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An Accurate and Computationally Efficient Explicit Friction Factor Model

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Abstract

The implicit Colebrook equation has been the standard for estimating pipe friction factor in a fully developed turbulent regime. Several alternative explicit models to the Colebrook equation have been proposed. To date, most of the accurate explicit models have been those with three logarithmic functions, but they require more computational time than the Colebrook equation. In this study, a new explicit non-linear regression model which has only two logarithmic functions is developed. The new model, when compared with the existing extremely accurate models, gives rise to the least average and maximum relative errors of 0.0025% and 0.0664%, respectively. Moreover, it requires far less computational time than the Colebrook equation. It is therefore concluded that the new explicit model provides a good trade-off between accuracy and relative computational efficiency for pipe friction factor estimation in the fully developed turbulent flow regime.

Keywords

Colebrook Equation, Explicit Models, Computational Time, Friction Factor, Complexity

1. Introduction

Friction factor estimation is important for modeling flows in pipes and is relevant in most engineering disciplines, for example: chemical, civil and mechanical. Over the years, the Colebrook equation [1] [2] has been widely used for pipe friction factor estimation in the fully developed turbulent regime. The equation is expressed as:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.71} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{1}$$

The major drawback of Colebrook equation is that it is implicit in friction factor (*f*). Therefore, it requires it-

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eration to obtain its solution. For simulations of long pipes and network of pipes, the Colebrook equation must be solved a huge number of times [3]. Therefore, an iterative solution to the Colebrook equation will be time consuming. The use of the Moody chart [4], as an alternative to the Colebrook equation, eliminates the requirement for iteration. However, it is a graphical tool and therefore not convenient for computer-based simulations. The quest for a fast, non-iterative and accurate model, as an alternative to the Colebrook equation, has given rise to various explicit friction factor models. These explicit models differ in their accuracies and relative computational efficiencies, depending on their degree of complexity.

In this work, a new explicit model was developed for estimating friction factor in the range for which the Colebrook equation is valid. The trade-off between model accuracy and relative computational efficiency has been considered.

The remaining sections of this paper are organized as follows: Section 2 reviews the available explicit friction factor models based on accuracy, complexity and relative computational efficiency. In Section 3, the development of the proposed model is presented while Section 4 reports the performance of the proposed model in comparison with those of the selected existing explicit models. In the final section, relevant conclusions are drawn based on the results obtained in this study.

2. Review of the Explicit Forms of the Colebrook Equation

2.1. Accuracy

The accuracies of the existing explicit models have been reported using common criteria such as the mean square error (MSE), percentage relative error and absolute error [5]-[8]. Model selection criteria (MSC) and Akaike information criterion (AIC) were used by Romeo, Royo and Monzon [9] for explicit model selection. These criteria were subsequently used by Genić *et al*. [7] and Yildrim [8] for comparison of several explicit models. Unfortunately, there is an apparent discrepancy in the MSC values reported [7] [9] for the same models. For example, the MSC values reported by Romeo, Royo and Monzon [9] and Genić *et al*. [7] for Moody [10] and Chen [11] models showed a wide contrast.

It has been shown that models with greater number of logarithmic functions are generally more accurate than those with lesser number of logarithmic functions, although the former require more computational time than the latter [6]. For instance, it is observed from works of Brkić [12], Winning and Coole [6] and Fang, Xu and Zhou [13], that the most accurate approximations are those by Zigrang and Sylvester [14], Serghides [15], Romeo, Royo and Monzon [9] and Buzzelli [16]. These models, with the exception of the model by Buzzelli [16], have three logarithmic functions (either natural logarithm or logarithm to base ten).

