Intelligent Information Management, 2019, 11, 87-101
https://www.scirp.org/journal/iim
ISSN Online: 2160-5920
ISSN Print: 2160-5912

State Estimation for Sound Environment System with Nonlinear Observation Characteristics by Introducing Wide-Sense Particle Filter

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How to cite this paper: Orimoto, H., Ikuta, A. and Hasegawa, K. (2019) State Estimation for Sound Environment System with Nonlinear Observation Characteristics by Introducing Wide-Sense Particle Filter. *Intelligent Information Management*, **11**, 87-101.
https://doi.org/10.4236/iim.2019.116008

Received: September 30, 2019
Accepted: November 3, 2019
Published: November 6, 2019

Abstract

In this study, a modified particle filter considering non-Gaussian properties of noises is proposed in a form applicable to real situation in sound environment system where the observation data are contaminated by the external noise (*i.e.*, background noise) of arbitrary probability distribution and measured in decibel scale. More specifically, a nonlinear observation model in decibel scale with a quantized level is first paid considered by introducing the additive property of energy variables (*i.e.*, sound intensity) in sound environment system. Next, a wide-sense particle filter of an expansion expression type is derived in a form suitable for the nonlinear observation characteristics and the signal processing considering higher-order correlation information between the specific signal and observation. Furthermore, the effectiveness of the proposed theory is confirmed by applying it to the observed data measured in real sound environment.

Keywords

Sound Environment, Nonlinear Observation, Non-Gaussian Distribution, Particle Filter

1. Introduction

In the real sound environment system, the observed data contains the effect of several fluctuation factors such as noises in addition to the specific signal. Furthermore, we often encounter the situation necessary to estimate reasonably only the specific signal based on the observed data by introducing some signal processing methods. For example, the background noise usually exists in real sound environment system and the effect of the background noise often has to be eliminated in order to evaluate the sound environment system. Therefore, it is very important to propose an estimation method of the specific signal based on the observed data contaminated by the background noise [1] [2]. Furthermore, the specific signal and the background noise usually show complex fluctuation of non-Gaussian distribution.

On the other hand, in order to estimate precisely the specific signal based on the noisy observation, some signal processing by use of digital computer is indispensable. Therefore, the observed analogue data have to be translated to digital ones at discrete time. However, many standard estimation methods proposed previously for stochastic systems are restricted only to a continuous level of the observation [3] [4] [5] [6] [7].

Though a few researches dealing with state estimation based on the quantized observation with discrete level have been proposed up to now, these have assumed Gaussian additive noise and have been restricted to linear estimator with state variables of Gaussian distribution [8] [9] [10]. Especially, the experimental confirmation has been confined to only the numerical simulation and the application to real state estimation problems has seldom been carried out. From the above viewpoint, in our previous study, a state estimation algorithm has been derived by introducing a difference operation to the non-differentiable nonlinear function expressing the quantized observation [11].

Though the particle filter has been proposed as a state estimation method for nonlinear stochastic systems with non-Gaussian noise [12] [13] [14] [15], there remain a number of problems such as the complexity of calculation in resampling process and the tremendous calculation time based on Monte Carlo simulation. Furthermore, how to describe the likelihood function reflecting non-Gaussian properties for the observed data still remains in the process of realization of the algorithm. Though a state estimation method has been proposed by applying the particle filter after expressing the quantized observation characteristic as a nonlinear system, Gaussian distribution has been assumed for the observation noise in the realization of the algorithm [16]. Furthermore, the validity of the estimation method has been confirmed by only numerical simulation applying the algorithm to simple mathematical models with Gaussian noise. The application of the method to real observed data has not been carried out.

In this paper, a modified particle filter for nonlinear systems considering non-Gaussian properties of specific signals, noises and observation data is proposed for the purpose of application to sound environment system. More specifically, a nonlinear observation model is introduced by considering the additive property of the energy variable (e.g., sound intensity) for the specific signal and external noise (*i.e.*, background noise), and the quantized observation in decibel scale. A particle filter is realized by introducing likelihood function in expansion expression. Next, a wide-sense particle filter of an expansion expression type is derived theoretically by considering not only the linear correlation between the specific signal and observation but also several nonlinear correlations. As the above result, the proposed method is suitable for the application to real sound environment and the estimation accuracy can be improved. The particle filters are used in many fields, because they can apply to many nonlinear stochastic systems with non-Gaussian noise. However, there are problems such as complexity of calculation and tremendous computation time. The proposed method can solve these problems to some extent and will help to improve the computational ability and accuracy of estimation. The effectiveness of the proposed algorithm is confirmed by applying it to the observed data measured in real sound environment under existence of background noise.

The remaining part of this paper is organized as follows: Section 2 introduces the nonlinear observation model. Section 3 summarizes the particle filter and introduces newly a likelihood function in expansion expression for the particle filter. In Section 4, a wide-sense particle filter with quantized observation is the proposed as a state estimation based on Bayes' theorem in expansion expression. Section 5 considers the prediction algorithm. In Section 6, experimental results applying the proposed method to sound environment verify the effectiveness of the theory. Finally, conclusions are summarized in Section 7.

