimals (AI) or on constructive elements (PI) (Robinson, 1960, chp. X).

He was also a great logician. By overcoming Aristotle's logic, he anticipated the framework of modern mathematical logic. In addition, his effort to build an alternative theory to the Newton's celebrated mechanics was, contrary to a common prejudice, a well-founded one. Indeed, Leibniz's use of non-classical logic is also apparent in the foundations of his mechanics where he showed that an empiricist approach can consistently produce, beyond general remarks on observational and experimental data, actual physical theories¹⁴ (Drago, 2001b; Drago, 2003).

He maintained that both metaphysics and mathematical infinitesimals have to be excluded, as inappropriate, from the foundations of a physical theory, because in

Table 1. The dichotomies, Leibniz's labyrinths and Kant's categories.

Leibniz's two labyrinths	L1 Labyrinth of the Infinity		L2 Labyrinth of the subjective behavior	
	Actual	Potential	"Law"	"Freedom"
	"The Mathematical antinomies": A1 and A2		"The Dialectical antinomies": A3 and A4	
Kant's four antinomies	A1 Finite or infinite Cosmos in time or space	A2 Finite or infinite divisibility of matter	A3 Either law or freedom	A4 Existence or not of a necessary being
	D1 Kind of Infinity		D2 Kind of Organization	
The two dichotomies	Potential	Actual	Classical Logic	Intuitionist Logic
	Constructive Mathematics	Classical Mathematics	Deductive theory	Problem-based

Legenda: A = Antinomy; D = Dichotomy; L = Labyrinth.

¹³For one more instance of DNP, notice that also the *coniectura* is defined by Cusanus as a DNP: "*assertio in alteritate veritatem uti est participans [= non in sua totalitate]*"; the meaning of its corresponding affirmative proposition, "*assertio in una totalitate*" is different (Drago, 2010).

¹⁴This fact suggests that a reconciliation of the empiricists and the idealists was actually suggested before Kant by the "rationalist" Leibniz.

physics one has to explain facts through facts, “without imaginary hypotheses” (Leibniz, 1677). He believed that the nature of an experimental proposition has to be contingent; this feature is defined by him (and also by Hume) as follows: “its corresponding contrary proposition is not contradictory”; thus, this kind of proposition is a DNP and as such it belongs to intuitionist logic.

He had laid down two DNPs as basic, “architectural” principles: Perpetual motion (i.e. a motion without an end) is impossible”; “No jumps in nature”. As a specific principle of the theory of mechanics he states the inertia principle in the following version: “In-difference of a body to rest and motion”. Leibniz’s reasoning proceeds through DNPs joined together to constitute *ad absurdum* proofs, usually based on the rejection of a motion without an end. His main goal is to satisfy the principle: “Our minds look for in-variants”, i.e. to obtain conservations and symmetries. In fact, he promoted more than any other the conservation of energy.

Unfortunately, Leibniz’s mechanics was not completed due to lack of a mathematical formalization of the principle of virtual velocities, suggested one year after his death by his follower J. Bernoulli.

However, Leibniz’s theory on the impact of elastic bodies was subsequently included and enlarged by Lazare Carnot through the mathematical formula of the principle of virtual velocities (Carnot, 1783). This theory was recently re-evaluated by (Gillispie, 1971; Grattan-Guinness, 1980). It constitutes a complete alternative theory to Newtonian mechanics not only in the notions and techniques, (Drago, 2004) but also in the basic choices (Drago, 1988); it is based on algebraic-trigonometric mathematics, hence PI only, and it is aimed at solving a basic problem—i.e. how to discover the in-variants of the motion of a system of bodies—; hence its organization is a PO. Remarkably, for the first time in the history of theoretical physics its mathematical technique was that of the symmetries (Drago, 1996; Drago, 2003).

