

Misconceptions of Universe Expansion, Accelerated Universe Expansion, and Their Sources. Virtual Reality of Inflationary Cosmology

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Abstract

In this work, we present our theory and principles of the mathematical foundations of Lobachevskian (hyperbolic) astrophysics and cosmology which follow from a mathematical interpretation of experimental data in a **Lobachevskian non-expanding Universe**. Several new scientific formulas of practical significance for astrophysics, astronomy, and cosmology are presented. A new method of calculating (from experimental data) the curvature of a Lobachevskian Universe is given, resulting in an estimated curvature- K on the order of 10^{-52} m^{-2} . Our model also estimates the radius of the non-expanding Lobachevskian Universe in a Poincare ball model as approximately 14.9 bly. **A rigorous theoretical explanation in terms of the fixed Lobachevskian geometry of a non-expanding Universe** is provided for experimental data acquired in the Supernova Project, showing an excellent agreement between experimental data and our theoretical formulas. We present new geometric equations relating brightness dimming and redshift, and employ them to fully explain the erroneous reasoning and erroneous conclusions of Perlmutter, Schmidt, Riess and the 2011 Nobel Prize Committee regarding “accelerated expansion” of the Universe. We demonstrate that experimental data acquired in deep space astrophysics when interpreted **in terms of Euclidean geometry** will result in illusions of space expansion: an **illusion of “linear space expansion”**—Hubble, and an **illusion of “accelerated (non-linear) space expansion”**—Perlmutter, Schmidt, Riess.

Keywords

Non-Expanding Universe, Hyperbolic Universe, Supernova Photometry, 2011 Nobel Physics Prize, Lobachevskian Geometry, Geometry of Geodesics

1. Motivation and Background

The motivation for the present paper is to give a **rational**, in terms of clearly defined mathematics, explanations of some deep space astrophysical data. By rational, we mean that we only use fixed Lobachevskian (hyperbolic) geometry and not the **pseudoscience of space expansion**.

This paper can be regarded as an instance of physical Lobachevskian geometry. For a reader new to Lobachevskian geometry, it would be beneficial to get some basic information on the subject from the following sources: [1]-[6].

The aim of the present work is twofold:

- 1) To give a rigorous mathematical analysis of data acquired in deep space astronomy and astrophysics.
- 2) To dismiss the claim of Perlmutter, Schmidt and Riess about the “**accelerated expansion of space**”, and to stop the “expanding Universe”.

We entirely reject the misconception of space expansion as having no experimental evidence. In a series of our work on applications of Lobachevskian geometry to physics, astrophysics, and cosmology [7]-[12], (see also Kadomtsev [13]), we proved that a non-expanding Lobachevskian Universe, being a Lorentz invariant entity, is able to explain in a coherent, Lorentz invariant way, all electromagnetic observable phenomena, including cosmological redshift, Doppler shifts, non-Euclidean intensity fading, space caused aberration, velocity caused aberration, polarization of light rotation, and CMB. Our exposition is done exclusively within the framework of Lobachevskian geometry and Maxwellian electromagnetics, which both are clearly defined and well understood.

1.1. Major Results of This Work

The major scientific results of this work are as follows:

- 1) A geometric brightness dimming parameter β (a first of its kind in science) is defined and introduced as a measure of light dimming in the Lobachevskian Universe.
- 2) An equation relating Lobachevski-von Brzeski cosmological redshift z and Lobachevski-von Brzeski intensity of light dimming parameter β is presented. Its linearized version gives a perfect fit to the experimental redshift/brightness data at low z acquired in the Supernova Project.
- 3) A method is proposed to calculate the curvature of the Lobachevskian Universe from the photometry data via the above parameter β . As an illustration, we estimate the Gaussian curvature K of Lobachevskian Universe to be $0.496 \times 10^{-52} \text{ m}^{-2}$.
- 4) The errors in reasoning and errors in data interpretation by Perlmutter, Schmidt, and Riess are explicitly demonstrated, as well as well as the misjudgment of the 2011 Royal Swedish Academy of Sciences, Nobel Prize Committee.
- 5) Via the geometry of geodesics (geometry of light rays), the sources of the illusions of “space expansion” and “accelerated space expansion” are explicitly shown and mathematically analyzed with direct relation to astrophysical data.

Agreement of geometry and physics is perfect.

6) The focusing properties of the Lobachevskian Universe are analyzed and favorably compared to astrophysical experimental data.

The content of this paper can be regarded as **physical Lobachevskian geometry**, which is actually a “translation” of statements and formulas of Lobachevskian geometry into the language of physics. This allows us to explain (in every detail) **all phenomena related to light propagation in Lobachevskian space** in a rigorous and logical manner, and in particular to explain **why the Supernova Project conclusions are wrong**.

1.2. Definitions

We work with **real geometries and real 3 dimensional spaces only**. By *space*, we mean a simply connected, Hausdorff, topological space.

Definition 1 A topological space X is a pair $X = (S, T)$ of a set S (finite or not) and a family T of open sets covering S , called a topology on X .

The aim of topology is to formalize an **intuitive notion of nearness** between the points of a set S . A topological space, with a topology induced by a metrical structure, is called a metric space.

Having a distance structure on a topological space, we can interpret experimental data solely in terms of distances and appropriate functions of distances. This is a **global look** on geometry which we follow in our exposition.

Definition 2 Lobachevskian (3 dimensional) real space is a simply connected, non-compact, locally compact, metric space of constant negative Gaussian curvature $K < 0$.

Definition 3 (Global). Euclidean space is the zero curvature, $K = 0$, limit of Lobachevskian space.

On a *local* scales, Lobachevskian geometry can be approximated (with an arbitrary degree of precision determined only by the sensitivity our instruments) by Euclidean geometry. This is the *linearization* (i.e. Euclideanization) of Lobachevskian geometry.

A Lobachevskian 3 dimensional Universe L_X^3 can be represented as $PSL(2C)/SU(2)$ quotient of the projective Lorentz group with respect to a stabilizer $SU(2)$. The same representation holds for Lobachevskian velocity space L_V^3 . The stabilizer $SU(2)$ in physics is regarded as an “origin” or “center” in homogeneous space L_X^3 , and as an “observer at rest”, “reference at rest” in homogeneous space L_V^3 . Note that the **stabilizer is defined up to an equivalence relation** with respect to translations given by a group element. This reflects the fact that homogeneous spaces in general, and L_X^3 and L_V^3 in particular, have **no fixed** “origin”. In regard to the Lobachevskian Universe this is known as the **Copernican principle**. With respect to velocity space, this is equivalent to saying that physics in all inertial systems is the same (*physics of Lobachevskian velocity space L_V^3 , in a representation of a Poincare ball, is known as Special Relativity, SR*). Spaces L_X^3 and L_V^3 are **isomorphic but not**

isometric. Physics in isomorphic spaces is qualitatively (but not quantitatively) the same, meaning it is isomorphic but not isometric. We also note that Maxwell's equations, which govern classical electromagnetic phenomena, are Lorentz invariant ($PSL(2C)$ invariant).

We want to be understood not only by experts but also by junior students in physics, astrophysics, astronomers and by all those with an interest in Nature. Thus, we use simple Lobachevskian geometry which can be easily comprehended. We use standard notation. The word "light" in this paper means any electromagnetic radiation over the entire EM spectrum. So, for instance radio waves as well as gamma rays are understood as light. **We limit our analysis to Lobachevskian physics of light.** Other effects imposed on light, e.g. by **gravitation, scattering, absorption** etc., are not discussed here; however in real life, their impact on experimental data at telescope has to be evaluated.

1.3. Relation between Euclidean and Lobachevskian Geometry

Note that first of all, Euclidean geometry is the zero curvature limit of Lobachevskian geometry and all formulas of Euclidean geometry can be obtained in this limit. But some formulas of Lobachevskian geometry have no Euclidean counterpart. In this sense, Lobachevskian geometry is richer in content than Euclidean geometry.

Regarding the notion of distance, we note that in all formulas of Lobachevskian geometry, "distance" d never enters **alone** but only as a product of curvature K times distance d , or as the ratio of distance d to characteristic constant κ , which sets the **absolute length scale** in Lobachevskian geometry. Thus the argument of hyperbolic functions is either the product $\sqrt{-K}d$ or the ratio $\frac{d}{\kappa}$ which we call **reduced or normalized distance**. In our view, **only dimensionless ratios, and dimensionless equations**, have significance in physics. The fact that sometimes you do not see K or κ only means that they are **set to unity**. Example: In Lobachevskian velocity space L_v^3 we use a system of units in which $c = \kappa = |K| = 1$.

In Euclidean physics distances are in arbitrary units and therefore they are **meaningless**. The unit of the meter for instance, as well as all physical units in Euclidean geometry and Euclidean physics are brought from the "outside" of the system. Contrary to Euclidean geometry, in Lobachevskian geometry there is a **natural unit of length** defined in the following way (other choices are possible [10]).

Definition 4 *The unit of length. We say that two parallel horospheres (horocycles in 2D) are at a **unit distance** $d = 1$ if the ratio of respective segments cut on them by two parallel geodesics is equal to e , the base of the natural logarithm.*

Another distinctive feature of Lobachevskian geometry, contrary to Euclidean geometry, is the **dependence of angles on distances**. This is usually expressed

as an absence of similar triangles in Lobachevskian geometry. In fact, distance in Lobachevskian space (and functions of it) carries **the same information** as an angle (and functions of it), a fact of profound importance for Lobachevskian astronomy, astrophysics, and cosmology. It follows that a metric in Lobachevskian space can be expressed **solely in angular terms**.

This feature of Lobachevskian space directly affects such basic astronomical data as **parallax**. Lobachevskian space also causes space induced (geometric) **aberration** absent in Euclidean space. It follows that deep space angular data cannot be interpreted in terms of Euclidean geometry.

Lobachevskian space is **more volumetric** than Euclidean space (of the same dimension). This fact directly affects photometric measurements and it is what all astronomers and astrophysicists have been ignorant of for the past 150 years. The volume of the Lobachevskian ball, $B_L^3(o, r) = 2\pi\kappa^3 \left(\sinh \frac{r}{\kappa} \cosh \frac{r}{\kappa} - \frac{r}{\kappa} \right)$ and its boundary, (the Lobachevskian sphere) $S_L^2 = 4\pi\kappa^2 \sinh^2 \frac{r}{\kappa}$, grow like e^r while Euclidean ball volume and Euclidean surface area grow like r^n (here $n = 3, 2$). In plain language, there is “more space” in an Lobachevskian Universe than in a Euclidean Universe. As we discuss in detail in section 4, it is precisely **this volumetric characteristic of Lobachevskian geometry—when interpreted via Euclidean geometry—that produces the illusion of space expansion**. The reader may want to check (as an exercise) that at the zero curvature limit, the surface of a two dimensional Lobachevskian sphere becomes the surface of a Euclidean sphere $S_E^2 = 4\pi r^2$.