Brkić [12], based on maximum relative error criterion, classified the existing explicit models as extremely accurate (error $\leq 0.14\%$), very accurate (error up to 0.5%), moderately accurate (error up to 1.5%), less accurate (error up to 5%), non advisable (error up to 25%) and extremely inaccurate (error $\geq 80\%$). Based on this classification, the performances of several explicit models were evaluated and their accuracies are summarized in **Table 1**. Yildrim [8] conducted a comparative review of 16 explicit models. In his work, friction factor data were generated by digitizing the Moody chart. The turbulent portion of the Moody chart is a graphical solution of the Colebrook equation. Hence, digitizing the Moody chart [4] may have introduced secondary errors in the overall analysis [12]. This view is supported by the error margin observed by Fang, Xu and Zhou [13]. Ghanbari, Farshad and Rieke [17] also digitized the Moody chart [4] when developing their model. They claim that the model is valid for Reynolds number (Re) between $2100 \leq \mathrm{Re} \leq 10^8$. It is not obvious how data was obtained for Reynolds number between 2100 and 3000 (*critical zone*), since the Moody chart does not contain Re values in this range.

2.2. Model Complexity and Computational Efficiency

In the bid to develop accurate explicit models, sometimes, simplicity is sacrificed for accuracy without consideration for the actual computational power of such model for massive numerical requirements [12] [18]. Generally, accuracy is obtained at the expense of additional computational complexity. To strike a balance between these two properties (complexity and accuracy), Zigrang and Sylvester [5] introduced the concept of complexity using friction factor models. Based on this concept, Brkić [12] computed the complexity and complexity index for 25 explicit models. His work revealed that models with three logarithmic functions or internal iterations were

Table 1. Existing explicit friction factor models.

Equation Number	Author [Reference]	Explicit Models	Applicable Range of Re and ε/D	Classification
2	Romeo, Royo and Monzon [9]	$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7065} - \frac{5.0272}{Re}\log\left(\frac{\varepsilon/D}{3.827} - \frac{4.567}{Re}\log\left(\left(\frac{\varepsilon/D}{7.7918}\right)^{0.9924} + \left(\frac{5.3326}{208.815+Re}\right)^{0.9345}\right)\right)\right)$	$3\times10^3 \le Re \le 1.5\times10^8$ $0 \le \varepsilon/D \le 5\times10^{-2}$	Extremely accurate
3	Serghides [15]	$f = \left(S_1 - \frac{(S_2-S_1)^2}{S_3 - 2\cdot S_2 + S_1}\right)^{-2}$, $S_1 = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{12}{Re}\right)$, $S_2 = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51\cdot S_1}{Re}\right)$, $S_3 = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51\cdot S_2}{Re}\right)$	Not specified	Extremely accurate
4	Chen [11]	$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7065\cdot D} - \frac{5.0452}{Re}\cdot\log_{10}\left(\frac{1}{2.8257}\left(\frac{\varepsilon}{D}\right)^{1.1098} + \frac{5.8506}{Re^{0.8981}}\right)\right)$	$4\times10^3 \le Re \le 4\times10^8$ $10^{-7} \le \varepsilon/D \le 5\times10^{-2}$	Very accurate
5	Buzzelli [16]	$\frac{1}{\sqrt{f}} = A - \left[\frac{A + 2\log_{10}\left(\frac{B}{Re}\right)}{1+\left(\frac{2.18}{B}\right)}\right]$; where $A - \frac{(0.774\ln(Re)) - 1.41}{\left(1+1.32\sqrt{\frac{\varepsilon}{D}}\right)}$, $B = \left(\frac{\varepsilon Re}{3.7D}\right) + 2.51A$	$3\times10^3 \le Re \le 3\times10^8$ $0 \le \varepsilon/D \le 5\times10^{-2}$	Extremely accurate
6	Zigrang and Sylvester [14]	$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7\cdot D} - \frac{5.02}{Re}\cdot\log_{10}\left(\frac{\varepsilon}{3.7\cdot D} - \frac{5.02}{Re}\cdot\log_{10}\left(\frac{\varepsilon}{3.7\cdot D} + \frac{13}{Re}\right)\right)\right)$	$4\times10^3 \le Re \le 10^8$ $4\times10^{-5} \le \varepsilon/D \le 5\times10^{-2}$	Extremely accurate
7	Barr [19]	$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{4.518\log\left(\frac{1}{7}Re\right)}{Re\left(1+\frac{1}{29}Re^{0.52}\left(\frac{\varepsilon}{D}\right)^{0.7}\right)}\right)$	Not specified	Very accurate
8	Fang, Xu, Zhou [13]	$f = 1.613\left[\ln\left(0.234\left(\frac{\varepsilon}{D}\right)^{1.1007} - \frac{60.525}{Re^{1.1105}} + \frac{56.291}{Re^{1.0712}}\right)\right]^{-2}$	$3\times10^3 \le Re \le 1.5\times10^8$ $0 \le \varepsilon/D \le 5\times10^{-2}$	Very accurate
9	Shacham [20]	$\frac{1}{\sqrt{f}} = -4\log\left[\frac{\varepsilon}{3.7D} - \frac{5.02}{Re}\log\left(\frac{\varepsilon}{3.7D} + \frac{14.5}{Re}\right)\right]$	$4\times10^3 \le Re \le 4\times10^8$	Moderately accurate
10	Sonnad and Goudar [21]	$\frac{1}{\sqrt{f}} = 0.8686\ln\left(\frac{0.4587Re}{S^{(S/S+1)}}\right)$; where $S = 0.124\cdot Re\cdot\frac{\varepsilon}{D} + \ln(0.4587\cdot Re)$	$4\times10^3 \le Re \le 10^8$ $10^{-6} \le \varepsilon/D \le 5\times10^{-2}$	Moderately accurate
11	Manadilli [23]	$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{3.70D} + \frac{95}{Re^{0.983}} - \frac{96.82}{Re}\right)$	$5.235\times10^3 \le Re \le 10^8$ $0 \le \varepsilon/D \le 5\times10^{-2}$	Less accurate
12	Ghanbari, Farshad and Rieke [17]	$f = \left[-1.52\log\left(\left(\frac{\varepsilon/D}{7.21}\right)^{1.042} + \left(\frac{2.731}{Re}\right)^{0.9152}\right)\right]^{-2.169}$	$2.1\times10^3 \le Re \le 10^8$ $0 \le \varepsilon/D \le 5\times10^{-2}$	Less accurate
13	Churchill [24]	$f = 8\left(\left(\frac{8}{Re}\right)^{12} + (A+B)^{-3/2}\right)^{1/12}$; where $A = \left[-2\log\left(\left(\frac{\varepsilon/D}{3.70}\right) + \left(\frac{7}{Re}\right)^{0.9}\right)\right]^{16}$, $B = \left(\frac{37530}{Re}\right)^{16}$	$Re > 0$ $0 \le \varepsilon/D \le 5\times10^{-2}$	Less accurate

