

Convergence Theorems for k -Strictly Pseudononspreading Multivalued in Hilbert Spaces

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Abstract

We introduce a k -strictly pseudononspreading multivalued in Hilbert spaces more general than the class of nonspreading multivalued. We establish some weak convergence theorems of the sequences generated by our iterative process. Some new iterative sequences for finding a common element of the set of solutions for equilibrium problem was introduced. The results improve and extend the corresponding results of Osilike Isiogugu [1] (Nonlinear Anal.74 (2011)) and others.

Keywords

Equilibrium Problem, K -Strictly Pseudononspreading Multivalued Mapping, Common Fixed Point

1. Introduction

Throughout this paper, we denote by \mathbb{N} and \mathbb{R} the sets of positive integers and real numbers, respectively. Let C be a nonempty closed subset of a real Hilbert space H . Let $N(C)$ and $CB(C)$ denote the family of nonempty subsets and nonempty closed bounded subsets of E , respectively. The Hausdorff metric on $CB(E)$ is defined by

$$H(A_1, A_2) := \max \left\{ \sup_{x \in A_1} d(x, A_2), \sup_{y \in A_2} d(y, A_1) \right\}$$

for $A_1, A_2 \in CB(C)$, where $d(x, A_1) = \inf \{ \|x - y\|, y \in A_1 \}$. An element $p \in E$ is called a fixed point of a multivalued mapping $T : C \rightarrow N(C)$ if $p \in T(p)$. The set of fixed points of a multivalued mapping T is

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represented by $F(T)$.

The multivalued mapping $T : C \rightarrow CB(C)$ is called nonexpansive if

$$H(Tx, Ty) \leq \|x - y\|, \quad \forall x, y \in C.$$

The multivalued mapping $T : C \rightarrow CB(C)$ is called quasi-nonexpansive if $F(T) \neq \emptyset$ and

$$H(Tx, Tp) \leq \|x - p\|, \quad \text{for } \forall x \in C \quad p \in F(T).$$

Iterative process for approximating fixed points (and common fixed points) of nonexpansive multivalued mappings have been investigated by various authors (see [2]-[5]).

Recently, Kohsaka and Takahashi (see [6] [7]) introduced an important class of mappings which they called the class of nonspreading mappings. Let C be a subset of Hilbert space H , they called a mapping $T : C \rightarrow C$ nonspreading if

$$2\|Tx - Ty\|^2 \leq \|Tx - y\|^2 + \|Ty - x\|^2 \quad \forall x, y \in C.$$

Lemoto and Takahashi [8] proved that $T : C \rightarrow C$ is nonspreading if and only if

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + 2\langle x - Tx, y - Ty \rangle \quad \forall x, y \in C.$$

Now, inspired by [6] and [7], we propose a definition as follows.

Definition 1.1 The multivalued mapping $T : C \rightarrow CB(C)$ is called nonspreading if

$$2\|u_x - u_y\|^2 \leq \|u_x - y\|^2 + \|u_y - x\|^2 \quad \text{for } u_x \in Tx, u_y \in Ty, \forall x, y \in C. \tag{1.1}$$

By Takahashi [8], We get also the multivalued mapping $T : C \rightarrow CB(C)$ is nonspreading if and only if

$$\|u_x - u_y\|^2 \leq \|x - y\|^2 + 2\langle x - u_x, y - u_y \rangle \quad \text{for } u_x \in Tx, u_y \in Ty, \forall x, y \in C. \tag{1.2}$$

Infact,

$$\begin{aligned} 2\|u_x - u_y\|^2 &\leq \|u_x - y\|^2 + \|u_y - x\|^2 \\ \Leftrightarrow 2\|u_x - u_y\|^2 &\leq \|u_x - x\|^2 + 2\langle u_x - x, x - y \rangle + \|x - y\|^2 \\ &\quad + \|u_x - x\|^2 + 2\langle x - u_x, u_x - u_y \rangle + \|u_x - u_y\|^2 \\ \Leftrightarrow 2\|u_x - u_y\|^2 &\leq 2\|u_x - x\|^2 + \|x - y\|^2 + \|u_x - u_y\|^2 + 2\langle u_x - x, x - u_x - (y - u_y) \rangle \\ \Leftrightarrow \|u_x - u_y\|^2 &\leq \|x - y\|^2 + 2\langle x - u_x, y - u_y \rangle \end{aligned}$$

