Scientific
Research

http://dx.doi.org/10.4236/ahs.2013.23017

Over and Undershot Waterwheels in the 18th Century. Science-Technology Controversy

Danilo Capecchi

Dipartimento di Ingegneria Strutturale e Geotecnica, Università La Sapienza, Rome, Italy
Email: danilo.capecchi@uniroma1.it

The present paper concerns the development of theory and experiments on water wheels in the 18th century. At that time, as a result of a growing demand for energy, a better understanding of the functioning of watermills, even at the theoretical level, was required in order to improve their efficiency. A hint about the evolution of the theory of wheels in the 19th century is reported also. We have tried to clarify the role played by some protagonists as Antoine Parent, Jean-Charles de Borda and John Smeaton. Their role has not been fully recognised even in contemporary studies. Then some considerations are developed on the relationships between science and technology on this particular subject, concluding that it was a happy and well-balanced marriage.

Keywords: Waterwheels; Hydraulic Machine; Science-Technology; History of Hydraulics

Introduction

Hydraulic machines can be of different kinds; for a synthetic description see (Singer, 1957; Strock & Teague, 1952; Cardwell, 1965; Syson, 1980). In the present paper reference is made to waterwheels only, for their widespread diffusion and since their development allows for an understanding of the science-technology relationship. Mainly two types of waterwheels exist: the undershot waterwheels and the overshot waterwheels, whose difference is shown in **Figure 1**. There is also another interesting wheel, intermediate between the two, the breast wheel, in which water enters from the mid points—or breast—of the wheel.

In the undershot machines, water flows beneath the wheel and hits blades or paddles evenly diffused around the periphery of the wheel. They are moved by the impulsion of the particles of water. In the overshot machines water is led above the wheel. Instead of blades, there are often buckets which get filled with water and move the wheel by means of the water gravity itself.

The waterwheel steadily evolved since its introduction 2000 years ago, to pump water and mill grain. It is not clear where it had its origin; it is clear however that it rapidly spread out as described by Roman, Greek and Chinese sources. There is evidence that the familiar vertical waterwheel developed within the Roman Empire and rapidly spread out; the undershot waterwheel was more common (Denny, 2003). Overshot wheels required a large head (2 - 10 m), therefore they were usually confined to hilly areas, or required extensive and expensive auxiliary constructions. On the other hand, undershot wheels, could operate with a small head (0.5 - 2 m), hence they could be located on small streams in flat areas, near to population centres. It is widely considered that the most dramatic industrial consequences of waterwheels occurred in the Middle Ages, when the scale of milling considerably increased with the de-velopment of large towns. Their considerable economic and social impact may be judged by the increased application of waterwheels. From grinding wheat and pumping water in antiquity, water powered mills evolved to forge iron, full cloth, saw wood and stone, and for metalworking and leather tanning (Denny, 2003: p. 194).

In the 18th century the waterwheels received new attention because of the rising of the manufacturing industry and its increasing request of energy. As soon as all the places suitable to install wheels run out, the only way to increase energy was to increase the efficiency, which called for an intervention of scientists and technicians. The problem of efficiency of waterwheels and their history in the 18th century has been the object of rather recent studies (Reynold, 1979; Cardwell, 1965, 1967), which also make general considerations on the role of the hydraulic energy in society. However, it must be said that in some cases these studies present some errors when interpreting the precise contribution of scientists. For example Parent is accused of committing faults he is actually not guilty of while de Borda is given undeserved merits.

The purpose of the present paper is to clarify these misunderstandings and also to present some reflections on the interaction between science and technology in this particular field, in the 18th century. The first point has been developed by considering the contributions on theory and practice of waterwheels published in the scientific journals of the time, mainly the *Mémoires* de l'Académie des science de Paris and the *Philosophical transactions*. A great attention was paid to explain the writings of the 18th century authors without judging them on the basis of the modern standards of mechanics and hydraulics. For the sake of simplicity, some concepts of mechanics of the 18th century were given for granted hoping the presentation to be comprehensible enough for a contemporary reader. The
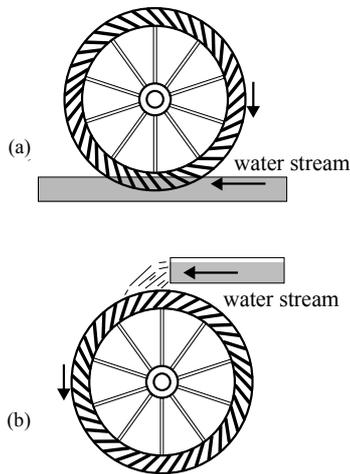
Figure 1.
Examples of undershot wheel (a) and overshot wheel (b).

second point concerning the relationship between science and technology is also based on the content of the mentioned scientific journals. However, the conclusion drawn in such a case are not comprehensive and a more targeted study would be necessary. This notwithstanding some indications can be drawn.

Science and Technology

At the beginning of the 18th century an increasing need of energy was felt by the emerging factories, especially in England. Before the introduction of steam the only way to get energy from nature was by means of the motion of water and air. Waterwheels were suitable machines to collect energy form the rivers. However the available streams of water were limited and an increase of energy could come only by improving the efficiency of the hydraulic machines.

Both scientists and engineers devoted themselves to the task. Hydrodynamics at the beginning of the century was scarcely developed. Only the theoretical results reported by Newton in the second book of his masterpiece, *Philosophiae naturalis principia mathematica* (Newton, 1972) and by Torricelli in his *Opera geometrica* (Torricelli, 1644: p. 265) and the experimental analysis of Edme Mariotte in his *Traité des eaux et autres corps fluides* (Mariotte, 1686) were of some help. In this situation scientists could consider very simplified models only. On the other side engineers, or at least some of them, were no longer practical men; they knew hydraulic quite enough and, mainly, had a scientific attitude versus experiments which were carried out using models of reduced size and accurate measurements. There were thus elements for science and technique to cooperate.

