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# Temperature-Profile/Lapse-Rate Feedback: A Misunderstood Feedback of the Climate System

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## ABSTRACT

This study shows that the heretofore assumed condition for no temperature-profile (TP)/lapse-rate feedback, $\Delta T(z) = \Delta T_s$ for all altitudes z, or $\Delta\left(\mathrm{d}T(z)/\mathrm{d}z\right) = 0$, in fact yields a negative feedback. The correct condition for no TP feedback is $\Delta T(z)/T(z) = \Delta T_s/T_s$ for all z, where $T_s$ is the surface temperature. This condition translates into a uniform increase (decrease) in lapse rate with altitude for an increase (decrease) in $T_s$. The temperature changes caused by a change in solar irradiance and/or planetary albedo satisfy the condition for no TP feedback. The temperature changes caused by a change in greenhouse gas concentration do not satisfy the condition for no TP feedback and, instead, yield a positive feedback.

**Keywords:** Climate Feedback; Feedback Analysis; Lapse-Rate Feedback

## 1. Introduction

Feedback due to changes in the vertical temperature profile has been called lapse-rate feedback [1]. It has been assumed that the condition for this feedback to be zero is that $\Delta T(z) = \Delta T_s$ for all altitudes *z*, where $T(z)$ is the temperature at z and $T_s$ is the surface temperature [2]. This condition is equivalent to $\Delta\left(\mathrm{d}T(z)/\mathrm{d}z\right) = 0$, that is, no change in lapse rate, hence the name lapse-rate feedback when $\Delta\left(\mathrm{d}T(z)/\mathrm{d}z\right) \neq 0$. Here we first use a one-layer atmospheric model, and then a multilayer atmospheric model, to show what we found 20 years ago [3], namely, that the correct answer for zero feedback is $\Delta \log T(z) = \Delta \log T_s$ or, equivalently,

$$\Delta T(z)/T(z) = \Delta T_s/T_s$$

for all *z*. When this condition is not satisfied, there is feedback. In particular, if $\Delta T(z) = \Delta T_s$ for all altitudes *z*, the feedback is negative. Since there is feedback when the lapse rate does not change, it is recommended that the name lapse-rate feedback be supplanted by the appellation temperature-profile (TP) feedback.

## 2. Feedback Analysis

This section is based on the feedback analysis of Schlesinger [4-6]. The net downward radiation at the top of the atmosphere (TOA) per unit area, *N*, is given by

$$N = \left(\frac{1-\alpha_p}{4}\right)S - \varepsilon_p \sigma T_s^4 \quad (1)$$

where S is the solar irradiance at TOA, $\alpha_p$ is the planetary albedo, $\sigma$ is the Stefan-Boltzmann constant, and $\varepsilon_p$ is the planetary emissivity. The change in N due to external radiative forcing *F*, say due to a change in S or the anthropogenic increase in the concentrations of greenhouse gases, can be written as

$$\Delta N = F + \left[\frac{\partial N}{\partial T_s} + \sum_j \frac{\partial N}{\partial I_j}\frac{\mathrm{d}I_j}{\mathrm{d}T_s}\right]\Delta T_s \quad (2)$$

where the second term on the right-hand side is the change in N due to the change in $T_s$ alone, and the third term is the change in N due to the change in internal quantities $I_j$—such as the temperature profile; water vapor amount; cloud amount, height and optical depth—through their dependence on $T_s$. From Equation (1) we can also write

$$\Delta N = F - \left[4\varepsilon_p \sigma T_s^3 + \left(\frac{S}{4}\frac{\Delta\alpha_p}{\Delta T_s} + \frac{\Delta\varepsilon_p}{\Delta T_s}\sigma T_s^4\right)\right]\Delta T_s \quad (3)$$

