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Distribution of Mass and Energy in Five General Cosmic Models

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Abstract

Distributions of the universe horizon distance and universe horizon volume were investigated in the light of five general cosmic models which were constructed in a previous study. Both distributions increase so slowly up to $t \approx 21.5444$ Myr, then they start raising very fast up to $t \approx 60$ Gyr. Afterwards, they increase again very slowly until $t \approx 124$ Gyr. Distributions of mass of radiation, matter and dark energy within the horizon volume of the universe were also studied in the five general cosmic models. The masses of both radiation and matter decrease gradually with time while the mass of dark energy increases. The mass of radiation prevailed in the early universe up to $t \approx 34627.5$ - 55916.2 yr, where it becomes equal to the mass of matter. Then the mass of matter dominated until $t \approx 9.4525$ - 10.0632 Gyr, where it becomes equal to the mass of dark energy. Thenceforward, the mass of dark energy prevails the universe. The cosmic space becomes approximately matter empty in the so far future of the universe.

Keywords

General Cosmic Models, Distribution of Mass and Energy

1. Introduction

In a previous study [1] the distribution of density parameters of radiation, matter and dark energy were investigated in details in five general cosmic models. Hence, it would be interesting to study the distributions of equivalent mass of radiation, mass of matter and equivalent mass of dark energy within the horizon volume of the universe in the general models.

Therefore, it is necessary to start this study by investigating the distributions of the horizon distance and horizon volume of the universe in the general models at different time intervals depending on the bases discussed in [2]. Description of methodology is given in Section 2 while algorithm would be illustrated in Section 3. Results and discussion are presented in Section 4. Conclusion is shown in Section 5.

2. Methodology

We have seen in [2] that the horizon distance and horizon volume of the universe at the present time are respectively

$$d_h(t_o)=\frac{c}{H_o}\int_0^1\frac{1}{a}\left[1-\Omega_{\Lambda,o}\left(1-a^2\right)+\Omega_{m,o}\left(\frac{1}{a}-1\right)+\Omega_{r,o}\left(\frac{1}{a^2}-1\right)\right]^{-\frac{1}{2}}\mathrm{d}a. \quad (1)$$

$$V_h(t_o)=\frac{8\pi}{3}d_h^3(t_o). \quad (2)$$

where $c,H_o,\Omega_{r,o},\Omega_{m,o},\Omega_{\Lambda,o}$ are all defined as in [1]. Thus the horizon distance of the universe at any given time is

$$d_h(t)=\frac{c}{H_o}\int_0^a\frac{1}{a}\left[1-\Omega_{\Lambda,o}\left(1-a^2\right)+\Omega_{m,o}\left(\frac{1}{a}-1\right)+\Omega_{r,o}\left(\frac{1}{a^2}-1\right)\right]^{-\frac{1}{2}}\mathrm{d}a. \quad (3a)$$

Consequently the change in the horizon distance of the universe in the time interval between two instants of scale factors a_1,a_2 is written as

$$\Delta d_h(t)=\frac{c}{H_o}\int_{a_1}^{a_2}\frac{1}{a}\left[1-\Omega_{\Lambda,o}\left(1-a^2\right)+\Omega_{m,o}\left(\frac{1}{a}-1\right)+\Omega_{r,o}\left(\frac{1}{a^2}-1\right)\right]^{-\frac{1}{2}}\mathrm{d}a. \quad (3b)$$

The horizon volume of the universe at any given time is

$$V_h(t)=\frac{8\pi}{3}d_h^3(t). \quad (4)$$

It is also obvious from [2] that the total density of the universe is given by

$$\rho(t)=\rho_{c,t}\Omega(t). \quad (5)$$

where

$$\Omega(t)=\rho_{c,t}\left(\Omega_{m,t}+\Omega_{r,t}+\Omega_{\Lambda,t}\right). \quad (6)$$

$$\rho_{c,t}=\frac{3H^2}{8\pi G}. \quad (7)$$

$$\Omega_{m,t}=\frac{\rho_{m,t}}{\rho_{c,t}}=\left(\frac{H_0}{H}\right)^2\frac{\Omega_{m,o}}{a^3}. \quad (8)$$

