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A Study of Dark Matter with Spiral Galaxy Rotation Curves. Part II

Bruce Hoeneisen

Universidad San Francisco de Quito, Quito, Ecuador
Email: bruce1@fnal.gov

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Abstract

In Part II of this study of spiral galaxy rotation curves we apply corrections and estimate all identified systematic uncertainties. We arrive at a detailed, precise, and self-consistent picture of dark matter.

Keywords

Dark Matter, Spiral Galaxies, Disk Galaxies, Majorana Neutrinos

1. Introduction

Dark matter in the core of spiral galaxies can exceed 10^7 times the mean dark matter density of the Universe. For this reason we have studied spiral galaxy rotation curves measured by the THINGS collaboration [1] with the hope of constraining the properties of dark matter [2]. In "Part I" of this study [2] we integrate numerically the equations that describe the mixture of two self-gravitating non-relativistic ideal gases, "baryons" and "dark matter". These equations require four boundary conditions: the densities $\rho_h(r_{\min})$ and $\rho_b(r_{\min})$ of dark matter and baryons at the first measured point $r_{\min}$, and the "reduced" root-mean-square radial velocities $\langle v_{rh}^2 \rangle'^{1/2}$ and $\langle v_{rb}^2 \rangle'^{1/2}$, defined as follows:

$$\langle v_{rh}^2 \rangle' \equiv \frac{\langle v_{rh}^2 \rangle}{1-\kappa_h}, \tag{1}$$

and similarly for baryons. $\langle v_{rh}^2 \rangle^{1/2}$ is the root-mean-square of the radial component of the dark matter particle velocities, and $0 \le \kappa_h \le 1$ describes dark matter rotation, see [2] for details. In the present analysis we take $\kappa_h = 0.15 \pm 0.15(\text{syst})$ [2]. The four boundary parameters are fit to minimize the χ^2 between the rotation curves $v_{\text{obs}}(r)$ and $v_b(r)$ measured by the THINGS collaboration [1], and the calculated rotation curves. The fits obtain rotation curves within the observational uncertainties. These fits are presented in Figures 1 to 10 of [2], and the fitted parameters are presented in Table 1 of [2].

In the present analysis we apply corrections and study all identified systematic uncertainties. We use the standard notation in cosmology as defined in [3], and the values of the cosmological parameters therein. Occasionally we use units with $\hbar = 1$ and $c = 1$ as is customary.

2. Corrections from $\rho_h(r_{\min})$ to $\rho_h(r \to 0)$

The first measured point $r_{\min}$ does not lie in the center of the spiral galaxy core, so we make a correction from $\rho_h(r_{\min})$ to $\rho_h(r \to 0)$ by numerical integration with the same equations and parameters described above. These corrections are presented in Table 1.

3. Measurement of the Adiabatic Invariant $v_{h\mathrm{rms}}(1)$

For each spiral galaxy we obtain the parameter

$$v_{h\mathrm{rms}}(1)^2 \equiv 3\langle v_{\mathrm{r}h}^2 \rangle \left(\frac{\Omega_c \rho_{\mathrm{crit}}}{\rho_h(0)} \right)^{2/3} \equiv \frac{3kT_h(1)}{m_h}. \tag{2}$$

$v_{h\mathrm{rms}}(1)$ is the dark matter particles root-mean-square velocity extrapolated to the present time with expansion parameter $a = 1$ in three dimensions, hence the factor 3. $T_h(1)$ is the temperature of dark matter of a homogeneous Universe at the present time. The parameter $v_{h\mathrm{rms}}(1)$ is invariant with respect to adiabatic expansion of the dark matter. Note that for an ideal "noble" gas

Table 1. Corrections from $\rho_h(r_{\min})$ [2] to $\rho_h(r \to 0)$. The statistical uncertainty is from the fit [2]. The systematic uncertainty is from the extrapolation from $r_{\min}$ to $r \to 0$.