Continued

14	Round [25]	$\frac{1}{\sqrt{f}}=1.8\log\left(\frac{\text{Re}}{0.135\cdot\text{Re}\cdot(\varepsilon/D)+6.5}\right)$	$4\times10^3 \le \text{Re}\le 10^8$ $0\le \varepsilon/D \le 5\times10^{-2}$	Non-advisable
15	Brkić [26]	$\frac{1}{\sqrt{f}}=-2\log\left(10^{-0.4343\beta}+\frac{\varepsilon}{3.71D}\right)$ where $\beta=\ln\frac{\text{Re}}{1.816\cdot\ln\left(\frac{1.1\cdot\text{Re}}{\ln(1+1.1\cdot\text{Re})}\right)}$	Not specified	Less accurate
16	Rao and Kumar [27]	$\frac{1}{\sqrt{f}}=2\log_{10}\left(\frac{(2\varepsilon/D)^{-1}}{\left(\frac{0.444+0.135\text{Re}}{\text{Re}}\right)\beta}\right)$ where $\beta=1-0.55e^{-0.33\left[\ln\left(\frac{\text{Re}}{6.5}\right)\right]^2}$	Not specified	Extremely inaccurate
17	Swamee and Jain [28]	$\frac{1}{\sqrt{f}}=-2\log_{10}\left(\left(\frac{\varepsilon}{3.7D}\right)^{1.11}+\frac{5.74}{\text{Re}^{0.9}}\right)$	$5\times10^3 \le \text{Re}\le 10^8$ $10^{-6}\le \varepsilon/D \le 5\times10^{-2}$	Less accurate
18	Vantankhah and Kouchakzadeh [29]	$\frac{1}{\sqrt{f}}=0.8686\ln\left(\frac{0.4587\text{Re}}{(G-0.31)^{G/G+0.9633}}\right)$ where $G=0.124\text{Re}(\varepsilon/D)+\ln(0.4587\text{Re})$	$5\times10^3 \le \text{Re}\le 10^8$ $10^{-6}\le \varepsilon/D \le 5\times10^{-2}$	Extremely accurate

