

Dynamic Harmonic Domain Modelling of Space Vector Based SSSC

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Abstract

This paper presents analytical frequency domain method for harmonic modeling and evaluation of Space Vector Pulse Width Modulation (SVPWM) based static synchronous series converter (SSSC). SVPWM is the best among all the PWM techniques. It gives a degree of freedom of space vector placement in a switching cycle. Dynamic modeling technique is used for space vector modulation (SVM) based voltage source converter that is adapted as a static synchronous series converter (SSSC) for harmonic analysis using dynamic harmonic domain. Performance of the SSSC is evaluated in dynamic harmonic domain simulation studies in MATLAB environment. The switching function spectra are necessary for harmonic transfer matrix which is calculated using Fourier series. This paper presents the analysis of harmonics for space vector based SSSC during steady state and dynamic condition.

Keywords

Dynamics Harmonic Domain, Voltage Source Converter (VSC), Space Vector Modulation (SVM), Static Synchronus Series Compensator (SSSC)

1. Introduction

Harmonic in power system caused by highly non-linear devices degrades its performance. Forced commutated VSCs are the main building block for low and medium power application. Due to recent development in the semiconductor technology and availability of high power switches e.g. Insulated Gate Bipolar Transistor (IGBT) and Gate Turn Transistor (GTO) have widespread acceptance in for high power VSC's, which are used for FACTS controllers. The power system harmonic analysis is the process of calculating the magnitude and phase of fundamental and higher order harmonic of system signals. The generation of harmonics in modern power system is due to large size of power converter. To reduce the harmonics in the system filter, modern switching

patterns are used. The increasing prevalence of flexible AC transmission system (FACTS) devices makes these accurate model devices essential. One attracting method for modeling the steady state performance of these devices is frequency domain analysis [1]. Harmonic phasor contains both positive and negative frequency terms for phase dependence. FACTS devices are characterized by their switching nature. One of the important FACTS devices is SSSC. SSSC is a series connected device that is able to provide active or reactive power support at the network location [2]. To understand the harmonics interaction between the SSSC based on VSC and utility system, there is a need of appropriate model.

VSC based SSSC acts as source of harmonic current injection into the system and also interacts with harmonic distortions present within the system. Various modulation techniques and topologies are used for SSSC to effectively reduce generated harmonics including PWM techniques, multi-module PWM techniques, selective harmonic elimination, and multi-level topology.

Previous work conducted in dynamic harmonic domain has been primarily focus on modeling PWM, multimodule, selective harmonic elimination based SSSC [4]-[8]. The power quality index can be assessed directly from the DHD modeling. Active, reactive, apparent powers as well as power factor are some important power quality indices. For the linear circuit the indices are defined in term of fundamental frequency whereas for nonlinear circuit, when nonlinear elements are present in a circuit such as electronics devices, given on basic of Fourier coefficient or given in terms of Total Harmonic Distortion (THD). For assessment of accurate power quality indices precise calculation of harmonic component is needed during transient period. Other authors have already made their contribution regarding modeling of FACTS devices and power system element using DHD method [7] [8].

There are two methods of DHD modeling. The first one is that direct mathematical mapping of all system equations and input is in frequency domain; and the other method that does not mapped all equations and input is in frequency domain gives more accurate result during transients. In second method, transient occurs due to change in circuit parameters. Second method is more suitable for calculation of instantaneous power quality indices, protection analysis and real time application all through it is violating causality and spurious dynamics result provided enough harmonic consideration [3].

To extend the results obtained by other authors in the modeling of SSSC FACTS device, the proposed SVPWM VSC based SSSC model is developed in order to be able to obtained the evolution in the of harmonic components of the SSSC signal. Dynamic harmonic analysis of PWM based SSSC is shown in [7] as well as multi-pulse SSSC which is shown in [8]. This paper presents the dynamic analysis of SVPWM based SSSC during steady state and dynamic condition. This gives the information about harmonic indices during transient condition also. This can be used for control system design for space vector based SSSC as space vector modulation scheme is more suitable for digital implementation. The paper is organized as follows. The second section provides fundamentals of DHD method. The third section is fundamental of space vector based pulse width modulation based voltage source converter; this arrangement is used in this paper. The fourth section deals with DHD modeling of SVPWM VSC used for SSSC.