Scientists were the first to be involved. But the results they found were useless from a practical point of view because they were far from the actual findings. For this reason the development of the hydraulic machines in the whole 18th century was greatly influenced by engineers that experimented different kinds of wheels, in particular overshot wheels and wheels with curved blades.

Difference between theory and practice was hardly accepted by the scientific community, thus the need to interpret the ex-

perimental results was pressing. But it took nearly a century from the first theoretical analysis, that of Parent in 1704 (see below), to reach a satisfactory interpretation of the hydraulic phenomena and to suggest a way to build more efficient machines thanks to the studies of French engineers, especially Poncelet (Poncelet, 1825, 1827).

Theoretical Studies

The first attempts to measure the efficiency of waterwheels were carried out by scientists and contributed to the development of hydrodynamics. For example, Edme Mariotte measured the force of water stream by means of a counterbalancing weight, drawing the conclusion that the force varies as the square of the velocity of impact (Mariotte, 1686: p. 205); a result which was already provided by Newton on theoretical basis (Newton, 1726: Part II, Theorem 27). A sophisticated analysis on the undershot wheels was soon given by Antoine Parent in 1704 (Parent 1745: pp. 116-123, 323-338).

Antoine Parent's Analysis of Undershot Wheels

In his memoir *Sur la plus grande perfection possible des machines* (Parent 1745) Parent considered the idealized system of wheels of **Figure 2**, deprived of any friction. A fluid (water) flows through a channel from left to right; it spins the large wheel CBD (**Figure 2**) as a consequence of the force exerted by the fluid in *B*. The rotatory motion of FGH is transmitted by means of teeth to the small wheel HMR, around whose axis a rope wraps and lifts a weight. The fluid flows through the channel EB with uniform velocity *V*; the larger wheel CBD rotates with constant angular velocity and the velocity at point *B* of the immersed blade is equal to *x*, so that the relative velocity of the water with respect to the blade is *V-x*. Parent wanted to calculate the weight *p* that the smaller wheel HMR is able to lift with velocity *u*.

He made the following assumptions: (a) the force exerted by the water flow on the blade in *B* is proportional to the square of the relative velocity between the blade and the water; (b) the Galilean principle of moments (the principle of virtual velocities) can be applied—as the friction is negligible—assuming that a steady state is reached in which the forces are balanced. There were also some assumptions not made explicit, although fundamental; (c) only one blade at a time is considered immersed in water; (d) the stream is considered to be perpendicular to the blade. Parent indicated with *P* the force exerted by the fluid in *B* with the blade at rest (then with a relative speed of the fluid equal to the absolute velocity *V*); and called *natural effect of the fluid* (effect naturelle) the product *PV* (Parent 1745: p. 326). The product *pu* of the weight *p* lifted by its velocity *u* is the (general) *effect of the fluid*. Assume with Parent: $B = AB$, $b = AH$; $C = LH$; $c = LI$, Q the weight *p* necessary to equilibrate *P*[1].

The relation between *P* and *Q* is given by the principle of virtual velocities as $Q = \alpha P$; where $\alpha = BC/bc$ measures the ratio of the virtual displacements of the blade *B* (horizontal) and the weight $p = Q$ (vertical). When the wheel CBD rotates,

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[1]Note that in his calculations Parent used the same symbol *P* to indicate both the suspended weight which equilibrates the wheel and the force exerted on the blade. We prefer to differentiate the symbols, retaining the symbol *P* for the force exerted on the blade and indicating the suspended equilibrating weight as *Q*.
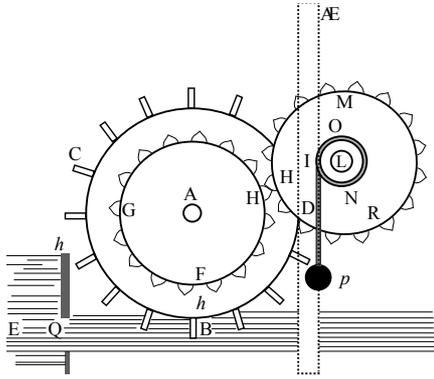
Figure 2.
Parent's undershot wheels

instead of P which is proportional to V^2, the force on the blade in B is P^*, lower than P and proportional to $(V\text{-}x)^2$, being $V\text{-}x$ the velocity of the water relative to the wheel. The weight p can be raised with uniform velocity, i.e. the weight to be equilibrated with P^* (because its motion is assumed to be uniform) is $p = \alpha P^*$; consequently:

$$Q : p = V^2 : (V - x)^2 \qquad (1)$$

which is an equation between x and p. Quite surprisingly for a modern, Parent chose to solve it with respect to x instead of p, which gives:

$$x = V \frac{\sqrt{Q} - \sqrt{p}}{\sqrt{Q}} \qquad (2)$$

With simple kinematical considerations, the velocity u of the weight p is obtained:

$$u = x \frac{bc}{BC} = V \frac{\sqrt{Q} - \sqrt{p}}{\sqrt{Q}} \frac{bc}{BC} \qquad (3)$$

and the effect of the fluid pu:

$$pu = V \frac{\sqrt{Q} - \sqrt{p}}{\sqrt{Q}} \frac{bc}{BC} p \qquad (4)$$

If the geometry of machine (bc and BC) and the absolute velocity V of the fluid are kept as constant, the effect of the machine only depends on p. Parent found the maximum value with the use of *Calculus*:

Art. V. If one now assumes B, C, b, c as constants and p is decreased, or decreased as far as possible, that is to say, we do it through all changes in size which is possible, the value that makes the machine to produce its greatest effect, there will be p variable in the general values of the effect of the preceding article, and taking the differential of the value, namely, $\overline{\sqrt{P} - \frac{2}{3}\sqrt{p}} \times V \frac{bc}{BC\sqrt{P}} dp$ with the purpose to equate it to zero (according to the method of the infinitesimals) it results the equality $\sqrt{P} = \frac{3}{2}\sqrt{p}$, i.e. $\frac{2}{3}\sqrt{P} = \sqrt{p}$, and finally $\left(\frac{4}{9}P = p\right)$ (Parent 1745: p. 331)[2].

From the value $p = 4/9\ Q$ which makes maximum the effect Parent obtained the maximum value of the effect itself simply

by substituting this value of p in the expression (4), also considering the equilibrium relation $Q = P\ BC/bc$:

$$pu = \frac{4}{27} PV \qquad (5)$$

i.e. the effect of the fluid is 4/27 of the natural effect. The optimal value of velocity can be obtained from Equation (2), resulting in $x = V/3$. Notice the all these values are independent of the machine geometry and of the fluid velocity.

If one wants to evaluate the efficiency of the machine, i.e. the ratio between dissipated *living force*[3] and work made in a second—as was done by many scientists and engineers of the 18th century, such as Smeaton, Daniel Bernoulli for instance—this can be made by reworking the Equation (5), assuming that the velocity V results from a downfall from the height H, such that $V^2 = 2gH$ (Torricelli's law). Being A the section of the vein of fluids (**Figure 3(a)**), γ the mass density of the fluid, $q = \gamma AV$ the flow (mass in a second), the force P and the product PV are respectively $P = \gamma AV^2 = 2g\gamma AH$ [4] and $PV = 2g\gamma AVH = 2gqH$; thus Equation (5) gives:

$$pu = \frac{8}{27} qgH \qquad (6)$$

But qgH is exactly the living force dissipated per unit of time ($1/2qV^2$) and pu the work made (per unit of time) by pu, thus the efficiency is 8/27. Parent however did not give this association probably because he could not write $P = \gamma AV^2$, as he only knew that P is proportional to AV^2. Indeed when considering the case of the fluid falling from the height H, he evaluated the effect assuming for P its static value $P = \gamma gH$, with γ the specific mass, g the acceleration of gravity and A the area of the fluid vein (see **Figure 3(b)**). The effect resulted then:

$$pu = \frac{4}{27} A\gamma gHV = \frac{4}{27} qgH \qquad (7)$$

i.e. one half of that obtained assuming for P its dynamic value $P = \gamma AV^2 = 2g\gamma AH$ (see **Figure 3(a)**). This is indeed an incongruence, because if Parent, as it seems from his reasoning, was considering a wheel immersed in a river then he should assume for P the dynamic value; if instead he was assuming that the wheel was placed in a channel having the same width of the blade, P is correctly evaluated by the static value, but the dynamic analysis leading to the Equation (5) is not tenable.

Apart from the last consideration that is not central, it can be said that Parent's approach is elegant and with no errors; its limitations are due to the idealization of the model. His results can be regained at ease using modern notations and concepts.

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[2]Art. V. Si l'on suppose donc maintenant B, C, b, c constant & que l'on diminue, ou que l'on augment p autant qu'il est possible; c'est à dire, qu'on le fasse passer par tous les changements de grandeur dont il est susceptible, afin de trouver sa valeur qui fasse produire à la Machine son plus grand effet on aura p variable dans le valeur générale de l'effet de l'article précèdent, & prenant la différentielle de cette valeur, savoir, $\overline{\sqrt{P} - \frac{2}{3}\sqrt{p}} \times V \frac{bc}{BC\sqrt{P}} dp$ afin de l'égale à zéro (selon la méthode des Infiniment petits) il résulte l'égalité $\sqrt{P} = \frac{3}{2}\sqrt{p}$, d'où l'on tire $\sqrt{P} = \frac{3}{2}\sqrt{p}$, & enfin $\left(\frac{4}{9}P = p\right)$. In this quotation, to be coherent with my symbols, P should be replaced by Q.
[3]Living force is the term used in the 18th century to indicate twice the kinetic energy; so the living force of a body with mass m and speed v is given by mv^2.
[4]The relation $P = \gamma AV^2$ is a classical results of dynamics of fluids.
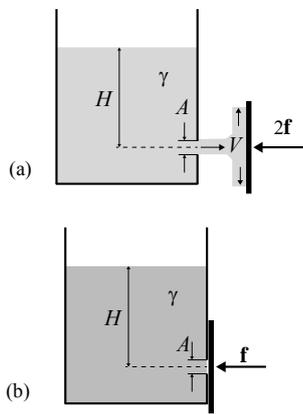
Figure 3.
Dynamic (a) and static (b) forces of a fluid.

To this purpose see (Denny, 2003) where also the analysis of the overshot wheel is reported. Although Parent's analysis was idealised, its results were adopted by many scientists of the 18th century such as John Theophilus Desaguliers (1683-1744), Colin Maclaurin (1698-1746) (Smeaton, 1776: pp. 452-455), Jean D'Alembert (1717-1783) and Leonhard Euler (1707-1783) included.