For the new equilibrium, $\Delta N = 0$, hence by Equation (2) we have

$$\Delta T_s = G_f F = \left(\frac{G_o}{1-f}\right)F \quad (4)$$

 

where

$$G_f = \frac{G_o}{1-f} \quad (5)$$

is the gain of the climate system with feedback,

$$f = G_o \sum_j \frac{\partial N}{\partial I_j}\frac{\mathrm{d}I_j}{\mathrm{d}T_s} \quad (6)$$

and $G_o$ is the gain of the climate system with zero feedback ($f = 0$). From Equation (3) with $\Delta N = 0$ and

$$\Delta\varepsilon_p = \Delta\alpha_p = 0$$

we obtain,

$$G_o = -\left(\frac{\partial N}{\partial T_s}\right)^{-1} = \frac{1}{4\varepsilon_p \sigma T_s^3} = \frac{T_s}{\left(1-\alpha_p\right) S} \quad (7)$$

the latter from Equation (1) with $N = 0$, that is, the equilibrium before the radiative forcing F is applied. For present-day conditions, prescribed in **Table 1** and calculated in **Table 2**, $G_o = 0.30\ \mathrm{K}/\mathrm{Wm}^{-2}$. Thus if the climate system had zero feedback, the temperature change due to a doubling of the $CO_2$ concentration would be $\left(\Delta T_{2x}\right)_o = G_o F_{2x} = 1.11^\circ\mathrm{C}$ for $F_{2x} = 3.71\ \mathrm{Wm}^{-2}$ [7].

From Equation (6) and Equations (2) and (3) with $\Delta N = 0$ it can be seen that

$$f = -G_o\left(\frac{S}{4}\frac{\Delta\alpha_p}{\Delta T_s} + \frac{\Delta\varepsilon_p}{\Delta T_s}\sigma T_s^4\right) \quad (8)$$

For the feedback we consider here, $\Delta\alpha_p = 0$. Thus, if $\varepsilon_p$ increases (decreases) with increasing (decreasing) $T_s$, $\Delta\varepsilon_p/\Delta T_s > 0$, the change in emissivity works to decrease (increase) $\Delta T_s$, hence as shown by Equation (8), $f < 0$. Conversely, if $\varepsilon_p$ decreases (increases) with increasing (decreasing) $T_s$, $\Delta\varepsilon_p/\Delta T_s < 0$, the change in emissivity works to increase (decrease) $\Delta T_s$, hence as shown by Equation (8), $f > 0$. Below we show that the heretofore assumed condition for zero feedback,

$$\Delta T(z) = \Delta T_s$$

for all altitudes $z$, actually yields a negative feedback, $f < 0$.

**Table 1. Prescribed quantities.**

| Quantity | Value |
|---|---|
| Stefan-Boltzmann constant, $\sigma$ | $5.67 \times 10^{-8}$ Wm$^{-2}$·K$^{-4}$ |
| Solar irradiance, $S$ | 1367 Wm$^{-2}$ |
| Planetary albedo, $\alpha_p$ | 0.3 |
| Longwave absorptance, $a$ | 0.8 |
| Solar absorptance, $b$ | 0.1 |
| Change in $R^{\uparrow}$ due to a $CO_2$ doubling | –3.71 Wm$^{-2}$ [7] |

## 3. Condition for No Temperature-Profile Feedback

Consider the one-layer atmosphere shown in **Figure 1** with temperature, $T_a$, infrared (IR) absorptance, $a$, and solar absorptance $b$. Energy balance at TOA and the surface gives

$$\left(1-a\right)\sigma T_s^4 + a\sigma T_a^4 = R^{\uparrow} = \sigma T_e^4 \equiv \varepsilon_p \sigma T_s^4 \quad (9)$$

$$\sigma T_s^4 - a\sigma T_a^4 = \left(1-b\right)\sigma T_e^4 \quad (10)$$

**Table 2. Calculated Quantities.**

| Quantity | Equation | Value |
|---|---|---|
| Planetary emissivity, $\varepsilon_p$ | (1) with $N = 0$ | 0.63 |
| Equivalent blackbody temperature, $T_e$ | (11) | 254.86 K |
| Surface temperature, $T_s$ | (12) | 285.89 K |
| Atmosphere temperature, $T_a$ | (13) | 245.01 K |
| Gain of the climate system without feedback, $G_o$ | (7) | 0.299 |
| Change in longwave absorptance due to a CO2 doubling, $\Delta a$ | $\Delta$ of (9) $\Delta a = \frac{-\Delta R^{\uparrow}}{\sigma T_S^4 - \sigma T_a^4}$ | 0.021 |
| Longwave absorptance after the CO2 doubling, $a'$ | $a' = a + \Delta a$ | 0.821 |
| Change in $\varepsilon_p$ due to a $CO_2$ doubling, $\left(\Delta\varepsilon_p\right)_e$ | (23) | –0.010 |
| $\Delta T_s/T_s$ | (26) | 0.0045 |
| $\Delta T_a/T_a$ | (27) | 0.0037 |
| $\Delta T_s$ | $T_S\left(\Delta T_S/T_S\right)$ | 1.29 K |
| $\Delta T_a$ | $T_a\left(\Delta T_a/T_a\right)$ | 0.91 K |
| Change in $\varepsilon_p$ due to change in temperatures, $\left(\Delta\varepsilon_p\right)_i$ | (15) | –0.0014 |
| Feedback due to change in temperatures, $f$ | (8) | 0.122 |