Galaxy	$r_{\min}$ [kpc]	$\rho_h(r_{\min})$ $[10^{-2} M_\odot\ \mathrm{pc}^{-3}]$	$\rho_h(r \to 0)$ $[10^{-2} M_\odot\ \mathrm{pc}^{-3}]$
NGC 2403	0.5	$7.5 \pm 1.4(\mathrm{stat})$	$10.3 \pm 1.4(\mathrm{stat}) \pm 0.8(\mathrm{syst})$
NGC 2841	4.0	$9.3 \pm 0.7(\mathrm{stat})$	$20.8 \pm 0.7(\mathrm{stat}) \pm 6.0(\mathrm{syst})$
NGC 2903	1.0	$14.6 \pm 2.1(\mathrm{stat})$	$14.7 \pm 2.1(\mathrm{stat}) \pm 0.7(\mathrm{syst})$
NGC 2976	0.1	$4.0 \pm 2.7(\mathrm{stat})$	$4.06 \pm 2.70(\mathrm{stat}) \pm 0.03(\mathrm{syst})$
NGC 3198	1.0	$4.5 \pm 0.8(\mathrm{stat})$	$5.3 \pm 0.8(\mathrm{stat}) \pm 1.0(\mathrm{syst})$
NGC 3521	1.0	$22.9 \pm 8.6(\mathrm{stat})$	$24.6 \pm 8.6(\mathrm{stat}) \pm 0.7(\mathrm{syst})$
NGC 3621	0.5	$2.6 \pm 0.5(\mathrm{stat})$	$2.93 \pm 0.50(\mathrm{stat}) \pm 0.10(\mathrm{syst})$
DDO 154	0.25	$1.3 \pm 0.3(\mathrm{stat})$	$1.36 \pm 0.30(\mathrm{stat}) \pm 0.10(\mathrm{syst})$
NGC 5055	1.0	$28.2 \pm 6.8(\mathrm{stat})$	$37.3 \pm 6.8(\mathrm{stat}) \pm 9.0(\mathrm{syst})$
NGC 7793	0.25	$8.0 \pm 1.6(\mathrm{stat})$	$8.98 \pm 1.6(\mathrm{stat}) \pm 0.5(\mathrm{syst})$

$T_h V^{\gamma-1} = \text{constant}$ with $\gamma = 5/3$. By "noble" we mean that collisions (if any) between dark matter particles do not excite internal degrees of freedom (if any) of these particles. Alternatively, Equation (2) can be understood as $v_h \propto 1/a$ for non-relativistic particles in an expanding Universe. At expansion parameter a when perturbations are still linear, and after dark matter becomes non-relativistic, the root-mean-square velocity of dark matter particles is

$$v_{h\text{rms}}(a) = \frac{v_{h\text{rms}}(1)}{a} \equiv \left(\frac{3kT_h(a)}{m_h}\right)^{1/2}. \tag{3}$$

Results are presented in **Table 2**. The average of $v_{h\text{rms}}(1)$ of 10 complete and independent measurements is

$$v_{h\text{rms}}(1) = 1.192 \pm 0.109(\text{tot})\,\text{km/s}. \tag{4}$$

This result is noteworthy since the 10 galaxies used for these measurements have masses spanning three orders of magnitude, and angular momenta spanning five orders of magnitude [2]. Note that the correction in **Table 1** has allowed us to include galaxy NGC 2841 in the average (this galaxy was excluded in [2] because the first measured point at $r_{\min}$ is at the edge of the galaxy core).

The expansion parameter $a_{h\text{NR}}$ at which dark matter becomes non-relativistic can be estimated from (3) as

Table 2. Presented are $\langle v_{rh}^2 \rangle'^{1/2}$ from **Table 1** of [2], and $v_{h\text{rms}}(1)$ defined in (2). $\rho_h(0)$ is taken from **Table 1**. $\kappa_h = 0.15 \pm 0.15(\text{syst})$ [2]. The statistical uncertainties of $\langle v_{rh}^2 \rangle'^{1/2}$ and $\rho_h(0)$ are correlated [2]. The systematic uncertainty includes contributions from **Table 1** and from κ_h. The χ^2 of these 10 measurements is $\chi^2 = 36.4$, so the total uncertainty of the average has been multiplied by $\left[36.4/(10-1)\right]^{1/2} = 2.0$, as recommended in [3].