more complex than the models which have two and less internal iterations

Winning and Coole [6] carried out a comparative review of 28 explicit friction factor models. They defined relative computational efficiency as the time taken by an explicit model to perform a task relative to the time taken by the Colebrook equation. The use of computational efficiency in their work clearly showed the impact of model complexity on the simulation time. They found that the models developed by Buzzelli [16] and Serghides [15] were the most accurate when ordered by absolute and relative errors, but when ordered by relative computational efficiencies, they ranked very low. The overall ranking reported was biased since it is not based on actual values of accuracy and relative computational efficiency. It was based on the number of available explicit models. If this number is altered, the values of the combined ranking may change.

Computational efficiency is observed to be dependent on the type of logarithmic function(s) contained in the reported models. The computation of the logarithm function in many computer languages is based on series expansion that requires several powers of arguments to be computed and added to each other [18]. Glustolisi [18] and co-worker state that the natural logarithm function executes faster than the logarithmic function to base ten. This is based on the fact that the convergence function used for its computation is quite fast. Therefore, the computation of the logarithm function to base ten in many computer languages is based on the computation of the natural logarithm [18]. It should be noted that an explicit equation which requires computational time longer than that of the Colebrook's equation defeats the aim of its development. An ideal explicit model should give a good trade-off between its accuracy and relative computational efficiency.

3. The Proposed Nonlinear Model

3.1. Data Generation

Using Microsoft Excel spread sheet, friction factor (*f*) data within an error limit of 10^{-9} were obtained from Equation (1) for Re values in the range $4\times10^3 \le \text{Re} \le 10^8$, using 1000 intervals in geometric order and (ε/D) value ranging from 10^{-6} to 0.05 using 28 intervals in arithmetic order. Thus, producing a matrix of 28,000 datasets for *f*, Re and (ε/D) was obtained for model.

3.2. Model Development

The plot of $\text{Re}\sqrt{f}$ against Re for the different relative roughness gives straight lines, as shown in **Figure 1**. Therefore, the model presented in this work is derived from the implicit Colebrook equation by substitution of the parameter, $\text{Re}\sqrt{f}$ with the equation of a straight line having Re as the abscissa. Then natural logarithm function was introduced to enhance the computational efficiency of the model as noted by Glustolisi [18]. After some rearrangements, the proposed new model was thus obtained as:

$$f=\left(-2\log_{10}\left(\frac{\varepsilon/D}{a}+\frac{b}{\mathrm{Re}}\left(\ln\left(\left(\frac{\varepsilon/D}{c}\right)^{d}+\left(\frac{e}{\mathrm{Re}+g}\right)\right)\right)\right)\right)^{h} \tag{19}$$

Using surface-fitting function in the MATLAB curve-fitting toolbox, coefficients *a*, *b*, *c*, *d*, *e*, *g*, and *h* with their parameter bounds were obtained at 95% confidence level (**Table 2**). The uncertainties associated with the estimated parameters, which are a measure of the reliability of the parameters, and consequently, a measure of the adequacy of the model, are reported in **Table 2**. A model which has parameter estimates with low levels of uncertainties (narrow intervals) is deemed to be good and adequate [30].

