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On Maxwell-Lorentz Equations in Dirac's Symmetrisation and Their Analogs for Gravitation and Space-Time

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Abstract

In further discussion on the Maxwell-Lorentz equations in Dirac's symmetrisation, I introduce the concept of magnetic monopole as an "act of electric current" in the 2nd equation (*i.e.* the analog of the "act of movement" in Classical Mechanics), I postulate a "magnetic displacement current" and a "magnetomotive force" in the 3rd and 4th equations, respectively (*i.e.* the analogs of the "electric displacement current" and of the "electromotive force" in the 4th and 3rd equations, respectively). As a consequence, I propose a generalised vision of the Electromagnetism in which inhomogeneous, microscopic, and relativistically linked equations describe the static and the oscillatory phenomena. Then, in the frame of Relativity, I propose analog microscopic equations to study the Gravitation and the Space-Time in terms of static and oscillatory phenomena: the static equations show the sources of newly defined vector fields (the generalised mass density as the source of the generalised mass field, the generalised time density as the source of the generalised space field, respectively), whereas the oscillatory equations describe the propagation of the gravitational waves and of the spatiotemporal waves, respectively. In other words, I propose to unify Electromagnetism, Gravitation, and Space-Time in terms of microscopic Maxwell-Lorentz-like equations in Dirac's symmetrisation, where the unifying trait is *c*. Finally, using the concepts of the proposed generalised Electromagnetism, I discuss the conservation in Electromagnetism and the interaction between matter and electromagnetic waves.

Keywords

Maxwell-Lorentz Equations, Dirac's Symmetrisation, Gravitation, Space-Time

1. Introduction

As well described by Thidé [1], the definitions of electric and magnetic fields were proposed such that they are closely related to the mechanical force between electric charges and between electric currents given by Coulomb's law and Ampère's law, respectively.

Just as in mechanics, in electrodynamics, it is frequently more useful to build the theory in terms of potentials rather than in terms of the electric and magnetic fields, especially in research related to radiation, relativity, and relativistically covariant electromagnetism. In addition, at the quantum level, electrodynamics is nearly exclusively formulated in terms of potentials. Furthermore, the use of potentials leads naturally to the special theory of relativity and to gauge field theories [1].

In this regard, under the Lorenz-Lorentz gauge condition [1], the electrodynamics is formulated via four scalar and inhomogeneous wave equations where the nonzero sources are the electric charge density and the electric current density.

In this work, as an alternative to the gauge approach, starting from the Maxwell-Lorentz equations in Dirac's symmetrisation, each of whom is characterized by a nonzero source [1], I propose a generalised vision of the Electromagnetism in which inhomogeneous, microscopic, and relativistically linked equations describe the static and the oscillatory phenomena (§3.1); in particular, instead of using potentials, I propose generalised definitions of the electric field, electric charge density, magnetic induction field, and electric current density.

The preceding generalisation is then extended, by analogy, to Gravitation and Space-Time.

2. Maxwell-Lorentz Equations in Dirac's Symmetrisation

Let me recall the Maxwell-Lorentz equations in Dirac's symmetrisation [1]:

$$\nabla \cdot \boldsymbol{E} = \frac{1}{\varepsilon_0}\rho^e \quad \text{with } \rho^e \text{ in } \left[\frac{\mathrm{A}}{\mathrm{m}^2}\frac{\mathrm{s}}{\mathrm{m}}\right] = \left[\frac{\mathrm{C}}{\mathrm{m}^3}\right], \tag{1}$$

$$\nabla \cdot \boldsymbol{B} = \mu_0 \rho^m \quad \text{with } \rho^m \text{ in } \left[\frac{\mathrm{A}}{\mathrm{m}^2}\right], \tag{2}$$

$$\nabla \times \boldsymbol{E} = -\mu_0\left(\boldsymbol{j}^m + \frac{1}{\mu_0}\frac{\partial \boldsymbol{B}}{\partial t}\right) \quad \text{with } \boldsymbol{j}^m \text{ in } \left[\frac{\mathrm{A}}{\mathrm{m}^2}\frac{\mathrm{m}}{\mathrm{s}}\right] = \left[\frac{\mathrm{C}}{\mathrm{m}^2}\frac{\mathrm{m}}{\mathrm{s}^2}\right], \tag{3}$$

$$\nabla \times \boldsymbol{B} = \mu_0\left(\boldsymbol{j}^e + \varepsilon_0\frac{\partial \boldsymbol{E}}{\partial t}\right) \quad \text{with } \boldsymbol{j}^e \text{ in } \left[\frac{\mathrm{A}}{\mathrm{m}^2}\right], \tag{4}$$

where ρ^e is the electric charge density, ρ^m is the magnetic charge density, $\boldsymbol{j}^e$ is the electric current density vector, and $\boldsymbol{j}^m$ is the magnetic current density vector.

The vector $\boldsymbol{j}^{e,dis} = \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$ (called "electric displacement current") is expressed in the same unit of $\boldsymbol{j}^e$, whereas the vector $\boldsymbol{j}^{m,dis} = \frac{1}{\mu_0}\frac{\partial \boldsymbol{B}}{\partial t}$ (postulated as "magnetic displacement current" in §2.3, according to [1]) is expressed in the same unit of $\boldsymbol{j}^m$.

In addition, ρ^e, ρ^m, $\boldsymbol{j}^e$, $\boldsymbol{j}^m$, $\boldsymbol{j}^{e,dis}$, and $\boldsymbol{j}^{m,dis}$ have a shared trait in their units: [A/m²].

2.1. First Maxwell-Lorentz Equation and Electric Monopole

Let me recall the electric monopole (Positive or Negative in [C]) associated to ρ^e [C/m³] (Equation (1), Figure 1). It generates an electric field $\boldsymbol{E}$ which is isotropic in the space, that is, without a preferential direction. $\boldsymbol{E}$ is directed outward from a positive charge q^+ and inward to a negative charge q^-.

2.2. Second Maxwell-Lorentz Equation and Magnetic Monopole

I propose a general structure for a magnetic monopole (North or South in [A]) associated to ρ^m [A/m²] (Equation (2), Figure 2). Let me recall the concept of "act of movement" in Classical Mechanics [2]: in a particle set, each particle is characterised by a velocity vector and the set of particle velocities is a velocity field $\boldsymbol{v}$ called "act of movement". As a consequence, if the moving particles carry an electric charge, I could introduce the corresponding act of movement of electric charges, that is, in other words, an "act of electric current" in [A].