$$\Omega_{r,t}=\frac{\rho_{r,t}}{\rho_{c,t}}=\left(\frac{H_0}{H}\right)^2\frac{\Omega_{r,o}}{a^4}. \quad (9)$$

$$\Omega_{\Lambda,t}=\frac{\rho_{\Lambda,t}}{\rho_{c,t}}=\left(\frac{H_0}{H}\right)^2\Omega_{\Lambda,o}. \quad (10)$$

$$H(t)=\frac{H_o}{a}\left[1-\Omega_{\Lambda,o}\left(1-a^2\right)+\Omega_{m,o}\left(\frac{1}{a}-1\right)+\Omega_{r,o}\left(\frac{1}{a^2}-1\right)\right]^{\frac{1}{2}}. \quad (11)$$

Hence, the total mass within the horizon volume of the universe at any given time is expressed as

$$M_h(t)=V_h(t)\rho(t). \quad (12)$$

The mass of matter $M_{m,t}$, the equivalent mass of radiation $M_{r,t}$ and the equivalent mass of dark energy

$M_{\Lambda,t}$ within the horizon volume of the universe at any given time are given by

$$M_{m,t} = M_h(t)\frac{\rho_{m,t}}{\rho(t)}$$

$$M_{m,t} = M_h(t)\frac{\Omega_{m,t}}{\Omega(t)}. \tag{13}$$

$$M_{r,t} = M_h(t)\frac{\Omega_{r,t}}{\Omega(t)}. \tag{14}$$

$$M_{\Lambda,t} = M_h(t)\frac{\Omega_{\Lambda,t}}{\Omega(t)}. \tag{15}$$

The cosmic time is given by Equation (16) in [1] as

$$t = \frac{1}{H_o}\int_0^a \left[1-\Omega_{\Lambda,o}\left(1-a^2\right)+\Omega_{m,o}\left(\frac{1}{a}-1\right)+\Omega_{r,o}\left(\frac{1}{a^2}-1\right)\right]^{-\frac{1}{2}} \mathrm{d}a. \tag{16a}$$

Thus the time interval between two instants with scale factors a_1, a_2 during the universe expansion is expressed as

$$\Delta t = \frac{1}{H_o}\int_{a_1}^{a_2} \left[1-\Omega_{\Lambda,o}\left(1-a^2\right)+\Omega_{m,o}\left(\frac{1}{a}-1\right)+\Omega_{r,o}\left(\frac{1}{a^2}-1\right)\right]^{-\frac{1}{2}} \mathrm{d}a. \tag{16b}$$

3. Algorithm

In determination of the distributions of $d_h(t), V_h(t), M_h(t), M_{m,t}, M_{r,t}$ and $M_{\Lambda,t}$ we use the following steps:

i) Set $t = 0, d_h = 0, K_1 = 1, K_2 = 2000$, then insert the value of $a_{\max} = 0.098$ for $t \le 0.5\ \text{Gyr}$, $a_{\max} = 10$ for $0.5 < t \le 50\ \text{Gyr}$ and $a_{\max} = 1000$ for $50 < t \le 124\ \text{Gyr}$.

ii) Compute $DA = \dfrac{a_{\max}}{DBLE(K_2)}$.

iii) Start general DO loop $I = K_1, K_2$ which includes the following sub steps:

iv) $a_1 = DA(I-1), a_2 = DAI$.

v) Calculate new value of cosmic time *t* numerically using(16-b), where $t = t + \Delta t$.

vi) Determinate new value of the universe horizon distance d_h numerically using (3-b) where, $d_h = d_h + \Delta d_h$.

vii) Obtain the corresponding values of $V_h, H, \rho_{c,t}, \Omega_{m,t}, \Omega_{r,t}, \Omega_{\Lambda,t}, \Omega(t), \rho(t), M_h(t), M_{m,t}, M_{r,t}, M_{\Lambda,t}$ using (4), (11), (7), (8), (9), (10), (6), (5), (12), (13), (14) and (15) respectively.

viii) Continue the general DO loop.