From the above equations on volume and surface area growth in Lobachevskian space, it follows:

Conclusion 5 *About brightness data: The apparent brightness of a luminous object immersed in Lobachevskian Universe analyzed in terms of Euclidean astronomy (geometry) will appear to be **dimmer than expected** and consequently it will cause the **illusion** of being “**more distant**”.*

As we show in Section 4, Conclusion 5 states exactly what Perlmutter, Schmidt and Riess recorded. Conclusion 5 also states what causes the illusion of lower brightness and larger distance, **misinterpreted** as the effect of “space expanding” and pushing the object further away.

Sometimes one may encounter the question which geometry, Euclidean or Lobachevskian, is the “true” geometry representing the physical world? Or which geometry is “better”? The latter question is not appropriate since, as a mathematical systems, both geometries are equally valid. The first question depends on the particular situation in which the applicability of one geometry is justified more than of the other. This is determined by the **sensitivity** of our instruments which **can or cannot** to detect any deviation from Euclidean geometry in the problem of interest [5]. When modeling of the physical world, when geometric effects due to the negative curvature of Lobachevskian geometry are beyond the detection ability (sensitivity) of our instruments, we use

Euclidean geometry instead of Lobachevskian geometry (which can be highly counterintuitive).

If by some “lucky accident” Lobachevskian (hyperbolic) geometry was developed ahead of Euclidean geometry, Euclidean geometry would follow **instantly** from it as its zero curvature limit. In the opposite direction, from Euclidean to Lobachevskian geometry, it took 2000 years to recover the **information lost** in that zero curvature limit. This asymmetry is quite impressive.

2. Mathematics of Cosmological Redshift in a Non-Expanding Lobachevskian Universe. Its Geometric Origin, Properties, and Applications to Real Astrophysical Data

Lobachevskian geometry is **inherently intertwined with electromagnetism**. It affects all of the characteristics of electromagnetic radiation which propagates in Lobachevskian space. Light (*i.e.* any EM radiation) propagating in Lobachevskian space will experience **space caused aberration** (an effect analogous to velocity caused aberration), and a change in its frequency, intensity, and polarization. It is therefore important to understand how Lobachevskian geometry affects the characteristics of electromagnetic radiation, something of fundamental importance to astronomical and astrophysical observations. In the following sections, we will discuss the two effects of utmost significance to astrophysics data: **cosmological redshift** and **apparent brightness**.

Light Frequency Shift as a Function of Distance and Curvature In Lobachevskian Geometry

The shape of the Universe, or the geometry of the Universe, due to ideas going back to Bernhard Riemann, can only be determined in an **experimental way**. It is well known in mathematics that the geometry of space can be recovered from studies of geodesics in the space in question. Since in physics, geodesics are interpreted as light rays (light = any EM radiation), and since almost all of the information about the Universe comes to us in the form of electromagnetic radiation, it is of utmost importance to understand **how information about the Universe is encoded in light rays, and what information is encoded in the light we receive from distant parts of the Universe**.

Due to its negative curvature, **parallel light rays in Lobachevskian space diverge at an exponential rate**. The reader should be aware that there is no mistake—indeed parallel geodesics diverge at exponential rate. The **rate of divergence** is given, due to Lobachevski, by the simple geometric relation (all ingenious formulas are simple):

$$\exp \frac{s}{\kappa} = \frac{l_2}{l_1} \quad (1)$$

Equation (1) is in our view one of the most important in all of mathematics. Regarding the physical world, Lobachevski’s Equation (1) is in fact the **law of cosmological redshift and of the Doppler shift as well**. Its linearized version

(5), **precisely recovers the original experimental Hubble data** [14]. However, this data was entirely and regrettably misinterpreted by Edwin Hubble (and the entire cosmological community thereafter) via the Doppler effect.

It is easy to see how in a few steps, physics can be extracted from geometry. Equation (1) tells us that two light rays separated by a distance l_1 at the source will be separated at a distance l_2 at the detector, $l_2 > l_1$. Distances l_1 and l_2 are measured along the horospheres orthogonal to light rays. Separation of horospheres is equal to s (which is the distance between the source and the detector measured as the geodesic parameter along the geodesic). Since on the horospheres the geometry is Euclidean, **distances l_1 and l_2 are measured according to Euclidean geometry**, while s is **Lobachevskian (hyperbolic) distance**. The constant κ sets the length scale in Lobachevskian space.

Given cosmological distances, we **cannot** “go” to the source, e.g. a Supernova 7-bly away, and to measure the separation between the two selected parallel geodesics. Instead we use **light itself as the gauge** plus Equation (1) (assuming that all atoms of the same sort in the Universe, under the same conditions, radiate the same spectrum). Therefore we select two geodesics separated at a source by a distance equal to some wavelength (green for instance) $l_1 = \lambda_1$. At the detector, accordingly to fundamental formula of Lobachevskian geometry (1), we will detect the separation of geodesics, $l_2 = \lambda_2 > \lambda_1$. Therefore:

$$\frac{r}{k} = \ln \frac{\lambda_2}{\lambda_1} = \ln \frac{\lambda_2 - \lambda_1 + \lambda_1}{\lambda_1} = \ln \left(\frac{\Delta\lambda}{\lambda_1} + 1 \right) = \ln(z + 1) \tag{2}$$

Equation (2) clearly shows **how** and **why** we see cosmological redshift. Representing the Lobachevskian Universe in a Poincare ball of radius R in Euclidean space, we obtain :

$$\frac{d}{R} = \tanh \frac{s}{\kappa} = \tanh(\ln(z + 1)) \tag{3}$$

where d is the Euclidean distance from the source (e.g. supernova) to the detector (telescope) in a Poincare ball model. Solving (3) for z , we get the geometric redshift $z = z_G$, **for all** $z_G = z \left(\frac{d}{R} \right)$, of light propagating in a Lobachevskian Universe.

$$z_G + 1 = \sqrt{\frac{1 + \frac{d}{R}}{1 - \frac{d}{R}}} \tag{4}$$

We call the geometric redshift (4) as Lobachevski-von Brzeski redshift $z_G = z_{LvB}$ since the basis for it is Lobachevskian geometry and it was discovered and reported by G. von Brzeski, and von Brzeski V., in the works we listed above.

One may easily check that a linearized form of (4) yields:

$$z_G \approx \sqrt{-Kd} \tag{5}$$

where $K = -\frac{1}{R^2}$ is the Gaussian curvature of Lobachevskian Universe.

To simplify the exposition, in all what follows, we use the following notation. We define K_- as the negative Gaussian curvature of Lobachevskian space. Since $0 > K_-$, $|K_-| = K > 0$, and we will use K (positive value) as such everywhere below.

The linear approximation (5) of our redshift Formula (4) (which holds for all z) recovers the original Hubble experimental data where distance is proportional to redshift.

Note that the velocity space in Einstein's SR is also a Lobachevskian 3-dimensional real space metrized by relative velocities. The signed distance in Lobachevskian velocity space is simply relative velocity. Its Poincare ball model has a constant Gaussian curvature $K = \frac{1}{c^2}$. Therefore the same effect of frequency redshift (relative velocity outward) **trivially** follows from (4) as:

$$z_v + 1 = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} \quad (6)$$

which is the common Doppler Shift (for the Doppler blue-shift, *i.e.* relative velocity inward). We call kinematic frequency shifts as Lobachevski-Doppler shifts $z_K = z_{ID}$. Its linearized version is:

$$z \approx \sqrt{K}v \quad (7)$$

The linear approximation (7) of (6) is routinely and intentionally misinterpreted in all literature via the Doppler effect as "proof" that Hubble observed receding galaxies [14]; see Appendix.

3. Taylor Series Approximation of Fundamental Equation of Lobachevskian Geometry Demonstrates the Illusion of Space Expansion

Since nearly all of the information we gather about the Universe comes to us via electromagnetic radiation—the **geometry of light rays, the geometry of geodesics** directly affects our observations. This is extremely important to realize since as our instrumentation methods get more and more powerful, we are able to collect **information about behavior of geodesics on cosmological scales**, information that was not accessible to astronomers of the past.

The geodesic light ray in a Lobachevskian Universe runs over arbitrarily large distances. To analyze its behavior **in terms of experimental science** we need to look at it in a "step by step" fashion in accordance with the development of astronomical observations which only in the last 100 - 200 years started to reach deep space ranges. We want to establish some qualitative relation between distance (geodesic parameter) from a luminous object to us and the power of our telescopes. Therefore we expand the geodesic light ray into a power series, and then **each term of the expansion will correspond to the increasing power of our instruments to collect data from increasingly distant objects**. This is shown in **Figures 1-6**.

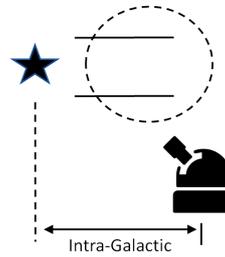


Figure 1. Pre-Hubble (intragalactic) local astronomy shown by dotted circle (region A). The linear size of a region in space in which we cannot detect any departure from the angles of a triangle summing to 180, or in which $\frac{I}{I_0} = \frac{1}{4\pi r^2}$ (apparent brightness), with error beyond the resolution of our instruments, has to be regarded as “local” or Euclidean, and justifies the use of Euclidean geometry in such a region.

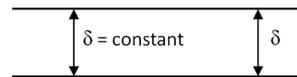


Figure 2. Above we show a “close up” of the geodesics in region A from **Figure 1**. Geodesics (light rays) are regarded as parallel Euclidean straight lines. This is the case of applicability of Euclidean astrophysics and Euclidean astronomy.

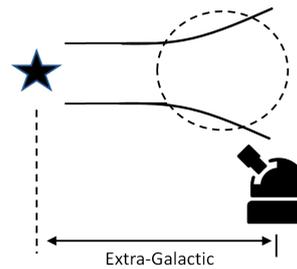


Figure 3. The above figure shows the misrepresentation of astrophysical and astronomical data due to Hubble and all thereafter for the past 100 years. This is the extragalactic distance case, region B. **The geometric spectral shifts are now within the detection power of diffraction gratings of telescopes.** Parallel rays are no longer parallel in the sense of Euclidean geometry. They are parallel in the sense of Lobachevskian geometry. Viewed in terms of Euclidean geometry, they create illusion of “linear space expansion” in the above approximation.

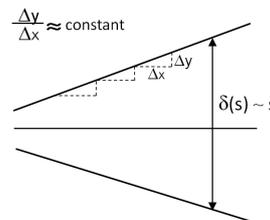


Figure 4. The figure above shows a close up of light rays (geodesics) in region B from **Figure 3**. This is the **weakly Lobachevskian case** interpreted **erroneously** as “**linear space expansion**”. The separation of geodesics in this case is a linear function of distance along the ray. The step function of Euclidean parallel segments Δx (pieces of Euclidean geodesics) has constant vertical step Δy . It creates an illusion of “linear space expansion”. **Space does not expand;** Lobachevskian geometry is fixed, and parallel geodesics spreads apart due to (1).