In particular, I propose a magnetic monopole composed of two separated acts of electric current in the form of two juxtaposed circular semi-loops (Figure 2). The magnetic monopole generates a magnetic induction field $\boldsymbol{B}$ which is isotropic in the space, that is, without a preferential direction. $\boldsymbol{B}$ is directed outward

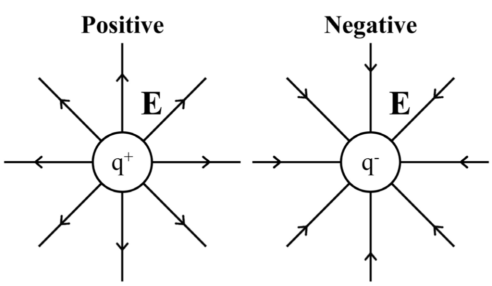

Figure 1. Electric monopoles.

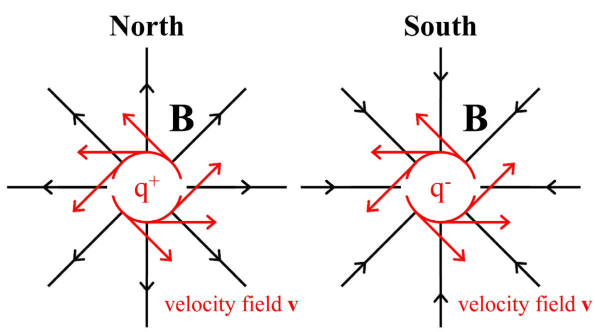

Figure 2. Magnetic monopoles with lines of force in the plane of the acts of movement and of the acts of electric current (red).

from a North anticlockwise pair of positive semi-loops (*i.e.* with positive charges q^+) and inward to a South anticlockwise pair of negative semi-loops (*i.e.* with negative charges q^-) (**Figure 2**).

2.3. Third Maxwell-Lorentz Equation and "Magnetic Displacement Current"

The third Maxwell-Lorentz equation is referred to the "electromotive force" (Equation (3)). In general, any kind of variation of $\boldsymbol{B}$ gives rise to $\nabla \times \boldsymbol{E}$.

In my opinion, $\boldsymbol{j}^m$ [(C/m²)(m/s²)] could be seen as related to the "closure displacement" of a magnetic monopole: the two acts of electric current accelerate towards each other, a loop of electric current is completed, and a magnetic dipole emerges (variation of $\boldsymbol{B}$) (**Figure 3**). In particular, $\boldsymbol{j}^m$ is orthogonal to the plane of the acts of electric current and directed as in **Figure 3**.

So, the vector $\boldsymbol{j}^{m,dis} = \frac{1}{\mu_0}\frac{\partial \boldsymbol{B}}{\partial t}$ (associated to a variation of $\boldsymbol{B}$) can be interpreted as having a displacement feature as $\boldsymbol{j}^m$ and, due to the analytical analogy with the electric displacement current $\boldsymbol{j}^{e,dis} = \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$, according to [1], it could be postulated as the "magnetic displacement current" (**Figure 4**).

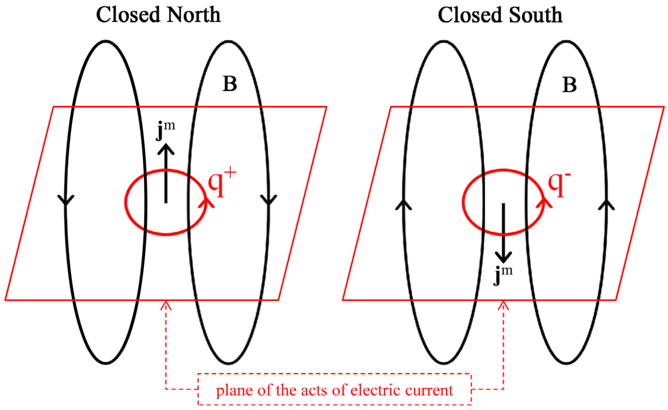

Figure 3. Magnetic dipoles emerging from the closure displacement of the magnetic monopoles.

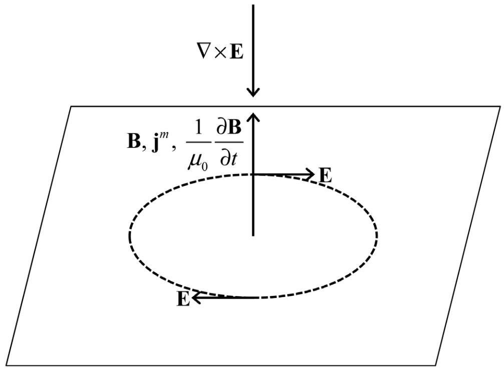

Figure 4. Electromotive force.

2.4. Fourth Maxwell-Lorentz Equation and "Magnetomotive Force"

The fourth Maxwell-Lorentz equation describes a phenomenon based on $\boldsymbol{j}^e$ and $\boldsymbol{j}^{e,dis}$ [A/m²] (Equation (4)). In analogy with $\boldsymbol{j}^m$, $\boldsymbol{j}^e$ is referred to the displacement of an electric monopole (or, under an equivalent viewpoint, related to an opening or closing electric dipole). As a consequence, all analogies have been enunciated and I could associate the fourth equation to a "magnetomotive force" where any kind of variation of $\boldsymbol{E}$ gives rise to $\nabla\times\boldsymbol{B}$ (**Figure 5**).

3. Further Symmetrisation

3.1. Generalised Electromagnetism

Combining the Equation (1) with the Equation (2) and combining the Equation (3) with the Equation (4), I obtain inhomogeneous and microscopic equations:

$$\nabla\cdot\boldsymbol{E}^{gen}=\nabla\cdot\overbrace{(\boldsymbol{E}+\mathrm{i}c\boldsymbol{B})}^{\boldsymbol{E}^{gen}\text{ in }\left[\frac{\mathrm{V}}{\mathrm{m}}\right]}=\underbrace{\frac{1}{\varepsilon_0}\overbrace{\left(\rho^e+\frac{\mathrm{i}}{c}\rho^m\right)}^{\rho^{gen,\boldsymbol{E}}\text{ in }\left[\frac{\mathrm{C}}{\mathrm{m}^3}\right]}}_{\text{in }\left[\frac{\mathrm{V}}{\mathrm{m}^2}\right]}=\frac{1}{\varepsilon_0}\rho^{gen,\boldsymbol{E}}, \tag{5}$$