2. Nonlinear Observation Model for Sound Environment System

Let us consider a stochastic environment system with the energy variables (e.g., sound intensity) of arbitrary distribution type, and express the system equation as:

$$x_{k+1} = Fx_k + Gu_k \tag{1}$$

where x_k denotes the specific signal energy at a discrete time k, and u_k is the random input with known statistics. Here, x_k and u_k are statistically independent of each other. Two parameters F and G are estimated by using an auto-correlation technique [1]. Furthermore, a nonlinear observation model is established by considering the additive property of energy variables and the quantized observation in decibel scale, as follows:

$$y_k = 10\log_{10}\left\{\left(x_k + v_k\right)/y_0\right\},\ \left(y_0 = 10^{-12}\left[\mathrm{W/m^2}\right]\right) \tag{2}$$

$$z_k = Q\left(y_k\right) \equiv g\left(x_k + v_k\right) \tag{3}$$

where y_k is the noisy observation in decibel scale contaminated by the additive background noise energy v_k. Though y_k is decibel variable with continuous level, the observation data are measured in a quantized level form suitable for the signal processing by use of a digital computer through A/D converter. The function $Q(\cdot)$ denotes a nonlinear function expressing the quantization mechanism and z_k is the quantized observation in decibel scale. Therefore, $g(\cdot)$ denotes a nonlinear function combining the nonlinearity of decibel observation with the quantized observation mechanism. In this study, a signal processing method to estimate the specific signal x_k is proposed on the basis of the quantized observation z_k contaminated by the background noise v_k.

In order to derive an algorithm to estimate the specific signal x_k based on the quantized observation z_k, Bayes' theorem is paid attention as a fundamental principle of the estimation.

$$P(x_k \mid Z_k) = P(x_k, z_k \mid Z_{k-1}) / P(z_k \mid Z_{k-1}) \tag{4}$$

where $Z_k \left(\equiv \{z_1, z_2, \cdots, z_k\}\right)$ is a set of observations until time *k*.

3. Particle Filter for Sound Environment System with Quantized Observation

3.1. Summary of Particle Filter

In this section, the well-known particle filter for nonlinear systems is summarized [12].

First, Equation (4) can be expressed as follows:

$$\begin{aligned} P(x_k \mid Z_k) &= \frac{P(z_k \mid x_k, Z_{k-1}) P(x_k \mid Z_{k-1})}{P(z_k \mid Z_{k-1})} \\ &= \frac{P(z_k \mid x_k) P(x_k \mid Z_{k-1})}{\int P(z_k \mid x_k) P(x_k \mid Z_{k-1}) \mathrm{d}x_k} \end{aligned} \tag{5}$$

By introducing M particles $X_{k|k-1} = \left[x_{k|k-1}^{(1)}, x_{k|k-1}^{(2)}, \cdots, x_{k|k-1}^{(M)}\right]$, the prior probability density function $P(x_k \mid Z_{k-1})$ can be expressed approximately as:

$$P(x_k \mid Z_{k-1}) \cong \frac{1}{M} \sum_{i=1}^{M} \delta\left(x_k - x_{k|k-1}^{(i)}\right) \tag{6}$$

where $\delta(\cdot)$ is Dirac delta function and $x_{k|k-1}^{(i)}\ (i = 1, 2, \cdots, M)$ are particles considered as elements of $P(x_k \mid Z_{k-1})$. Furthermore, the posterior probability function $P(x_k \mid Z_k)$ is also expressed approximately by use of the delta function in terms of M particles: $X_{k|k} = \left[x_{k|k}^{(1)}, x_{k|k}^{(2)}, \cdots, x_{k|k}^{(M)}\right]$, where $x_{k|k}^{(i)}\ (i = 1, 2, \cdots, M)$ are particles considered as elements of $P(x_k \mid Z_k)$.

Next, using the property of delta function, the denominator of the right hand of Equation (5), which is expressed as C_k, can be derived as follows:

$$C_k \cong \int P(z_k \mid x_k) \frac{1}{M} \sum_{i=1}^{M} \delta\left(x_k - x_{k|k-1}^{(i)}\right) \mathrm{d}x_k = \frac{1}{M} \sum_{i=1}^{M} \alpha_k^{(i)} \tag{7}$$

with

$$\alpha_k^{(i)} \equiv P\left(z_k \mid x_k = x_{k|k-1}^{(i)}\right), \ (i = 1, 2, \cdots, M) \tag{8}$$

Equation (8) expresses the likelihood function of x_k when the observation z_k is obtained. From Equation (6) and Equation (7), Equation (5) can be expressed as

$$P(x_k \mid Z_k) \cong P(z_k \mid x_k) \frac{1}{C_k M} \sum_{i=1}^{M} \delta\left(x_k - x_{k|k-1}^{(i)}\right) \tag{9}$$

From the above equation, the following relationship is derived.

$$\begin{aligned}\Pr\left(x_k = x_{k|k-1}^{(i)} \mid Z_k\right) &\cong \frac{1}{C_k M} P\left(z_k \mid x_k = x_{k|k-1}^{(i)}\right) \\ &= \frac{\alpha_k^{(i)}}{\sum_{i=1}^{M} \alpha_k^{(i)}} \equiv \tilde{\alpha}_k^{(i)}, \ \left(i = 1, 2, \cdots, M\right)\end{aligned} \tag{10}$$

Therefore, the cumulative distribution for Equation (10) can be given as follows:

$$F_{k|k}\left(x\right) \cong \sum_{i=1}^{M} \tilde{\alpha}_k^{(i)} I\left(x - x_{k|k-1}^{(i)}\right) \tag{11}$$

where the function $I(\cdot)$ denotes unit step function defined as

$$I\left(x-a\right) = \begin{cases} 1 & \left(x \ge 0\right) \\ 0 & \left(x < 0\right) \end{cases} \tag{12}$$

Through resampling procedure, Equation (11) can be rewritten as

$$F_{k|k}\left(x\right) \cong \frac{1}{M} \sum_{i=1}^{M} I\left(x - x_{k|k}^{(i)}\right) \tag{13}$$