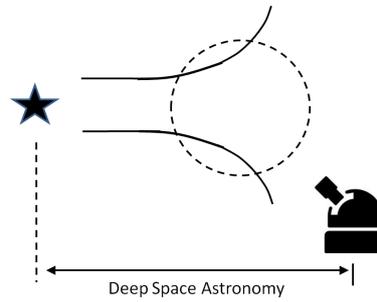


Figure 5. Deep space astronomy (5 - 10 billion light years), which we call region C, corresponds to the quadratic approximation of the power series (10). Geodesics are now parabolas, and the redshift parameter $B(s) = s$ —the speed at which parallel geodesics spread apart—is a linear function of geodesic parameter (distance).

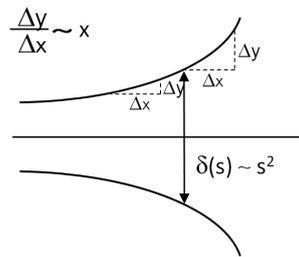


Figure 6. 2011 Physics Nobel Prize for “accelerated space expansion”. Using powerful telescopes, the exponential deviation of geodesics in deep space astronomy is clearly seen. The separation of geodesics is now proportional to the square of the distance along the geodesic. This is the strongly Lobachevskian case, given by redshift parameter $B(s) = s$. The separation of geodesics in this approximation is a quadratic function of distance, which is an elementary definition of uniformly accelerated motion. The above **illusion was interpreted by Perlmutter, Schmidt, and Riess as “accelerated space expansion”** and endorsed by the Swedish Royal Academy Nobel Prize Committee in 2011 [16].

Again we start from the fundamental equation of Lobachevskian geometry:

$$e^{\frac{s}{\kappa}} = \frac{\delta_2}{\delta_1} \tag{8}$$

where δ_1, δ_2 are (Euclidean) distances measured along two horospheres separated by geodesic (Lobachevskian) distance s , and κ is the characteristic constant setting the length scale in the Lobachevskian Universe.

The characteristic constant κ is related to the Gaussian curvature K as κ^{-2} . Since the characteristic constant κ (and consequently the curvature K is so far unknown, we set $\kappa = K = 1$, which is a generic (arbitrary) value and has no effect on what follows. We estimate the value of K in section 5. We also set $\delta_1 = 1 = \lambda_s$ and $\delta_2 = \delta$. Therefore the geodesic (light ray) is given by $\gamma(s) = e^s$. Thus, the **distance δ is simply the detected wavelength λ_d measured in units of source wavelength**, which means $\delta = 1 + z$, where z is fractional increase in a wavelength or redshift.

Thus (8) now reads:

$$e^s = \delta = 1 + z \tag{9}$$

In Equation (9) **all symbols, namely: geodesic parameter s , separation of geodesics $\delta(s)$ and cosmological redshift $z(s)$, are dimensionless.** They are pure, non-negative real numbers. Our Equation (9) **clearly represents cosmological redshift as a geometric law of Nature**, valid for all z .

Equation (9) may be interpreted in the following way. The dimensionless distance $\delta(s)$, in terms of (geometrical) redshift measured from the reference geodesic $\gamma_o(s)$ to the geodesic $\gamma(s)$ along the horosphere passing through s , is equal to e^s . Recall that the reference geodesic $\gamma_o(s)$ and the geodesic $\gamma(s)$ are **parallel**. They belong to the same equivalence class $[\gamma]$ defined by the point at the boundary at infinity $\partial L_3(\infty)$ in Lobachevskian Universe.

The following is important in order to understand the essence of the erroneous interpretation done by Perlmutter, Schmidt, and Riess, as well as by the Nobel Prize Committee awarding its 2011 prize for for that erroneous conclusion.

We analyze (9) as follows. First we write the expansion of a geodesic in a Lobachevskian Universe (9) in a power series (10). Next, by differentiating the power series term by term we obviously end up with the same exponential function (11). But now the the RHS of (11) is equal to the **rate of change of distance $\delta(s)$ with a real parameter s** . We see that $\frac{d\delta(s)}{ds}$ is the **speed $v(s)$** at which the parallel geodesics (parallel light rays) are spreading apart. Note that both expansions (10) and (11) **start at a star location from which geodesic parameter is counted**.

$$e^s = 1 + a + \frac{1}{2}s^2 + \frac{1}{6}s^3 + \dots = \delta(s) = 1 + z \quad (10)$$

$$\frac{de^s}{ds} = 0 + 1 + s + \frac{1}{2}s^2 + \dots = \frac{d\delta(s)}{ds} = v(s) = z' \quad (11)$$

In (11) $z' = \frac{dz}{ds}$ is the rate of change of the redshift with geodesic parameter (distance along geodesic). Equations (10) and (11) explain the most important events in the entire history of astronomy.

In accordance with (11) we introduce the following definition of **cosmological redshift parameter $B(s)$** :

Definition 6 (von Brzeski) *The redshift parameter $B(s)$ is the numerical value of the local slope of a geodesic light ray in Lobachevskian space / Lobachevskian Universe. It is a non-negative real number which classifies the physical geometry applicable to astrophysical measurements and data.*

Remark 7 *Our definition of redshift above has far reaching significance beyond cosmology. It applies to an entire new area of physics. New age meta-materials with sufficiently large engineered $B(s)$ will enable X-ray and γ -ray **frequency down conversion**. This will allow us to see images from the interior of atomic nuclei and images of the interiors of stars and galactic nuclei where γ and X-rays are generated.*

3.1. From Ancient Times to Pre-Hubble. First Term in the Power Series: Euclidean Astronomy With No “Expansion of Space”

This period is characterized by observations conducted by the naked eye and weak telescopes, limited to what we call *region A* in space. In a region A, the distance between the object and observer is “insignificant”, meaning its linear size can be as large as 10^5 and perhaps 10^6 light years [5]. Thus we approximate e^s by the first term in (10), and the RHS of (10) immediately tells us that redshift in region A (due to geometry) is trivially equal to zero.

$$\gamma(a) = e^s \approx 1 = \text{const.} = 1 + z \Rightarrow z = 0, \text{ trivially} \quad (12)$$

The speed of geodesic separation, which is given by the von Brzeski redshift parameter $B(s)$ corresponding to the first term in (10) and shown by the first term in (11), is equal to zero.

$$B(s) = \frac{d(\text{const.})}{ds} = 0 = v(s) \quad (13)$$

Equation (13) states: a light ray in Lobachevskian space, in an approximation via the first term of the power series (10), is a **constant function**. This is a straight Euclidean line $\gamma = 1$, *i.e.* a Euclidean geodesic.

Conclusion 8 *In approximation (12), light rays (exponentials) in Lobachevskian Universe are replaced by Euclidean straight lines.*

Equation (13) states: **the speed of separation $B(s)$ of Euclidean straight lines in an approximation corresponding to the first term of power series (12) is zero, $B(s) = 0$.**

Conclusion 9 *A separation speed of zero, $B(s) = 0$, implies that geodesics are not spreading apart, which means that the distance between them is constant, or that they are parallel in the sense of Euclidean geometry.*

Conclusions (8) and (9) state that the **geometry of space** recovered from expansions (10, 11) in approximation (12) is Euclidean, which is in full agreement with the statement that locally (*i.e.* region A), Lobachevskian geometry may be approximated by Euclidean geometry.

In fact, using solely a geometric argument, we proved the following theorem:

Theorem 10 *von Brzeski G., von Brzeski V. Euclidean geometry of space cannot change the frequency of light propagating in it. Cosmological redshift due to Euclidean geometry of space is impossible.*

- *Classical view:* An electromagnetic wave propagating in Euclidean space has the same frequency k_0 and polarization (since $k_0 = \pm \bar{k}$) at any point in its path.
- *Quantum view:* A photon propagating in Euclidean space has the same energy $\hbar k_0$ and momentum $\hbar \bar{k}$ at any point in its path.

Conclusion 11 *If in a region of space regarded as Euclidean, spectral shifts are recorded, then their origin is due to relative velocity (Doppler), gravity, scattering and perhaps to other unaccounted factors.*

The size of a region A is determined (at a fixed curvature) only by the sensitivity of our instrumentation. Today, with modern diffraction gratings with

a resolving power of 10^6 , region A means we operate in a domain of 10^5 light-years (size of the galaxy) and perhaps as much as 10^7 light-years in linear size. As long as astronomical observations were conducted within our galaxy, **no systematic** spectral shifts have been recorded and this is why **do not see “space expansion” within our galaxy**. Given that apparatus which records spectral shifts (diffraction grating) has a limited resolving power, and the spectral shift is a function of the distance d times Gaussian curvature \sqrt{K} , cosmological distances are needed (at constant curvature) to make it possible for diffraction gratings to “see” it. Make a diffraction grating with a resolving power, say 10^{15} and you will see cosmological redshift in our galaxy, contrary to what cosmologists tell us. Make a diffraction grating with a resolving power of 10^{20} and you will record cosmological redshift in your own backyard.

The above analysis shows (see **Figure 1** and **Figure 2**) that on galactic distance scales in a Lobachevskian Universe, the von Brzeski redshift parameter $B(s)=0$, which implies Euclidean geometry. **This is the reason that we do not see any illusion of “expanding space” on galactic scales.**

3.2. Post-Hubble Epoch. Better Telescopes See Now See the Second Term in the Power Series

This is the period of the illusion of “linear space expansion”. It spans the time period from Hubble’s mistake on extra galactic redshift [14] to the erroneous Supernova project conclusions by Perlmutter, Schmidt, Riess [15].

In this period, improved telescopes are able to acquire light from more distant objects in space. This corresponds to approximating the light ray e^s by the first expansion term in (10) which is s :

$$\gamma(s) = e^s \approx 1 + s = \delta(s) = z + 1 \Rightarrow z = s \quad (14)$$

We see that in approximation (14) **redshift z is proportional to the geodesic parameter (the distance along geodesic)**. This is **exactly what Hubble observed**, but not what is advocated in all literature as the “Hubble distance velocity law”. Does that mean that space is undergoing expansion and/or inflation? **Of course not**. However, Hubble did not know Lobachevskian geometry and he misinterpreted the linear approximation of the piece of the exponential geodesic in Lobachevskian universe as a **straight Euclidean geodesic, plus “linear inflation”**. The reader should be aware that the proportionality of measured redshift versus distance (as measured by Hubble), which results from approximation (14), is **notoriously misrepresented** in all literature as “proof” that Hubble experimentally observed receding galaxies and space inflation; see e.g. Perlmutter [15] for that misrepresentation. We return again to Edwin Hubble’s mistake in Section 8.