$$\begin{aligned}\nabla\times\boldsymbol{B}^{gen}&=\nabla\times\overbrace{\left(\boldsymbol{B}+\frac{\mathrm{i}}{c}\boldsymbol{E}\right)}^{\boldsymbol{B}^{gen}\text{ in }[\mathrm{T}]}=\underbrace{\mu_0\overbrace{\left[\left(\boldsymbol{j}^e-\frac{\mathrm{i}}{c}\boldsymbol{j}^m\right)+\left(\varepsilon_0\frac{\partial\boldsymbol{E}}{\partial t}-\frac{\mathrm{i}}{c}\frac{1}{\mu_0}\frac{\partial\boldsymbol{B}}{\partial t}\right)\right]}^{\boldsymbol{j}^{gen,\boldsymbol{B}}\text{ in }\left[\frac{\mathrm{A}}{\mathrm{m}^2}\right]}}_{\text{in }\left[\frac{\mathrm{T}}{\mathrm{m}}\right]}\\&=\underbrace{\mu_0\overbrace{\left[\left(\boldsymbol{j}^e-\frac{\mathrm{i}}{c}\boldsymbol{j}^m\right)+\left(\boldsymbol{j}^{e,dis}-\frac{\mathrm{i}}{c}\boldsymbol{j}^{m,dis}\right)\right]}^{\boldsymbol{j}^{gen,\boldsymbol{B}}\text{ in }\left[\frac{\mathrm{A}}{\mathrm{m}^2}\right]}}_{\text{in }\left[\frac{\mathrm{T}}{\mathrm{m}}\right]}=\mu_0\boldsymbol{j}^{gen,\boldsymbol{B}},\end{aligned} \tag{6}$$

where the complex vector $\boldsymbol{E}^{gen}=\boldsymbol{E}+\mathrm{i}c\boldsymbol{B}\in\mathbb{C}^3$ is the generalised electric field [V/m] (**Figure 6**) ($\boldsymbol{E}^{gen}$ is also known as the Riemann-Silberstein vector $\boldsymbol{G}$ in Majorana formalism [1] [3] [4]), the complex scalar $\rho^{gen,\boldsymbol{E}}\in\mathbb{C}$ is the generalised electric charge density [C/m³] associated to the generalised electromagnetic

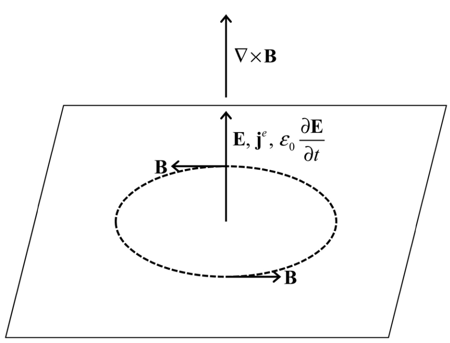

Figure 5. Magnetomotive force.

monopole $\aleph_1^{gen} \in \mathbb{C}$ [C], the complex vector $\boldsymbol{B}^{gen} = \boldsymbol{B} + \frac{\mathrm{i}}{c}\boldsymbol{E} \in \mathbb{C}^3$ is the generalised magnetic induction field [T] (**Figure 7**), and the complex vector $\boldsymbol{j}^{gen,\boldsymbol{B}} \in \mathbb{C}^3$ is the generalised electric current density [A/m^2] associated to the generalised electromagnetic dipole $\aleph_2^{gen} \in \mathbb{C}^3$ [C].

In addition, because of $\|\boldsymbol{E}\| = c\|\boldsymbol{B}\|$, I obtain:

$$\begin{aligned}\left\|\boldsymbol{E}^{gen}\right\| &= \sqrt{\boldsymbol{E}^{gen} \cdot \overline{\boldsymbol{E}^{gen}}} = \sqrt{(\boldsymbol{E} + \mathrm{i}c\boldsymbol{B})(\boldsymbol{E} - \mathrm{i}c\boldsymbol{B})} \\ &= \sqrt{\|\boldsymbol{E}\|^2 + c^2\|\boldsymbol{B}\|^2} = \sqrt{2}\|\boldsymbol{E}\|,\end{aligned} \tag{7}$$

$$\begin{aligned}\left\|\boldsymbol{B}^{gen}\right\| &= \sqrt{\boldsymbol{B}^{gen} \cdot \overline{\boldsymbol{B}^{gen}}} = \sqrt{\left(\boldsymbol{B} + \frac{\mathrm{i}}{c}\boldsymbol{E}\right)\left(\boldsymbol{B} - \frac{\mathrm{i}}{c}\boldsymbol{E}\right)} \\ &= \sqrt{\|\boldsymbol{B}\|^2 + \frac{1}{c^2}\|\boldsymbol{E}\|^2} = \sqrt{2}\|\boldsymbol{B}\|,\end{aligned} \tag{8}$$

$$\overbrace{\|\boldsymbol{G}\| = \left\|\boldsymbol{E}^{gen}\right\|}^{\text{static}} = \sqrt{2}\|\boldsymbol{E}\| = \sqrt{2}c\|\boldsymbol{B}\| = c\overbrace{\left\|\boldsymbol{B}^{gen}\right\|}^{\text{oscillatory}}. \tag{9}$$

The Equation (5) describes the static feature of Electromagnetism, whereas the Equation (6) the oscillatory one: the relativistic link, via c, between these features is shown in the Equation (9).

Some considerations. Maxwell wrote: "light and magnetism are affections of the same substance" [5]. Moreover, because of based on static $\boldsymbol{G}$ (Equation (9)) [3] [4], the photon wave function is a controversial concept and it cannot have all the properties of Schrödinger's probabilistic wave functions [3]. In addition, in the Equation (6), the displacement currents behave like a current density flowing in free space: their "existence has far-reaching physical consequences as it predicts that such physical observables as electromagnetic energy, linear momentum, and angular momentum can be transmitted over very long distances, even through empty space" [1].

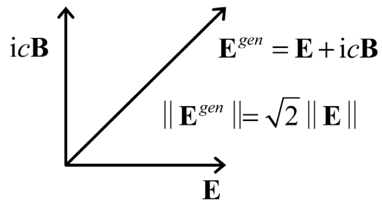

Figure 6. Generalised electric field $\boldsymbol{E}^{gen}$ [V/m].

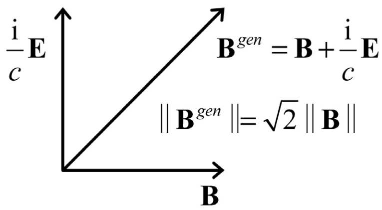

Figure 7. Generalised magnetic induction field $\boldsymbol{B}^{gen}$ [T].