3.3. Performance Criteria

1)
$$\text{Absolute error}=\left|f_{\text{Colebrook}}-f_{\text{explicit}}\right| \tag{20}$$

2)
$$\text{Mean square error (MSE)}=\frac{\sum_{i=1}^{i=N}\left(f_{\text{Colebrook}}-f_{\text{explicit}}\right)^2}{N} \tag{21}$$

3)
$$\text{Relative error}=\frac{\left|f_{\text{Colebrook}}-f_{\text{explicit}}\right|}{f_{\text{Colebrook}}}\times 100 \tag{22}$$

4)
$$\text{Mean relative error (MRE)}=\frac{1}{N}\sum_{i=1}^{i=N}\frac{\left|f_{\text{Colebrook}}-f_{\text{explicit}}\right|}{f_{\text{Colebrook}}}\times 100 \tag{23}$$

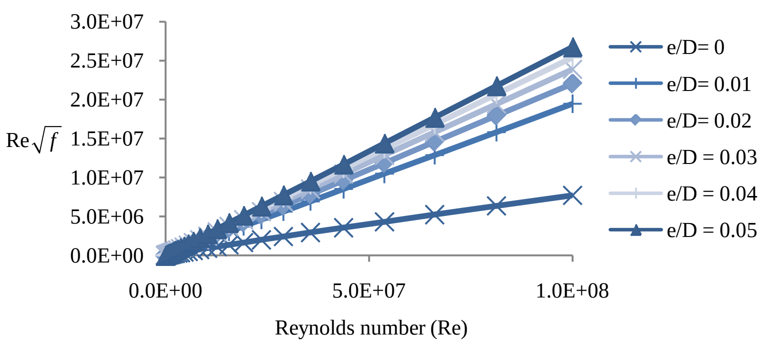

Figure 1. Graph of Re against $\mathrm{Re}\sqrt{f}$.

Table 2. Parameters of the new model.

Coefficient	Value	Parameter Bound	Absolute Relative Uncertainty (%)
a	3.71	(3.71, 3.71)	0
b	−1.975	(−1.975, −1.975)	0
c	3.93	(3.93, 3.93)	0
d	1.092	(1.092, 1.092)	0
e	7.627	(7.626, 7.628)	0.01311
g	395.9	(395.6, 396.2)	0.076
h	−2	(−2, −2)	0

5) Relative Computational efficiency: According to Winning and Coole [6], relative computational efficiency is the ratio of the time required by the explicit model to perform a task to the time required by the Colebrook equation to perform the same task. It means that a model with relative computational efficiency value greater than one (1.0) will require more time than the Colebrook equation to perform a particular task and vice-versa for a model with a value less than one (1.0).

Ten million friction factor calculations were performed using the available explicit models in the ranges of Re and ε/D for which the Colebrook equation is valid. These calculations were performed four times and the average was recorded for each of the explicit model. For this analysis, f values for the Colebrook equation were determined using the method developed by Clamond [3] because of its speed of convergence. The relative computational efficiency was thereafter determined based on the approach proposed by Winning and Coole [6]. The results are as shown in **Table 4**.

3.4. Model Accuracy, Adequacy and Computational Efficiency

It is observed from **Table 3** that the new model (for this study), having the least mean relative and maximum relative errors of 0.0025% and 0.0664%, respectively, is more accurate than the selected extremely accurate models. In addition to the high accuracy of the new model from this study, its parameters are observed to have very low uncertainties ≤ 0.076% (see **Table 2**). This indicates that the parameters are known precisely. Consequently, the model is deemed very accurate and adequate for predicting friction factor.

It is observed from **Table 4** that all the existing extremely accurate models, with the exception of Buzzelli [16] equation, have relative computational efficiencies greater than one (1.0). This is not unexpected, given their complexity with respect to the number of logarithmic functions contained in the models. On the contrary, relative computational efficiency values of less than one have been reported in the work of Winning and Coole [6] for all the extremely accurate models. These values are disputable considering the complexity of these models (in terms of the numbers of logarithmic functions). Our findings show that the Buzzelli [16] model is almost two times faster than the Serghides [15], Romeo, Royo and Monzon [9], Zigrang and Sylvester [14] models. The Buzzelli [16] model has only two logarithmic functions, a combination of logarithm to base ten and the natural logarithm functions. The Buzzelli's [16] model, based on the analysis in this study, is the best existing model in terms of accuracy and relative computational efficiency. However, it is found that that the new model is 39 and 1.9 times (in terms of mean and maximum relative errors, respectively) more accurate than the Buzzelli [16] model (see **Table 3**). Interestingly, the new model has two logarithmic functions and a higher accuracy (see **Figure 2** for error distribution). It has approximately the same relative computational efficiency as the Buzzelli [16] model, which has only two logarithmic functions. Thus, the new model is regarded as a superior model to the existing extremely accurate explicit models.