2. Dynamic Harmonic Domain Basic

The system given by Ordinary Differential Equations (ODE) can be transform to an alternative arrangement called the Dynamic Harmonic Domain, based on the approximation of the system by Fourier series over a period of the fundamental frequency [9].

Linear periodic system (LTP) is converted into linear time domain (LTI) system using dynamic harmonic domain (DHD).

Consider a LTP system is given by

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$

$$y(t) = C(t)x(t) + D(t)u(t)$$
(1)

where A(t) is given as

$$A(t) = a_{-h} e^{-jh\omega_0 t} + \dots + a_{-1} e^{-j\omega_0 t} + a_0 + a_1 e^{j\omega_0 t} + \dots + a_h e^{jh\omega_0 t}$$
(2)

where h is the highest harmonics of interest and ω_0 is the fundamental frequency of the system.

Equation (1) can be represented in the Fourier series, the ordinary differential equation to an alternative DHD

representation. Therefore

$$\dot{X} = (A - S)X + BU$$

$$Y = CX + DU.$$
(3)

Variable given in the Equation (1) can transform into the vectors in the Equation (3) with their coefficient related to the harmonic components of their instantaneous signals as,

$$X = \left[X_{-h}(t) \cdots X_{-1}(t) X_{0}(t) X_{1}(t) \cdots X_{h}(t) \right]^{1}.$$

S is the matrix of differential and given by

$$S = \operatorname{diag} \left[-jh\omega_0 \cdots - j\omega_0 \ 0 \ j\omega_0 \cdots jh\omega_0 \right].$$

The matrices A, B, C and D has Toeplitz structure and their time domain counterpart such that

$$A = \begin{bmatrix} A_0 & A_{-1} & \cdots & A_{-h} & & & \\ A_1 & \ddots & \ddots & \ddots & \ddots & \\ \vdots & \ddots & \ddots & \ddots & \ddots & A_{-h} & & \\ A_h & \ddots & \ddots & A_0 & A_{-1} & \ddots & \ddots & \\ & \ddots & \ddots & A_1 & A_0 & \ddots & \ddots & A_{-h} \\ & & A_h & \ddots & \ddots & \ddots & \ddots & \vdots \\ & & & \ddots & \ddots & \ddots & \ddots & \ddots & A_{-1} \\ & & & & A_h & \dots & A_1 & A_0 \end{bmatrix}$$

The steady state solution is obtained directly from Equation (3), considering ($\dot{X} = 0$) gives,

$$X = (S - A)^{-1} BU$$

$$Y = CX + DU.$$
(4)

The matrices A, B, C and D are constant and input U is also constant. The solution of X and Y is obtained from Equation (4). The solution of Equation (4) can be used to initialize the DHD simulation in time domain.

3. Space Vector Modulation

Principles of SVPWM

SVPWM is based in such a way that there are only two independent variables in a three-phase voltage system. We use orthogonal coordinates to represent the 3-phase voltage in the phasor diagram. A three-phase voltage vector represented by complex space vector as in Equation (1) neglecting zero sequence components [10] [11]

$$\begin{bmatrix} V_{\alpha} \\ V_{\beta} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}$$
(5)

SVM use the combinations of switching states to approximate the locus of V_{ref} . In α - β plane a hexagon centred at origin of $\alpha\beta$ plane, identifies the space vectors shown in Figure 1. Space vector plane of Figure 1 is divide into six sectors. Each sector covers the space corresponding to 60°.