4. Results and Discussion

The distribution of the universe horizon distance in the general models up to $t = 0.5\ \text{Gyr}$ is shown in **Figure 1(a)**. The distributions of all models coincide on each other until $t \approx 21.5444\ \text{Myr}$. Then, the distributions of models A, B and C coincide on each other and get upper than the coincided distributions of models D and E. The universe horizon distance increases quite slowly with time in all general models up to $t \approx 21.5444\ \text{Myr}$, hence it stats raising very fast. The distribution of the universe horizon distance in the general models in the range $t = 0.5\text{ - }50\ \text{Gyr}$ is illustrated in **Figure 1(b)**. The distributions of all models coincide on each other up to $t \approx 1.1111\ \text{Gyr}$. Afterwards, the distributions of models A, B and C coincide on each other and become higher than the coincided distributions of models D and E. In all general models the universe horizon distance distributions increase very fast with time. The distribution of the universe horizon distance in the general models in the range $t = 50\text{ - }124\ \text{Gyr}$ is presented in **Figure 1(c)**. The distributions of models A, B and C are close to each other and lie upper than the distributions of models E and D. The increase in the universe horizon distance gets very small with increasing time in all general models.

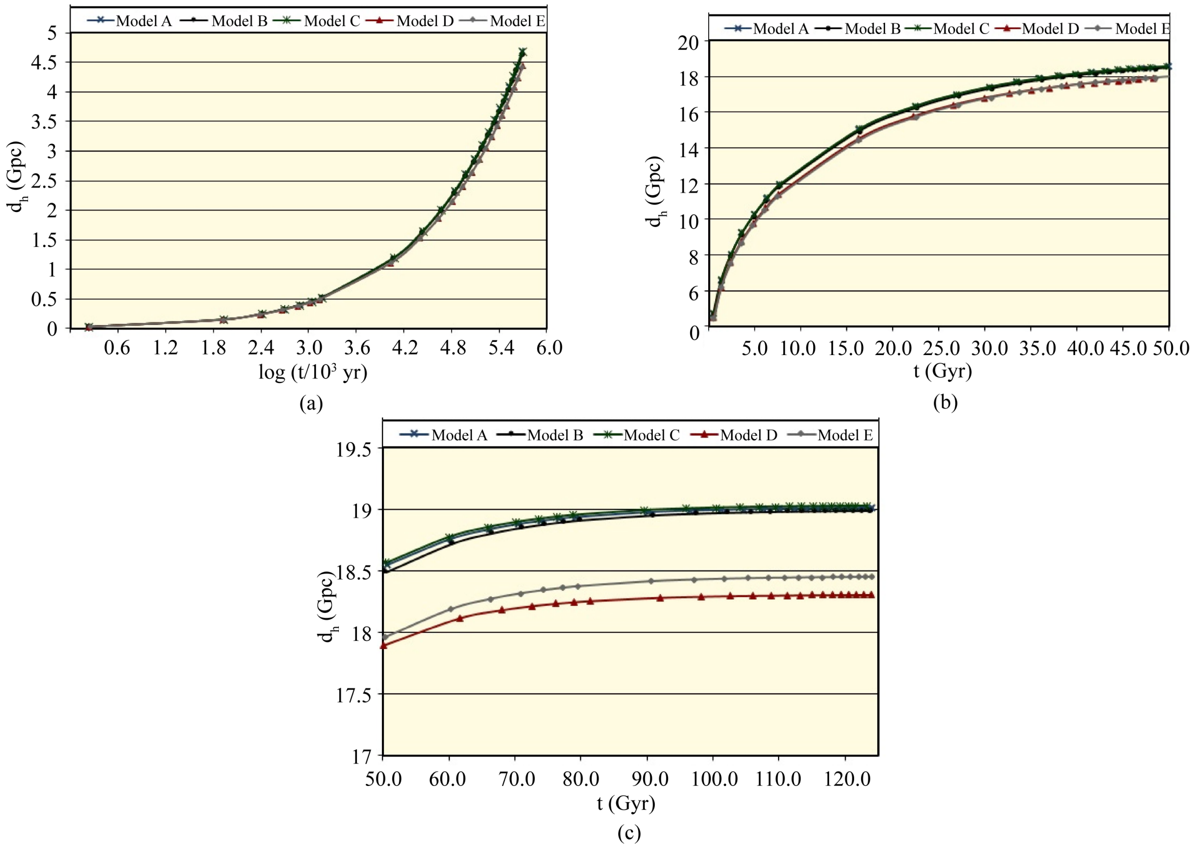

Figure 1. The distribution of the universe horizon distance in the general cosmic models (a) up to t = 0.5 Gyr; (b) in the range t = 0.5 - 50 Gyr; (c) in the range t = 50 - 124 Gyr.

Table 1 shows the universe horizon distances in the general models at special times. These times are the time of radiation-matter mass equivalence t_{rm}, the time of matter-dark energy mass equivalence $t_{m\Lambda}$, the present time $t_o = 13.7 \pm 0.2$ Gyr and the time $t_n = 124$ Gyr.

The results illustrated in **Figures 1(a)-(c)** are supported by those displayed in **Figures 2(a)-(c)** which show the distributions of the universe horizon volume in the general models in the ranges up to t = 0.5 Gyr, t = 0.5 - 50 Gyr and t = 50 - 124 Gyr respectively. **Table 2** presents the universe horizon volumes in the general models at the special times $t_{rm}, t_{m\Lambda}, t_o$ and t_n.