Using the particles: $X_{k|k} = \left[x_{k|k}^{(1)}, x_{k|k}^{(2)}, \cdots, x_{k|k}^{(M)}\right]$ obtained from Equation (13), the estimate $\hat{x}_k$ of x_k can be obtained as follows:

$$\hat{x}_k = \frac{1}{M} \sum_{i=1}^{M} x_{k|k}^{(i)} \tag{14}$$

3.2. Particle Filter for Sound Environment System by Introducing Likelihood Function in Expansion Expression

The quantized observation in Equation (3) can be expressed by introducing a quantized noise ε_k as follows:

$$z_k = Q\left(y_k\right) = y_k + \varepsilon_k = 10\log_{10}\left\{\left(x_k + v_k\right)/y_0\right\} + \varepsilon_k \tag{15}$$

Considering Equation (2), the likelihood function of $x_k = x_{k|k-1}^{(i)}$ for y_k is given as

$$P\left(y_k \mid x_k = x_{k|k-1}^{(i)}\right) = P_v\left(10^{y_k/10-y_0} - x_{k|k-1}^{(i)}\right) \tag{16}$$

where $P_v(\cdot)$ denotes the probability density function of the background noise v_k. The statistical orthogonal expansion series [17] defined by

$$P_v\left(v_k\right) = N\left(v_k; \bar{v}_k, R_k\right) \sum_{n=0}^{\infty} B_n \frac{1}{\sqrt{n!}} H_n\left(\frac{v_k - \bar{v}_k}{\sqrt{R_k}}\right) \tag{17}$$

$$\bar{v}_k \equiv \left\langle v_k \right\rangle, \ R_k \equiv \left\langle \left(v_k - \bar{v}_k\right)^2 \right\rangle, \ B_n \equiv \left\langle \frac{1}{\sqrt{n!}} H_n\left(\frac{v_k - \bar{v}_k}{\sqrt{R_k}}\right) \right\rangle$$

$$N\left(x; \mu, \sigma^2\right) \equiv \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{\left(x-\mu\right)^2}{2\sigma^2}\right\} \tag{18}$$

is adopted as an expression considering non-Gaussian distribution. Here $\langle\cdot\rangle$

denotes an averaging operation on variables and $H_n(\cdot)$ is a Hermite polynomial with the nth order. Therefore, the likelihood function of x_k defined by Equation (8) is expressed as

$$\alpha_k^{(i)} = \left\langle P_v\left(10^{(z_k+\varepsilon_k)/10-y_0} - x_{k|k-1}^{(i)}\right)\right\rangle_{\varepsilon_k} \tag{19}$$

The averaging operation on ε_k in the above equation can be evaluated by use of the probability distribution of the quantized noise ε_k such as a uniform distribution. Then, the estimate $\hat{x}_k$ of x_k can be obtained from Equation (14) by use of particles $x_{k|k}^{(i)}$ calculated from Equation (19).

4. Wide-Sense Particle Filter for Sound Environment System with Quantized Observation

4.1. State Estimation Based on Bayes' Theorem in Expansion Expression

In order to express Equation (4) in a form reflecting hierarchically linear and nonlinear correlations between the specific signal x_k and the quantized observation z_k, by expanding the conditional probability density function $P(x_k, z_k \mid Z_{k-1})$ in a statistical orthogonal expansion series, the following expression is derived [1] [2].

$$P(x_k \mid Z_k) = \frac{P_0(x_k \mid Z_{k-1})\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{mn}\varphi_m^{(1)}(x_k)\varphi_n^{(2)}(z_k)}{\sum_{n=0}^{\infty} A_{0n}\varphi_n^{(2)}(z_k)} \tag{20}$$

$$A_{mn} \equiv \left\langle \varphi_m^{(1)}(x_k)\varphi_n^{(2)}(z_k) \mid Z_{k-1}\right\rangle \tag{21}$$

The above two functions $\varphi_m^{(1)}(x_k)$ and $\varphi_m^{(1)}(x_k)$ are orthonormal polynomials of degrees m and n with weighting functions $P_0(x_k \mid Z_{k-1})$ and $P_0(z_k \mid Z_{k-1})$ describing the dominant part of the actual fluctuation. Based on Equation (20), the estimate of the polynomial function $f_M(x_k)$ of x_k with Mth order can be derived as follows.

$$\begin{aligned}\hat{f}_M(x_k) &\equiv \left\langle f_M(x_k) \mid Z_k\right\rangle \\ &= \sum_{m=0}^{M}\sum_{n=0}^{\infty} C_{Mm}A_{mn}\varphi_n^{(2)}(z_k) \Big/ \sum_{n=0}^{\infty} A_{0n}\varphi_n^{(2)}(z_k)\end{aligned} \tag{22}$$

where C_{Mm} is an appropriate constant satisfying the following equality:

$$f_M(x_k) = \sum_{m=0}^{M} C_{Mm}\varphi_m^{(1)}(x_k) \tag{23}$$

4.2. Realization of Wide-Sense Particle Filter for Sound Environment System

Though the particle filter is useful for the state estimation problem of non-linear systems, this filter needs very complicated algorithm and a large number of computational times based on Monte Carlo simulation and the resampling procedure. In this section, a hybrid algorithm combining the analytical formula for state estimation with Monte Carlo simulation by use of particles is proposed.