3.2. Generalised Gravitation

From Dirac's symmetrisation, I obtain the Equation (9), that is:

$$\left\|\boldsymbol{E}^{gen}\right\|^2 = \left\|\boldsymbol{B}^{gen}\right\|^2 c^2. \tag{10}$$

In my opinion, the gravitational analog of Equation (10) is the relativistic law

$$\mathscr{E} = Mc^2. \tag{11}$$

As a consequence, I can explore the path toward the gravitational analog of the Maxwell-Lorentz equations in Dirac's symmetrisation. In particular, I rewrite the Equation (11) in analogy with the Equation (9) and the Equation (10):

$$\left\|\boldsymbol{O}^{gen}\right\|^2 = c^2 \left\|\boldsymbol{Q}^{gen}\right\|^2, \tag{12}$$

$$\overbrace{\left\|\boldsymbol{O}^{gen}\right\|}^{\text{static}} = c\overbrace{\left\|\boldsymbol{Q}^{gen}\right\|}^{\text{oscillatory}}, \tag{13}$$

where, developing the analogy Electromagnetism/Gravitation, the complex vector $\boldsymbol{O}^{gen} = \boldsymbol{O} + \mathrm{i}c\boldsymbol{Q} \in \mathbb{C}^3$ is the generalised mass field [$\sqrt{\mathrm{J}}$] and describes the static feature of Gravitation, the vector $\boldsymbol{O} \in \mathbb{R}^3$ is the mass field [$\sqrt{\mathrm{J}}$], the vector $\boldsymbol{Q} \in \mathbb{R}^3$ is the mass induction field [$\sqrt{\mathrm{kg}}$], the complex vector $\boldsymbol{Q}^{gen} = \boldsymbol{Q} + \frac{\mathrm{i}}{c}\boldsymbol{O} \in \mathbb{C}^3$ is the generalised mass induction field [$\sqrt{\mathrm{kg}}$] and describes the oscillatory feature of Gravitation. The relativistic link, via c, between the static and the oscillatory features of Gravitation is shown in the Equation (13).

So, in analogy with the Equation (5) and the Equation (6), the generalised gravitational Maxwell-Lorentz equations in Dirac's symmetrisation are:

$$\nabla \cdot \boldsymbol{O}^{gen} = \nabla \cdot \overbrace{\left(\boldsymbol{O} + \mathrm{i}c\boldsymbol{Q}\right)}^{\boldsymbol{O}^{gen} \text{ in } \left[\sqrt{\mathrm{J}}\right]} = \underbrace{\frac{1}{\varepsilon_g}\overbrace{\left(\rho^o + \frac{\mathrm{i}}{c}\rho^q\right)}^{\rho^{gen,\boldsymbol{O}} \text{ in } \left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]}}_{\text{in } \left[\frac{\sqrt{\mathrm{J}}}{\mathrm{m}}\right]} = \frac{1}{\varepsilon_g}\rho^{gen,\boldsymbol{O}}, \tag{14}$$

$$\begin{aligned} \nabla \times \boldsymbol{Q}^{gen} &= \nabla \times \overbrace{\left(\boldsymbol{Q} + \frac{\mathrm{i}}{c}\boldsymbol{O}\right)}^{\boldsymbol{Q}^{gen} \text{ in } \left[\sqrt{\mathrm{kg}}\right]} = \underbrace{\mu_g \overbrace{\left[\left(\boldsymbol{j}^o - \frac{\mathrm{i}}{c}\boldsymbol{j}^q\right) + \left(\varepsilon_g \frac{\partial \boldsymbol{O}}{\partial t} - \frac{\mathrm{i}}{c}\frac{1}{\mu_g}\frac{\partial \boldsymbol{Q}}{\partial t}\right)\right]}^{\boldsymbol{j}^{gen,\boldsymbol{Q}} \text{ in } \left[\frac{\mathrm{kg}}{\mathrm{s \cdot m}^2}\right]}}_{\text{in } \left[\frac{\sqrt{\mathrm{kg}}}{\mathrm{m}}\right]} \\ &= \underbrace{\mu_g \overbrace{\left[\left(\boldsymbol{j}^o - \frac{\mathrm{i}}{c}\boldsymbol{j}^q\right) + \left(\boldsymbol{j}^{o,dis} - \frac{\mathrm{i}}{c}\boldsymbol{j}^{q,dis}\right)\right]}^{\boldsymbol{j}^{gen,\boldsymbol{Q}} \text{ in } \left[\frac{\mathrm{kg}}{\mathrm{s \cdot m}^2}\right]}}_{\text{in } \left[\frac{\sqrt{\mathrm{kg}}}{\mathrm{m}}\right]} = \mu_g \boldsymbol{j}^{gen,\boldsymbol{Q}}, \end{aligned} \tag{15}$$

where the scalar $\rho^o \in \mathbb{R}$ is the local mass density [kg/m^3], the scalar $\rho^q \in \mathbb{R}$ is the local mass current density [kg/s·m^2], the complex scalar $\rho^{gen,\mathbf{O}} \in \mathbb{C}$ is the generalised mass density [kg/m^3] associated to the generalised gravitational monopole $\daleth_1^{gen} \in \mathbb{C}$ [kg], the vector $\boldsymbol{j}^o \in \mathbb{R}^3$ is the tangent mass current density [kg/s·m²], the vector $\boldsymbol{j}^q \in \mathbb{R}^3$ is the circular mass current density [kg/s²·m], the vector $\boldsymbol{j}^{o,dis} = \varepsilon_g \dfrac{\partial \boldsymbol{O}}{\partial t} \in \mathbb{R}^3$ is the tangent mass displacement current [kg/s·m²], the vector $\boldsymbol{j}^{q,dis} = \dfrac{1}{\mu_g}\dfrac{\partial \boldsymbol{Q}}{\partial t} \in \mathbb{R}^3$ is the circular mass displacement current [kg/s²·m], the complex vector $\boldsymbol{j}^{gen,\boldsymbol{Q}} \in \mathbb{C}^3$ is the generalised mass current density [kg/s·m²] associated to the generalised gravitational dipole $\daleth_2^{gen} \in \mathbb{C}^3$ [kg], ε_g is the vacuum gravitational permittivity [$\sqrt{\text{kg}} \cdot \text{s}/\text{m}^3$], μ_g is the vacuum gravitational permeability [$\text{s} \cdot \text{m}/\sqrt{\text{kg}}$]. In particular, "local" has the same geometric meaning as in ρ^e and ρ^m, "tangent" has the same geometric meaning as in $\boldsymbol{j}^e$ and $\boldsymbol{j}^{e,dis}$, and "circular" has the same geometric meaning as in $\boldsymbol{j}^m$ and $\boldsymbol{j}^{m,dis}$ (**Figures 1-5**).