Definition 1.2 The multivalued mapping $T : C \rightarrow CB(C)$ is called k -strictly pseudononspreading if there exists $k \in [0, 1)$ such that

$$\|u_x - u_y\|^2 \leq \|x - y\|^2 + k\|x - u_x - (y - u_y)\|^2 + 2\langle x - u_x, y - u_y \rangle, \quad \text{for } u_x \in Tx, u_y \in Ty, \forall x, y \in C. \tag{1.3}$$

Observe that suppose T is k -strictly pseudononspreading with $F(T) \neq \emptyset$, and $p \in F(T) \Rightarrow Tp = \{p\}$, then

$$\|u_x - p\|^2 \leq \|x - p\|^2 + k\|x - u_x\|^2 \quad x \in C, u_x \in Tx, p \in F(T).$$

Clearly every nonspreading multivalued mapping is k -strictly pseudononspreading multivalued mapping. The following example shows that the class of k -strictly pseudononspreading mappings is more general than the class of nonspreading mappings.

Example (see [1] page 1816 Example 1), Let \mathcal{R} denote the reals with the usual norm. Let $T : \mathcal{R} \rightarrow \mathcal{R}$ be defined for each $x \in \mathcal{R}$ by

$$Tx = \begin{cases} x, & x \in (-\infty, 0) \\ -2x, & x \in [0, \infty) \end{cases}$$

The equilibrium problem for $\phi: E \times E \rightarrow R$ is to find $x \in E$ such that $\phi(x, y) \geq 0, \forall y \in E$. The set of such solution is denoted by $EP(\phi)$. Given a mapping $T: E \rightarrow CB(E)$, let $\phi(x, y) = \langle x, y \rangle$ for all $y \in E$. The $x \in EP(\phi)$ if and only if $x \in E$ is a solution of the variational inequality $\langle Tx, y \rangle \geq 0$ for all $y \in E$.

Numerous problems in physics, optimization, and economics can be reduced to find a solution of the equilibrium problem. Some methods have been proposed to solve the equilibrium problem see, for instance, Blum and Oettli [9], Combettes and Hirstoaga [10], Li and Li [11], Giannessi, Maugeri, and Pardalos [12], Moudafi and Thera [13] and Pardalos, Rassias and Khan [14], Ceng *et al.* [15]. In the recent years, the problem of finding a common element of the set of solutions of equilibrium problems and the set of fixed points of single-valued nonexpansive mappings in the framework of Hilbert spaces has been intensively studied by many authors.

In this paper, inspired by [1] we propose an iterative process for finding a common element of the set of solutions of equilibrium problem and the set of common fixed points of k -strictly pseudononspreading multivalued mapping in the setting of real Hilbert spaces. We also prove the strong and weak convergence of the sequences generated by our iterative process. The results presented in the paper improve and extend the corresponding results in [1] and others.

2. Preliminaries and Lemma

In the sequel, we begin by recalling some preliminaries and lemmas which will be used in the proof.

Lemma 2.1 Let H be a real Hilbert space, for all $x, y \in H$ and $t \in [0, 1]$, then the following well known results hold:

- (i) $\|x + y\|^2 \leq \|x\|^2 + 2\langle x + y, y \rangle$.
- (ii) $\|tx + (1-t)y\|^2 = t\|x\|^2 + (1-t)\|y\|^2 - t(1-t)\|x - y\|^2$
- (iii) If $\{x_n\}$ is a sequence in H which converges weakly to $z \in H$ then

$$\limsup_{n \rightarrow \infty} \|x_n - y\|^2 = \limsup_{n \rightarrow \infty} \|x_n - z\|^2 + \|z - y\|^2$$

Let C be a nonempty closed convex subset of a real Hilbert space H . The nearest point projection $P_C: H \rightarrow C$ defined from H onto C is the function which assigns to each $x \in H$ its nearest point denoted by $P_C x$ in C . Thus $P_C x$ is the unique point in C such that

$$\|x - P_C x\| \leq \|x - y\|, \quad \forall y \in C.$$

It is known that for each $x \in H$

$$\langle x - P_C x, y - P_C x \rangle \leq 0, \quad \forall y \in C.$$

Lemma 2.2 (see [5]) Let C be a nonempty closed convex subset of a real Hilbert space H . Let $P_C: H \rightarrow C$ be the metric projection of H onto C . Let $\{x_n\}$ be a sequence in C and let $\|x_{n+1} - u\| \leq \|x_n - u\|$ for all $u \in C$. Then $\{P_C x_n\}$ converges strongly.