For speed of geodesic separation in approximation (14) we have:

$$\gamma'(s) = e^s \approx 1 = \nu(s) = B(s) = \frac{dz}{ds} \quad (15)$$

which reads that speed of geodesic separation is **constant**, and the **redshift z is**

proportional to distance since $\frac{dz}{ds} = B(s) = \text{const}$, and this is exactly what Hubble measured with his telescope.

3.3. From the Mistake of Perlmutter, Schmidt, Riess to Present. More Powerful Optics Now See the Third Term in the Power Series

This is the period of the illusion of “non-linear, accelerated space expansion”, in which we discuss where the conclusion of “accelerated expansion” came from.

Progress in technology resulted in advanced telescopes able to see the third term of a power series (10), *i.e.* able to acquire data from objects located in deep space, say 7×10^9 light years (e.g. the 2011 Nobel Prize case). That region is denoted in **Figure 5** as region C, and corresponds to approximating the geodesic e^s by a quadratic polynomial, with the third, dominant term s^2 :

$$\gamma(s) = e^s \approx 1 + s + \frac{1}{2}s^2 = \delta(s) = 1 + z \Rightarrow z = s + \frac{1}{2}s^2 \quad (16)$$

From approximation (16), we see that separation δ is speeding up in a **non-linear fashion, and the redshift now is also a non-linear function of distance.**

Again, the speed of separation $B(s)$ is given by (11) and it is shown on **Figure 6**.

$$\gamma'(s) = e^s \approx 1 + s = z' = v(s) = B(s) = s \quad (17)$$

We know that if the speed (rate of change) is proportional to the geodesic parameter, such motion is called **uniformly accelerated motion**. In region C, the slope $B(s)$ appears closer to an exponential function, and the distance between the reference light ray and the ray parallel to it **blows up very rapidly**. The Euclidean distance between the steps increases in **non-linear (seemingly “accelerated”) fashion**.

There is **no principal difference** at all in tracing the geodesic light ray $\gamma(s)$ in a Lobachevskian Universe or in drawing (tracing) the geodesic $\gamma(s)$ on a piece two dimensional space (a sheet of paper)—the **flat**—by a pencil. (The flat, or totally geodesic surface, is an analogue of the Euclidean plane. It has the property that if a geodesic has two common points with the flat then the geodesic lies entirely in the flat.) We hope that nobody will claim that geodesics are spreading because the piece of paper is expanding.

However, the “official” version of cosmological redshift which is currently in all sources on cosmology, presented here by Nobel Prize Laureate Saul Perlmutter, states: “...*the redshift, is a very direct measurement of the relative expansion of the universe, because as the universe expands the wavelengths of the photons traveling to us stretch exactly proportionately—and that is the redshift*” [15].

We strongly disagree with the above definition of frequency shifts recorded at the telescope. As we explained already (see Equations (4) and (6)), the recorded shift in frequency (ignoring other factors) is a **mixture of unknown**

proportions of a systematic geometric redshift (4) and kinematic, random blue / red Doppler shift. This is shown in detail from classical, quantum, and topological points of view in the Appendix.

Putting the fundamental error of space expansion at the base of all of their data reduction process, Perlmutter, Schmidt and Riess automatically nullified the entire conclusion of their subsequent work. This will be clearly seen below, where we calculate the relation between photometry and redshift in a Lobachevskian Universe.

4. Photometric Data Analysis in Supernova Project and the Sources of Its Failed Interpretation

In this chapter we shall explain, in full detail, why the conclusions about photometric data collected in Supernova Project by Perlmutter, Schmidt and Riess, and by Swedish Nobel Prize Committee as well, are wrong. Their conclusions are based on an incorrect light intensity analysis from deep space objects.

The light intensity recorded at a telescope (apparent brightness) coming from a luminous object immersed in a Lobachevskian Universe is due to the two causes. One is analogous to down-shift in frequency (redshift). Here we present the second, namely the volumetric characteristics of Lobachevskian geometry which are different to their Euclidean counterparts.

1) The spread of an object's total light flux (luminosity) I_0 over a Lobachevskian sphere of Lobachevskian radius r given by (18) below:

$$I(\kappa, r) = \frac{I_0}{4\pi\kappa^2 \sinh^2 \frac{r}{\kappa}} \quad (18)$$

Note that the surface of a Lobachevskian sphere grows like e^r so the **apparent brightness will be substantially less than the apparent brightness expected in Euclidean astronomy**. This factor alone explains why Perlmutter, Schmidt, and Riess recorded 25% lower brightness than expected in the Euclidean astronomy they used [15] (no space inflation needed).

2) Non-Euclidean—Lobachevskian geometric dimming (decrease in intensity)—see von Brzeski G., von Brzeski, V. [8]—is analogous to a geometric decrease in frequency or geometric redshift. Geometric dimming of brightness is a monotonically decreasing function of Lobachevskian distance asymptotically approaching zero as the distance increases to infinity. Points at an infinite distance from any point in the Lobachevskian Universe belong to the boundary at infinity of the Lobachevskian Universe. **We will never see** any object at the boundary at infinity in Lobachevskian Universe due to this effect alone. It manifests itself as a dark background—**the dark night sky**.

3) The apparent brightness I and apparent color (frequency) are both affected by geometric and kinematic factors (as discussed in the Appendix). In general, in a Lobachevskian Universe the **apparent color** (frequency) and the **apparent brightness** (intensity) are both functions of distance d and relative velocity v (at

fixed constant negative curvature). A star moving toward us will appear bluish and brighter, while a star moving away will appear reddish and dimmer [8] provided that the distances are the “same”. The behavior of light in a Lobachevskian Universe is affected by the negative curvature of a large scale vacuum and by the the negative curvature of Lobachevskian velocity space. Scattering, dust, etc., are not of interest to us.

Now we show in detail the nature of the fallacious reasoning and errors of Perlmutter, Schmidt, Riess, and the Swedish Royal Academy, which led to “accelerated space expansion”.

We recall that the surface of a sphere in a Lobachevskian Universe is given as

$$S_L^2 = 4\pi\kappa^2 \sinh^2 \frac{r}{\kappa} \quad (19)$$

where r is the Lobachevskian radius of the sphere and κ is the characteristic length constant setting the **absolute length scale** related to the (negative) Gaussian curvature as $\frac{1}{\kappa^2} = K$.

We expand (19) in a power series around zero (or origin/center of the sphere) where the light source (e.g. supernova) is located. This is because we want to see what brightness will look like with respect to each term in the expansion. Note that in the realm of the Lobachevskian Universe, “around zero” might be a few hundred light years or so. Since $\sinh(0) = 0$, the expansion will have only even-power (2, 4, 6, ...) terms. Thus, we have:

$$S_L^2 = 4\pi\kappa^2 \sum \frac{1}{n!} \frac{d^n \sinh^2 \frac{r}{\kappa}}{dr^n} \quad (20)$$

$$= 4\pi\kappa^2 \left[\frac{1}{2} \frac{2r^2}{\kappa^2} \left(\cosh \frac{2r}{\kappa} \right) \Big|_{r=0} + \frac{1}{24} \frac{r^4}{\kappa^4} \left(\cosh \frac{2r}{\kappa} \right) \Big|_{r=0} + \dots \right]$$

$$S_L^2 \approx 4\pi r^2 + \frac{4\pi}{3} r^4 K + \frac{8\pi}{45} r^6 K^2$$

$$\text{higher terms depending on distance } r \text{ and curvature } K \quad (21)$$

The reader should note that all the terms in expansion (21) have dimension of m^2 , *i.e.* surface area. From (21), we see that in the lowest (quadratic) approximation (what the Nobel Prize Committee calls “nearby supernovae”), the **curvature cancels out**, and the Lobachevskian sphere S_L^2 becomes the Euclidean sphere $4\pi r^2$. Therefore, in this (local approximation) case **we are entitled to use Euclidean geometry in all photometric formulas**. The following fact is of fundamental importance to the whole of astronomy, astrophysics and cosmology when interpreting photometric (and other) data: **Euclidean geometry works faithfully at “nearby” distances**, or where **locally** Lobachevskian geometry can be approximated by Euclidean geometry via the **linearization** mentioned above.

Measuring brightness data from nearby supernovae accordingly to Euclidean photometry (as the first term in (21) shows) **was justified**. However using the Euclidean geometry on distances large enough fails because at such distances,

our instruments recorded a light intensity corresponding to the surface of Lobachevskian sphere, *i.e.* including the second and following terms in (21). If Perlmutter, Schmidt, and Riess had properly interpreted Lobachevskian data, then all the “surprises” would disappear and agreement between experimental data and Lobachevskian geometry of the Universe would be perfect.

However that was not the case, and in his paper [15], Perlmutter argues that “while light traveled billions of years to reach the telescope, the (Euclidean) Universe was **constantly expanding**, so the sphere $S = 4\pi R^2$ expanded as well”. As a result, at the time when the light reached the telescope, the density of light for a larger, **expanded Euclidean sphere** would obviously be lower. However, as it turned out, the present value of “universe expansion” was enough to fit the data. Therefore, Perlmutter arbitrarily accelerated the entire infinite universe to obtain a fit to the $\frac{1}{4\pi r^2}$ law of Euclidean photometry.

Also let us recall the announcement of the Royal Swedish Academy [16] regarding 2011 Nobel Prize in Physics: “*By comparing the brightness of distant, far away supernovae with the brightness of nearby supernovae, the scientists discovered that the far away supernovae were **about 25% too faint**. They were **too far away**. The Universe was accelerating. And so the discovery is fundamental and a milestone for cosmology. And a challenge for generations of scientists to come*”.

It amazing that wording of the Royal Swedish Academy and the Nobel Prize Committee echoes our Conclusion 5 of this work, with one essential difference however. We proved that an observer, in a Lobachevskian universe, will record an anomalous (in terms of Euclidean photometry) light dimming and being ignorant of hyperbolic photometry laws, he/she will wrongly interpret the result as an effect of arbitrarily increased distance. The Nobel Prize Committee, due to their ignorance of Lobachevskian geometry, takes this illusion as a real thing.

4.1. Calculation of the Curvature of the Lobachevskian Universe

So far our geometric analysis was done in general terms which allowed us to make correct qualitative conclusions but no numerical results have been presented. In this section, employing the brightness data from the Supernova project, we estimate the curvature and the radius of the visible Lobachevskian Universe in a Poincare ball model.

In the history of mankind, after the discovery of Lobachevskian geometry, there were several attempts to measure the curvature of space, by Lobachevski himself and by Gauss as well. Lobachevski calculated the curvature of space from the parallax of Sirius, and Gauss tried to find the angular defect of a triangle made by three church towers in his neighborhood. Both attempts, albeit correct in principle, were unsuccessful since as we have shown above, local geometry is apparently flat and the instrumental methods used by Lobachevski and Gauss were not sufficiently sensitive.