Jean-Charles de Borda's Analysis

Jean Charles de Borda (1733-1799), in the *Memoire sur les roues hydrauliques* of 1767 (de Borda, 1767), much later than Parent's one, when hydrodynamics had become a mature science, reconsidered the problem of the efficiency of the water wheels. He studied several situations. Besides the classical undershot wheel with plane blades he also studied a wheel with curved blades and an overshot wheel. Here the first case is referred to, while the latter is discussed in one of following sections. The case of the wheel with curved blades is not discussed because it was too difficult a problem for de Borda to solve and was satisfactory solved only in the 19th century (Poncelet, 1825, 1827; Coriolis, 1831, 1835).

De Borda derived the behaviour for the undershot wheel with plane blades starting from the analysis of the wheel having a vertical axis. Since a detailed presentation of de Borda results would be too long, his result will be summarized adapting them to Parent's problem and symbols, also considering that de Borda's text contains many misprints. Moreover as he used two different approaches, one based on the principle of living force[5] the other based on D'Alembert principle (D'Alembert, 1758: pp. 73-75), which however gives the same result, for the same reason of economy the latter only is referred.

At the beginning de Borda assumed, as Parent did, that the single blade was moving with velocity x (de Borda's symbol V), impacted orthogonally by a fluid stream with velocity V (de Borda's symbol B). But the hydrodynamic context is different; while Parent assumed the force on the blade resulting from the friction in a medium, de Borda assumed an impact of the water on the blade moving in a narrow channel as large as the blade. According to D'Alembert principle the impact force of the blade and water is proportional to the lost motion given by the relative velocity $(V-x)$. If q (de Borda's symbol E) is the flow of

the fluid (mass for second) the quantity of motion lost in an interval of time Δt is given by $q(V–x) \Delta t$. The quantity of motion acquired by the weight p is given instead by $p\Delta t$. For the equilibrium, the use of the principle of virtual velocities leads to the relation: $pbc\Delta t = q(V − x)BC\Delta t$ and the effect pu, considering that $u = x\ bc/BC$, is given by $pu = qx(V − x)$, which has its maximum for $x = V/2$ with a value:

$$pu = \frac{1}{4}qV^2 \qquad (8)$$

Assuming that V *is* generated by a downfall of water H, it is $V^2 = 2gH$ and the previous relation becomes:

$$pu = \frac{1}{2}qgH \qquad (9)$$

Thus the efficiency of the undershot machine would be 1/2 (i.e. 50%), that is a much higher value (twice) than that found by Parent. Notice that Equation (9) is ours, obtained completing de Borda reasoning; however somewhere in his memoir (de Borda, 1767: p. 284) he explicitly said that the theoretical maximum efficiency of the undershot machine is 1/2. Adding that in practice this result is never reached, he stated that the lower value 3/8 should be assumed (de Borda, 1767: p. 285).

In a comment de Borda tried to justify his result which is different from Parent's one:

What my solution says is contrary to what has been said so far by the mathematicians who worked on the matter who all found that to produce the greatest impact on a paddle wheel, it should be left to the paddles one third of the velocity of the fluid that hits them, and here I show what this result is based on. It is considered but one paddle on this wheel AB, against which the force is sought of the shock of the fluid; it was found by calling B the velocity of the fluid and V that of the paddle, that the shock was proportional to $(B − V)^2$ and as the effect of the impeller is necessarily proportional to the speed of the blades multiplied by the shock of the fluid, the effect of the wheel is given by $V\ (B − V)^2$, from which it is obtained for the *maximum* $V = 1/3\ B$; but it was observed that the movement in question, the action of the water is not exerted against an isolated blade, but against several blades at a time, and that these blades closing all the breading of the small canal and removing from the fluid the velocity that this has more than that, the amount lost by the fluid, and therefore the shock experienced by the paddle movement is no longer proportional to the square of the difference in fluid velocities and pallets, but only to the difference in the speed; from which it follows that the effect is represented by $V\ (B − V)$, and not by $V\ (B − V)^2$; now matching $V\ (B − V)$ to a *maximum*, we find $V = 1/2\ B$ (de Borda, 1767: pp. 273-274)[6].

A reader, not only a modern one, cannot but remain confused by this argumentation. It seems unrelated from the analysis summarised above, the correctness of which is not discussed here. There are neither theoretical nor experimental argumentations justifying the assertion that when there is more than a blade immersed in the water the force of impact should vary as $(V − x)$ instead as $(V − x)^2$. Why this comment when de Borda has theoretically already proved that for a single blade the force of impact varies as $(V − x)$? This is a mystery, that becomes still greater by noticing that de Borda had good reason not to care-

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[5]In modern term the principle of conservation of mechanical energy.

fully scrutinise and accept his theory, as it was in good agreement with experience, at least as far as efficiency and velocity of the wheel are concerned (see Smeaton's comments, hereafter).

In the secondary literature it is sometimes argued that Parent made calculation errors, and the true efficiency of the undershot wheel is 8/24 instead of 4/27 (Cardwell, 1967; Reynolds, 1979). De Borda would instead have found exact results if he had corrected the error due to the approximation in considering a wheel at the time and a factor two which Parent had neglected:

> In 1767, Borda published a short paper correcting the two main errors of Parent and harmonising theory with experiment (Cardwell, 1967: p. 212).

Parent's theory is actually correct if properly intended. The difference with de Borda's one depends on the different hydraulic context assumed by the two scientists.

Empirical Investigations

### John Smeaton Investigations on Undershot Wheels

The first systematic experiments on waterwheels were probably those of the English engineer John Smeaton (1724-1792) who in 1759 published *An experimental enquiry concerning the natural powers of water and wind to turn mills, and other machines, depending on a circular motion*, before de Borda's memoir. He compared under and overshot wheels.