Galaxy	$\langle v_{rh}^2 \rangle'^{1/2}$ [km/s]	$v_{h\text{rms}}(1)$ [km/s]
NGC 2403	101 ± 3	$1.103 \pm 0.083(\text{stat}) \pm 0.088(\text{syst})$
NGC 2841	220 ± 3	$1.900 \pm 0.047(\text{stat}) \pm 0.232(\text{syst})$
NGC 2903	142 ± 3	$1.377 \pm 0.095(\text{stat}) \pm 0.106(\text{syst})$
NGC 2976	129 ± 177	$1.921 \pm 3.061(\text{stat}) \pm 0.144(\text{syst})$
NGC 3198	104 ± 3	$1.417 \pm 0.112(\text{stat}) \pm 0.139(\text{syst})$
NGC 3521	153 ± 10	$1.250 \pm 0.227(\text{stat}) \pm 0.095(\text{syst})$
NGC 3621	126 ± 5	$2.092 \pm 0.202(\text{stat}) \pm 0.159(\text{syst})$
DDO 154	36.5 ± 3.7	$0.783 \pm 0.137(\text{stat}) \pm 0.062(\text{syst})$
NGC 5055	144 ± 4	$1.024 \pm 0.091(\text{stat}) \pm 0.113(\text{syst})$
NGC 7793	85.5 ± 5.0	$0.977 \pm 0.115(\text{stat}) \pm 0.076(\text{syst})$
Average		$1.192 \pm 0.109(\text{tot})$

$$a_{h\mathrm{NR}} \approx \frac{v_{h\mathrm{rms}}(1)}{c}. \tag{5}$$

There are threshold factors of O(1) presented in Section 5.

4. Dark Matter Mass m_h

We consider the scenario with dark matter dominated by a single type of particle (plus anti-particle) of mass m_h. The mass density of a non-relativistic gas of fermions or bosons with chemical potential μ can be written as [4]

$$\rho_h = \left\langle v_{rh}^2 \right\rangle^{3/2} \frac{N_{f,b} m_h^4}{(2\pi)^{3/2} \hbar^3} \Sigma_{f,b}, \tag{6}$$

where the sums are

$$\Sigma_{f,b} = \frac{e^{\mu'}}{1^{3/2}} \mp \frac{e^{2\mu'}}{2^{3/2}} + \frac{e^{3\mu'}}{3^{3/2}} \mp \frac{e^{4\mu'}}{4^{3/2}} + \cdots, \tag{7}$$

where $\mu' \equiv \mu/(kT_h)$, with upper signs for fermions, and lower signs for bosons. The sums for fermions and bosons are $\Sigma_f = 0.76515$ and $\Sigma_b = 2.612$ for chemical potential $\mu = 0$. N_f (N_b) is the number of fermion (boson) degrees of freedom. From (2) and (6) we obtain

$$m_h = \left[\frac{(6\pi)^{3/2} \Omega_c \rho_{\mathrm{crit}} \hbar^3}{v_{h\mathrm{rms}}(1)^3 N_{f,b} \Sigma_{f,b}} \right]^{1/4}. \tag{8}$$

Note that the measured m_h is independent of $\Omega_c \rho_{\mathrm{crit}}$, see (2). From (4) and (8) we obtain

$$m_h = (53.5 \pm 3.6(\mathrm{tot})\,\mathrm{eV}) \cdot \left(\frac{2}{N_f} \frac{0.76515}{\Sigma_f} \right)^{1/4}, \tag{9}$$

for fermions, and

$$m_h = (46.8 \pm 3.2(\mathrm{tot})\,\mathrm{eV}) \cdot \left(\frac{1}{N_b} \frac{2.612}{\Sigma_b} \right)^{1/4}, \tag{10}$$

for bosons. Note that we have obtained these results directly from the fits to the spiral galaxy rotation curves, with no input from cosmology. The uncertainties in (9) and (10) include all statistical and systematic uncertainties listed in Table 1 and Table 2.