The following successful calculation of the negative curvature of the Universe from experimental data, resulting in credible numbers, is **first such calculation in science**.

Looking at expansion (20), we have learned that indeed the Lobachevskian sphere is larger than a Euclidean sphere of the same numerical radius, as we see in **Figure 7**. Now we rewrite the expansion in geometric form, which tells us about the physics of each term. We write expansion (20) as power series in terms of *curvature*, $K = \frac{1}{\kappa^2}$:

$$S_L^2 = \sum K^i S_i = K^0 4\pi r^2 + K^1 \frac{4}{3}\pi r^4 + K^2 \frac{8}{45}\pi r^6 + \dots \text{higher order terms} \quad (22)$$

We see that the coefficient of K to the 0th power is simply the Euclidean sphere, which again tells us that Lobachevskian geometry appears to be flat locally, (no detectable curvature on local scales). As we move further away from the source of light, the curvature begins to be noticeable, and consecutive coefficients of powers of K show how much the area of the sphere grows relative to the Euclidean sphere. We also see that each term has dimension of length squared m^2 .

Given Equation (19), we now define a quantity entirely analogous to the Lobachevski-von Brzeski geometric redshift, which may be considered as the relative excess of length of the received wavelength (far from source), with respect to the wavelength close to the source. This change in wavelength, as we stated in (4), is a function of distance and curvature. This time however, we define the relative excess in area as ratio of the surface areas of a sphere **far** from the source (Lobachevskian sphere) relative to a sphere **close** to the source (Euclidean sphere). Obviously this ratio, call it $\frac{\text{DISTANT}}{\text{CLOSE}}$, by photometry rules, is an inverse of the brightness ratio. We see that Definition 12 below

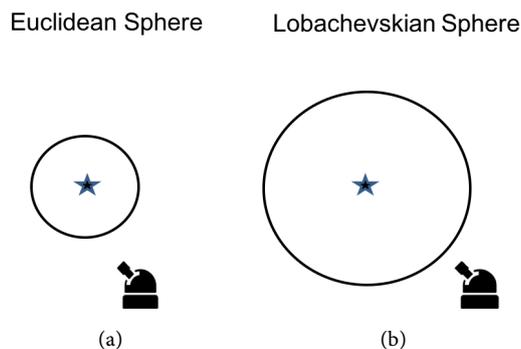


Figure 7. Here we see two spheres of the same numerical radius r . The one denoted by A is the Euclidean sphere. Its surface grows as r^2 . The one denoted by B is the Lobachevskian sphere S_L^2 . Its surface grows as e^r . Space is neither expanding nor expanding in accelerated fashion; however the geometry of space which on local scales (small distances) appears to be Euclidean, is Lobachevskian on a global scale; see power series (10). The total luminous energy output of a distant source spread over the Lobachevskian sphere (whose volume is larger than its Euclidean counterpart) will result in “lower apparent brightness” than expected from Euclidean geometry.

exactly fits with the data acquisition scheme in Supernova Project; see [15].

Definition 12. von Brzeski G., von Brzeski V.. *The Lobachevski-von Brzeski brightness dimming parameter β is the inverse ratio of the DISTANT source apparent brightness to the CLOSE source apparent brightness, provided that luminosities of both are the same. In other words, β is the ratio of surfaces of the Lobachevskian sphere to the Euclidean sphere.*

$$\beta + 1 = \frac{4\pi\kappa^2 \sinh^2 \frac{r}{\kappa}}{4\pi r^2} \tag{23}$$

Accordingly to Definition 12, expanding the Lobachevskian sphere in a power series with respect to curvature $K = \frac{1}{\kappa^2}$, we have:

$$\beta + 1 = 1 + K \frac{1}{3} r^2 + K^2 \frac{2}{45} r^4 + \dots \tag{24}$$

Equation (24) is quite analogous to our geometric redshift Formula (4). The geometric brightness dimming parameter β may be regarded as the two dimensional analogue of redshift. This is not surprising: geometry works well—redshift which is an increase in length [m], increases as length squared [m²] (area) in the case of a surface.

Now we take the first and second terms (terms linear in K) in approximation (24) and apply it to the 0.75 brightness $\frac{\text{DISTANT}}{\text{CLOSE}}$ ratio recorded in the Supernova Project. We also assume that DISTANT brightness corresponds to 3bly (billion light years). Thus, we have $\frac{1}{0.75} = \frac{4}{3}$, and :

$$\frac{4}{3} + 1 \approx 1 + \frac{1}{3} K r^2 \tag{25}$$

As the below calculation shows, using the Supernova project data, the **negative Gaussian curvature** K of the Lobachevskian Universe (in units of m⁻²) has the following very reasonable value:

$$-K = 0.496 \times 10^{-52} \text{ m}^{-2} \tag{26}$$

and the radius of the Lobachevskian Universe

$$R = \frac{1}{\sqrt{-K}} = 14.9 \times 10^9 \text{ light-years} \tag{27}$$

or **14.9 billion light years**.

The reader should note that the above very crude calculations nevertheless produce very reasonable numbers which cannot be ignored. The mathematical framework is correct but there is uncertainty in data due to several reasons. For instance we do not know the contribution to the measured brightness from kinematics (see the Appendix). True data will have to be harvested from a large number of equidistant sources (like a shell) which will **average out** kinematic brightness due to the random orientation of relative velocities.

We are also confident that calculations based on our geometric formula for cosmological redshift (4) will result in similar numbers, but redshift data needs to be genuine dimensionless data from instrumentation, not data that has been

manipulated in terms of units of velocity. Nevertheless we invite all astrophysicists, astronomers, and all those with access to relevant data to work on mathematical applications of Lobachevskian geometry.

A precise determination of the curvature of the Lobachevskian Universe is a hard task. But if this were already done, then diffraction gratings and photometers could be promptly calibrated in units of distance and distances to all luminous objects in the universe will be known instantly, **provided that** all other factors (kinematic, scattering, etc.) are accounted for.

4.2. Lobachevski-von Brzeski Cosmology Versus Big Bang Accelerated Space Expansion

Perlmutter, Schmidt, and Riess's reasoning and conclusions about an accelerating expansion of the Universe can easily be traced to the 100-year old Hubble error linking redshift and the apparent recession velocity of galaxies, expressed via the **fictitious relation** called the "Hubble distance velocity law". See discussion of this in Section 8.

Being embedded in a Lobachevskian Universe, the high redshift supernovae obviously showed lower than expected (from Euclidean geometry) brightness—see (19) and (21), therefore exhibiting an illusion of being further in Euclidean space. From the brightness-redshift relation (again in the Euclidean geometry of Big Bang cosmology), it follows that the dimmer the object, the higher the redshift. In the virtual world terminology of an "expanding Universe" redshift is given by another **quantity called the Hubble constant H** , which in reality has nothing with the fiction of space expansion; see section 8.

We now demonstrate how the redshift-brightness dimming parameter relation works. We write the Lobachevski-von Brzeski redshift Equation (3) in terms of the curvature of Lobachevskian Universe K as it follows from fundamental formula of Lobachevskian geometry (1):

$$\sqrt{K}d = \frac{d}{\kappa} = \ln(z+1) \quad (28)$$

and we again write the equation for Lobachevski-von Brzeski geometric brightness dimming parameter β :

$$\beta + 1 = \frac{4\pi\kappa^2 \sinh^2 \frac{d}{\kappa}}{4\pi d^2} = \frac{\sinh^2 \frac{d}{\kappa}}{\frac{d^2}{\kappa^2}} = \left(\frac{\sinh \frac{d}{\kappa}}{\frac{d}{\kappa}} \right)^2 \quad (29)$$

Eliminating K from (28) and (29) we get a **very important** Lobachevski-von Brzeski Equation (30) between β and z (the first of its kind in science), provided that kinematic (and other) factors are not present or are known and accounted for.

$$\beta + 1 = \left(\frac{\sinh(\ln(z+1))}{\ln(z+1)} \right)^2 \quad (30)$$

Equation (30), discovered by us, for relative intensity down shift β versus spectral down shift (redshift) z , expressed in terms of pure real numbers, is **an**

exact result. It represents a true law of Nature which reflects geometric relations between space and an electromagnetic field in a Lobachevskian Universe. This is how Nature works, and may serve as an illustration to what Eugene Wigner calls “*an unreasonable effectiveness of mathematics in natural sciences*”.

We can easily get a linearized version (small redshifts—weakly Lobachevskian case) of Equation (30). Since:

$$\sinh z = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!} = z + \frac{z^3}{3!} + \frac{z^5}{5!} + \dots \quad (31)$$

and since for small z :

$$\ln(z+1) \approx z \quad (32)$$

we have a linearized version of our general Equation (30) as:

$$\beta = \frac{1}{3}z^2 \quad (33)$$

Note that in (33), β is proportional to z squared, which is consistent with the fact that β is the relative change in (surface) area whereas z is the relative change in wavelength (*i.e.* length).

It is interesting to note that the same result can be obtained in another way. We note that the inverse of (29) is in fact the inverse of the expansion of $\frac{x}{\sinh x}$ in the power series with coefficients called Bernoulli numbers B_n :

$$\frac{x}{\sinh x} = 1 - (2^2 - 2) \frac{B_1}{2!} x^2 + (2^4 - 2) \frac{B_2}{4!} x^4 - \dots \quad (34)$$

In (34), B_n are Bernoulli numbers. For instance the first three are: $B_1 = \frac{1}{6}$, $B_2 = \frac{1}{30}$, $B_3 = \frac{1}{42}$, and so on. The numbers B_n were introduced to mathematics by Jacob Bernoulli Sr. in the 18th century. In the mid-20th century, they found an unexpected application in topology, in the theory of characteristic classes. Our case shows an unexpected applicability of Bernoulli numbers in Lobachevskian geometry and Lobachevskian astrophysics.

As an exercise, the reader may obtain the same result as (33), by noting that for small β , $\frac{1}{1+\beta} \approx 1 - \beta$ and taking into account only the first two terms of (34) *i.e.* the linearized, or weakly Lobachevskian case.

The astrophysicist **familiar** with Lobachevskian geometry will see in this the signature of the negative curvature of the Lobachevskian Universe. The astrophysicist **ignorant** of Lobachevskian geometry will claim that the rate of expansion was lower in the past than the present, $H_{\text{past}} < H_{\text{present}}$, since he associates expansion with redshift. Consequently, he will also claim that that expansion in the present is higher than in the past, $H_{\text{present}} > H_{\text{past}}$, thus the **expansion of the Euclidean Universe is accelerating**. This is exactly what Saul Perlmutter, Schmidt, and Riess claim [15].

Equation (30) and its linearized form (33), which describe the brightness dimming vs. redshift relation, along with the plot in **Figure 8** should become the standard tools for astrophysicists for interpreting deep space astrophysical data.