Smeaton's attention to water wheels was due to the demand of English industry for an improvement of the efficiency of existing water wheels. Being not convinced of Parent's results he performed numerous experiments on the model shown in **Figure 4**, where ABCD is a reservoir which collects water for recirculation after its action on the waterwheel. Water is pumped out of the waterwheel via a hand pump (MN is the handle of the pump, L the pump rod) to another reservoir DE. The water in DE was maintained at a constant level by observing the graduated rod FG, while the water released on the wheel was controlled by the rod HI. A rope connected to the axle of the wheel in O and led through the pulleys P and Q raised a pan of weights, R, used for measuring the wheel's output. The apparatus could be adapted to test overshot wheels as shown by the dotted line in the cross-sectional view.

Smeaton defined the *original power* of the water as the product between the quantity of water expounded in a given time and the height that water comes down from. The *effect* of

the machine is the sum of the weight raised by the action of this water and the weight necessary to overcome the friction, multiplied by the height the weight will be raised to in a given time. The efficiency is the ratio between effect and power (Smeaton, 1759: p. 106-107). In one of his experiments where the power was 3970 pounds × inches in a minute (the product of the flow of 264.7 lb of water multiplied by the height of fall of 15 inches), by varying the raised weight, he found that the maximum effect corresponded to 1266 pounds × inches in a minute (the product of a weight of 9.375 lb raised to and height of 135 inches), for an efficiency of 1266/3970 = 32%, greater than that provided by Parent (25%) but lower than that provided by de Borda (50%). The ratio between the velocity of the blades of the wheel and the velocity of water was often greater than that foreseen by Parent, arriving in some cases close to 1/2 instead of 1/3. Also the weight raised was much greater, (3/4) instead of 4/9 of the equilibrating weight (Smeaton, 1759: p. 115). Smeaton justified the difference between theory and experiment as a consequence of different assumptions:

> It must be remembered, therefore, that, in the present case, the wheel was not placed in an open river, where the natural current, after it has communicated its impulse to the float, has room on all fides to escape, as the theory supposes; but in a conduit or rate, to which the float being adapted, the water cannot otherwise escape than by moving along with the wheel. It is observable, that a wheel working in this manner, as soon as the water meets the float, receiving a sudden check, it rises up against the float, like a wave against a fixed object; insomuch that when the sheet of water is not a quarter of an inch thick before it meets the float, yet this sheet will act upon the whole surface of a float, whose height is 3 inches; and consequently was the float no higher than the thickness of the sheet of water, as the theory also supposes, a great part of the force would have been 10 ft, by the water's dashing over the float (Smeaton, 1759: pp. 113-114).
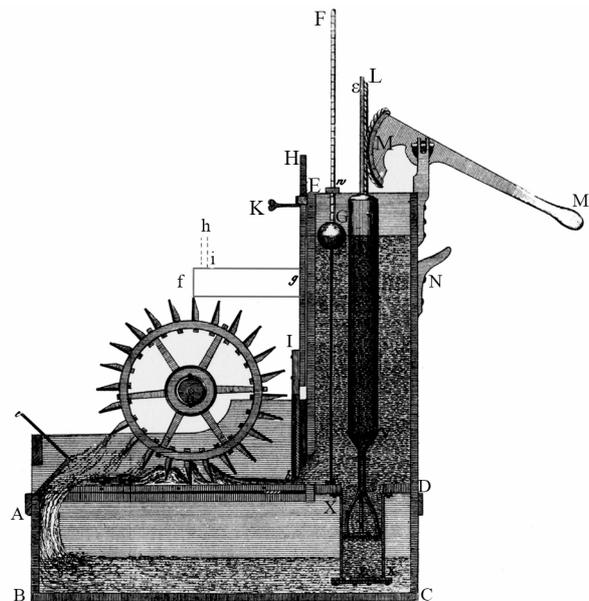
Figure 4.
Smeaton's experimental set (Smeaton, 1756: p. 102).

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[6]Ce que ma solution vient de me donner, est contraire à ce qu'ont dit jusqu'à présent les Géomètres qui ont travaillé fur cette matière en effet, tous ont trouvé, que pour faire produire à une roue à palettes le plus grand effet possible, il ne fallait laisser prendre aux palettes que le tiers de la vitesse du fluide qui les frappait, & voici sur quoi ce résultat était fondé. On ne considérait dans cette roue qu'une seule palette A B, contre laquelle en cherchait la force du choc du fluide; on trouvait, en appelant B la vitesse du fluide & V celle des palettes, que le choc était proportionnel à $(B - V)^2$ & comme l'effet de la roue est nécessairement proportionnel à la vitesse des palettes multipliée par le choc du fluide, on avait l'effet de la roue représenté par $V(B - V)^2$ d'où on tirait pour le *maximum V = 1/3 B*; mais il fallait observer que dans le mouvement dont il s'agit, l'action de l'eau ne s'exerce pas contre une palette isolée, mais contre plusieurs palettes à la fois, & que ces palettes fermant tout le panage du petit canal & ôtant au fluide la vitesse qu'il a de plus qu'elles, la quantité du mouvement perdu par ce fluide, & par conséquent le choc qu'éprouvent les palettes, n'est plus proportionnel au carré de la différence des vitesses des fluides & des palettes, mais seulement à la différence de ces vitesses d'où il suit que l'effet est représenté par $V(B - V)$, & non pas par $V(B - V)^2$; or égalant $V(B - V)$ à un *maximum*, on trouve $V = \frac{1}{2}B$.