From the Equation (14) and the Equation (15), I obtain the gravitational Maxwell-Lorentz equations in Dirac's symmetrisation:

$$\nabla \cdot \boldsymbol{O} = \frac{1}{\varepsilon_g}\rho^o \quad \text{with } \rho^o \text{ in} \left[\frac{\text{kg}}{\text{s}\cdot\text{m}^2}\frac{\text{s}}{\text{m}}\right] = \left[\frac{\text{kg}}{\text{m}^3}\right], \tag{16}$$

$$\nabla \cdot \boldsymbol{Q} = \mu_g \rho^q \quad \text{with } \rho^q \text{ in} \left[\frac{\text{kg}}{\text{s}\cdot\text{m}^2}\right], \tag{17}$$

$$\nabla \times \boldsymbol{O} = -\mu_g \left(\boldsymbol{j}^q + \frac{1}{\mu_g}\frac{\partial \boldsymbol{Q}}{\partial t}\right) \quad \text{with } \boldsymbol{j}^q \text{ in} \left[\frac{\text{kg}}{\text{s}\cdot\text{m}^2}\frac{\text{m}}{\text{s}}\right] = \left[\frac{\text{kg}}{\text{m}^2}\frac{\text{m}}{\text{s}^2}\right] = \left[\frac{\text{kg}}{\text{s}^2\cdot\text{m}}\right], \tag{18}$$

$$\nabla \times \boldsymbol{Q} = \mu_g \left(\boldsymbol{j}^o + \varepsilon_g \frac{\partial \boldsymbol{O}}{\partial t}\right) \quad \text{with } \boldsymbol{j}^o \text{ in} \left[\frac{\text{kg}}{\text{s}\cdot\text{m}^2}\right], \tag{19}$$

with

$$\|\boldsymbol{O}\| = c\|\boldsymbol{Q}\|, \tag{20}$$

$$\begin{aligned} \left\|\boldsymbol{O}^{gen}\right\| &= \sqrt{\boldsymbol{O}^{gen} \cdot \overline{\boldsymbol{O}^{gen}}} = \sqrt{(\boldsymbol{O} + \mathrm{i}c\boldsymbol{Q})(\boldsymbol{O} - \mathrm{i}c\boldsymbol{Q})} \\ &= \sqrt{\|\boldsymbol{O}\|^2 + c^2\|\boldsymbol{Q}\|^2} = \sqrt{2}\|\boldsymbol{O}\|, \end{aligned} \tag{21}$$

$$\begin{aligned} \left\|\boldsymbol{Q}^{gen}\right\| &= \sqrt{\boldsymbol{Q}^{gen} \cdot \overline{\boldsymbol{Q}^{gen}}} = \sqrt{\left(\boldsymbol{Q} + \frac{\mathrm{i}}{c}\boldsymbol{O}\right)\left(\boldsymbol{Q} - \frac{\mathrm{i}}{c}\boldsymbol{O}\right)} \\ &= \sqrt{\|\boldsymbol{Q}\|^2 + \frac{1}{c^2}\|\boldsymbol{O}\|^2} = \sqrt{2}\|\boldsymbol{Q}\|, \end{aligned} \tag{22}$$

and

$$\varepsilon_g \mu_g = \frac{1}{c^2}. \tag{23}$$

In addition, ρ^o, ρ^q, $\rho^{gen,\boldsymbol{O}}$, $\boldsymbol{j}^o$, $\boldsymbol{j}^q$, $\boldsymbol{j}^{o,dis}$, $\boldsymbol{j}^{q,dis}$, and $\boldsymbol{j}^{gen,\boldsymbol{Q}}$ have a shared trait in their units: [kg/s·m²].

Some considerations. I think: "gravitational light and mass induction field are affections of the same substance" (Equation (15)). In addition, in the Equation (15), the mass displacement currents behave like a mass current density flowing in free space: their existence has far-reaching physical consequences as it predicts that the gravitational effect can be transmitted over very long distances, even through empty space.

Combining the Equations (9), (13), and (20), I obtain the relativistic unification of Electromagnetism and Gravitation:

$$\frac{\|\boldsymbol{E}\|}{\|\boldsymbol{B}\|}=\frac{\|\boldsymbol{E}^{gen}\|}{\|\boldsymbol{B}^{gen}\|}=\frac{\|\boldsymbol{O}^{gen}\|}{\|\boldsymbol{Q}^{gen}\|}=\frac{\|\boldsymbol{O}\|}{\|\boldsymbol{Q}\|}=c. \tag{24}$$

Examples

Let me recall the Gauss's law for gravity:

$$\nabla\cdot\boldsymbol{g}=-4\pi G\rho^{o}, \tag{25}$$

where $\boldsymbol{g}$ is the gravitational field [m/s^2] and G is the universal gravitational constant [6]. Combining the Equation (16) and the Equation (25), I obtain the differential relation between the gravitational field $\boldsymbol{g}$ and the mass field $\boldsymbol{O}$:

$$\nabla\cdot\boldsymbol{g}=-4\pi G\rho^{o}=-4\pi\varepsilon_g G\nabla\cdot\boldsymbol{O}=-\frac{4\pi}{\mu_g c^2}G\nabla\cdot\boldsymbol{O}. \tag{26}$$

Let me recall the Newton's law for gravity:

$$\boldsymbol{g}=-G\frac{M}{r^2}\boldsymbol{u}_r, \tag{27}$$

where M is a point mass [kg], r is the distance from M [m], and $\boldsymbol{u}_r$ is the radial unit vector. Because of the Equation (25) and the Equation (27) can be obtained one from the other, from the Equation (26), I can write:

$$\boldsymbol{O}=\frac{1}{4\pi\varepsilon_g}\frac{M}{r^2}\boldsymbol{u}_r=\frac{\mu_g c^2}{4\pi}\frac{M}{r^2}\boldsymbol{u}_r, \tag{28}$$

that is, $\boldsymbol{O}$ for a point mass is the mathematical analog of $\boldsymbol{E}$ for a point electric charge (**Figure 8**). Moreover, using the Equation (26) and the Equation (28), I obtain the gravitational potential energy U_g [J]:

$$\begin{aligned} U_g &=-4\pi\varepsilon_g Grm\|\boldsymbol{O}_M\|=-\frac{4\pi}{\mu_g c^2}Grm\|\boldsymbol{O}_M\| \\ &=\left(4\pi\varepsilon_g\right)^2 Gr^3\boldsymbol{O}_M\cdot\boldsymbol{O}_m=\left(\frac{4\pi}{\mu_g c^2}\right)^2 Gr^3\boldsymbol{O}_M\cdot\boldsymbol{O}_m \\ &=-G\frac{Mm}{r}, \end{aligned} \tag{29}$$

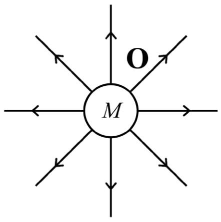

Figure 8. Gravitational monopole.

where m is the point mass attracted by M [kg], r is the distance between M and m [m], and $\boldsymbol{O}_M$ and $\boldsymbol{O}_m$ are the mass fields arising from M and m, respectively.