We present the following properties of a k -strictly pseudononspreading multivalued mapping.

Lemma 2.3 Let C be a nonempty closed convex subset of a real Hilbert space H , and let $T: C \rightarrow CB(C)$ be a k -strictly pseudononspreading multivalued mapping. If $F(T) \neq \emptyset$, and $p \in F(T) \Rightarrow Tp = \{p\}$, then it is closed and convex.

Proof. Let $\{x_n\} \subseteq F(T)$ and $x_n \rightarrow x \in C$ (as $n \rightarrow \infty$). Since $\|u_x - x\| \leq \|u_x - u_{x_n}\| + \|x_n - x\|$ and

$$\|u_x - u_{x_n}\|^2 \leq \|x_n - x\|^2 + k\|u_x - x\|^2 \leq (\|x_n - x\| + \sqrt{k}\|u_x - x\|)^2,$$

we have $0 \leq \|u_x - x\| \leq \frac{2}{1-\sqrt{k}}\|x_n - x\| \rightarrow 0$ (as $n \rightarrow \infty$). Hence $x \in F(T)$.

Next let $z = tp + (1-t)q$, where $p, q \in F(T)$ and $0 \leq t \leq 1$, we have

$$\begin{aligned} \|z - u_z\|^2 &= \|t(p - u_z) + (1-t)(q - u_z)\|^2 = t\|(p - u_z)\|^2 + (1-t)\|q - u_z\|^2 - t(1-t)\|p - q\|^2 \\ &\leq t[\|p - z\|^2 + k\|z - u_z\|^2] + (1-t)[\|q - z\|^2 + k\|z - u_z\|^2] - t(1-t)\|p - q\|^2 = k\|z - u_z\|^2, \quad u_z \in Tz, \end{aligned}$$

Thus $\|z - u_z\| = 0$ and hence $z \in F(T)$. This complete the proof of Lemma 2.3 \square

Lemma 2.4 Let C be a nonempty closed convex subset of a real Hilbert space H , and let $T : C \rightarrow CB(C)$ be a k -strictly pseudononspreading multivalued mapping. If $F(T) \neq \emptyset$, and $p \in F(T) \Rightarrow Tp = \{p\}$, then $I - T$ is demiclosed at 0.

Proof. Let $\{x_n\}$ be a sequence in C which $x_n \rightharpoonup p$ and $x_n - u_{x_n} \rightarrow 0$ (as $n \rightarrow \infty$).

Since $x_n \rightharpoonup p$, it is bounded. For each $x \in H$ define $\phi : H \rightarrow [0, +\infty)$ by

$$\phi(x) = \limsup_{n \rightarrow \infty} \|x_n - x\|^2.$$

Then from Lemma 2.1 we obtain

$$\phi(x) = \limsup_{n \rightarrow \infty} \|x_n - p\|^2 + \|p - x\|^2 = \phi(p) + \|p - x\|^2 \quad \forall x \in H,$$

and so $\phi(u_p) = \phi(p) + \|p - u_p\|^2$ (where $u_p \in Tp$).

In addition,

$$\begin{aligned} \phi(u_p) &= \limsup_{n \rightarrow \infty} \|x_n - u_p\|^2 = \limsup_{n \rightarrow \infty} \|u_{x_n} - u_p\|^2 \\ &\leq \limsup_{n \rightarrow \infty} \|x_n - p\|^2 + k \|p - u_p\|^2 \\ &= \phi(p) + k \|p - u_p\|^2. \end{aligned}$$

We obtain $(1-k)\|p - u_p\|^2 = 0$. Thus $\|p - u_p\| = 0$ and hence $p \in F(T)$. This complete the proof of Lemma 2.4. \square