In a subsequent paper Smeaton reassumed his experimental results, rhetorically exaggerating the difference between experimental and theoretical findings, asserting also that for large wheel (as the wheels of actual mills), the efficiency is greater arriving up to 50%:

I have found be the commonly received doctrine among theoretical mechanics [...] that, where the velocity of water is double, the adjutage or aperture of the sluice remaining the same the effect is eight time; that is not as the square but as the cube of the velocity [...].

For if that conclusion were true, only 4/27 of the water expended could be raised back again to the height of the reservoir from which it had descended, exclusively of all kinds of friction, &c. which would make the actual quantity raised back again still less; that is, less than one-seventh of the whole; whereas it appears from Table I of the said volume (Smeaton, 1759), that in some of the experiments here related, even upon the small scale on which they were tried, the work done was equivalent to the raising back again about one quarter of the water expended; and in large works the effect is still greater, *approaching towards half*, *which seems to be the limit for the undershot mills*, *as the whole would be the limit for the overshot mills* [emphasis added][...].

The velocity also of the wheel, which according to M. Parent's determination, adopted by Desaguliers and Maclaurin, ought to be no more than one-third of that of the water, varies at the *maximum* in the above mentioned experiments of table, between one third and one half but in all the cases there related, in which the most work is performed in proportion to the water expended and which approach the nearest to the circumstances of great works, when properly executed the *maximum* lies much nearer to one half than one third (Smeaton, 1776: pp. 456-457).

Antoine de Parcieux's and John Smeaton's Experiments on Overshot Wheels

The overshot waterwheel received no attention by the scientists probably because there was the spread opinion that they had the same efficiency as the undershot ones (Reynolds, 1979: p. 274). This was the opinion of Leonhard Euler also, who in his work of 1754 denied that the overshot wheel had any advantage over the undershot ones (Euler, 1754: p. 198). Bernard Forest de Belidor (Belidor, 1782: p. 286) referred that an undershot wheel is six times more efficient that an overshot one, while Desaguliers on the contrary affirmed that a "well-made overshot mill" may be ten times more efficient than an undershot wheel (Desaguliers, 1744: p. 532).

The first known study on the subject was that of Antoine de Parcieux (1703-1768) which is usually classified as an engineer though member of the Académie des sciences de Paris. The interest of de Parcieux derived by the desire of Madame de Pompadour to have current water from the small river the Blaise in Crésy, raising to a height of 50 meters. Because of the small flow of the river, an undershot wheel would not have been able to satisfy the request.

De Parcieux was brought to think that the efficiency of the overshot wheels should be higher than that of the current undershot wheels by assimilating the water, that descends and works as an engine for the water that should be raised, to two weights which are located on two opposite sides of a pulley and are connecters by a rope.

I soon saw that I could get a much better use of water weight, considering it as weights which falling raise others: but how has one to take the wheel (de Parcieux, 1759: p. 607)[7].

He said to have made experiments with a pulley using as power a weight of 96 ounces which raised weights of 24, 32, 40, etc. ounces registering the amount these weights rise in one second, i.e. the velocity in the first part of motion. The velocity ranged from 85 inches per second for a weight of 24 ounces to 20 inches per second for a weight of 72 ounces (de Parcieux, 1754: p. 609).

On the basis of his results de Parcieux suggested a simple experiment by imagining two waterwheels equal to each other but with their buckets inclined in opposite directions. The wheel receiving the falling water was able to raise water in the other wheel under the condition that the raised water was less than that falling one. And, in the same way as in a pulley, the speed of the wheels will depend on the ratio between falling and raising weight. In the limit, if the wheels rotate very slowly, the amount of raised water will be equal to that fallen, and the efficiency of the overshot waterwheel will reach 100%.

The explanation, on the greater efficiency of wheels that rotate slowly actually has no weight. The experiment of the pulley is of course truthful, but here accelerated motions are concerned. In the case of the waterwheel there is instead a stationary motion. In this situation it can be shown that the velocity of the wheel, at least ideally, has no influence on the efficiency. The greater efficiency actually registered for the overshot wheels that rotate slowly depends on the construction methods and operation. In (Denny 2003) the reasons for the efficiency of the overshot wheels to decrease with the increasing of speed are illustrated.

With his apparatus, Smeaton was able to experiment an overshot wheel with water flowing from the tape indicated with *fg* in **Figure 4**. He found that using the same wheel with plane blades, the efficiency was double than that of the undershot wheels and confirmed the results obtained by de Parcieux, that the efficiency of the wheel increased by slowing its speed.

Smeaton was convinced that most of the difference between over and under wheels were due to the loss of living force of the water in the latter case associated to its change in shape during impact. He also proposed an (unsatisfactory) explanation for the increase of efficiency of the overshot wheel by slowing the speed of the wheel, assuming that at the higher speed the efficiency of the water pressure was lower.

When the velocity is *greater* [water] does not press so much upon the bucket as when it is less, the power of the water to produce effects will be greater in the less velocity than in the greater: and hence we are led to this general rule, *that*, caeteris paribus, the less the velocity of the wheel, the greater will be the effect thereof (Smeaton, 1759: p. 133).

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[7]Je vis bientôt que je pouvais tirer un bien meilleur parti du poids de l'eau, en la considérant comme des poids qui en descendant, en enlèveraient d'autres: mais quelle vitesse falloir il faire prendre à la roue.

In the subsequent years Smeaton performed many experiments on the impact of non-elastic bodies assuming that the loss of living force in the impact was due to a change of shape of the bodies. The following quotations resumes Smeaton's ideas about the energy (modern term) required to change the shape of a body.