As a consequence, the historical development of classical physics may be characterised through, on one hand, the paradigmatic role played by the pair of choices AO and IA—represented by the Newton’s theory, and, on the other hand, the subordinate role played by the pair of choices IP and OP—represented by Leibniz-L. Carnot’s mechanics and moreover some other commonly under evaluated theories (classical chemistry, thermodynamics, etc.)¹⁵.

I conclude that Leibniz had a command of the “hard” science of his time to the extent that he decisively improved it, albeit in an alternative direction to that of Newton’s science.

7. The Decisive Role Played by Leibniz in the History of Modern Philosophy of Knowledge

In the history of philosophy of the knowledge Leibniz was the first philosopher

¹⁵All that is confirmed by the birth of special relativity, which was commonly considered to be an alternative theory to Newton’s mechanics. As remarked in the above, its choices are the opposite ones to Newton’s. Moreover, it may be obtained also by a direct generalization (fn. 12) of L. Carnot’s mechanics (Drago, 2001b; Scarpa, 2002).

who had the opportunity to ponder upon the incompatible foundations of scientific theories, i.e. the two different foundations of calculus and the two different foundations of mechanics (i.e. Newton's and his beginnings of a new formulation). Probably this opportunity suggested to Leibniz the discovery of “*two famous labyrinths* where our reason very often goes astray”, (Leibniz, 1875-90, vol. VI, 29/H 53, my Italics). The labyrinth of either law or freedom presents in subjective terms the basic choices concerning the organization; i.e. “law” represents the external, logical laws one obeys in order to deduce consequences from a few fixed principles of an AO theory; and “freedom” represents the liberty of a scientist looking for a new method capable of solving a stated problem of a PO theory.

Leibniz partially formalized the former labyrinth in mathematical terms through the two possible foundations of calculus. Moreover, he closely approached a logical formalization of the latter labyrinth because he recognized “two great principle of our reasoning”, i.e. *the principle of non-contradiction*—clearly governing the classical logic of an AO—, and *the principle of sufficient reason*.

Let us examine the latter principle, which is variously interpreted. It is a DNP: *Nothing exists without reason, ... or that every truth has its a priori proof... although it is not always in our power to achieve this analysis* (Leibniz, 1686)¹⁶.

In this presentation Leibniz admirably adds an explanation to the first proposition; this proposition is not equivalent to the corresponding affirmative one—which follows the former one—, because of the lack of evidence for the latter; hence Leibniz states that in the former proposition the law of double negation fails, which is the characteristic feature of non-classical logic:¹⁷

¹⁶On the basis of this principle and his recognition of the four basic choices a previous paper reconstructed in modern terms Leibniz's *Scientia generalis*, or the “science of science” (Drago, 1994).

¹⁷By means of a comparison of the original texts of all past PO theories I showed that a PO theory develops through DNPs composing a chain of *ad absurdum* proofs; the final conclusion is a decidable DNP, $\neg\neg U$. An author of a PO theory then makes use of the corresponding affirmative predicate, from which he draws deductively all possible consequences, to be tested by experiment. This translation of the final DPN, $\neg\neg U$, into the corresponding affirmative predicate U —then used as an additive hypothesis for a further deductive development to be verified against reality (Drago, 2012a)—is not justified by any author; yet, it is apparent that there is no other justification than an appeal to the principle of sufficient reason; i.e. an author of a PO theory implicitly believes that through the previous reasoning he has already accumulated all the possible evidence for stating the affirmative translation of the concluding DNP. Hence, the PSR translates this predicate into U , whose logical consequences may be tested against experimental data. In 1962 Markov suggested, although implicitly, the constraints for a correct application of PSR to a predicate (Markov, 1962: p. 5); this one has to be 1) the conclusion of an *ad absurdum* proof, and 2) decidable. It is easily verified that both constraints are satisfied in a PO physical theory. In such a way the application of Leibniz's principle constitutes a translation from intuitionist to classical logic; (Drago, 2012a); as such, it is the inverse translation of what is called the “negative translation” from classical predicate logic into intuitionist predicate logic (Troelstra & van Dalen, 1988: p. 50ff.). Hence, the nature of this principle is not to establish as universally valid its DNP (“Nothing is without reason”), but to allow, in specific cases (decidability as first), the translation of a DNP—representing a theoretical surmise—into its corresponding affirmative proposition, to be considered as a useful hypothesis for deductively obtaining new derivations (Drago, 2017b).