3. Main Results

Theorem 3.1 Let C be a nonempty closed convex subset of a real Hilbert space H , and let $T : C \rightarrow CB(C)$ be a k -strictly pseudononspreading multivalued mapping with $F(T) \neq \emptyset$ and $p \in F(T) \Rightarrow Tp = \{p\}$. Let $\beta \in [k, 1)$ and $\{\alpha_n\}$ be a real sequence in $[0, 1)$ such that $\lim_{n \rightarrow \infty} \alpha_n = 0$. Let $\{x_n\}$ and $\{z_n\}$ be sequences generated initially by an arbitrary element $x_1 \in C$ and then by

$$\begin{cases} x_{n+1} = \alpha_n x_n + (1 - \alpha_n) y_n \\ y_n = \beta x_n + (1 - \beta) u_{x_n} \quad u_{x_n} \in Tx_n, n \geq 1 \\ z_n = \frac{1}{n} \sum_{i=1}^n x_i \quad n \geq 1 \end{cases}$$

Then, the sequences $\{z_n\}$ converge weakly to $z \in F(T)$, where $z = \lim_{n \rightarrow \infty} P_{F(T)} x_n$.

Proof. Let $T^\beta := \beta I + (1 - \beta)T$

First, We claim that $F(T^\beta) = F(T)$.

Indeed, if $p \in F(T)$, then

$$u_p^\beta = (\beta I + (1 - \beta)T)p = \beta p - (1 - \beta)u_p = p, \quad u_p^\beta \in T^\beta p,$$

this implies $F(T^\beta) = F(T)$ and $p \in T^\beta p \Rightarrow T^\beta p = \{p\}$ Next, for $\forall x, y \in C$ we have

$$\begin{aligned} \|u_x^\beta - u_y^\beta\|^2 &= \|\beta(x - y) + (1 - \beta)(u_x - u_y)\|^2 \\ &= \beta \|x - y\|^2 + (1 - \beta) \|u_x - u_y\|^2 - \beta(1 - \beta) \|(x - u_x) - (y - u_y)\|^2. \end{aligned} \tag{3.1}$$

By (1.3) and (3.1), we obtain

$$\|u_x^\beta - u_y^\beta\|^2 \leq \|x - y\|^2 + \frac{2}{1-\beta} \langle x - u_x^\beta, y - u_y^\beta \rangle. \tag{3.2}$$

Observe also that for each $p \in F(T)$

$$\begin{aligned} \|x_{n+1} - p\| &= \|\alpha_n(x_n - p) + (1-\alpha_n)(u_{x_n}^\beta - p)\| \\ &\leq \alpha_n \|x_n - p\| + (1-\alpha_n) \|u_{x_n}^\beta - p\| \leq \|x_n - p\|, \end{aligned}$$

hence $\{x_n\}$ is bounded. By Lemma 2.1 and (3.2), we obtain

$$\begin{aligned} \|x_{n+1} - u_y^\beta\|^2 &= \|\alpha_n(x_n - u_y^\beta) + (1-\alpha_n)(u_{x_n}^\beta - u_y^\beta)\|^2 \\ &= \alpha_n \|x_n - u_y^\beta\|^2 + (1-\alpha_n) \|u_{x_n}^\beta - u_y^\beta\|^2 - \alpha_n(1-\alpha_n) \|x_n - u_{x_n}^\beta\|^2 \\ &\leq \alpha_n \|x_n - u_y^\beta\|^2 + (1-\alpha_n) \left[\|x_n - y\|^2 + \frac{2}{1-\beta} \langle x_n - u_{x_n}^\beta, y - u_y^\beta \rangle \right]. \end{aligned} \tag{3.3}$$

Since

$$\|x_n - y\|^2 = \|x_n - u_y^\beta\|^2 + \|u_y^\beta - y\|^2 + 2 \langle x_n - u_y^\beta, u_y^\beta - y \rangle, \tag{3.4}$$

it follows from (3.3) and (3.4) that

$$\begin{aligned} \|x_{n+1} - u_y^\beta\|^2 &\leq \|x_n - u_y^\beta\|^2 + \|u_y^\beta - y\|^2 + 2 \langle x_n - u_y^\beta, u_y^\beta - y \rangle \\ &\quad - 2\alpha_n \langle x_n - u_y^\beta, u_y^\beta - y \rangle + \frac{2}{1-\beta} \langle x_n - x_{n+1}, u_y^\beta - y \rangle. \end{aligned} \tag{3.5}$$