The distribution of mass and energy within the universe horizon volume of the universe in any general model up to $t = 0.5$ Gyr is exhibited in **Figure 3(a)**. The distributions of both radiation and matter decrease gradually with time and intersect at the time $t_{rm} = 34627.5 - 55916.2$ yr as shown in **Table 3**. On the other hand, the distribution of dark energy increases gradually until it intersects with the radiation distribution at the time $t_{r\Lambda} = 0.5166 - 0.5839$ Gyr as seen in **Table 4**. The distribution of total mass coincides with that of radiation up to $t \approx 5843.4141$ yr. Afterwards, the two distributions diverge from each other. However, the distribution of the total mass coincides on the distribution of matter from the time $t \approx 857695.9$ yr onwards.

The distribution of mass and energy within the universe horizon volume of the universe in any general model in the range $t = 0.5 - 50$ Gyr is displayed in **Figure 3(b)**. It is obvious that the distributions of matter and radiation decrease gradually with time and the former lies above the later. The distribution of dark energy increases with time and intersects with the distribution of matter at $t = 9.4525 - 10.0632$ Gyr as illustrated in **Table 5**. The distribution of the total mass coincides on the distribution of the matter up to $t = 4.5714$ Gyr, then they diverge from each other. Furthermore, the distribution of the total mass coincides on the distribution of the dark energy from $t = 18.2857$ Gyr onwards. Masses of radiation, matter and dark energy within the universe horizon volume in the general models at the present time are illustrated in **Table 6**.

The distribution of mass and energy within the universe horizon volume in any general model in the range

Table 1. Horizon distances of the universe in the general cosmic models at special times.

Model	$d_h(t_{rm})$/Mpc	$d_h(t_{m\Lambda})$/Gpc	$d_h(t_o)$/Gpc	$d_h(t_n)$/Gpc
A	118.1520	12.7667	14.2969	19.0103
B	112.1480	12.7586	14.1588	18.9849
C	118.0740	12.7812	14.2780	19.0274
D	98.0130	12.3367	13.8666	18.3062
E	94.3170	12.4269	13.8070	18.4510