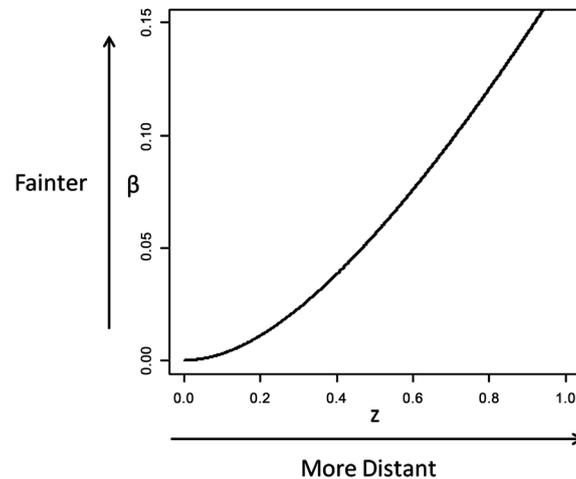


Figure 8. The above figure shows dimming β versus redshift z in a Lobachevskian Universe according to our Equation (29). As the plot clearly shows, there is a non-linear relationship between the dimming parameter β and redshift z for a given increase in redshift, there is an even greater increase in dimming. This non-linear relationship was proclaimed by Perlmutter, Schmidt, Riess [15] and 2011 Nobel Prize Committee as the signature of “accelerated space expansion”.

5. Hubble Constant and Lobachevskian Geometry

5.1. Hubble Constant as Measure of Gaussian Curvature of Non Expanding Lobachevskian Universe

Lets take a closer look at the so-called “Hubble constant” H . We now show that instead of being a measure of “space expansion” (which we believe does not exist), it is a measure of the Gaussian curvature of the Lobachevskian Universe.

The “Hubble Constant” did not appear out of nothing. It came from real data acquired by Hubble [14] in a Lobachevskian Universe. However, due to Hubble’s conceptual error (see Section 8), this data were misinterpreted in terms of velocity $z = \frac{1}{c}v$ (7) instead of in terms of space $z = \frac{1}{R}d$ (5). We will show the true meaning of the Hubble constant in **terms of a non-expanding Lobachevskian Universe**.

Definition 13 *The Hubble constant H , in units of inverse length, is the inverse of radius $\frac{1}{R}$ of the Poincare ball model of the Lobachevskian Universe.*

Its inverse $\frac{1}{H}$ is the radius of R of Poincare ball model of the Lobachevskian Universe. Its square H^2 is the Gaussian curvature of the Lobachevskian Universe understood as a Poicare ball model of Lobachevskian geometry.

First of all, we need to say that the physical unit of velocity/distance, assigned to H , is wrong. It is an intentional manipulation intended to give a false impression that H is related to the non-existent phenomenon of the expansion of the Universe. The falsification can be easily seen, since all quantities in physics must be expressed in **irreducible** units in a given system of units. Thus, m/s/m is reduced to s^{-1} , or equivalently, since c is a universal constant, it can be expressed

as inverse length. Thus the true irreducible dimension of H is either s^{-1} or m^{-1} ; we prefer m^{-1} . Therefore, as it is seen, the Hubble constant H , having true units of inverse length, **does not represent any process of expansion**, contrary to what Big Bang cosmology has been insisting since the beginning of the 20th century.

For instance, we take its “present value” as $75 \text{ k}\cdot\text{ms}^{-1}/\text{Mpc}$. Since a megaparsec is such a huge distance that we cannot intuitively comprehend, we convert everything to meters (since $c=1$). Taking $1 \text{ Mpc} = 3.086 \times 10^{22} \text{ m}$ and $c = 3 \times 10^8 \text{ m}\cdot\text{s}^{-1}$, $1 \text{ bly} = 1 \text{ billion light-years} = 9.46 \times 10^{24} \text{ m}$, we get for $H = 75$:

$$H_{75} = 0.802 \times 10^{-26} \text{ m}^{-1} \quad (35)$$

$$\frac{1}{H} = R = 13 \text{ bly} \quad (36)$$

and for the Gaussian curvature of the Lobachevskian Universe in terms of the Hubble constant we get:

$$K = H^2 = 0.64 \times 10^{-52} \text{ m}^{-2} \quad (37)$$

For $H = 65$:

$$H_{65} = 0.64 \times 10^{-26} \text{ m}^{-1} \quad (38)$$

$$\frac{1}{H} = R = 16 \text{ bly} \quad (39)$$

and for the Gaussian curvature of the Lobachevskian Universe in terms of Hubble constant we get:

$$K = H^2 = 0.44 \times 10^{-52} \text{ m}^{-2} \quad (40)$$

It should be noted that in the Poincare ball model of the Lobachevskian Universe, the radius R determines the **range of visibility, or horizon**. The actual Lobachevskian Universe is **infinite**. The actual radius and its model radius should not be misunderstood.

Comparing our results (26) and (27) following from **Lobachevski-von Brzeski theoretical cosmology** (in static Lobachevskian Universe) and from properly interpreting existing data, we see that the curvature we calculated is consistent with the real world: it is on the order of 10^{-52} m^{-2} . For the Gaussian curvature of the Lobachevskian Universe, we have (in units of 10^{-52} m^{-2}):

$$0.44(H_{65}) < 0.496(\text{vonBrzeski}) < 0.64(H_{75}) \quad (41)$$

And for the radius of the Poincare ball model, in **units of billions of light years**, we have:

$$15.8(H_{65}) > 14.9(\text{vonBrzeski}) > 12.9(H_{75}) \quad (42)$$

The **fit** between our theoretical calculations (25) based on a non-expanding Lobachevskian Universe cosmology and existing experimental data is **excellent**. We proved that our model of a **non-expanding Lobachevskian Universe works** and delivers **correct numbers**. This is the way astrophysics and cosmology should follow. The Hubble constant belongs to Lobachevskian geometry and we indeed

live in Lobachevskian world.

5.2. A Global Look at the Lobachevskian Universe. A Concave Lens Model. Virtual Reality of Inflationary Cosmology

So far we have shown that Lobachevskian geometry applied to space yields logical and mathematically consistent representations of all phenomena concerned with the propagation of light on all range of distance.

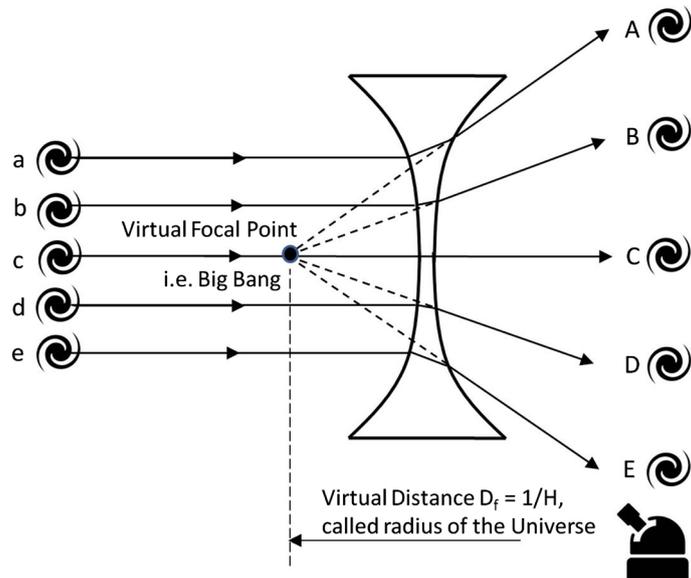
Nevertheless we have still not shown the virtual nature of the **Big Bang**. If a Lobachevskian geometry of space were unable to show that the Big Bang in reality never existed, this would be a substantial disadvantage. Fortunately the power of Lobachevskian geometry easily shows virtual nature (*i.e.* mirage) of the Big Bang, and all of the virtual science associated with it.

It is well known that a geodesic (light ray) passing in the vicinity of a massive body will be deflected **inward due to the local positive curvature of space**. This effect in geometrical optics is the same as deflecting geodesics (light rays) by a **positively curved** convex lens. In the same fashion, a geodesic will be **continuously deflected outward due to the global negative curvature** in a Lobachevskian Universe. In geometrical optics this effect is the same as deflecting geodesics (light rays) by a **negatively curved**—concave lens. **Lobachevskian lensing** is a global effect, while gravitational lensing is a local effect. Thus one can think of gravitation as a **local convex lens**, and about Lobachevskian Universe as a **globally concave lens**. An observer who interprets his observations in terms of Euclidean geometry will be convinced that the object **located at a real position a is at virtual (non-existing) position A** . This illusion or mirage of “seeing” an object at a virtual position A is caused by an ignorance of the existing geometry of space between the object and an observer.

Figure 9 shows this in detail: between the distant Universe and us the observers, a Lobachevskian space of constant negative curvature $-K$ is “inserted”, which spreads apart parallel geodesics (light rays), and all optical phenomena **act in the exact same way as in a concave lens with the negative curvature**.

We recall what Saul Perlmutter said about “looking backward in time with Hubble constant present value” [15]. Looking backward in time, Perlmutter **reversed the sign of the geodesic parameter** and ran the light rays back with the “present Hubble constant”. Obviously, the backward-running virtual rays (showed as dashed lines in **Figure 9**) all **converge in one point**, which is the **virtual focus of concave lens**.

The distance from the **virtual point** of the Big Bang to an observer is called the Hubble radius of the Universe. The virtual time resulting from the division of virtual distance by the Hubble constant is called the “age of the Universe”, another virtual entity. So when Perlmutter Schmidt and Riess were “looking backward in time”, all they were seeing was the **mirage of a virtual reality and the mirage of a virtually expanding Universe**, a mirage produced by Lobachevskian geometry. The above analysis also shows the non-applicability of the notion of time in regard to the entire Universe.



Figuer 9. In this figure we show the essence of the mirage of the Big Bang and expanding space. Galaxies, located a fixed positions *a - e* are observed by a telescope (lower right). Due to the negative curvature of space between the telescope and galaxies, represented here by a concave lens of cosmic proportions, they will *seemingly* spread apart to positions *A - E*, causing an illusion of space expansion. The solid lines in the figure above represent **real light rays**; the **dashed lines are virtual rays**, *i.e.* the “looking backwards in time” projections of the real light rays. These virtual rays converge to a virtual focal point, known otherwise as the Big Bang. The distance in units of time to that imaginary point of BB is called the the “age of the universe”. Its inverse, $1/D_f = H$, is the Hubble Constant in the standard Big Bang treatment.

How can we interpret the virtual focal distance? For a convex lens, the lens power—the inverse of the real focal distance—is a measure of the rate of convergence of **real** geodesics. For a concave lens, the lens power—the inverse of a virtual focal distance—is a measure of the rate of divergence of real geodesics whose virtual back-projection converges to a **virtual focal point**.

We end this section with a mini vocabulary from the world of inflationary cosmology.