The distinct possible switching states of the 2-level VSC are represented as eight voltage vectors, out of which six are active states $(V_1 - V_6)$ and two are null states (V_0, V_7) . The active states contribute output line voltage as $+V_{dc}$ or $-V_{dc}$, where as null states does not contribute any output voltage for VSC. The eight voltage vectors are tabulated in **Table 1** denotes ON state of the switch and 0 denotes OFF state of the switch.

The reference vector is synthesized by the three adjacent switching vectors. For example, when V_{ref} falls into sector I as shown in Figure 2, it can be synthesized by V_1 , V_2 and V_0 . The volt second balance equation is

$$V_{ref}T_s = V_1T_1 + V_2T_2 + V_0T_0$$
(6)



Figure 2. Representation of reference vector.

Table 1. 2-level inverter voltage vectors voltage vectors.						
S. No.	S_a	S_b	S _c -	Line to neutral voltage		
				V _{an}	V_{bn}	V_{cn}
1	1	0	0	V_{dc}	0	0
2	1	1	0	V_{dc}	V_{dc}	0
3	0	1	0	0	V_{dc}	0
4	0	1	1	0	V_{dc}	V_{dc}
5	0	0	1	0	0	V_{dc}
6	1	0	1	V_{dc}	0	V_{dc}
7	1	1	1	V_{dc}	V_{dc}	V_{dc}
8	0	0	0	0	0	0

where T_s is the period of the switching cycle, T_1 and T_2 are the switching times of the vectors V_1 and V_2 . T_1 and T_2 are calculated as

$$T_{1} = \frac{V_{ref}}{\sqrt{\frac{2}{3}}} T_{s} \frac{\sin(60 - \theta)}{\sin 60}$$
(7)

$$T_2 = \frac{V_{ref}}{\sqrt{\frac{2}{3}}} T \frac{\sin \theta}{\sin 60}$$
(8)

$$T_0 = T_s - T_1 - T_2 (9)$$

Similar calculation is applied to sector II to V_1 Vector V_8 can be used in place of V_7 . The choice is based on the requirement to minimize average number of switching per cycle.

The maximum value of V_{ref} is obtain when $\theta = 30^{\circ}$ and V_{ref} is given by

$$\operatorname{Max} V_{ref} = \cos 30^{\circ} \cdot \sqrt{\frac{2}{3}} V_{dc} \tag{10}$$

This is the maximum value of line to line voltage injected by the converter. The maximum magnitude of V_{ref}

is also the radius of circle inscribed in th hexagon shown in **Figure 1**. The Square wave converter generates a space vector of magnitude $\sqrt{6}/\pi$ V_{dc} the maximum value of the modulation index as

$$m_{\rm max} = \frac{\pi}{\sqrt{6} \cdot \sqrt{2}} = 0.907.$$
 (11)

4. Dynamic Harmonic Domain Modelling of SVPWM Based VSC

SSSC is important series FACT device which inject voltage into the transmission line. It is combination of voltage source converter, capacitor and coupling transformer. Primary of transformer is connected to the VSC and secondary is connected to the transmission line. If the voltage of VSC is in quadrature of line current then some reactive power exchange is take place and if voltage is not in quadrature of line current active power exchange is take place. DHD model of SSSC is presented in [Bharat] considering selective harmonic elimination method. In preceding discussion we extent this model considering space vector modulation techniques. **Figure 3** shows the equivalent circuit of SSSC connected between the transmission line. AC side of SSSC shows the sinusoidal three phase voltage source connected in series. $R_e + jX_e$ shows the resistance and impedance of coupling transformer. the three-phase voltages and currents on the AC side of the SSSC are $V_{ABCl}(t)$ and $i_{ABC}(t)$, respectively, and can be expressed in terms of the DC side voltage $v_{dc}(t)$, DC side current $i_1(t)$ and the switching functions as

$$V_{ABC}(t) = p_{S}(t) \cdot v_{dc}(t)$$

$$i_{1}(t) = q_{S}(t) \cdot i_{ABC}(t)$$
(12)