A non-relativistic non-degenerate ideal gas has

$$\frac{\mu}{kT_h} = -\ln\left(\frac{v}{v_Q} \right), \tag{11}$$

where $v \equiv V/N$ is the volume per particle, and $v_Q \equiv \left[2\pi\hbar^2/(m_h kT_h) \right]^{3/2}$ is the "quantum volume". For a non-degenerate ideal gas, $v/v_Q \gg 1$ so the chemical potential μ is negative, and increases logarithmically with particle concentration. Fermi-Dirac or Bose Einstein degeneracy sets in as $\mu \to 0$. Note that in an adiabatic expansion $\mu/(kT_h)$ is constant.

Fitting spiral galaxy rotation curves, we obtain limits $m_h > 16\,\text{eV}$ for fermions, and $m_h > 45\,\text{eV}$ for bosons, at 99% confidence [2]. Equivalently, from (9) and (10), we obtain $\Sigma_f \lesssim 96$ for $N_f = 2$, and $\Sigma_b \lesssim 3.1$ for $N_b = 1$.

5. Transition from Ultra-Relativistic to Non-Relativistic Dark Matter

Consider dark matter in statistical equilibrium with chemical potential μ and temperature T_h. This assumption is justified by the observed Boltzmann distribution of the dark matter [2]. We apply periodic boundary conditions in an expanding cube of volume a^3V. The comoving number density of dark matter particles is [4]:

$$n_h a^3 = \frac{N_{f,b}}{(2\pi\hbar)^3}\int_0^\infty 4\pi p^2 \mathrm{d}p \frac{1}{\exp\left[\left(\sqrt{m_h^2c^4 + p^2c^2/a^2} - m_hc^2 - \mu\right)\Big/(kT_h)\right] \pm 1}. \quad (12)$$

The last factor is the average number of fermions (upper sign) or bosons (lower sign) in an orbital of momentum p/a.

Now let dark matter decouple while ultra-relativistic, and assume no self-annihilation. Then $n_h a^3$ is conserved. In an adiabatic expansion, e.g. collisionless dark matter, the number of dark matter particles in an orbital is constant so μ and T_h adjust accordingly. The problem has one degree of freedom, so we choose, without loss of generality, $\mu' \equiv \mu/(kT_h)$ constant. $T_h \propto 1/a$ in the ultra-relativistic limit ($kT_h \gg mc^2$), and $T_h \propto 1/a^2$ in the non-relativistic limit ($kT_h \ll mc^2$). (In the transition between these two limits T_h is momentum dependent.) Let us define $x \equiv pc/(akT_h)$, and $y^2 \equiv p^2/(2m_ha^2kT_h)$. In the ultra-relativistic limit

$$n_h a^3 = A_{f,b}N_{f,b}\left(\frac{kaT_h}{\hbar c}\right)^3, \quad A_{f,b} = \frac{1}{2\pi^2}\int_0^\infty \frac{x^2\mathrm{d}x}{\exp[x-\mu'] \pm 1}. \quad (13)$$

In the non-relativistic limit

$$n_h a^3 = \Sigma_{f,b}N_{f,b}\left(\frac{m_hka^2T_h}{2\pi\hbar^2}\right)^{3/2}, \quad \Sigma_{f,b} = \frac{4}{\pi^{1/2}}\int_0^\infty \frac{y^2\mathrm{d}y}{\exp\left[y^2-\mu'\right] \pm 1}, \quad (14)$$

as in (6). The intercept of these two asymptotes *defines* $a_{h\mathrm{NR}}$ and $T_{h\mathrm{NR}} \equiv T_h(a_{h\mathrm{NR}}) = T_h(1)/a_{h\mathrm{NR}}^2$:

$$m_hc^2 = 2\pi\left(\frac{A_{f,b}}{\Sigma_{f,b}}\right)^{2/3} kT_{h\mathrm{NR}}, \quad (15)$$

$$a_{h\mathrm{NR}} = \left(\frac{2\pi}{3}\right)^{1/2}\left(\frac{A_{f,b}}{\Sigma_{f,b}}\right)^{1/3}\frac{v_{h\mathrm{rms}}(1)}{c}. \quad (16)$$