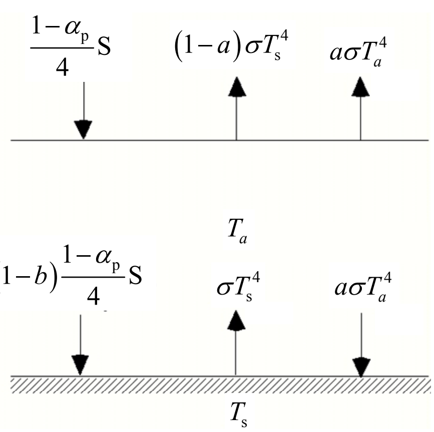


**Figure 1. Schematic diagram of the solar and infrared fluxes in a one-layer atmosphere.**

 

where $R^{\uparrow}$ is the outgoing longwave radiation at TOA, and

$$T_e = \left[\frac{\left(1-\alpha_p\right) S}{\left(4\sigma\right)}\right]^{1/4} \tag{11}$$

is the equivalent blackbody temperature of the planet.

Solving for $T_s$ and $T_a$ yields

$$T_s = \left(\frac{2-b}{2-a}\right)^{1/4} T_e \tag{12}$$

$$T_a = \left[\frac{a+b\left(1-a\right)}{a\left(2-a\right)}\right]^{1/4} T_e \tag{13}$$

From Equation (9)

$$\varepsilon_p = \left(1-a\right) + a\left(\frac{T_a}{T_s}\right)^4 = \tau + a\left(\frac{T_a}{T_s}\right)^4 \tag{14}$$

where $\tau = 1-a$ is the IR transmittance of the atmosphere. For fixed a,

$$\left(\Delta\varepsilon_p\right)_i = 4a\left(\frac{T_a}{T_s}\right)^4 \left[\frac{\Delta T_a}{T_a} - \frac{\Delta T_s}{T_s}\right] \tag{15}$$

where subscript *i* denotes "internal" in contrast to "external", for example, by changing the concentration of $CO_2$. From Equation (8) with fixed $\alpha_p$, the necessary and sufficient condition for no TP feedback is $\Delta\varepsilon_p = 0$, which by Equation (15) requires

$$\frac{\Delta T_a}{T_a} = \frac{\Delta T_s}{T_s} \text{ or, equivalently, } \Delta \log T_a = \Delta \log T_s . \tag{16}$$

This result is readily generalizable to an atmosphere with an arbitrary number of layers K by writing

$$\varepsilon_p = \tau_s + \sum_{k=1}^{K} a_k \left(\frac{T_k}{T_s}\right)^4 \tag{17}$$

where $T_k$ is the temperature of layer k, $a_k = \tau_{k-1/2} - \tau_{k+1/2}$, and $\tau_j$ is the transmittance from level *j* to TOA. For fixed $\tau_j$,

$$\left(\Delta\varepsilon_p\right)_i = 4\sum_{k=1}^{K} a_k \left(\frac{T_k}{T_s}\right)^4 \left[\frac{\Delta T_k}{T_k} - \frac{\Delta T_s}{T_s}\right] \tag{18}$$

Thus the sufficient condition for no TP feedback, $\left(\Delta\varepsilon_p\right)_i = 0$, is

$$\frac{\Delta T_k}{T_k} = \frac{\Delta T_s}{T_s} \text{ for all } k \text{ or, equivalently,} \quad \Delta \log T_k = \Delta \log T_s \text{ for all } k \tag{19}$$

For all practical purposes this is also the necessary condition for no TP feedback.