In addition, according to the solution proposed by Heaviside [1], using the Equation (28), I can write the gravitational analog of the Biot-Savart law for a point electric charge:

$$\boldsymbol{Q} = \frac{\mu_g}{4\pi}\frac{M}{r^2}\boldsymbol{v}\times\boldsymbol{u} = \frac{1}{4\pi\varepsilon_g c^2}\frac{M}{r^2}\boldsymbol{v}\times\boldsymbol{u} = \frac{1}{c^2}\boldsymbol{v}\times\boldsymbol{O}, \tag{30}$$

where $\boldsymbol{v}$ is the constant velocity of M (with $\|\boldsymbol{v}\| \ll c$) and $\boldsymbol{u}$ is the unit vector pointing from the non-retarded position of M to the position where $\boldsymbol{Q}$ is being measured (**Figure 9**).

3.3. Generalised Space-Time

I can enrich the Equation (24) introducing a relativistic link, via c, for the Space-Time:

$$\overbrace{\left\|\boldsymbol{X}^{gen}\right\|}^{\text{static}} = c\overbrace{\left\|\boldsymbol{T}^{gen}\right\|}^{\text{oscillatory}}, \tag{31}$$

where, developing the analogy Electromagnetism/Gravitation/Space-Time, the complex vector $\boldsymbol{X}^{gen} = \boldsymbol{X} + \mathrm{i}c\boldsymbol{T} \in \mathbb{C}^3$ is the generalised space field [m] and describes the static feature of Space-Time, the vector $\boldsymbol{X} \in \mathbb{R}^3$ is the space field [m], the vector $\boldsymbol{T} \in \mathbb{R}^3$ is the time induction field [s], the complex vector $\boldsymbol{T}^{gen} = \boldsymbol{T} + \frac{\mathrm{i}}{c}\boldsymbol{X} \in \mathbb{C}^3$ is the generalised time induction field [s] and describes the oscillatory feature of Space-Time. In other words, the relativistic link, via c, between the static and the oscillatory features of Space-Time is shown in the Equation (31).

So, in analogy with the Equations (5) and (6), (14) and (15), the generalised spatiotemporal Maxwell-Lorentz equations in Dirac's symmetrisation are:

$$\nabla\cdot\boldsymbol{X}^{gen} = \nabla\cdot\overbrace{\left(\boldsymbol{X}+\mathrm{i}c\boldsymbol{T}\right)}^{\boldsymbol{X}^{gen}\text{ in }[\mathrm{m}]} = \frac{1}{\varepsilon_t}\underbrace{\overbrace{\left(\rho^x + \frac{\mathrm{i}}{c}\rho^t\right)}^{\rho^{gen,\boldsymbol{X}}\text{ in }\left[\frac{\mathrm{s}}{\mathrm{m}^3}\right]}}_{\text{in }\left[\frac{\mathrm{m}}{\mathrm{m}}\right]} = \frac{1}{\varepsilon_t}\rho^{gen,\boldsymbol{X}}, \tag{32}$$

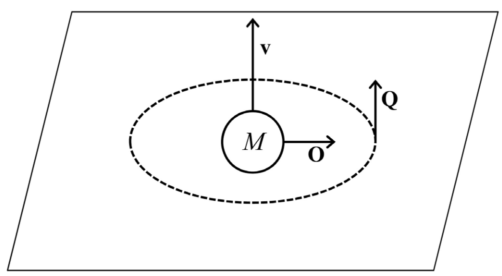

Figure 9. Gravitational dipole ($\boldsymbol{O}$ and $\boldsymbol{Q}$ lie on the plane perpendicular to $\boldsymbol{v}$).

$$\nabla \times \boldsymbol{T}^{gen} = \nabla \times \overbrace{\left(\boldsymbol{T} + \frac{\mathrm{i}}{c} \boldsymbol{X} \right)}^{\boldsymbol{T}^{gen} \text{ in } [\mathrm{s}]} = \underbrace{\mu_t \overbrace{\left[\left(\boldsymbol{j}^x - \frac{\mathrm{i}}{c} \boldsymbol{j}^t \right) + \left(\varepsilon_t \frac{\partial \boldsymbol{X}}{\partial t} - \frac{\mathrm{i}}{c} \frac{1}{\mu_t} \frac{\partial \boldsymbol{T}}{\partial t} \right) \right]}^{\boldsymbol{j}^{gen,\boldsymbol{T}} \text{ in } \left[\frac{1}{\mathrm{m}^2}\right]}}_{\text{in } \left[\frac{\mathrm{s}}{\mathrm{m}}\right]}$$
$$= \underbrace{\mu_t \overbrace{\left[\left(\boldsymbol{j}^x - \frac{\mathrm{i}}{c} \boldsymbol{j}^t \right) + \left(\boldsymbol{j}^{x,dis} - \frac{\mathrm{i}}{c} \boldsymbol{j}^{t,dis} \right) \right]}^{\boldsymbol{j}^{gen,\boldsymbol{T}} \text{ in } \left[\frac{1}{\mathrm{m}^2}\right]}}_{\text{in } \left[\frac{\mathrm{s}}{\mathrm{m}}\right]} = \mu_t \boldsymbol{j}^{gen,\boldsymbol{T}}, \tag{33}$$