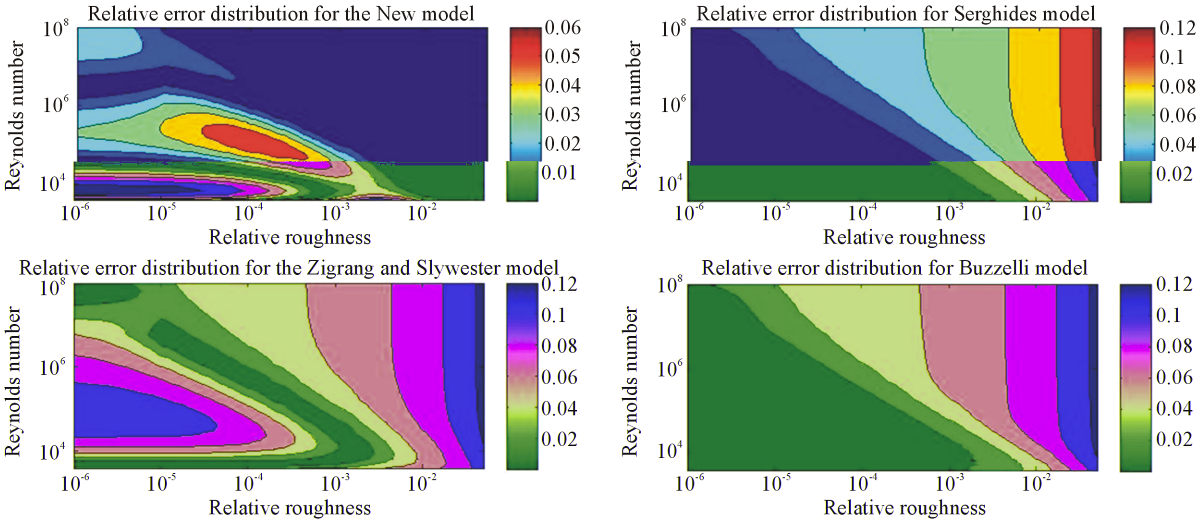

Figure 2. Relative error distribution for this study, Zigrang and Sylvester [14], Serghides [15] and Buzzelli [16] models when compared with the implicit Colebrook equation.

Table 3. Explicit models ordered by maximum relative error.

Equation number	Reference	Absolute Error			MSE	Percentage Relative Error (%)		
		Minimum	Maximum	Average		Minimum	Maximum	Mean
19	This study	3.176E−12	2.306E−05	7.868E−07	4.662E−12	6.730E−09	0.0664	0.0025
2	Serghides [15]	6.465E−08	8.965E−05	5.377E−05	3.446E−09	1.620E−04	0.1255	0.0978
3	Buzzelli [16]	9.740E−13	8.977E−05	5.438E−05	3.511E−09	9.544E−09	0.1255	0.0990
4	Zigrang and Sylvester [14]	3.152E−08	8.965E−05	5.444E−05	3.474E−09	8.454E−05	0.1255	0.1011
18	Vantankhah and Kouchakzadeh [29]	7.882E−11	9.517e−05	2.158E−05	9.836e−10	2.625E−07	0.1332	0.0614
5	Romeo, Royo and Monzon [9]	3.692E−06	6.382E−05	2.449E−05	7.188E−10	2.490E−02	0.1462	0.0477
6	Chen [11]	2.858E−08	1.258E−04	3.456E−05	1.743E−09	1.029E−04	0.3596	0.0709
7	Barr [19]	1.047e−09	3.281E−04	5.207E−05	5.010E−09	2.387E−06	0.5089	0.0942
8	Fang, Xu, Zhou [13]	3.101E−08	4.612E−04	8.178E−05	1.095E−08	7.320E−05	0.5997	0.1645
9	Shacham [20]	2.044E−09	3.464E−04	5.659E−05	4.034E−09	6.200E−06	0.8679	0.1254
10	Sonnad and Goudar [21]	6.593E−06	3.961E−04	8.527E−05	1.093E−08	7.413E−02	0.9926	0.1697
11	Haaland [22]	9.660E−09	7.309E−04	1.713E−04	3.736E−08	2.128E−05	1.2910	0.3241
12	Manadilli [23]	6.261E−09	1.863E−03	2.898E−04	2.159E−07	1.945E−05	2.5827	0.5485
13	Ghanbari, Farshad and Rieke [17]	1.740E−09	2.000E−03	2.657E−04	2.121E−07	1.399E−04	2.7744	0.7810
16	Brkić [26]	5.781E−07	2.178E−03	2.854E−04	2.733E−07	8.089E−04	2.9427	0.5403
14	Churchill [24]	1.529E−07	2.025E−03	3.019E−04	2.864E−07	1.518E−03	3.2178	0.5746
18	Swamee and Jain [28]	1.254E−07	2.479E−03	3.333E−04	3.159E−07	1.271E−03	3.436	0.6300
15	Round [25]	1.551E−08	6.000E−03	2.600E−03	1.033E−05	2.219E−04	8.3383	4.4466
17	Rao and Kumar [27]	5.631E−09	3.991E−02	1.480E−03	1.651E−05	1.195E−05	85.479	5.5086