From Equation (19), $\Delta T_k / \Delta T_s = T_k / T_s$ for all *k*. Thus, for no TP feedback the change in temperature with altitude, $\Delta T_k$, parallels the undisturbed temperature profile, $T_k$. **Figure 2** shows $\Delta T_k / \Delta T_s = T_k / T_s$ for the US Standard Atmosphere. It is seen that for no TP feedback, $\Delta T_k / \Delta T_s = T_k / T_s$ decreases with increasing altitude in the troposphere and increases with altitude in the stratosphere. A similar decrease then increase is needed in the mesosphere and thermosphere, respectively. However, as can be seen from Equation (18), these regions are of less importance than the troposphere and stratosphere because of their smaller absorptance, $a_k$. **Figure 2** shows that the temperature changes required for no TP feedback is less than the heretofore assumed uniform temperature change by as much as 25% at the tropopause and lower stratosphere.

Now suppose $\Delta T_k = \Delta T_s$ for all k, as heretofore assumed for no feedback. Then Equation (18) yields

$$\frac{\left(\Delta\varepsilon_p\right)_i}{\Delta T_s} = 4\sum_{k=1}^{K} a_k \left(\frac{T_k}{T_s}\right)^4 \left[\frac{T_s - T_k}{T_k T_s}\right] \tag{20}$$

Because $T_s > T_k$ for at least the part of the atmosphere where $a_k \left(T_k / T_s\right)^4$ is largest, namely, the troposphere and lower stratosphere, $\left(\Delta\varepsilon_p\right)_i / \Delta T_s > 0$, hence by Equation (8) the feedback is negative, $f < 0$, for a uniform temperature change and thus no change in lapse rate.

How must the lapse rate change for there to be no TP feedback? From the definition of lapse rate,

$$\Gamma = \left(T_k - T_{k-1}\right) / \left(z_{k-1} - z_k\right)$$

where *z* is altitude and k increases from TOA to the surface, it is straightforward to show from Equation (19) that for no TP feedback,

$$\frac{\Gamma'}{\Gamma} = 1 + \frac{\Delta T_s}{T_s} \tag{21}$$

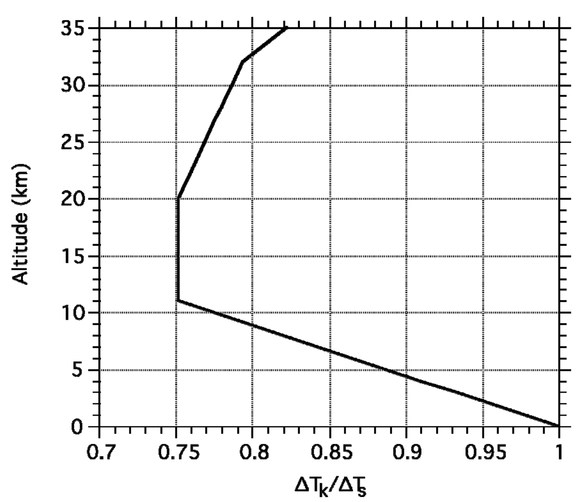


**Figure 2. The vertical profile of $\Delta T(z)/\Delta T_S$ for zero TP feedback for the US Standard Atmosphere.**

where $\Gamma' = \left[\left(T_k + \Delta T_k\right) - \left(T_{k-1} + \Delta T_{k-1}\right)\right] / \left(z_{k-1} - z_k\right)$.

Thus for no TP feedback the lapse rate must increase uniformly with altitude for surface warming, $\Delta T_s > 0$, and must decrease uniformly with altitude for surface cooling, $\Delta T_s < 0$. These changes in lapse rate are not large—for a 3˚C global temperature change they are about 1% of the undisturbed lapse rate.

## 4. Application to Solar Forcing

We now return to the one-layer atmosphere of **Figure 1**. From Equations (11)-(13) for fixed a and b it is straightforward to show that

$$\frac{\Delta T_a}{T_a} = \frac{\Delta T_s}{T_s} = \frac{\Delta T_e}{T_e} = \frac{1}{T_e} \Delta \left\{ \left[ \frac{\left(1-\alpha_p\right) S}{4\sigma} \right]^{1/4} \right\} \tag{22}$$

Thus the response of the atmospheric temperature to a change in $T_a$, either through a change in the solar irradiance or planetary albedo or both, satisfies the requirement for no TP feedback.