The well-known Gaussian distribution is adopted as $P_0(x_k \mid Z_{k-1})$ and $P_0(z_k \mid Z_{k-1})$, because this probability density function is the most standard one.

$$P_0(x_k \mid Z_{k-1}) = N(x_k; x_k^*, \Gamma_{x_k}) \tag{24}$$

$$x_k^* \equiv \langle x_k \mid Z_{k-1} \rangle, \quad \Gamma_{x_k} \equiv \left\langle (x_k - x_k^*)^2 \mid Z_{k-1} \right\rangle \tag{25}$$

$$P_0(z_k \mid Z_{k-1}) = N(z_k; z_k^*, \Omega_{z_k}) \tag{26}$$

$$z_k^* \equiv \langle z_k \mid Z_{k-1} \rangle, \quad \Omega_{z_k} \equiv \left\langle (z_k - z_k^*)^2 \mid Z_{k-1} \right\rangle \tag{27}$$

Then, the orthonormal functions with two weighting probability density functions in Equation (24) and Equation (26) can be given in the Hermite polynomial:

$$\varphi_m^{(1)}(x_k) = \frac{1}{\sqrt{m!}} H_m\left(\frac{x_k - x_k^*}{\sqrt{\Gamma_{x_k}}}\right) \tag{28}$$

$$\varphi_n^{(2)}(z_k) = \frac{1}{\sqrt{n!}} H_n\left(\frac{z_k - z_k^*}{\sqrt{\Omega_{z_k}}}\right) \tag{29}$$

Therefore, considering especially two cases of $f_1(x_k) = x_k$ and $f_2(x_k) = (x_k - \hat{x}_k)^2$, estimates for mean and variance are given as

$$\begin{aligned} \hat{x}_k &\equiv \langle x_k \mid Z_k \rangle \\ &= \frac{\sum_{n=0}^{\infty} \{A_{0n}C_{10} + A_{1n}C_{11}\} \frac{1}{\sqrt{n!}} H_n\left(\frac{z_k - z_k^*}{\sqrt{\Omega_{z_k}}}\right)}{\sum_{n=0}^{\infty} A_{0n} \frac{1}{\sqrt{n!}} H_n\left(\frac{z_k - z_k^*}{\sqrt{\Omega_{z_k}}}\right)} \quad \left(C_{10} = x_k^*, C_{11} = \sqrt{\Gamma_{x_k}}\right) \end{aligned} \tag{30}$$

$$\begin{aligned} P_{x_k} &\equiv \langle (x_k - \hat{x}_k) \mid Z_k \rangle \\ &= \frac{\sum_{n=0}^{\infty} \{A_{0n}C_{20} + A_{1n}C_{21} + A_{2n}C_{22}\} \frac{1}{\sqrt{n!}} H_n\left(\frac{z_k - z_k^*}{\sqrt{\Omega_{z_k}}}\right)}{\sum_{n=0}^{\infty} A_{0n} \frac{1}{\sqrt{n!}} H_n\left(\frac{z_k - z_k^*}{\sqrt{\Omega_{z_k}}}\right)} \end{aligned} \tag{31}$$

$$\left(C_{20} = \Gamma_{x_k} + (x_k^* - \hat{x}_k)^2, C_{21} = 2\sqrt{\Gamma_{x_k}}(x_k^* - \hat{x}_k), C_{22} = \sqrt{2}\Gamma_{x_k}\right)$$

Furthermore, by considering a case of $f_{N_1}(x_k) = \left(1/\sqrt{N_1!}\right) H_{N_1}\left((x_k - \hat{x}_k)/\sqrt{P_k}\right)$, the estimate for the expansion coefficient reflecting the non-Gaussian property of the specific signal x_k can be obtained as follows:

$$\hat{a}_{N_1} \equiv \frac{1}{\sqrt{N_1!}} \left\langle H_{N_1}\left(\frac{x_k - \hat{x}_k}{\sqrt{P_k}}\right) \mid Z_k \right\rangle$$

$$= \frac{\sum_{n=0}^{\infty}\left\{A_{0n}C_{N_1 0} + A_{1n}C_{N_1 1} + \cdots + A_{N_1 n}C_{N_1 N_1}\right\}\frac{1}{\sqrt{n!}}H_n\left(\frac{z_k - z_k^*}{\sqrt{\Omega_{z_k}}}\right)}{\sum_{n=0}^{\infty}A_{0n}\frac{1}{\sqrt{n!}}H_n\left(\frac{z_k - z_k^*}{\sqrt{\Omega_{z_k}}}\right)} \tag{32}$$

where $C_{N_1 l}\left(l = 0,1,\cdots,N_1\right)$ are coefficients satisfying the following equality:

$$\frac{1}{\sqrt{N_1!}}H_{N_1}\left(\frac{x_k - \hat{x}_k}{\sqrt{P_k}}\right) = \sum_{l=0}^{N_1}C_{N_1 l}\frac{1}{\sqrt{l!}}H_l\left(\frac{x_k - x_k^*}{\sqrt{\Gamma_{x_k}}}\right) \tag{33}$$

Considering Equation (3) and statistical independence between x_k and v_k, two parameters z_k^* and Ω_{z_k}, and the expansion coefficients A_{mn} in the estimation algorithm of Equations (30)-(32), are given as

$$\begin{aligned} z_k^* &= \left\langle g\left(x_k + v_k\right) \mid Z_{k-1}\right\rangle \\ &= \iint g\left(x_k + v_k\right)P\left(x_k \mid Z_{k-1}\right)P_v\left(v_k\right)\mathrm{d}x_k\mathrm{d}v_k \end{aligned} \tag{34}$$

$$\begin{aligned} \Omega_{z_k} &= \left\langle\left(g\left(x_k + v_k\right) - z_k^*\right)^2 \mid Z_{k-1}\right\rangle \\ &= \iint\left(g\left(x_k + v_k\right) - z_k^*\right)^2 P\left(x_k \mid Z_{k-1}\right)P_v\left(v_k\right)\mathrm{d}x_k\mathrm{d}v_k \end{aligned} \tag{35}$$