Summing (3.5) from $n = 1$ to n , and dividing by n we obtain

$$\begin{aligned} \frac{1}{n} \|x_{n+1} - u_y^\beta\|^2 &\leq \frac{1}{n} \|x_1 - u_y^\beta\|^2 + \frac{2}{1-\beta} \left\langle \frac{x_1 - x_{n+1}}{n}, -y + u_y^\beta \right\rangle + \|u_y^\beta - y\|^2 \\ &\quad + 2 \langle z_n - u_y^\beta, u_y^\beta - y \rangle - \frac{2}{n} \sum_{i=1}^n \alpha_i \langle x_i - u_y^\beta, u_y^\beta - y \rangle. \end{aligned} \tag{3.6}$$

Since $\{x_n\}$ is bounded, then $\{z_n\}$ is also bounded. Thus there exists a subsequence $\{z_{n_j}\}$ of $\{z_n\}$ such that $z_{n_j} \rightharpoonup w$ (as $j \rightarrow \infty$). we also have

$$\begin{aligned} \frac{1}{n_j} \|x_{n_j+1} - u_y^\beta\|^2 &\leq \frac{1}{n_j} \|x_1 - u_y^\beta\|^2 + \frac{2}{1-\beta} \left\langle \frac{x_1 - x_{n_j+1}}{n_j}, -y + u_y^\beta \right\rangle + \|u_y^\beta - y\|^2 \\ &\quad + 2 \langle z_{n_j} - u_y^\beta, u_y^\beta - y \rangle - \frac{2}{n_j} \sum_{i=1}^{n_j} \alpha_i \langle x_i - u_y^\beta, u_y^\beta - y \rangle. \end{aligned} \tag{3.7}$$

As $j \rightarrow \infty$ we obtain from (3.7) that

$$0 \leq \|u_y^\beta - y\|^2 + 2 \langle w - u_y^\beta, u_y^\beta - y \rangle. \tag{3.8}$$

Since $y \in H$ was arbitrary, setting $y = w$ in (3.8) we have

$$0 \leq \|u_w^\beta - w\|^2 - 2 \|u_w^\beta - w\|^2,$$

from which it follows that $w \in F(T^\beta) = F(T)$. Since $F(T)$ is closed and convex by Lemma 2.3, thus we can define the projection $P_{F(T)}$.

From Lemma 2.2, $\{P_{F(T)}x_n\}$ converges strongly. Let $\lim_{n \rightarrow \infty} P_{F(T)}x_n = z$.

Next we show that $z = w$.

Since $\{x_n\}$ and $\{P_{F(T)}x_n\}$ are bounded, there exists $M > 0$ such that $\|x_n - P_{F(T)}x_n\| \leq M$ ($\forall n \geq 1$), then we obtain by $\langle w - P_{F(T)}x_n, x_n - P_{F(T)}x_n \rangle \leq 0$

$$\begin{aligned} \langle w - z, x_n - P_{F(T)}x_n \rangle &= \langle w - P_{F(T)}x_n, x_n - P_{F(T)}x_n \rangle + \langle P_{F(T)}x_n - z, x_n - P_{F(T)}x_n \rangle \\ &\leq \langle P_{F(T)}x_n - z, x_n - P_{F(T)}x_n \rangle \leq \|P_{F(T)}x_n - z\| \cdot \|x_n - P_{F(T)}x_n\| \\ &\leq M \|P_{F(T)}x_n - z\| \end{aligned} \quad (3.9)$$

Summing (3.9) from $n=1$ to n_j , and dividing by n_j we obtain

$$\left\langle w - z, z_{n_j} - \frac{1}{n_j} \sum_{i=1}^{n_j} P_{F(T)}x_i \right\rangle \leq \frac{M}{n_j} \sum_{i=1}^{n_j} \|P_{F(T)}x_i - z\|. \quad (3.10)$$

Since $z_{n_j} \rightarrow w$ as $j \rightarrow \infty$, and $\lim_{n \rightarrow \infty} P_{F(T)}x_n = z$, we have

$$\langle w - z, w - z \rangle = \|w - z\|^2 \leq 0.$$

Hence $w = z$, so, the sequences $\{z_n\}$ converge weakly to $z \in F(T)$, where $z = \lim_{n \rightarrow \infty} P_{F(T)}x_n$. This complete the proof of Theorem 3.1. \square

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