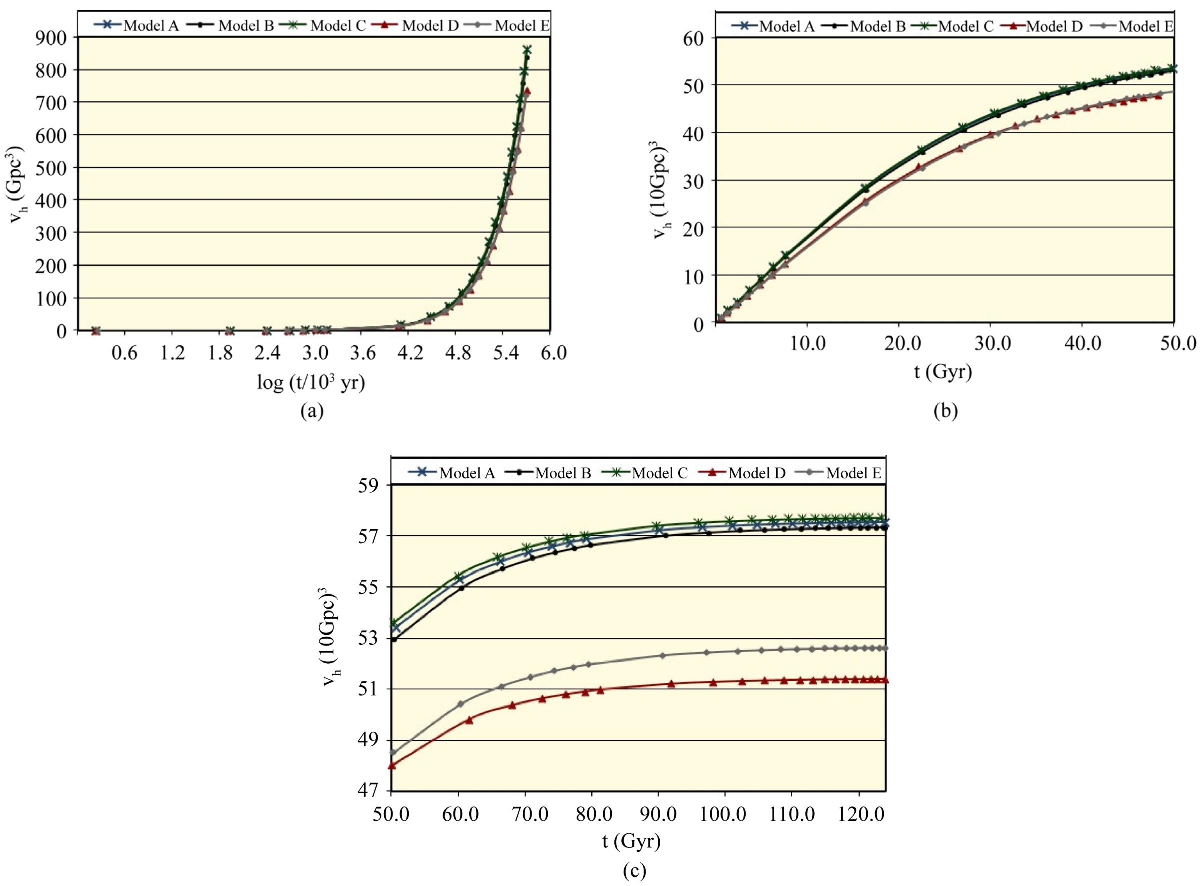

Figure 2. The distribution of the universe horizon volume in the general cosmic models (a) up to t = 0.5 Gyr; (b) in the range t = 0.5 - 50 Gyr; (c) in the range t = 50 - 124 Gyr.

Table 2. Horizon volumes of the universe in the general cosmic models at special times.

Model	$V_h(t_{rm})$/Mpc^3	$V_h(t_{m\Lambda})/(10\text{ Gpc})^3$	$V_h(t_o)/(10\text{ Gpc})^3$	$V_h(t_n)/(10\text{ Gpc})^3$
A	13820.0	17.4322	24.4819	57.5555
B	11820.0	17.3990	23.7792	57.3248
C	13790.0	17.4916	24.3849	57.7111
D	7890.0	15.7295	22.3372	51.3939
E	7030.0	16.0772	22.0504	52.6234