$$\begin{aligned} A_{mn} &= \left\langle\frac{1}{\sqrt{m!}}H_m\left(\frac{x_k - x_k^*}{\sqrt{\Gamma_{x_k}}}\right)\frac{1}{\sqrt{n!}}H_n\left(\frac{g\left(x_k + v_k\right) - z_k^*}{\sqrt{\Omega_{z_k}}}\right) \mid Z_{k-1}\right\rangle \\ &= \frac{1}{\sqrt{m!}}\frac{1}{\sqrt{n!}}\iint H_m\left(\frac{x_k - x_k^*}{\sqrt{\Gamma_{x_k}}}\right)H_n\left(\frac{g\left(x_k + v_k\right) - z_k^*}{\sqrt{\Omega_{z_k}}}\right)P\left(x_k \mid Z_{k-1}\right)P_v\left(v_k\right)\mathrm{d}x_k\mathrm{d}v_k \end{aligned} \tag{36}$$

The conditional probability density function $P\left(x_k \mid Z_{k-1}\right)$ in Equations (34)-(36) can be expressed as:

$$P\left(x_k \mid Z_{k-1}\right) = N\left(x_k; x_k^*, \Gamma_{x_k}\right)\sum_{m=0}^{\infty}A_{m0}\frac{1}{\sqrt{m!}}H_m\left(\frac{x_k - x_k^*}{\sqrt{\Gamma_{x_k}}}\right) \tag{37}$$

Furthermore, as the probability density function $P_v\left(v_k\right)$ of the background noise v_k, the expansion expression of Equation (17) is adopted. Two first terms of the probability density functions in Equation (17) and Equation (37) are expressed approximately as

$$N\left(x_k; x_k^*, \Gamma_{x_k}\right) \cong \frac{1}{M}\sum_{i=1}^{M}\delta\left(x_k - x_{Gk|k-1}^{(i)}\right) \tag{38}$$

$$N\left(v_k; \overline{v}_k, R_k\right) \cong \frac{1}{M}\sum_{i=1}^{M}\delta\left(v_k - v_{Gk}^{(i)}\right) \tag{39}$$

by introducing particles: $X_{Gk|k-1} = \left[x_{Gk|k-1}^{(1)}, x_{Gk|k-1}^{(2)}, \cdots, x_{Gk|k-1}^{(M)}\right]$ and $V_{Gk} = \left[v_{Gk}^{(1)}, v_{Gk}^{(2)}, \cdots, v_{Gk}^{(M)}\right]$ considered as elements of $N\left(x_k; x_k^*, \Gamma_{x_k}\right)$ and $N\left(v_k; \overline{v}_k, R_k\right)$. Therefore, Equations (34)-(36) can be given as follows:

$$z_k^* = \frac{1}{M}\sum_{i=1}^{M}g\left(x_{Gk|k-1}^{(i)} + v_{Gk}^{(i)}\right)\sum_{m=0}^{\infty}A_{m0}\frac{1}{\sqrt{m!}}H_m\left(\frac{x_{Gk|k-1}^{(i)} - x_k^*}{\sqrt{\Gamma_{x_k}}}\right)$$

$$\cdot \sum_{n=0}^{\infty} B_n \frac{1}{\sqrt{n!}} H_n \left(\frac{v_{Gk}^{(i)} - \bar{v}_k}{\sqrt{R_k}} \right) \tag{40}$$

$$\begin{aligned} \Omega_{z_k} = &\frac{1}{M} \sum_{i=0}^{M} \left(g\left(x_{Gk|k-1}^{(i)} + v_{Gk}^{(i)} \right) - z_k^* \right)^2 \sum_{m=0}^{\infty} A_{m0} \frac{1}{\sqrt{m!}} H_m \left(\frac{x_{Gk|k-1}^{(i)} - x_k^*}{\sqrt{\Gamma_{x_k}}} \right) \\ &\cdot \sum_{n=0}^{\infty} B_n \frac{1}{\sqrt{n!}} H_n \left(\frac{v_{Gk}^{(i)} - \bar{v}_k}{\sqrt{R_k}} \right) \end{aligned} \tag{41}$$

$$\begin{aligned} A_{mn} = &\frac{1}{M} \sum_{i=0}^{M} H_m \left(\frac{x_{Gk}^{(i)} - x_k^*}{\sqrt{\Gamma_{x_k}}} \right) H_n \left(\frac{g\left(x_{Gk}^{(i)} + v_{Gk}^{(i)} \right) - z_k^*}{\sqrt{\Omega_{z_k}}} \right) \\ &\cdot \sum_{m=0}^{\infty} A_{m0} \frac{1}{\sqrt{m!}} H_m \left(\frac{x_{Gk|k-1}^{(i)} - x_k^*}{\sqrt{\Gamma_{x_k}}} \right) \sum_{n=0}^{\infty} B_n \frac{1}{\sqrt{n!}} H_n \left(\frac{v_{Gk}^{(i)} - \bar{v}_k}{\sqrt{R_k}} \right) \end{aligned} \tag{42}$$

5. Prediction Algorithm

Considering Equation (1), the prediction step necessary to perform the recursive estimation of the specific signal is given as follows:

$$\begin{aligned} \left\langle x_{k+1}^i \mid Z_k \right\rangle &= \left\langle \left(Fx_k + Gu_k \right)^i \mid Z_k \right\rangle \\ &= \sum_{j=0}^{i} \binom{i}{j} F^j \left\langle x_k^j \mid Z_k \right\rangle G^{i-j} \left\langle u_k^{i-j} \right\rangle \end{aligned} \tag{43}$$

By using a relationship of Hermite polynomial:

$$x^m = \sum_{r=0}^{[m/2]} (2r-1)!! \binom{m}{2r} H_{m-2r}(x) \tag{44}$$

the function $\left\langle x_k^j \mid Z_k \right\rangle$ in Equation (43) can be evaluated by use of the estimates $\hat{x}_k, P_{x_k}$ and $\hat{a}_{N_1}\left(N_1 = 3, 4, \cdots, j\right)$. Therefore, by combining the estimation algorithms in Equations (30)-(32) with the prediction algorithm in Equation (43), the recurrence estimation of x_k can be achieved.