In sum, Leibniz recognized both choices of the dichotomy PI/AI and he partially formalized them. He also recognized the choices of the dichotomy AO/PO, yet in a partial way only; that is, he made use of, but not recognized both the DNPs and the model of development of a PO theory. Moreover, he was undecided between PI and AI and also between PO and AO and he wavered in his use of the two kinds of logic; so that he interpreted the two dichotomies as unresolved *labyrinths*.

In a retrospective view, Leibniz's main shortage was to have given insufficient attention to Cusanus (also because the main book of the latter, *De non Aliud*, was edited in the late 19th Century), who had introduced, much more than the two great principles, the two faculties of the human mind, in particular the *intellectus* arguing in a declared non-Aristotelian logic. One more shortage was unsystematic way of developing his programs¹⁸.

8. Conclusion

The analysis developed in the previous sections leads to the conclusion that in a retrospective view of the history of modern Western philosophy of knowledge, the crucial event is represented by the emergence of the two dichotomies. Unfortunately, no philosopher understood how to deal with them.

When one examines more precisely this emergence one sees that, whereas Leibniz has the merit of having introduced an approximation of the dichotomies, i.e. the two labyrinths, yet he remains undecided about their roles in the philosophy of knowledge; Kant abandoned their generality by applying them to a specific case, the World; moreover, he added (dubious) proofs for each of the alternatives as seen from either the empiricist viewpoint or the metaphysical viewpoint; finally he erroneously considered these proofs to be in mutual contradiction. In such a way he invited philosophers to recognize them as contradictions and then promptly dismiss them, precisely as a diligent scholar has to do with all contradictions; the result was the exclusion of the two labyrinths from the reflections of subsequent philosophers. As a final result he obstructed the activity of human reason, which later had to restrict itself to a formalist approach to reality.

The subsequent historical account of the philosophy of knowledge was above all of an idealistic nature to the extent that it saw such a history as a progressive development. It favors Kant's system because his a priori transcendental categories are considered a preparation for the later idealist philosophies, mainly Hegel's, which is considered by most scholars to be the greatest advance in philosophy. Two centuries of further development of philosophy have denied such an optimistic view of philosophical development. Indeed, despite the alleged "reconciliation" of empiricism and idealism achieved by Kant, some crucial divergences arose after him, i.e. divergences among various philosophers, divergences

¹⁸One reason was suggested by de Santillana (1960: p. 242): "The distinction [between physics and idealism] is so sharply traced as that between the evil and the blessed water. Now, what happened in Leibniz is that the evil entered into the water and heated it to boil."

among scientists (e.g. positivists or not) and a divergence between these two groups; and these divergences all continued to grow.

In the second half of the 20th Century, however, the developments of Mathematics and mathematical Logic suggested that these labyrinths together represent two independent dimensions of human research, capable of developing two systematic approaches to reality

The analysis of the above Section 5 shows that a century before Kant, *Leibniz had offered-through, on one hand, the formal recognition of the two kinds of infinity and, on the other hand, the consistent use of DNPs of non-classical logic, the two philosophical-logical principles and the two labyrinths of human reason—the best anticipation of these two dichotomies and hence of the foundations of science. He was in any case the last and in fact only scientist to manage in a valid and creative way, the relationship between the foundations of modern science and the philosophy of knowledge. For this reason the present analysis ought to be entitled “Leibniz vindicatus”*.

I conclude that it is necessary to abandon the current story of the philosophy of knowledge and to recover the relation of Philosophy with Science by starting from its apex, the main valid suggestions of Leibniz’s thinking.

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