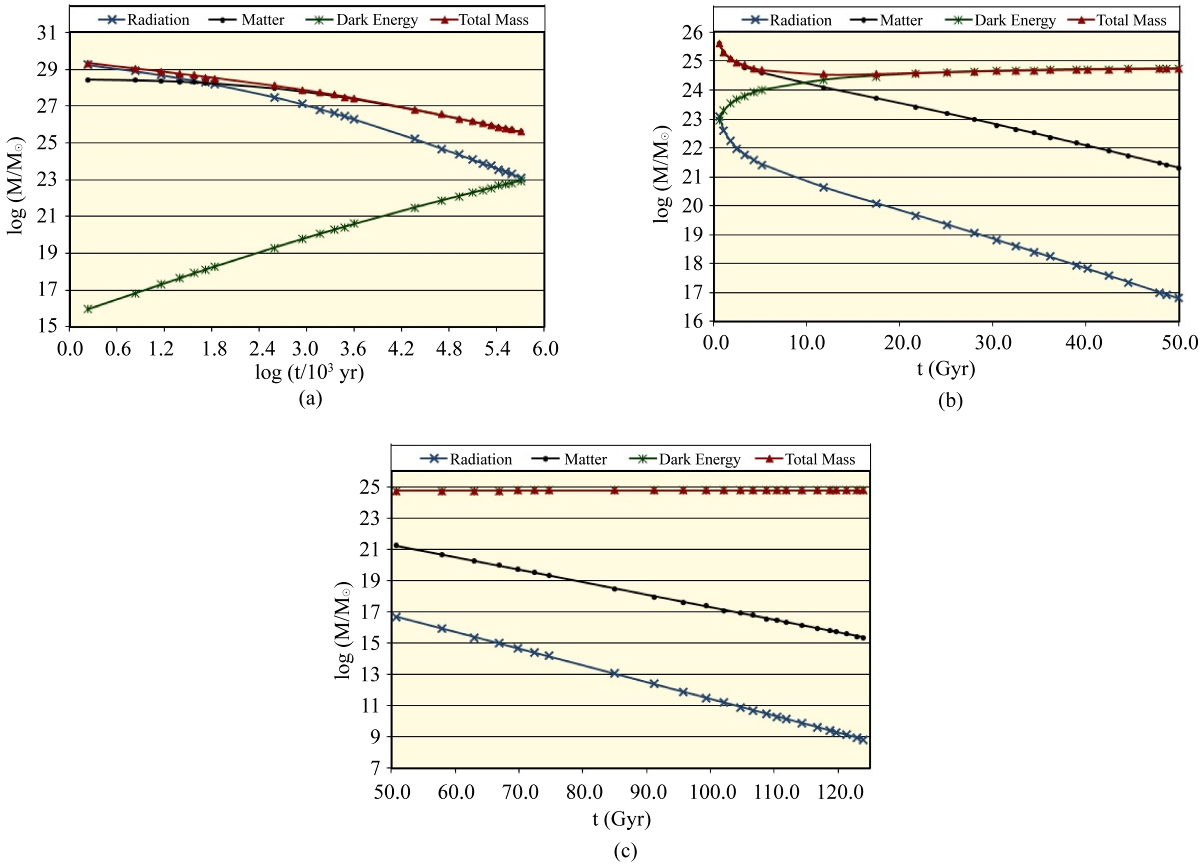

Figure 3. The distribution of mass and energy within the universe horizon volume in any general cosmic model (a) up to t = 0.5 Gyr; (b) in the range t = 0.5 - 50 Gyr; (c) in the range t = 50 - 124 Gyr.

Table 3. Cosmic times at which $M_r(t) = M_m(t)$ within the universe horizon volume in the general cosmic models.

Model	$t_{rm}/10^3$ yr	$\text{Log}(M_{rm}/M_\odot)$	$\text{Log}(M_\Lambda/M_\odot)$
A	55.9162	28.2526	18.1498
B	49.6665	28.2876	18.0712
C	55.8428	28.2529	18.1562
D	38.4783	28.3338	17.9566
E	34.6275	28.3753	17.8515

Table 4. Cosmic times at which $M_r(t) = M_\Lambda(t)$ within the universe horizon volume in the general cosmic models.