 $v_{ABC1}(t)$ and $i_{ABC}(t)$ are three phase voltage and current vectors given by:

$$V_{ABC1}(t) = \begin{bmatrix} v_{A1}(t) & v_{B1}(t) & v_{C1}(t) \end{bmatrix}$$
$$i_{ABC}(t) = \begin{bmatrix} i_A(t) & i_B(t) & i_C(t) \end{bmatrix}^{\mathrm{T}}.$$

In Equation (12), $p_s(t)$ and $q_s(t)$ are transformation vectors, which are given by:

$$p_{s}(t) = \begin{bmatrix} S_{ab}(t) & S_{bc}(t) & S_{Ca}(t) \end{bmatrix}^{1}$$
$$q_{s}(t) = \begin{bmatrix} S_{ab}(t) & S_{bc}(t) & S_{Ca}(t) \end{bmatrix}$$

The state equation can be written for SSSC are:

$$\frac{\mathrm{d}V_{dc}\left(t\right)}{\mathrm{d}t} = \frac{1}{C}i_{1}\left(t\right) = \frac{1}{C}q_{s}\left(t\right)i_{ABC}\left(t\right) \tag{13}$$



Figure 3. Static series synchronus compensator (SSSC).

$$R_e i_{ABC}\left(t\right) + L_e \frac{\mathrm{d}i_{ABC}\left(t\right)}{\mathrm{d}t} = V_{ABC}\left(t\right) - V_{ABC1}\left(t\right) \tag{14}$$

$$\frac{di_{ABC}(t)}{dt} = -\frac{R_{e}}{L_{e}}i_{ABC}(t) + \frac{1}{L_{e}}V_{ABC}(t) - \frac{1}{L_{e}}p_{s1}(t)V_{dc}(t).$$
(15)

The linear time periodic Equations (13) & (15) in matrix form will be written as:

$$\begin{bmatrix} \frac{di_{ABC}(t)}{dt} \\ \frac{dV_{dc}(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_e}{L_e} & -\frac{1}{L_e} p_{s1}(t) \\ \frac{1}{C} q_s(t) & 0 \end{bmatrix} \begin{bmatrix} i_{ABC}(t) \\ V_{dc}(t) \end{bmatrix} + \frac{1}{L_e} \begin{bmatrix} V_{ABC}(t) \\ 0 \end{bmatrix}.$$
(16)

The Equation (16) can be transfer to linear time invariant equation considering theory of DHD analysis as:

$$\begin{bmatrix} \dot{I}_{ABC}(t) \\ \dot{V}_{dc}(t) \end{bmatrix} = \begin{bmatrix} -\frac{R_e}{L_e} U_1 - D(jh\omega_0) & -\frac{1}{L_e} P_s(t) \\ \frac{1}{C} Q_s(t) & -D(jh\omega_0) \end{bmatrix} \begin{bmatrix} i_{ABC}(t) \\ V_{dc}(t) \end{bmatrix} + \frac{1}{L_e} \begin{bmatrix} V_{ABC}(t) \\ 0 \end{bmatrix}.$$
(17)

The initial condition are obtained by considering derivatives of state variable as zero gives:

$$\begin{bmatrix} i_{ABC}(t) \\ V_{dc}(t) \end{bmatrix} = -\frac{1}{L_e} \begin{bmatrix} -\frac{R_e}{L_e} U_1 - D(jh\omega_0) & -\frac{1}{L_e} P_s(t) \\ \frac{1}{C} Q_s(t) & -D(jh\omega_0) \end{bmatrix}^{-1} \begin{bmatrix} V_{ABC}(t) \\ 0 \end{bmatrix}.$$
(18)

Switching Vectors in Harmonic Domain

The General switching function is obtained in time domain for space vector modulation. The harmonic content in switching function is given by Fourier series.