To obviate this, those of the old opinion seriously set about proving, that the bodies might change their figure, without any loss of motion in either of the striking bodies.

[…]

On the other hand, if it can be shown that the figure of a body can be changed, without a *power*, then, by the same law, we might be able to make a *forge hammer* work upon a mass of soft iron, without any other power than that necessary to overcome the friction resistance, and original *vis inertiae*, of the parts of the machine to be put in motion: for, as no progressive motion is given the mass of iron by the hammer (it being supported by the anvil). no power call be expended that way; and if none is lost to the hammer from changing the figure of the iron, which is the only effect produced, then the whole power must reside in the hammer, and it would jump back again, to the place from which it fell, just in the same manner as if it fell upon a body perfectly elastic, upon which, if it did fall, the case would really happen: the power, therefore to work the hammer would be the same whether, it fell upon an elastic or non-elastic body; an idea so very contrary to all experience (Smeaton, 1782: pp. 342-343).

Thanks to Smeaton, at least in Great Britain, the overshot wheels became dominant, and contrasted the success of steam machines.

However much Mr. Smeaton's valuable observations may have been disregarded by authors, they have not been lost to practical men... [As a result of his experiments] he determined to apply the water, in all cases, so that it should act more by its weight, and less by its impulse; and the advantage gained by that improved construction was found to be fully equal to his expectation. It was afterwards so generally adopted and improved upon by himself and by other engineers in this country, that although undershot water-wheels were, about fifty years ago, the most prevalent, they are now rarely to be met with; and wherever economy of power is an object, no new ones are made (Reynold, 1979: p. 291).

In his *Hydrodinamica* of 1738 Daniel Bernoulli reinterpreted Parent's result calculating in 8/27 the theoretical efficiency related to the living force of the flowing water before the impact upon the blades of the wheel. Bernoulli concluded that the small efficiency of the undershot wheels had to be ascribed to the fact that part of the water living force is lost to keep still the high speed of the water flowing after that the impact against the paddles of the wheel has occurred (Bernoulli, 1738: p. 193). Johann Albrecht Euler—a son of Leonhard—in a memoir submitted in 1754 for a prize competition, which he actually won, analysed separately undershot, gravity and reaction wheels (Euler, 1754). For the undershot wheel he found out Parent's result, i.e. an efficiency equal to 8/27 (Euler, 1754: p. 12). For the gravity wheel (the overshot ones of **Figure 5**) Euler con-

cluded that if the buckets were large enough to collect all the water of the stream and if the diameter of the wheel was equal to the height of the fall, the efficiency of the gravity wheel would be 100%.

Smeaton did not know Daniel Bernoulli's work, neither probably Johann Albrecht Euler's. Equally, at least in 1759, Smeaton did not know de Borda's analysis of the overshot wheels which, though correct, needed to be interpreted.

De Borda, considering a very idealized wheel whose buckets did not leave water, drew the conclusion that an overshot wheel as in **Figure 6**, where the stream of water MN is tangent to the wheel, reaches its maximum effect when $BH = 0$ and the wheel rotates with zero velocity, confirming that the efficiency of this kind of wheel increases by lowering the speed of rotation. Actually de Borda was not precise. Indeed, after a substantially correct analysis, he concluded that the effect of the overshot wheel, using Parent's symbols is given by (de Borda, 1770: p. 281):

$$pu = q\left[\left(\sqrt{2gh} - x\right) + g\frac{H-h}{x}\right]x \qquad (10)$$

with $h = BH$ and $H = BE$, with reference to **Figure 6**. He correctly concluded that the maximum of the efficiency is reached for $h = 0$; but incorrectly that x should be zero. Indeed for $h = 0$ the previous relation should be rewritten as:

$$pu = q\left[g\frac{H}{x}\right]x = qgH \qquad (11)$$

Figure 5.
Euler's gravity wheel (Euler, 1754: Tab. II)

Figure 6.
de Borda's wheel (de Borda, 1767: p. 286).

which indicates an efficiency of 100% and the independence of the velocity x of the wheel. However, at the end of his paper (de Borda, 1770: p. 286), De Borda noticed that the efficiency is in fact substantially independent of x, as its change with x is rather small.

Further Development in the 19th Century

In the 19th century the development of the hydraulic machines was brought in the frame of applied mechanics, where theory and practice were carried out by the same people, the modern engineers, determining a great improvement in the efficiency of all kind of machines. A prominent role was played by the military engineers of the *École de applications de l'Artillerie et du Génie*, in particular Jean Victor Poncelet (1788-1867) and Arthur Jules Morin (1795-1880).

These engineers were so much involved in mathematics and physics as to consider themselves more scientists than practitioners; for instance they addressed their memories to the Académie des science instead to technological journals. They made extensive use of experiments, but not as much as they should to verify the effectiveness of the general mechanical theories behind their designs. The experiments instead had two main purposes. From the one hand they had to highlight some minor defects of the machines to be corrected after a theoretical review of the problem; on the other hand they should furnish numerical values of selected correction factors which allowed transition from theoretical to practical formulas. This was not due to errors in the theory but to simplified assumptions. For example very often the conservation of living forces—or the work—was assumed and then no friction was taken into account; its effect was evaluated by performing experiments under different operating conditions and arranging tables of correction factors.