Model	$t_{r\Lambda}$ / Gyr	$\text{Log}(M_{r\Lambda}/M_\odot)$	$\text{Log}(M_m/M_\odot)$
A	0.5839	23.0153	25.5410
B	0.5736	22.9834	25.5375
C	0.5808	23.0202	25.5445
D	0.5166	22.9407	25.5350
E	0.5259	22.8906	25.5216

$t = 50 - 124\ \text{Gyr}$ is exhibited in **Figure 3(c)**. Again the distribution of both matter mass and radiation mass decrease with time and the former is higher than the later. The distributions of dark energy mass and total mass coincide on each other. Masses of radiation, matter and dark energy within the universe horizon volume in the general models at t_n are given in **Table 7**.

Table 8 shows the equivalent number of the Coma-like clusters to the mass of matter within the universe horizon volume $N_{COMA}(t)$ in the general models at the special times $t_{rm}, t_{m\Lambda}, t_o$ and t_n. It is obvious that this content of matter strongly decreases with time such that the cosmic space becomes almost matter empty in the far future of the universe.

5. Conclusion

In this article distributions of the universe horizon distance and universe horizon volume were determined in the five general cosmic models which were established previously. The two distributions were found increasing slowly up to $t \approx 21.5444\ \text{Myr}$, hence they raise appreciably fast up to $t = 60\ \text{Gyr}$, then they increase again so slowly until $t = 124\ \text{Gyr}$. Distributions of mass of radiation, matter and dark energy within the universe horizon volume were also investigated in the five general models. The masses of radiation and matter are decreasing with time although the mass of dark energy is increasing. The mass of radiation was dominant in the early

Table 5. Cosmic times at which $M_m(t) = M_\Lambda(t)$ within the universe horizon volume in the general cosmic models.

Model	$t_{m\Lambda}$ / Gyr	$\text{Log}(M_{m\Lambda}/M_\odot)$	$\text{Log}(M_r/M_\odot)$
A	9.4650	24.2507	20.8831
B	9.6930	24.2393	20.8339
C	9.4525	24.2594	20.8937
D	9.6289	24.2564	20.7974
E	10.0632	24.2109	20.7030

Table 6. Masses of radiation, matter and dark energy within the universe horizon volume in the general cosmic models at $t = t_o$.

Model	$\text{Log}(M_r/M_\odot)$	$\text{Log}(M_m/M_\odot)$	$\text{Log}(M_\Lambda/M_\odot)$
A	20.4531	23.9625	24.3973
B	20.4324	23.9683	24.3785
C	20.4570	23.9663	24.4086
D	20.4035	23.9954	24.3981
E	20.3684	23.9921	24.3328

Table 7. Masses of radiation, matter and dark energy within the universe horizon volume in the general cosmic models at $t = t_n$.

Model	$\text{Log}(M_r/M_\odot)$	$\text{Log}(M_m/M_\odot)$	$\text{Log}(M_\Lambda/M_\odot)$
A	8.8400	15.3480	24.7694
B	8.9797	15.4781	24.7571
C	8.7546	15.2847	24.7779
D	8.1326	14.8863	24.7706
E	8.8615	15.4585	24.7258

Table 8. Equivalent number of the Coma-like clusters to the mass of matter within the universe horizon volume in the general cosmic models at special times.

Model	t_{rm}	$t_{m\Lambda}$	t_o	t_n
A	8.9448×10^{12}	8.9057×10^{8}	4.5864×10^{8}	1.1142
B	9.6955×10^{12}	8.6750×10^{8}	4.6480×10^{8}	1.5034
C	8.9510×10^{12}	9.0859×10^{8}	4.6267×10^{8}	0.9631
D	10.7838×10^{12}	9.0234×10^{8}	4.9473×10^{8}	0.3848
E	11.8651×10^{12}	8.1259×10^{8}	4.9099×10^{8}	1.4370

universe up to $t = 34627.5 - 55916.2$ yr, where it becomes equivalent to the mass of matter. Afterwards, the mass of matter prevailed until $t = 9.4525 - 10.0632$ Gyr, where it becomes equal to the mass of dark energy. From this time onwards the mass of dark energy dominates the universe. The cosmic space gets approximately matter empty in the very remote future of the universe.

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