For $\mu = 0$, we obtain for fermions $A_f = 0.09135$, $\Sigma_f = 0.76515$, $m_hc^2 = 1.523kT_{h\mathrm{NR}}$, and $a_{h\mathrm{NR}} = 0.7126v_{h\mathrm{rms}}(1)/c$; and for bosons $A_b = 0.1218$, $\Sigma_b = 2.612$, $m_hc^2 = 0.8139kT_{h\mathrm{NR}}$, and $a_{h\mathrm{NR}} = 0.5209v_{h\mathrm{rms}}(1)/c$. Einstein condensation sets in at $\mu = 0$.

For $\mu/(kT_h) = -1.5$ we obtain for fermions $A_f = 0.0220$, $\Sigma_f = 0.2074$, $m_h c^2 = 1.409 kT_{h\mathrm{NR}}$, and $a_{h\mathrm{NR}} = 0.6852 v_{h\mathrm{rms}}(1)/c$; and for bosons $A_b = 0.02328$, $\Sigma_b = 0.2432$, $m_h c^2 = 1.315 kT_{h\mathrm{NR}}$, and $a_{h\mathrm{NR}} = 0.6620 v_{h\mathrm{rms}}(1)/c$.

For $\mu/(kT_h) = -10.0$ we obtain for both fermions and bosons $A_{f,b} = 4.6 \times 10^{-6}$, $\Sigma_{f,b} = 4.5 \times 10^{-5}$, $m_h c^2 = 1.366 kT_{h\mathrm{NR}}$, and $a_{h\mathrm{NR}} = 0.6747 v_{h\mathrm{rms}}(1)/c$.

In summary, from the measured adiabatic invariant $v_{h\mathrm{rms}}(1)$ we obtain m_h and $a_{h\mathrm{NR}}$ with (8) and (16) respectively. The ratio T_h/T of dark matter-to-photon temperatures, after e^+e^- annihilation while dark matter is still ultra-relativistic, is

$$\frac{T_h}{T} = \frac{1}{2\pi}\left(\frac{\Sigma_{f,b}}{A_{f,b}}\right)^{2/3} \frac{a_{h\mathrm{NR}} m_h c^2}{kT_0}, \tag{17}$$

where the photon temperature is $T = T_0/a$. Note that T_h/T is proportional to $v_{h\mathrm{rms}}(1)^{1/4}$, and is proportional to $1/T_0$. The intercept of the two asymptotes that we implemented allows direct comparison of (17) with T_h/T in Table 7 of [2].

6. Results for the Case $\mu = 0$

We now specialize to the case of zero chemical potential $\mu = 0$ corresponding, in particular, to equal numbers of dark matter particles and anti-particles, or to Majorana sterile neutrinos [5], or to dark matter that was once in diffusive equilibrium with the Standard Model sector. We obtain from the measured adiabatic invariant $v_{h\mathrm{rms}}(1)$:

$$m_h = [53.5 \pm 3.6(\mathrm{tot})] \cdot \left(\frac{2}{N_f}\right)^{1/4} \mathrm{eV}, \tag{18}$$

$$a_{h\mathrm{NR}} = [2.83 \pm 0.26(\mathrm{tot})] \times 10^{-6}, \tag{19}$$

$$\frac{T_h}{T} = [0.423 \pm 0.010(\mathrm{tot})] \cdot \left(\frac{2}{N_f}\right)^{1/4} \tag{20}$$

for fermions, or

$$m_h = [46.8 \pm 3.2(\mathrm{tot})] \cdot \left(\frac{1}{N_b}\right)^{1/4} \mathrm{eV}, \tag{21}$$

$$a_{h\mathrm{NR}} = [2.07 \pm 0.19(\mathrm{tot})] \times 10^{-6}, \tag{22}$$

$$\frac{T_h}{T} = [0.507 \pm 0.012(\mathrm{tot})] \cdot \left(\frac{1}{N_b}\right)^{1/4} \tag{23}$$

for bosons. These uncertainties are valid for the considered scenario and include statistical uncertainties and all identified systematic uncertainties listed in **Table 1** and **Table 2**. Systematic uncertainties unknown at present may be needed in the future.