After some preparatory work (Belhoste, 1994) Poncelet prepared the *Mémoire sur les roues verticales à palettes courbes mues par en dessous, suivi d'expériences sur les effets mécaniques de ces roues* concerning the undershot waterwheels. It was presented before the Académie des science in 1824 and published in the *Annales de Chimie et Physique* in 1825, with minor revisions (Poncelet, 1825); an improved version was published in 1827 (Poncelet, 1827). Poncelet's purpose was to satisfy Lazare Carnot (1753-1823) (and de Borda) requirements for an efficient machine: avoiding loss of living force by impact and by the release of water with significant speed. He reached his goal by assuming curved and inclined blades as shown in **Figure 7**; probably not a new idea, but a good idea that was not pursued with the due firmness:

> The idea to substitute curved blades to plane blades of the old systems seemed so naturel and simple that one can think that its arose to everyone; so I did not attribute a great merit to it. But because the simplest ideas are often those which found the most difficulties to be accepted, I did not want limit myself to theoretical speculations (Poncelet: pp. 144-145)[8].

[8]L'idée de substituer des palettes courbes aux palettes droites de l'ancien système parait si naturelle et simple, qu'il y a lieu de croire qu'elle sera venue à plus d'une personne; aussi n'ai-je pas la prétention de lui attribuer un grand mérite; mais, comme les idées les plus simples son fort souvent celles qui rencontrent le plus de difficultés à être admises, je n'ai pas voulu m'en tenir à des aperçus purement théoriques.

With these devices the undershot wheels could reach, at least theoretically, an efficiency of 100%. The water wheel as proposed by Poncelet is now known as *Poncelet wheel*. Later studies and experiments highlighted some weakness of Poncelet wheels (Belhoste, 1994), which however spread out and for a long time were competing with the water turbine introduced by Benoit Fourneyron (1802-1867) proposed to replace the waterwheel around 1830 (Fourneyron 1840). Contrary to the common belief, turbines did not replace the waterwheels and their design was only in the syllabus of engineering faculties at least until 1940. They disappeared only after the Second World War. Today new attention is paid to properly designed water-wheels, both undershot and overshot, as an economical solution to get energy from water streams with low head (Muller, 2004).

Conclusion

In the early 18th century, as a result of the growing demand for energy, a better understanding of the functioning of the waterwheels, even at the theoretical level, was required in order to improve their efficiency. At the time there was a widespread belief among scientists that it did not really matter which type of wheel to study because the effectiveness would have to be the same for all types of wheels, probably because at an intuitive level one could think that the living force (the kinetic energy) of water is communicated always in the same way.

Despite the theoretical knowledge of hydraulics was very limited, in 1704 Antoine Parent could write a very interesting work on the hydraulics of the undershot wheels that preserved its value for most of the century. Parent's result was not in a perfect agreement with practice and some differences persisted in comparison to experimental research. Smeaton work was for sure of much interest. He finally pointed out what should have been clear to everyone, that is that the overshot wheels could have much higher efficiency, up to twice, than undershot, at least as regards the wheels used in the 18th century.

Meanwhile hydrodynamics was developing thanks to the theoretical work of Leonhard and Johann Euler and D'Alembert, who had also made it possible theoretical investigation of the operation of water wheels. But their theoretical work written on the subject had a very limited impact on the technological development of the wheels, mainly because they were not read by engineers, in particular, they were not read by Smeaton and were soon forgotten. Thus the legacy the experiments and the theoretical speculations of the 18th century left to the 19th century consists mainly of the two points concerning the optimization of the efficiency of water wheels:

a) the impact of the water upon the paddles of the wheel should be avoided;

b) the wheel must move so that the water is unloaded with the minimum possible speed.

These conclusions were collected by Lazare Carnot (Carnot 1786) who based his theory of machines on the conservation of living force and impact for insensible degrees. The problem of the efficiency of waterwheels was dealt with in a different fashion in the 19th century by French engineers, such as Morin and Poncelet. They clarified that the efficiency does not depend on the kind of wheel (over or undershot) but on the way they are designed.

If one had to summarize in a few words the role of science in the technology of water-wheels, he would be attempted to say that it was modest, almost none at all as claimed by Reynolds

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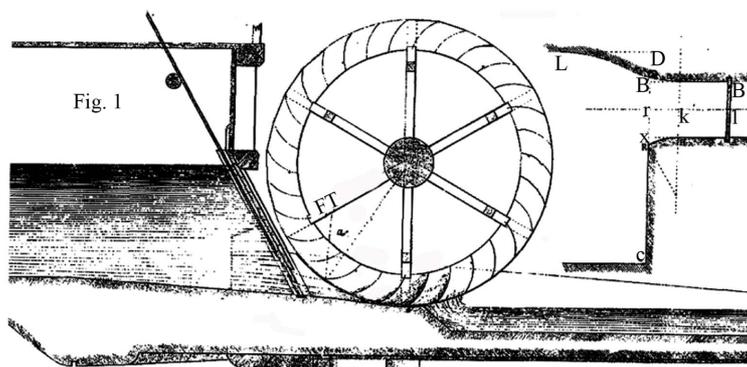

Figure 7.
Poncelet wheel (Poncelet, 1827: Planche 1)

(1979) and Rupert Hall (1961). In our opinion there was instead a fruitful interaction between science and technology. In fact, the application of rational mechanics based on a high formalization had a limited impact, which instead will have a profound influence on the technology of the 19th century. On the contrary, less formalized theoretical considerations had a decisive role, despite their high degree of idealization, such as those of Parent and de Borda. The hydrodynamical studies aimed at assessing the pressure of fluids also had decisive importance. Moreover, considering De Parcieux's and Smeaton's peers as foreign to science, as was done by some historians asserting the low influence of science on technology, is certainly debatable and not shared by all. For example, Musson and Robinson (1969) considered Smeaton's contribution as an example of the direct application of science to technology.

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