**Figure 3** shows the profile of

$$\left(\Delta T_k / T_k\right) / \left(\Delta T_s / T_s\right)$$

in response to a 2% increase in the solar irradiance calculated by our 26-layer stratosphere/troposphere radiative-convective model [3] with the convective adjustment turned off and no temperature dependence of the infrared transmittances. It is seen that

$$\left(\Delta T_k / T_k\right) / \left(\Delta T_s / T_s\right) = 1$$

for all 26 layers, hence TP feedback is zero for solar forcing.

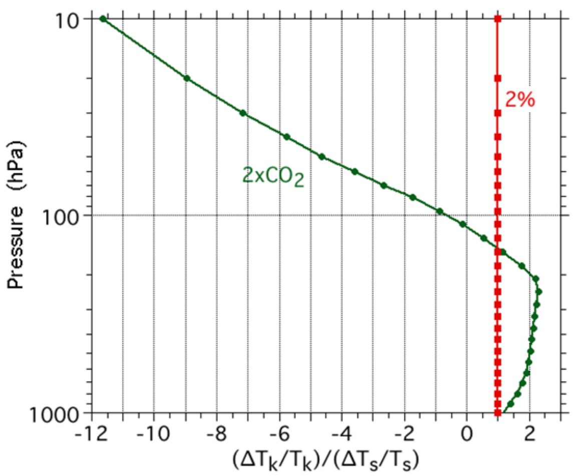


**Figure 3. Vertical profile of ($\Delta T_k/T_k$)/($\Delta T_s/T_s$) versus pressure simulated for a 2% increase in solar irradiance and a $CO_2$ doubling by a 26-layer stratosphere/troposphere radiative-convective model [3] with the convective adjustment turned off and no temperature dependence of the longwave transmittances.**

## 5. Application to Infrared Forcing

In this section we show that condition (16) for no TP feedback is not satisfied for radiative forcing in the infrared, such as from changing the concentration of greenhouse gases, which changes $a$ to $a' = a + \Delta a$. By Equation (14) this will change the planetary emissivity by

$$\left(\Delta \varepsilon_p\right)_e = -\left[1 - \left(\frac{T_a}{T_s}\right)^4\right] \Delta a \tag{23}$$

where the subscript e denotes an "external" change.

The new equilibrium is given by Equations (9) and (10) with $a$ replaced by $a'$, $T_s$ by $T_s + \Delta T_s$, and $T_a$ by $T_a + \Delta T_a$. This yields after using the binomial expansion and linearizing, $\left(1 + \Delta x/x\right)^4 \cong 1 + 4\,\Delta x/x$,

$$\left[1 - a'\right] T_s^3 \Delta T_s + a' \, T_a^3 \Delta T_a = \frac{\Delta a}{4}\left(T_s^4 - T_a^4\right) \tag{24}$$

$$T_s^3 \Delta T_s - a' \, T_a^3 \Delta T_a = \frac{\Delta a}{4} T_a^4 \tag{25}$$

Solving these Equations yields

$$\frac{\Delta T_s}{T_s} = \frac{\Delta a}{4\left(2 - a'\right)} \tag{26}$$

$$\frac{\Delta T_a}{T_a} = \frac{\Delta a}{4a'}\left[\frac{\left(T_s/T_a\right)^4}{2 - a'} - 1\right] \tag{27}$$

From Equations (26) and (27), together with Equations (12) and (13), we obtain

$$\frac{\Delta T_a / T_a}{\Delta T_s / T_s} = \frac{1}{a'}\left[\frac{a\left(2 - b\right)}{a + b\left(1 - a\right)} - \left(2 - a'\right)\right] \tag{28}$$

Setting Equation (28) equal to unity, assuming $a' \cong a$ and solving yields two solutions, $b$ = 0 and $a$ = 2, the former having an unphysical solar absorptance and the latter an unphysical infrared absorptance. Consequently, for the case of infrared forcing, the condition required for zero TP feedback given by Equation (16) cannot be satisfied.