$$S_a(\omega t) = \sum_{n=-h}^{h} S_a e^{jn\omega_0 t}$$
⁽¹⁹⁾

$$S_b(\omega t) = \sum_{n=-h}^{h} S_b e^{jn\omega_0 t}$$
⁽²⁰⁾

$$S_{c}(\omega t) = \sum_{n=-h}^{h} S_{c} e^{jn\omega_{0}t}$$
⁽²¹⁾

where S_a , S_b , S_c are switching function obtained by using SVPWM algorithm. The line switching vector is defined as

$$S_{ab} = S_a - S_b$$

$$S_{bc} = S_b - S_c$$

$$S_{ca} = S_c - S_a$$
(22)

The switching vector for harmonic domain is defined as

$$S_{1} = \begin{bmatrix} S_{ab} \\ S_{bc} \\ S_{ca} \end{bmatrix}, \quad S_{2} = \begin{bmatrix} S_{ab} & S_{bc} & S_{ca} \end{bmatrix}$$
(23)

5. Simulation Result

In order to access dynamic harmonics response including power quality indices, the per-phase inductive reac-

tance and resistance of the coupling transformer and the capacitance of the dc capacitor are $R = 0.04 \Omega$, L = 0.2 mH and $C = 5000 \mu$ F, respectively.

Under steady state conditions the bus per phase voltages V_R and V_S in volts at 50 Hz are

$$vR_{a}(t) = \sin \omega_{0}t, \quad vS_{a}(t) = 0.8 \sin \omega_{0}t$$
$$vR_{b}(t) = \sin(\omega_{0}t - 120^{\circ}), \quad vS_{b}(t) = 0.8 \sin(\omega_{0}t - 1200)$$
$$vR_{c}(t) = \sin(\omega_{0}t + 120^{\circ}), \quad vs_{c}(t) = 0.8 \sin(\omega_{0}t + 120^{\circ})$$

Assume disturbances in the voltages starting at 0.04 seconds and lasting for 0.005 seconds. During disturbances voltages on the V_S bus is half of the original value. The simulation was started at $t_0 = 0$ seconds with final time $t_f = 0.1$ seconds and an integration time step is 0.001 s. 50 harmonics are considered. System is simulated using MATLAB software.

Figure 4 exhibits the dynamic behaviour of only the 1st, 3rd, 5th and 7th harmonic components with time for the phase-a current of the SSSC and DC capacitor voltage magnitudes. It clearly shows that behaviour of these variables from the onset of the disturbance until its ending, and their post-disturbance variations. The effective-ness of SSSC in bringing the system back to original steady state is also shown in these plots.

SSSC's dynamic power quantities of all the three phases are shown in **Figure 5**. Dynamic power quantities of the SSSC such as RMS current Irms (t), apparent power S(t), active power P(t), reactive power Q(t), and distortion power D(t) in all the three phases are of interest for power quality assessments and control. Their changes with time during and after the disturbance are shown in the plots.

The total harmonic distortion measurement is important information for assessment of the device under different conditions. The total harmonic distortions (THD) of the voltage and current of the SSSC in all the three phases are shown in **Figure 6**. These plots show that, during the disturbance, the THD changes considerably due to the dynamic response of the harmonic components. The voltage and the current THD of the SSSC in all three phases are shown in **Figure 6**.

6. Conclusions

This paper presents the space vector based switching strategy for a SSSC that utilizes the voltage source converter to minimize the harmonic at the point of common contact. The linear time periodic equation is converted into linear time invariant system which is done using dynamic harmonic domain for calculation of harmonic interference in the system during transient and evaluated based dynamic harmonic domain algorithms using MATLAB code.



The proposed model is used to calculate harmonic interference produced by space vector based SSSC during

Figure 4. SSSC voltage and current. (a) Current at SSSC terminal phase A; (b) Voltage on DC capacitor.



Figure 5. SSSC terminal electric quantities. (a) RMS current; (b) Apparent power; (c) Distorted power; (d) RMS voltage on capacitor; (e) Active power; (f) Reactive power.



Figure 6. THD in voltage and current. (a) THD in voltage; (b) THD in current

steady state and during disturbances. This gives accurate result and information about harmonics indices during transient operation of system as compared to time domain simulation which is important for designing control system for the system.

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