These results can be compared with expectations in Table 7 of [2] (and its extensions for other N_f and N_b). Note that T_h/T is proportional to $v_{h\mathrm{rms}}(1)^{1/4}$, and proportional to $1/T_0$, so it is highly significant that the measured $v_{hrms}(1)$ obtains $T_h/T \approx 0.4$ for $\mu = 0$. A different measured $v_{hrms}(1)$, or a different T_0, would have lead to the conclusion that $\mu \neq 0$ and/or dark matter was never in thermal equilibrium with the Standard Model sector. In conclusion, the measured value of $v_{hrms}(1)$ is strong evidence that $\mu = 0$ and that dark matter was in thermal equilibrium with the Standard Model sector at some time in the early history of the Universe.

Measurements with individual spiral galaxies for the case of fermions with $N_f = 2$, e.g. sterile Majorana neutrinos, are presented in Table 3.

7. Additional Systematic Uncertainties?

Non-spherical spiral galaxies: Equations (3) to (6) of [2] are valid in general. So long as the numerical integration is along a radial direction in the plane of the galaxy, with $\nabla P_h = \hat{\boldsymbol{e}}_r \, \mathrm{d}P_h/\mathrm{d}r$ and $\nabla \cdot \boldsymbol{g}_h = \left(1/r^2\right)\mathrm{d}\left(r^2 g_h\right)/\mathrm{d}r$, and similarly for baryons, there is no approximation, and no systematic uncertainty is needed.

Mixing of dark matter: So long as dark matter is assumed collisionless, the adiabatic invariant $v_{hrms}(1)$ should be exactly conserved, so we assign no systematic uncertainty to Equation (3).

New studies may require additional systematic uncertainties. However, at present we do not identify any.

Table 3. Measurements of the expansion parameter $a_{h\mathrm{NR}}$ at which dark matter becomes non-relativistic, the dark matter particle mass m_h, and the ratio of temperatures T_h/T of dark matter-to-photons after e^+e^- annihilation and before dark matter becomes non-relativistic. In this table the particles of dark matter are assumed to be fermions with $N_f = 2$ and $\mu = 0$. The 1σ total uncertainties include the statistical and systematic uncertainties of $v_{hrms}(1)$ in Table 2. The χ^2's are 36.4, 40.8, and 40.4 respectively, for 10 - 1 degrees of freedom, so the uncertainties of the averages have been multiplied by $\left[\chi^2/(10-1)\right]^{1/2}$, as recommended in [3].

Galaxy	$10^6 \times a_{h\mathrm{NR}}$	m_h [eV]	T_h/T
NGC 2403	2.62 ± 0.29	56.7 ± 4.6	0.415 ± 0.011
NGC 2841	4.52 ± 0.56	37.7 ± 3.5	0.476 ± 0.015
NGC 2903	3.27 ± 0.34	48.0 ± 3.7	0.439 ± 0.011
NGC 2976	4.57 ± 7.28	37.4 ± 44.7	0.477 ± 0.190
NGC 3198	3.37 ± 0.42	47.0 ± 4.4	0.442 ± 0.014
NGC 3521	2.97 ± 0.59	51.6 ± 7.6	0.428 ± 0.021
NGC 3621	4.97 ± 0.61	35.1 ± 3.2	0.487 ± 0.015
DDO 154	1.86 ± 0.36	73.3 ± 10.5	0.381 ± 0.018
NGC 5055	2.43 ± 0.34	59.9 ± 6.3	0.408 ± 0.014
NGC 7793	2.32 ± 0.33	62.1 ± 6.6	0.403 ± 0.014
Average	2.83 ± 0.26	46.1 ± 3.3	0.432 ± 0.010