1) *Big Bang*: The “Big Bang” (BB) is simply the virtual focal point *f* of Lobachevskian negatively curved space in a concave lens representation.

2) *Hubble radius of the Universe*: The Hubble radius of the Universe is the distance D_f to the virtual focal point *f* expressed in units of length in Lobachevskian negatively curved space in a concave lens representation.

3) *Hubble age of the Universe*: The Hubble age of the Universe is the distance D_f to the the virtual focal point *f* expressed in units of time in Lobachevskian negatively curved space in a concave lens representation ($c \equiv 1$ is a universal constant).

4) *Hubble constant*: The Hubble constant is a real number assigned to the inverse of a virtual focal distance $\frac{1}{D_f}$ of negatively curved Lobachevskian

space in concave lens representation. It is in units of inverse length if from 2) and is in units of inverse time if from 3). In optical terminology it is the power of the lens. The power varies depending on the experiment set up.

5) *Hubble flow*: The Hubble flow is the illusion of galaxies apparently spreading apart, see galaxies labeled A, B, C, D, E in **Figure 9**.

6) *Hubble distance velocity law*: The Hubble distance velocity law is a **fictitious relation** resulting from a wrong interpretation of extragalactic redshift via linear Doppler (7) by Hubble, and not by (5).

5.3. The Sources of Difficulties in Nailing Down the Hubble Constant

The misinterpretation of the Hubble constant due to its identification with $R^{-1} \frac{dR}{dt}$ of the Friedman solution of Einstein's equations, in units of Mpc^{-1} (km/s), has no explanation on why the 100-year-old effort to nail down this pivotal quantity for GR based cosmology has so far failed. We now provide an explanation.

The concave lens model of the Lobachevskian Universe is quite useful in showing the nature of the difficulties in experimentally determining the curvature $K = H^2$ of the Lobachevskian Universe. Recall that in Section 1.3, we mentioned that the argument of the hyperbolic function is never a distance alone, but a product $d = \sqrt{K}r$ which we called reduced or normalized distance. In terms of the Hubble constant, this is $d = Hr$ (we use H only in units of inverse length). It follows that even though H (curvature K) stays constant, d is a variable changing from one experiment to another, depending on the quality of telescopes.

Thus we observe the effects of the **variable power of a concave lens** (Hubble constant) inserted between us and the distant luminous object; see **Figures 10-12**. This indeterminacy (noise), or jitter, imposed on the Hubble constant is equivalent to the jitter imposed on the radius of the Universe $R = \frac{1}{H} = \frac{1}{\sqrt{K}}$,

understood as a radius of a Poincare ball model. A rough estimation due to inequality (42) is $R_{\max} - R_{\min} \sim 3 \text{ bly}$, which is about $\frac{3}{15} \sim 20\%$. This results in a “fuzzy” radius of $15 \pm 1.5 \text{ bly}$.

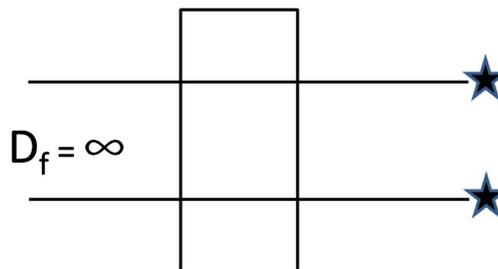


Figure 10. Intragalactic astrophysics. Parallel slab: virtual focal distance $D_f = \infty$. “Hubble constant” $H = 1/\infty = 0$. Euclidean space—no expansion; see also **Figure 1** and **Figure 2**.

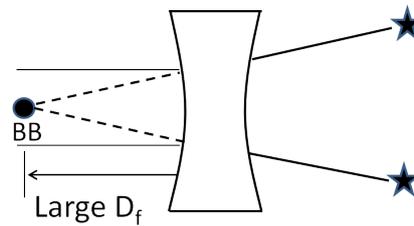


Figure 11. Extragalactic astrophysics. Weakly Lobachevskian case: virtual focal distance D_f is large. “Hubble constant” $H = 1/D_f$ is small. Space causes illusion of linear expansion; see also **Figure 3** and **Figure 4**.

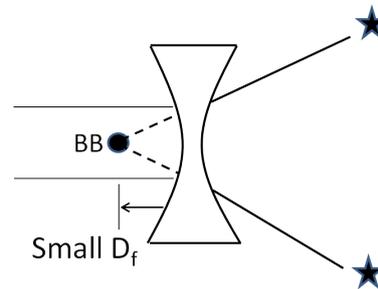


Figure 12. Deep space astronomy. Strongly Lobachevskian case: virtual focal distance D_f is small. “Hubble constant” $H = 1/D_f$ is large. Space causes illusion of accelerated space expansion; see also **Figure 5** and **Figure 6**.

This is the reason that some authors in the literature are trying to replace the Hubble constant with a “Hubble parameter”.

5.4. “Dark Energy” or Negative Curvature of Lobachevskian Universe?

The concept of “**dark energy**” is a copycat version of Einstein’s General Relativity (GR) for the negative curvature case. Its essence is as follows.

If the source of positively-bent space, or positive curvature, accordingly to GR is mass-energy density, then by analogy with real baryonic matter, there must be some, mysterious “dark matter energy density” pushing matter away and acting like some kind of anti-gravity, thus imposing a negative curvature $K < 0$ on space, and causing the mirage of space expansion. Lobachevskian geometry **does not require any “dark energy”**, and we doubt there is **any detectable “dark energy”** in the Universe. The physical large scale vacuum is, simply speaking, negatively-bent. That’s all. However those who love “dark energy” may identify it with the negative curvature of the Lobachevskian Universe.

6. Our Predictions Of Phenomena to Be Discovered in Astrophysics

Based on our previous papers and analysis of the interdependence of Lobachevskian geometry with electromagnetism, we predict the discovery of the following phenomena. These are already mathematically described in our papers listed above.

1) Lobachevskian space caused astronomical parallax. This parallax is different than the parallax in Euclidean geometry, since in Lobachevskian space an angle depends on distance and in Lobachevskian space the sum of the angles in a triangle is less than π .

2) Lobachevskian space caused aberration of light [10].

3) Lobachevskian space caused change in polarization of light (magnitude and direction) or polarization rotation [8].

4) Lobachevskian space caused dimming of light, an effect analogous to cosmological geometric redshift, but affecting the intensity of light [8].

5) Detection of gravitational waves (GW), if any, due to the scattering of an electromagnetic wave on the variable local curvature of space, inflicted by GW and manifested by a periodicity of redshift and intensity of light described in [8]. The search and detection of GW proposed by us is based on the search of **both amplitude and frequency modulated electromagnetic wave** coming from some regions of interest in deep space.

6) Down conversion to the visible spectrum of X-ray and γ -rays based on geometric downshift of frequency of an electromagnetic wave (photons) in Lobachevskian meta-materials.

As we explained above, the above effects will be noticeable only at **appropriately large distances** when the negative curvature of Lobachevskian Universe will come into play. We invite astrophysicists to look for these effects since they significantly will improve our understanding of the Universe.

7. Summary

We note that Hubble, Perlmutter, Schmidt and Riess, as well as all other astronomers over the ages, have all been observing precisely the same reality: the Lobachevskian Universe, however at different scales determined by power of their telescopes. This progression is shown in **Figures 1-6**.

1) Hubble in 1922 saw the second, **linear term** s in the power series expansion (10) of the geodesic light ray e^s in the Lobachevskian Universe. Being ignorant of Lobachevskian geometry, he could not understand (as all astronomers and astrophysicists of 20th century) that as the first man on the Earth, he directly experimentally experienced the Lobachevskian geometry of the Universe.

2) At the end of 20th century, Perlmutter, Schmidt and Riess, having better instruments, were able to look a bit further than Hubble. It follows that their telescopes were able to see the third, **quadratic term** $\frac{1}{2}s^2$ of the expansion in power series (10) of the geodesic light ray e^s . **Being ignorant of Lobachevskian geometry** (Perlmutter, Schmidt, and Riess **could not understand Lobachevskian data** coming out of their instruments. It is remarkable that the Nobel Prize Committee in 2011 endorsed the $\frac{1}{2}s^2$ quadratic term in series expansion (10) of a light ray in **static Lobachevskian geometry** and the Lobachevsky-von Brzeski geometric dimming factor as “the discovery of accelerating expansion of

the Universe through observation of distant supernovae”. In fact, photometric data of distant supernovae acquired by Perlmutter, Schmidt, and Riess **proved the Lobachevskian geometry of the Universe**.

3) The next term in power series expansion (10) is $\frac{1}{6}s^3$, and if we continue with the same “approach” to cosmology we witnessed over the past 100 years, we can expect the “discovery” of “**superluminal accelerated expansion of the Universe**” exceeding any number of times the speed of light. It seems that our prediction may unfortunately be correct since by the latest reports on June 3rd, 2016 jointly by NASA and ESA, published in The Guardian by Tim Radford, “acceleration of space” is going up to 9% faster than found a few years earlier by Perlmutter, Schmidt, and Riess (data from Hubble space telescope).

The illusions in the perception of space and motion, e.g flat Earth and the illusory movement of the Sun around the Earth were common in the domain of science. It worth noting that to justify the apparent movement of the Sun around the Earth, a sophisticated system of epicycles was developed, similar to the system of so-called cosmological parameters developed on purpose to justify Big Bang inflationary cosmology. The geocentric system was abolished by Polish astronomer, Nicolas Copernicus. The Flat Earth society however still exists worldwide.

8. Historical Notes

Lobachevskian geometry is a result the intellectual struggle of the brightest mathematical minds in history, a struggle to prove the Euclid’s 5th postulate about parallels. It was realized that abandoning the 5th postulate results in a new geometrical system with a richer content than Euclidean geometry. Many mathematicians contributed, but it was Nikolay Ivanovich Lobachevski 1792-1856, a Russian mathematician and geometer of Polish ancestry who put it in its final shape. Even for Lobachevski himself, the result was so unusual that he called it “imaginary geometry”. William K. Clifford called Lobachevski the **Copernicus of Geometry** due to the revolutionary character of his work. It is remarkable how few physicists are familiar with the content of Lobachevskian geometry more than 150 years after its publication.

It was just an **unfortunate coincidence** that at the time Einstein developed his theory of General Relativity, Edwin Hubble carried out astronomical observations of extragalactic nebulae [14] and he experimentally found that: redshift (in region B see **Figure 3** and **Figure 4**) is proportional to distance: $z = c_1 d$, c_1 is in units of inverse distance. **Linear** Doppler shift $z = c_2 v$, c_2 is in units of inverse velocity. From those **two separately, but logically unrelated, true relations**, a **wrong conclusion** $v = \frac{c_1}{c_2} d$ was drawn. The ratio $\frac{c_1}{c_2}$ has units of velocity per distance, *and it was quickly identified with the so called “Hubble constant H ”*; see also [7]. The incorrect relation resulted from a **wrong**

assumption that experimentally recorded redshift is solely due to the linear Doppler effect $z = \sqrt{K}v = \frac{v}{c}$ (7) was called the “Hubble distance velocity law”:
 $v = \frac{c_1}{c_2}d = Hd$ with $H \equiv \frac{c_1}{c_2}$. In the Appendix, we show the mathematical nature of different sources of redshift and their implications on experimental data.