6. Application to Sound Environment

In order to examine the practical usefulness of the proposed state estimation method with nonlinear observation characteristics, the proposed algorithms were applied to the actual sound environmental data. The road traffic noise was adopted as an example of a specific signal with a complex fluctuation form. Applying the proposed estimation method to actually observed data contaminated by background noise and quantized with 1 dB width and 2 dB width roughly, the fluctuation wave form of the specific signal was estimated. The statistics of the specific signal and the background noise used in the experiment are shown in Table 1 and Table 2 respectively.

Figure 1 and Figure 2 show the estimation results of the fluctuation wave form of the specific signal by applying the algorithm proposed in Sect. 3 (with

Table 1. Mean and standard deviation of the specific signal (in W/m^2).

Data	Data 1	Data 2	Data 3	Data 4	Data 5
Mean Value	2.23×10^{-4}	3.44×10^{-4}	3.25×10^{-4}	3.82×10^{-4}	3.71×10^{-4}
Standard Deviation	1.47×10^{-4}	2.13×10^{-4}	2.65×10^{-4}	3.19×10^{-4}	3.56×10^{-4}

Table 2. Mean and standard deviation of the background noise (in W/m^2).

Data	Data 1	Data 2	Data 3	Data 4	Data 5
Mean Value	2.50×10^{-4}	2.49×10^{-4}	2.44×10^{-4}	2.47×10^{-4}	2.48×10^{-4}
Standard Deviation	1.08×10^{-5}	9.61×10^{-6}	9.49×10^{-6}	1.05×10^{-5}	9.26×10^{-6}

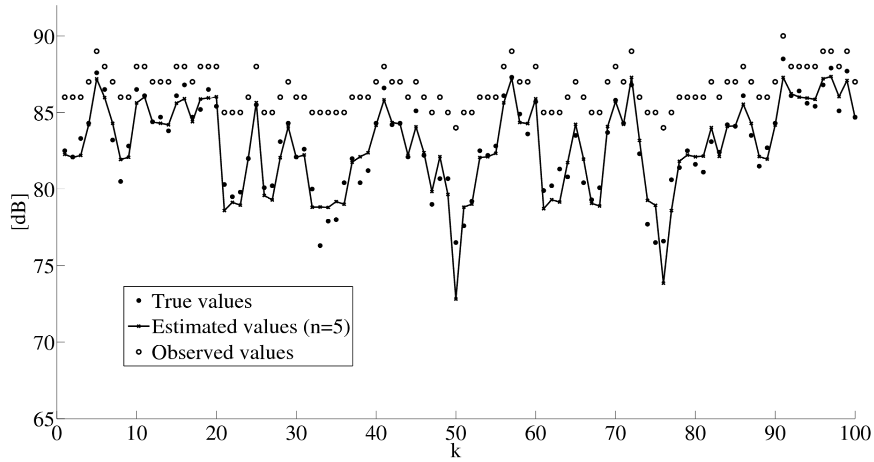

Figure 1. Estimation results by applying the proposed method in Sect. 3 based on the quantized observation data with 1 dB width.

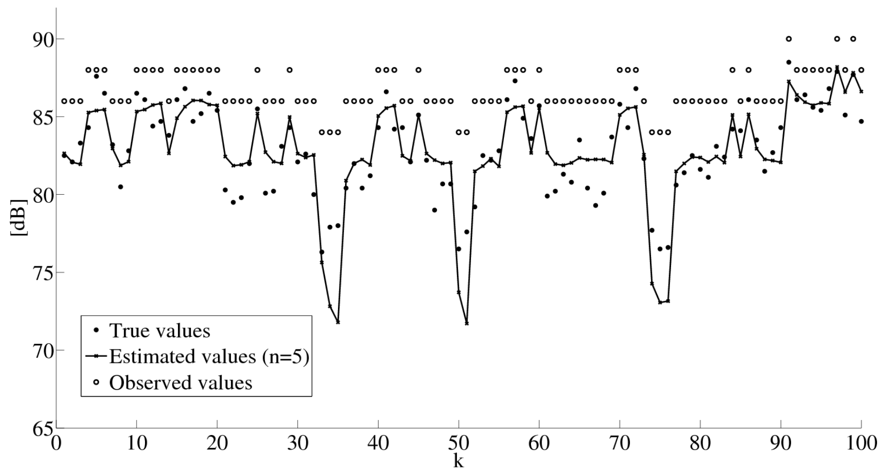

Figure 2. Estimation results by applying the proposed method in Sect. 3 based on the quantized observation data with 2 dB width.

$n = 5$ in Equation (17)) to Data 1. In these figures, the horizontal axis shows the discrete time k of the estimation process, and the vertical axis expresses the sound level taking a logarithmic transformation of energy-scaled variables, because the actual sound environment usually is evaluated on decibel scale. The estimates of the proposed method show good agreement with the true values.