Table 4. Computational efficiencies of the proposed and existing explicit models.

Equation number	Author [Reference]	Simulation time 1 (s)	Simulation time 2 (s)	Simulation time 3 (s)	Simulation time 4 (s)	Mean Simulation time (s)	Relative Computational efficiency
13	Ghanbari, Farshad and Rieke [17]	2.0018	2.0018	1.9825	2.1063	2.0231	0.3776
3	Buzzelli [16]	2.0160	1.9813	2.0708	2.1527	2.0552	0.3836
19	This study	2.0828	2.0082	2.0730	2.1669	2.0827	0.3883
11	Haaland [22]	2.0833	2.0080	2.2566	2.0259	2.0935	0.3907
16	Brkić [26]	2.0687	2.1409	2.1994	2.1103	2.1298	0.3974
12	Manadilli [23]	2.1511	1.9947	2.3941	2.0287	2.1421	0.3998
18	Rao and Kumar [27]	2.2285	2.4316	2.1779	2.0146	2.2131	0.4130
15	Round [25]	2.5790	2.1902	2.2479	2.1139	2.2828	0.4260
10	Sonnad and Goudar [21]	2.7362	2.71272	2.7368	2.7202	2.7264	0.5088
9	Shakham [20]	3.921	4.1258	4.1667	4.1146	4.082	0.7650
6	Chen [11]	4.3073	4.1367	3.9977	4.1984	4.1601	0.7763
7	Barr [19]	4.5447	4.2223	4.1907	4.3816	4.3348	0.8090
5	Romeo, Royo and Monzon [9]	6.0524	6.2378	6.0786	6.0439	6.1032	1.1390
2	Serghides [15]	6.0707	6.1784	6.1017	6.1326	6.1334	1.1447
4	Zigrang and Sylvester [14]	6.8008	6.0883	6.9692	6.3390	6.5493	1.2222

4. Conclusion

A new explicit model is developed for predicting friction factor in the range for which the Colebrook equation is valid. Until now, the best predictions are obtained with models having three logarithmic functions. The new simple model having only two logarithmic functions and maximum relative error of 0.0664% in this study is found to be more accurate than the selected existing extremely accurate models. Moreover, the relative computational efficiency (0.3883) of the new model is in close agreement with that (0.3836) of the Buzzelli [16] which was adjudged as the best existing model in this work. Therefore, the new model provides a good trade-off between accuracy and relative computational efficiency. Thus it is superior model to the existing explicit models for estimating pipe friction factor in the fully developed turbulent flow regime.

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Nomenclature

f	Darcy friction factor [dimensionless]
D	Internal pipe diameter [m]
ε	Pipe absolute roughness [m]
ε/D	Relative roughness (dimensionless)
Re	Reynolds number (dimensionless)
a-h	The new model parameters (Equation (19))
s	Time in seconds