where the scalar $\rho^x \in \mathbb{R}$ is the local time density [s/m³], the scalar $\rho^t \in \mathbb{R}$ is the local time current density [1/m²], the complex scalar $\rho^{gen,\boldsymbol{X}} \in \mathbb{C}$ is the generalised time density [s/m³] associated to the generalised spatiotemporal monopole $\mathfrak{I}_1^{gen} \in \mathbb{C}$ [s], the vector $\boldsymbol{j}^x \in \mathbb{R}^3$ is the tangent time current density [1/m²], the vector $\boldsymbol{j}^t \in \mathbb{R}^3$ is the circular time current density [1/s·m], the vector $\boldsymbol{j}^{x,dis} = \varepsilon_t \frac{\partial \boldsymbol{X}}{\partial t} \in \mathbb{R}^3$ is the tangent time displacement current [1/m²], the vector $\boldsymbol{j}^{t,dis} = \frac{1}{\mu_t} \frac{\partial \boldsymbol{T}}{\partial t} \in \mathbb{R}^3$ is the circular time displacement current [1/s·m], the complex vector $\boldsymbol{j}^{gen,\boldsymbol{T}} \in \mathbb{C}^3$ is the generalised time current density [1/m²] associated to the generalised spatiotemporal dipole $\mathfrak{I}_2^{gen} \in \mathbb{C}^3$ [s], ε_t is the vacuum temporal permittivity [s/m³], μ_t is the vacuum temporal permeability [s·m]. In particular, "local" has the same geometric meaning as in ρ^e and ρ^m, "tangent" has the same geometric meaning as in $\boldsymbol{j}^e$ and $\boldsymbol{j}^{e,dis}$, and "circular" has the same geometric meaning as in $\boldsymbol{j}^m$ and $\boldsymbol{j}^{m,dis}$ (**Figures 1-5**).

From the Equation (32) and the Equation (33), I obtain the spatiotemporal Maxwell-Lorentz equations in Dirac's symmetrisation:

$$\nabla \cdot \boldsymbol{X} = \frac{1}{\varepsilon_t} \rho^x \quad \text{with } \rho^x \text{ in } \left[\frac{1}{\mathrm{m}^2} \frac{\mathrm{s}}{\mathrm{m}} \right] = \left[\frac{\mathrm{s}}{\mathrm{m}^3} \right], \tag{34}$$

$$\nabla \cdot \boldsymbol{T} = \mu_t \rho^t \quad \text{with } \rho^t \text{ in } \left[\frac{1}{\mathrm{m}^2} \right], \tag{35}$$

$$\nabla \times \boldsymbol{X} = -\mu_t \left(\boldsymbol{j}^t + \frac{1}{\mu_t} \frac{\partial \boldsymbol{T}}{\partial t} \right) \quad \text{with } \boldsymbol{j}^t \text{ in } \left[\frac{1}{\mathrm{m}^2} \frac{\mathrm{m}}{\mathrm{s}} \right] = \left[\frac{\mathrm{s}}{\mathrm{m}^2} \frac{\mathrm{m}}{\mathrm{s}^2} \right] = \left[\frac{1}{\mathrm{s} \cdot \mathrm{m}} \right], \tag{36}$$

$$\nabla \times \boldsymbol{T} = \mu_t \left(\boldsymbol{j}^x + \varepsilon_t \frac{\partial \boldsymbol{X}}{\partial t} \right) \quad \text{with } \boldsymbol{j}^x \text{ in } \left[\frac{1}{\mathrm{m}^2} \right], \tag{37}$$

with

$$\|\boldsymbol{X}\| = c \|\boldsymbol{T}\|, \tag{38}$$

$$\|\boldsymbol{X}^{gen}\| = \sqrt{\boldsymbol{X}^{gen} \cdot \overline{\boldsymbol{X}^{gen}}} = \sqrt{(\boldsymbol{X} + \mathrm{i}c\boldsymbol{T})(\boldsymbol{X} - \mathrm{i}c\boldsymbol{T})}$$
$$= \sqrt{\|\boldsymbol{X}\|^2 + c^2 \|\boldsymbol{T}\|^2} = \sqrt{2} \|\boldsymbol{X}\|, \tag{39}$$

$$\left\|\boldsymbol{T}^{gen}\right\| = \sqrt{\boldsymbol{T}^{gen} \cdot \overline{\boldsymbol{T}^{gen}}} = \sqrt{\left(\boldsymbol{T} + \frac{\mathrm{i}}{c}\boldsymbol{X}\right)\left(\boldsymbol{T} - \frac{\mathrm{i}}{c}\boldsymbol{X}\right)}$$
$$= \sqrt{\left\|\boldsymbol{T}\right\|^2 + \frac{1}{c^2}\left\|\boldsymbol{X}\right\|^2} = \sqrt{2}\left\|\boldsymbol{T}\right\|, \tag{40}$$

and

$$\varepsilon_t \mu_t = \frac{1}{c^2}. \tag{41}$$

In addition, ρ^x, ρ^t, $\rho^{gen,\boldsymbol{X}}$, $\boldsymbol{j}^x$, $\boldsymbol{j}^t$, $\boldsymbol{j}^{x,dis}$, $\boldsymbol{j}^{t,dis}$, and $\boldsymbol{j}^{gen,\boldsymbol{T}}$ have a shared trait in their units: [1/m^2].

Some considerations. I think: "spatiotemporal light and time induction field are affections of the same substance" (Equation (33)). In addition, in the Equation (33), the time displacement currents behave like a time current density flowing in free space: their existence has far-reaching physical consequences as it predicts that the spatiotemporal light can be transmitted over very long distances, even through empty space. According to the preceding view, the time is the source of the space (Equation (32)) and the spatiotemporal waves are able to expand and to contract the Space-Time (Equation (36) and Equation (37)): a temporal variation of the time induces a spatial variation of the space, whereas a temporal variation of the space induces a spatial variation of the time.

Combining the Equations (24), (31), and (38), I obtain the relativistic unification of Electromagnetism, Gravitation, and Space-Time:

$$\frac{\left\|\boldsymbol{E}^{gen}\right\|}{\left\|\boldsymbol{B}^{gen}\right\|} = \frac{\left\|\boldsymbol{E}\right\|}{\left\|\boldsymbol{B}\right\|} = \frac{\left\|\boldsymbol{O}^{gen}\right\|}{\left\|\boldsymbol{Q}^{gen}\right\|} = \frac{\left\|\boldsymbol{O}\right\|}{\left\|\boldsymbol{Q}\right\|} = \frac{\left\|\boldsymbol{X}^{gen}\right\|}{\left\|\boldsymbol{T}^{gen}\right\|} = \frac{\left\|\boldsymbol{X}\right\|}{\left\|\boldsymbol{T}\right\|} = c. \tag{42}$$

Example

For a black hole, according to the viewpoint of a distant observer, the negative temporal variation of the time causes the positive spatial variation of the space, that is, the spatial compression/contraction of the space or, in other words, the accumulation of space into the space (Equation (36)). For the viewpoint of a distant observer, the spatial compression/contraction of the space is the negative temporal variation of the space which causes the negative spatial variation of the time, that is, the spatial traction/expansion of the time or, in other words, the rarefaction of time in the space (Equation (37)). For the viewpoint of a distant observer, the spatial traction/expansion of the time is the negative temporal variation of the time (Equation (36)): so, the positive feedback loop of the black hole (spatiotemporal hole) is closed.