Furthermore, the estimation algorithm proposed in Sect. 4 was applied to the observation data. In this estimation, the finite number of expansion coefficients $A_{mn}(m,n \le 2)$ was used for the simplification of the estimation algorithm. The estimated results of two cases by applying the proposed algorithm to the quantized data with 1 dB and 2 dB widths are shown in **Figure 3** and **Figure 4**.

For comparison, the estimation results calculated by using our previous method [11] and standard method are also shown in **Figure 5** and **Figure 6**. Since Kalman's filtering theory has been widely used in the field of stochastic system, the extended Kalman filter [5] was also applied to the observation data as a trail by using observation model shown in Equation (15). The results by our previous method show relatively good estimation. On the other hand, there are great

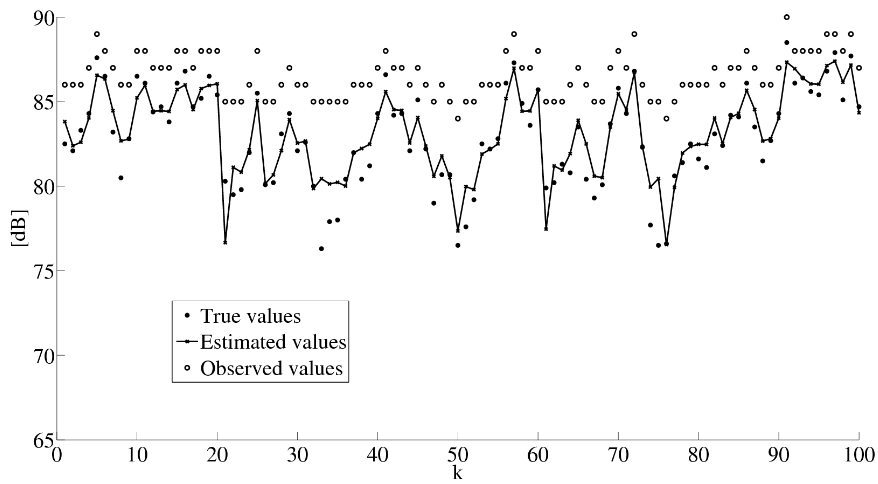

Figure 3. Estimation results by applying the proposed method in Sect. 4 based on the quantized observation data with 1 dB width.

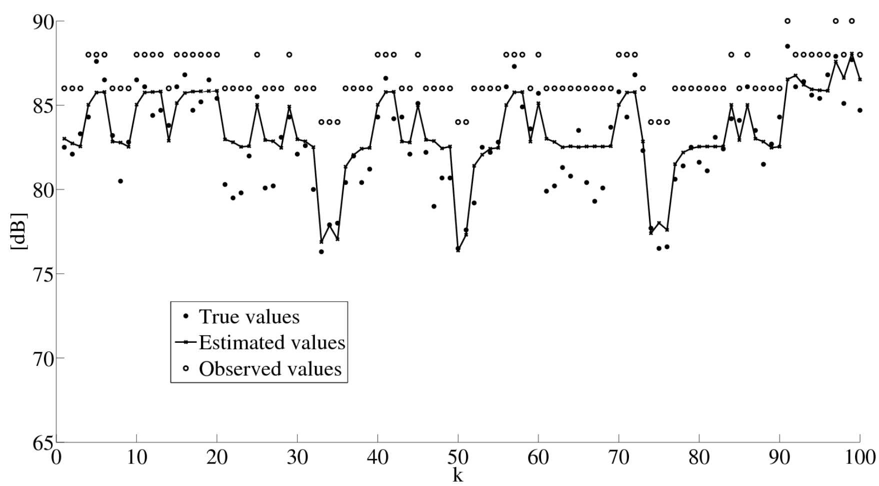

Figure 4. Estimation results by applying the proposed method in Sect. 4 based on the quantized observation data with 2 dB width.

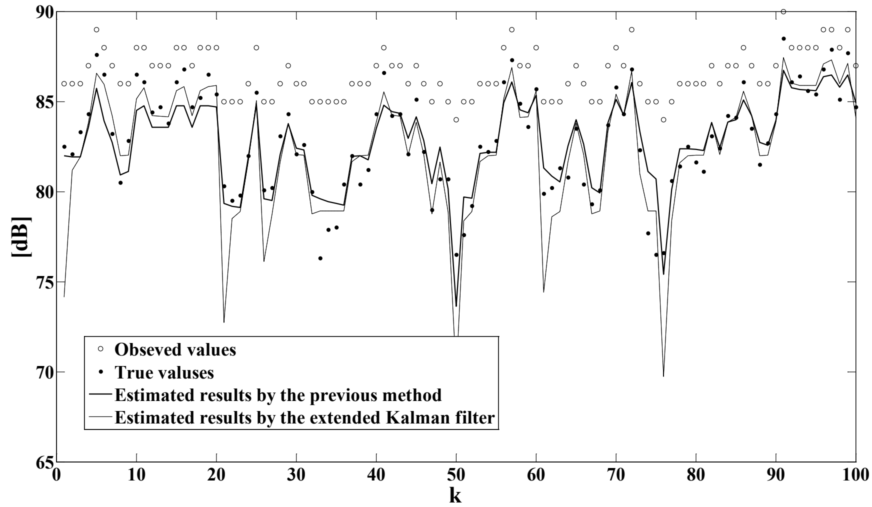

Figure 5. Estimation results by applying our previous method and the extended Kalman filter based on the quantized observation data with 1 dB width.