The values of $\Delta T_a/T_a$, $\Delta T_s/T_s$, $\left(\Delta \varepsilon_p\right)_i$ and $f$ calculated for a $CO_2$ doubling for the prescribed values shown in **Table 1** are presented in **Table 2**. It can be seen that the change in the temperature profile yields a negative $\left(\Delta \varepsilon_p\right)_i$ which is the same sign as the change in $\left(\Delta \varepsilon_p\right)_e$ due to the doubling of the $CO_2$ concentration. As a result the feedback is positive and rather large.

This positive TP feedback increases the surface temperature change by 16% from its zero feedback value of 1.11 K to 1.29 K. It occurs even in the absence of a stratosphere, which the one-layer atmospheric model does not possess. The presence of a stratosphere would all the more result in TP feedback because the sign of the stratospheric temperature changes induced by a change in greenhouse-gas concentration is opposite to the sign of the tropospheric temperature changes, thereby not satisfying condition (19) for no TP feedback.

**Figure 3** shows the profile of $(\Delta T_k/T_k)/(\Delta T_s/T_s)$ in response to a doubling of the $CO_2$ concentration calculated by our 26-layer stratosphere/troposphere radiative-convective model [3] with the convective adjustment turned off and no temperature dependence of the long-wave transmittances. It is seen that

$$(\Delta T_k/T_k)/(\Delta T_s/T_s) \neq 1$$

for all layers, hence the TP feedback is not zero. As shown in **Table 2**, $f = 0.122$

## 6. Conclusions

This study has shown the following: 1) the heretofore assumed condition for no temperature-profile (TP)/lapse-rate feedback, $\Delta T(z) = \Delta T_s$ for all altitudes $z$, which gives no change in lapse rate, $\Delta(\mathrm{d}T(z)/\mathrm{d}z) = 0$, in fact yields a negative feedback; 2) the correct condition for no TP feedback is $\Delta T(z)/T(z) = \Delta T_s/T_s$ for all $z$; 3) this condition translates into a uniform increase (decrease) in lapse rate with altitude for an increase (decrease) in surface temperature; 4) the temperature changes caused by a change in solar irradiance and/or planetary albedo satisfy the condition for no TP feedback; and 5) the temperature changes caused by a change in greenhouse gas concentration do not satisfy the condition for no TP feedback and, instead, yield a positive feedback.

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## REFERENCES

[1] R. D. Cess, "Global Climate Change: An Investigation of Atmospheric Feedback Mechanisms," *Tellus*, Vol. 27, No. 3, 1975, pp. 193-198. doi:10.1111/j.2153-3490.1975.tb01672.x

[2] A. Arking, "The Radiative Effects of Clouds and Their Impact on Climate," *Bulletin of the American Meteorological Society*, Vol. 72, No. 6, 1991, pp. 795-813. doi:10.1175/1520-0477(1991)072<0795:TREOCA>2.0.CO;2

[3] C. B. Entwistle, "Analysis of the Nature of Zero Feedback in the Climate System Using a Multilayer Radiative-Convective Model," M.S. Thesis, University of Illinois at Urbana-Champaign, Urbana and Champaign, 1992, p. 201.

[4] M. E. Schlesinger, "Feedback Analysis of Results from Energy Balance and Radiative-Convective Models," In: M. C. MacCracken and F. M. Luther, Eds., *The Potential Climatic Effects of Increasing Carbon Dioxide*, U. S. Department of Energy, 1985, pp. 280-319.

[5] M. E. Schlesinger, "Quantitative Analysis of Feedbacks in Climate Model Simulations of $CO_2$-Induced Warming," In: M. E. Schlesinger, Ed., *Greenhouse-Gas-Induced Climatic Change*: *A Critical Appraisal of Simulations and Observations*, Elsevier, Amsterdam, 1988, pp. 653-737.

[6] M. E. Schlesinger, "Quantitative Analysis of Feedbacks in Climate Model Simulations," In A. Berger, R. E. Dickinson and J. W. Kidson, Eds., *Understanding Climate Change*, American Geophysical Union, Washington DC 1989, pp. 177-187. doi:10.1029/GM052p0177

[7] G. Myhre, E. J. Highwood, K. P. Shine and F. Stordal, "New Estimates of Radiative Forcing Due to Well Mixed Greenhouse Gases," *Geophysical Research Letters*, Vol. 25, No. 14, 1998, pp. 2715-2718. doi:10.1029/98GL01908

 