The quantity resulting from those irresponsible manipulations was eagerly identified with the $\frac{dR}{dt}R^{-1}$ term of the Friedman solution units of inverse time [1/s], or energy, and was called the Hubble constant.

The FRW metric and Einstein’s equations are two logically and mathematically independent concepts. It is **not granted a priori** that their fusion will produce a meaningful result. The combination of the FRW metric (with a time dependent term) and Einstein’s equations has led to an **un-physical solution** containing the mirage of “space expansion”, characterized by the entity resulting from the error—the “Hubble constant” and associated wording.

Contrary to quantum mechanics, Einstein’s GR did not result in some “breakthrough” in physics, neither on a micro scale nor on a global scale. On the micro scale, after a 100 years of intense effort, we still do not have the quantum theory of gravity and it is not clear is this possible at all. On the global scales, the questionable applicability of Einstein’s general relativity to cosmology has not met expectations either. It is entirely unclear why a differential equation which in mathematics is considered to be a **local object** is suitable to describe the Universe in its **infinite wholeness**? Einstein’s General Theory of Relativity (GR), while applicable to a star or galaxy, is entirely unsuitable as a tool for analyzing the Universe as a whole. It also should be said that any partial differential Equation (PDE) is mostly void unless initial and boundary conditions are not specified upfront. Initial and boundary conditions for the entire Universe (if any) are known perhaps to only God, if one exists. It is not surprising that in conjunction with the FRW metric, GR produced bizarre **non-physical effects** and conclusions of “space expansion”, Big Bang, “space inflation”, “dark energy”, “accelerated space expansion”.

We demonstrated in several ways that **Lobachevskian space when interpreted in terms of Euclidean space will cause the illusion of “expansion of the Euclidean space”**. We represent it again, from ancient times up to present, in **Figure 13**, and reader can see himself/herself “expansion of space”.

The illusions in the perception of space and motion (e.g. a flat Earth and Ptolemy’s geocentric system) are not something unusual in the domain of science. The development of rational science may be regarded as a struggle to overcome false ideas and illusions, one after the other.

If Edwin Hubble and his fellow astrophysicists at the time knew the above geometrical formula of cosmological redshift (4) in its linear approximation (5), he obviously would have seen that it is exactly what he experimentally measured,

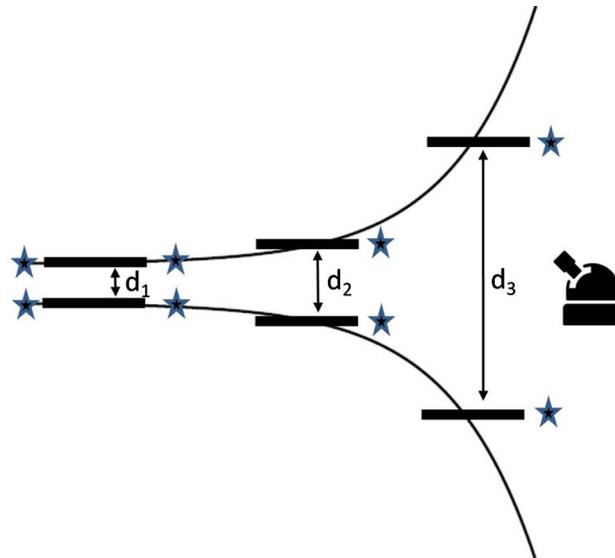


Figure 13. The above figure again shows the illusions that result when Lobachevskian geometry is interpreted in terms of Euclidean (flat) geometry. Starting as parallel at the star, the distance the geodesics first spread apart in linear fashion, and then in non-linear fashion. This is the whole essence of the misunderstanding of space “expansion” and accelerated space “expansion”. The newest “trick” in the idea of “space expansion” is the so-called “**metric expansion**”, when the flat Euclidean Universe (space) expands into itself. The Lobachevskian geodesics (solid thin lines) are replaced by piecewise parallel (in a Euclidean sense) **local** straight line geodesics (thick segments). The Universe (space) is flat and **apparently** expanding in an accelerated fashion $d_3 > d_2 > d_1$. This is analogous to the Flat Earth scenario, in which geodesics on the positively curved surface of the Earth were replaced by Euclidean geodesic-straight lines.

i.e. redshift z is a linear function of distance d : $z = \sqrt{K}d$.

If that had happened, there would never have been a Big Bang, and there never would have been any “space expansion”: no “linear space expansion”, no “non-linear space expansion”, no “metric space expansion”, no “accelerated space expansion, and no “dark energy”. The geometry of space on large scales, recovered from the geometry of geodesics, is strongly Lobachevskian, and on local scales can be approximated with arbitrary precision by the Euclidean geometry. “Global scales” and “local scales”, as we explained in the text, are determined by the power of our instruments. Nevertheless we see also the presence of matter in the Universe in various states of aggregation, and in constant chaotic motion. From that point of view the Universe is a **dynamical system** which can be studied by relevant exact mathematical methods.

Acknowledgements

This work is dedicated to two great scientists in the history of mankind: Russian geometer Nikolay Ivanovich Lobachevski and Polish astronomer Nicolas Copernicus.

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Appendix

Probability and Certainty in Physics

Looking at Equations (4) and (6), we see that we have **two maps from Lobachevskian geometry into the physical world**:

- 1) Equation (4) maps **distance** in position space **to redshift**.
- 2) Equation (6) maps signed **distance** in velocities space, (*i.e.* relative velocity), **to redshift or to blue shift** depending on orientation of relative velocity.

Below we give three equivalent interpretations of the situation shown in **Figure 14**.

1) **Interpretation via classical physics.** From the point of view of classical physics we have a (typical) situation of phase space mapping. The phase space of the Universe is a direct sum of the Lobachevskian position space L_X^3 and the Lobachevskian velocity space L_V^3 . Geometry which describes the phase space properties is **symplectic geometry**. The total measurable redshift at the telescope is a mixture of geometric and kinematic contributions and is described by some **probability measure** on the phase space in question. In the case of **local astronomy**, Lobachevskian position space L_X^3 is approximated by the **Euclidean space** R_E^3 .

2) **Interpretation via quantum mechanics.** An interesting interpretation of the cosmological frequency shift (and apparent brightness as well) in a Lobachevskian Universe (as we mention in [7]) is due to quantum mechanics (QM). We can regard the two components of the cosmological frequency shift as two **pure states**: the frequency shift z_G due to Lobachevskian geometry of space **alone**, and the frequency shift of kinematic origin **alone**, as the pure state z_K . Looking at **Figure 14**, a quantum mechanical physicist will say: **Figure 14** shows a typical example of a **quantum mixed state** of redshift recorded at the telescope: $|Z_{tot}\rangle = p_G|Z_G\rangle + p_K|Z_K\rangle$.

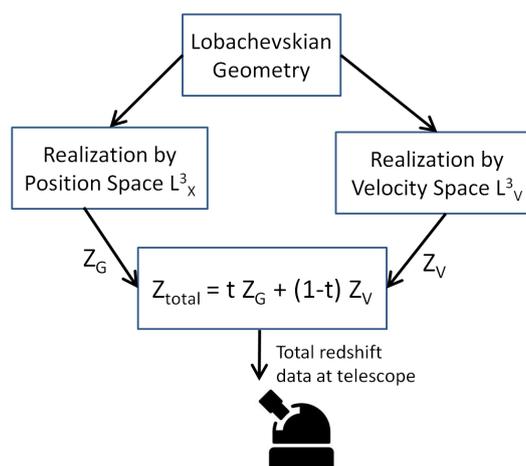


Figure 14. The above figure shows the composite nature of redshift recorded at a telescope due to geometric space-caused redshift and kinematic redshift. Hubble's mistake was to assume that $t=0$ and that $Z_{total} = Z_V$, *i.e.* the observed redshift is only due to the (pure-state) Doppler shift.

Unfortunately we do not have knowledge of how the mixed state was prepared, and we have no way of knowing the real numbers (probability amplitudes) p_G and p_K . This is known as **information loss** for a QM system being in a mixed state.

3) **Topological, homotopy interpretation.** Looking at **Figure 14**, a topologist will say: I see a definition of a **homotopy** [4]. Two maps z_G and z_K are continuously related on a segment $t \in [0,1]$ by homotopy H . What you call a (pure) geometrical shift z_G and a (pure) kinematic shift z_K are in our language **fixed ends of a homotopy**. Call them, if you wish, as QM **pure states** or **von Neumann zero entropy states**. For a topologist (reader can check this as an exercise), the homotopy H relation fHg between two maps f and g is an **equivalence relation** R . Thus for $f = z_G$ and for $g = z_K$, $z_G R z_K$, and mathematics says that spectral shift data acquired at telescope carries no information on how to discriminate between equivalent maps, and as a consequence **we do know what we see**, $z_{\text{total}} = tz_G + (1-t)z_K$, $t \in [0,1]$. In other words, mathematics says that **we do not know where point** z_{total} is located on the homotopy segment connecting z_G and z_K .

Note that the **brightness data at telescope can be analyzed in an analogous way**. We leave that to the reader.

It is well known that two variables related by a Fourier transform are called conjugate. Such conjugate variables for instance are, position and velocity, position and momentum, and so on. In quantum mechanics, the precision with which the momentum and position for a quantum object can be determined is governed by the Heisenberg uncertainty principle.

In classical physics, the precision of a simultaneous determination of position and velocity of an object is limited too. For instance, in radar techniques the position and velocity of an object are determined via the use of electromagnetic waves, and we have the same situation of indeterminacy. It is expressed by the so-called **radar ambiguity function**. The ambiguity function, like the Heisenberg inequality, sets mutual limits at which canonically conjugate variables can be determined. As in radar technology, which can be regarded as an “active astrophysics” with a manmade signal, a transmitted narrow pulse (assembled with large number of frequencies) results in **good distance**, but **poor velocity** data. The wide pulse of a single frequency results in **good velocity** but **poor distance** data; see an excellent mathematical analysis of radar techniques by Walter Schempp [17].

In application to astrophysics and astronomy the same reasoning applies. It tells us, for instance, that the precision in determining the distance to some galaxy and its velocity has a mutual limit in the sense that a better resolution in distance will result in more ambiguity in its relative (to us) velocity. There is a limit D such that $\Delta x \Delta v \geq D$. This is a mathematical constraint and no “perfect instrumentation” or “perfect measurements techniques” can overcome it.