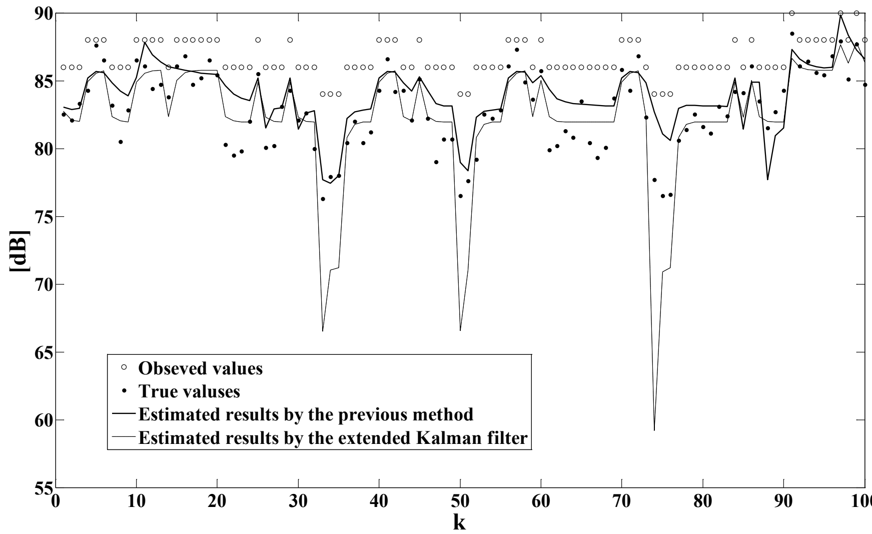

Figure 6. Estimation results by applying our previous method and the extended Kalman filter based on the quantized observation data with 2 dB width.

discrepancies between the estimates based on the standard type dynamical estimation method (*i.e.*, extended Kalman filter), particularly in the estimation of the lower level values of the fluctuation. For Data 2 - Data 5, the same results as Data 1 were obtained.

The squared sums of the estimation error are shown in Table 3 and Table 4. From Table 3 in the case of quantized observation data with 1 dB, it can be found numerically that the more accurate estimation results are obtained by considering the higher-order expansion terms in Equation (17) of the estimation algorithm in Sect. 3. Furthermore, it is obvious that the proposed method in Sect. 4 is more useful than our previous method [11] and the extended Kalman filter. Furthermore, in the case of the quantized observation data with 2 dB, the estimate results by the proposed method in Sect. 4 shows more accurate estimation than the results of other methods.

Though two methods in Sects. 3 and 4 show almost the same accurate estimation, the computation time of two methods is quite different. The comparison of the computation times between two methods is shown in Table 5. The estimation

Table 3. Comparison for root mean squared error of the estimation results based on the quantized observation data with 1 dB width (in dB).

Data		Data 1	Data 2	Data 3	Data 4	Data 5
	n					
Method in Sect. 3	0	1.108	0.9520	1.122	1.130	1.099
	3	1.051	0.8640	1.099	1.159	1.179
	4	1.017	0.8770	0.8600	1.123	1.192
	5	0.9740	0.8490	0.8450	1.087	1.080
Method in Sect. 4		1.128	1.120	1.656	1.629	2.343
Previous Method		1.212	0.9890	1.189	1.217	1.462
Extended Kalman Filter		1.866	1.092	1.303	1.257	2.261

Table 4. Comparison for root mean squared error of the estimation results based on the quantized observation data with 2 dB width (in dB).

Data		Data 1	Data 2	Data 3	Data 4	Data 5
	n					
Method in Sect. 3	0	1.829	1.518	1.878	2.404	2.311
	3	1.559	1.472	1.871	2.189	2.211
	4	1.693	1.438	1.862	2.219	2.447
	5	1.722	1.441	1.858	2.093	2.193
Method in Sect. 4		1.394	1.148	1.811	1.872	2.244
Previous Method		2.056	1.727	2.144	2.083	2.671
Extended Kalman Filter		2.964	1.751	1.875	2.225	3.356

Table 5. Average computation times for $M = 100$ (in s).

Quantized Width (dB)	n	Method in Sect. 3	Method in Sect.4
1	0	0.5791	0.01040
	3	0.7668	
	4	0.9958	
	5	1.3720	
2	0	0.6160	0.01070
	3	0.7820	
	4	1.0030	
	5	1.3980	

algorithm in Sect. 3 needs computation cost from 55.68 times (in the case of $n=0$ in Equation (17)) to 131.9 times (in the case of $n=5$) as compared with the algorithm in Sect. 4. Therefore, the method in Sect. 4 is more advantageous than the method in Sect. 3 by considering the computation cost.

From the above results, it can be concluded that the proposed method in Sect. 4 is most effective among all four methods.

7. Conclusions

In this study, state estimation method for a sound environment system with nonlinear observation characteristics has been theoretically proposed on the basis of Bayes' theorem by introducing a wide-sense particle filter. More specifically, two types of the recursive algorithm to estimate the specific signal have been derived based on the quantized level observation matched for the signal processing by use of a digital computer. Furthermore, the validity and effectiveness of the proposed theory have been experimentally confirmed by applying it to the real environmental noise data in sound environment.

The proposed approach is still at the early of study, and there are left a number of practical problems to be continued in the future. For example, the proposed method has to be applied to many other actual data of sound environment. Furthermore, the proposed theory has to be extended to more complicated situations involving multi-signal sources, and an optimal number of expansion terms in the proposed estimation algorithm of expansion type have to be found.

Acknowledgements

The authors are grateful to Mr. Takuya Komatsu for his help during this study. This work was supported in part by the fund from the Grant-in-Aid for Scientific Research No. 15K06116 from the Ministry of Education, Culture, Sports, Science and Technology-Japan.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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