8. Conclusions

A numerical integration obtains rotation curves for spiral galaxies [2]. This integration requires four parameters (boundary conditions). These parameters are obtained by a fit that minimizes the χ^2 between the observed [1] and calculated rotation curves. The fits for ten spiral galaxies, as well as the fitted parameters, are presented in Reference [2]. The fits are in agreement with observations within observational uncertainties. Two of the measured parameters, that are of interest to the present analysis, are $\rho_h(r_{\min})$ and $\langle v_{rh}^2 \rangle'^{1/2}$, and are presented in **Table 1** and **Table 2**. From these two parameters we calculate the adiabatic invariant $v_{h\text{rms}}(1)$ defined in (2). Measurements of $v_{h\text{rms}}(1)$ for ten spiral galaxies are presented in **Table 2**. We obtain an average

$$v_{h\text{rms}}(1) = 1.192 \pm 0.109(\text{tot})\,\text{km/s}. \tag{24}$$

This result is remarkable considering that the ten galaxies span three orders of magnitude in mass, and five orders of magnitude in angular momenta [2].

We consider dark matter that is dominated by a single type of particle of mass m_h. We assume that dark matter decoupled from the Standard Model sector and from self-annihilation while still ultra-relativistic. Then from $v_{h\text{rms}}(1)$ we obtain directly the expansion parameter at which dark matter becomes non-relativistic:

$$a_{h\text{NR}} \approx \frac{v_{h\text{rms}}(1)}{c}, \tag{25}$$

up to a threshold factor of O(1) presented in Section 5. From the adiabatic invariant $v_{h\text{rms}}(1)$ we also obtain the mass m_h of dark matter particles, as a function of the chemical potential μ, with no input from cosmology, see (8).

The fits to spiral galaxy rotation curves allow us to set lower bounds to the dark matter particle mass m_h [2], and upper bounds to the dark matter chemical potential μ, that are not much greater than zero.

To proceed, we need to know the chemical potential μ of dark matter. We consider the scenario with $\mu = 0$ which is appropriate for equal numbers of dark matter particles and anti-particles, or Majorana sterile neutrinos [5], or dark matter that was once in diffusive equilibrium with the Standard Model sector. The upper bound to μ, obtained from the spiral galaxy rotation curves, is close to zero. A negative chemical potential would imply a dark matter temperature while ultra-relativistic higher than the temperature of the Standard Model sector, which seems implausible. In any case we proceed assuming $\mu = 0$, and obtain the results (18) to (23).

The ratio T_h/T is proportional to $v_{h\text{rms}}(1)^{1/4}$, and proportional to $1/T_0$, so the result $T_h/T \approx 0.4$ is highly significant. A different measured adiabatic invariant $v_{h\text{rms}}(1)$, or a different T_0, could have obtained T_h/T orders of magnitude different from unity, so the measurement $T_h/T \approx 0.4$ is strong evidence that dark matter was once in thermal equilibrium with the Standard Model sector, and gives added support to the scenario $\mu \approx 0$.

We compare the measured T_h/T and m_h with expectations, see Table 7 of [2] (and extensions with other N_f and N_b), and find one very good match: fermion dark matter with $N_f = 2$ that decoupled in the approximate temperature range from the confinement-deconfinement transition to m_s, that suggests Majorana sterile neutrino dark matter [2]; and one marginal match for a boson with $N_b = 3$ that decoupled in the temperature range from m_π to m_c.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] de Blok, W.J.G., *et al.* (2008) High-Resolution Rotation Curves and Galaxy Mass Models from THINGS. *The Astronomical Journal*, **136**, 2648-2719. https://doi.org/10.1088/0004-6256/136/6/2648

[2] Hoeneisen, B. (2019) A Study of Dark Matter with Spiral Galaxy Rotation Curves. *International Journal of Astronomy and Astrophysics*, **9**, 71-96. https://doi.org/10.4236/ijaa.2019.92007

[3] Tanabashi, M., *et al.* (Particle Data Group) (2018) The Review of Particle Physics. *Physical Review D*, **98**, Article ID: 030001.

[4] Hoeneisen, B. (1993) Thermal Physics. Mellen Research University Press, San Francisco.

[5] Srivastava, Y.N., Widom, A. and Swain, J. (1997) Thermodynamic Equations of State for Dirac and Majorana Fermions. hep-ph/9709434. http://cds.cern.ch/record/334578/files/9709434.pdf