4. Conservation

Generalised Electromagnetism

Using the formula $\nabla \cdot (\boldsymbol{E} \times \boldsymbol{B}) = \boldsymbol{B}\nabla \times \boldsymbol{E} - \boldsymbol{E}\nabla \times \boldsymbol{B}$, the Equation (3), and the Equation (4), I obtain:

$$\nabla\cdot\overbrace{\left(\frac{1}{\mu_0}\boldsymbol{E}\times\boldsymbol{B}\right)}^{\boldsymbol{S^{EB}}\text{ in}\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]}=-\boldsymbol{E}\left(\boldsymbol{j}^e+\varepsilon_0\frac{\partial\boldsymbol{E}}{\partial t}\right)-\boldsymbol{B}\left(\boldsymbol{j}^m+\frac{1}{\mu_0}\frac{\partial\boldsymbol{B}}{\partial t}\right)$$
$$=-\left(\boldsymbol{j}^e\boldsymbol{E}+\boldsymbol{j}^m\boldsymbol{B}\right)-\left(\boldsymbol{j}^{e,dis}\boldsymbol{E}+\boldsymbol{j}^{m,dis}\boldsymbol{B}\right) \tag{43}$$
$$=-\left(\boldsymbol{j}^e\boldsymbol{E}+\boldsymbol{j}^m\boldsymbol{B}\right)-\frac{\partial}{\partial t}\overbrace{\left(\frac{1}{2}\varepsilon_0\boldsymbol{E}^2+\frac{1}{2}\frac{1}{\mu_0}\boldsymbol{B}^2\right)}^{U^{\boldsymbol{EB}}\text{ in}\left[\frac{\mathrm{J}}{\mathrm{m}^3}\right]},$$

where $\boldsymbol{S^{EB}}=\frac{1}{\mu_0}\boldsymbol{E}\times\boldsymbol{B}\in\mathbb{R}^3$ is the electromagnetic linear momentum density (also known as the electromagnetic Poynting vector) [W/m^2] and $U^{\boldsymbol{EB}}=\frac{1}{2}\varepsilon_0\boldsymbol{E}^2+\frac{1}{2}\frac{1}{\mu_0}\boldsymbol{B}^2\in\mathbb{R}$ is the electromagnetic field energy density [J/m^3] [1] [7].

According to [8], there are two linear conductivity laws:

$$\boldsymbol{j}^e=\sigma^e\boldsymbol{E}, \tag{44}$$

$$\boldsymbol{j}^m=\sigma^m\boldsymbol{B}, \tag{45}$$

where σ^e [S/m] and σ^m [S·m/s^2] are the electric conductivity and the magnetic conductivity, respectively, with the following relativistic link to grant the conservation:

$$\sigma^m=c^2\sigma^e, \tag{46}$$

whereas I can postulate analog formulas for the displacement currents and the related displacement conductivities:

$$\boldsymbol{j}^{e,dis}=\sigma^{e,dis}\boldsymbol{E}, \tag{47}$$

$$\boldsymbol{j}^{m,dis}=\sigma^{m,dis}\boldsymbol{B}, \tag{48}$$

$$\sigma^{m,dis}=c^2\sigma^{e,dis}. \tag{49}$$

As a consequence, using the Equations (7)-(9) and (44)-(49), from the Equation (43), I obtain:

$$\nabla\cdot\boldsymbol{S^{EB}}=-\left(\sigma^e+\sigma^{e,dis}\right)\left\|\boldsymbol{E}^{gen}\right\|^2=-\left(\sigma^m+\sigma^{m,dis}\right)\left\|\boldsymbol{B}^{gen}\right\|^2, \quad 50)$$

useful to study the interaction between the matter and the electromagnetic waves, whereas, using the Equations (7), (9), (44)-(46), and (49), from the Equations (43) and (50), I obtain:

$$\frac{\partial U^{\boldsymbol{EB}}}{\partial t}=\sigma^{e,dis}\left\|\boldsymbol{E}^{gen}\right\|^2=\sigma^{m,dis}\left\|\boldsymbol{B}^{gen}\right\|^2, \tag{51}$$

suitable to study the transmission of the electromagnetic energy.

Now, according to [7], I recall the Poynting theorem:

$$\frac{\partial U^{matter}}{\partial t}+\frac{\partial U^{\boldsymbol{EB}}}{\partial t}+\nabla\cdot\boldsymbol{S}^{matter}+\nabla\cdot\boldsymbol{S^{EB}}=0, \tag{52}$$

where $\boldsymbol{S}^{matter} \in \mathbb{R}^3$ is the mechanical Poynting vector [W/m^2] (related to the act of movement and to the kinetic energy) and $U^{matter} \in \mathbb{R}$ is the mechanical energy density [J/m^3] (related to the kinetic energy).

As a consequence, using the Equations (50)-(52), I obtain:

$$\frac{\partial U^{matter}}{\partial t} + \nabla \cdot \boldsymbol{S}^{matter} = -\frac{\partial U^{\boldsymbol{EB}}}{\partial t} - \nabla \cdot \boldsymbol{S}^{\boldsymbol{EB}} = \sigma^e \left\| \boldsymbol{E}^{gen} \right\|^2 = \sigma^m \left\| \boldsymbol{B}^{gen} \right\|^2, \tag{53}$$

useful for describing the effects of the electromagnetic waves on the matter, also on the cells [9].

Finally, using the definition of $\boldsymbol{S}^{\boldsymbol{EB}}$ and the Equations (7)-(9) and (53), I can write:

$$\left\| \boldsymbol{S}^{\boldsymbol{EB}} \right\| = \frac{1}{2} \varepsilon_0 c \left\| \boldsymbol{E}^{gen} \right\|^2 = \frac{1}{2} \frac{c}{\mu_0} \left\| \boldsymbol{B}^{gen} \right\|^2, \tag{54}$$

$$\frac{\partial U^{matter}}{\partial t} + \nabla \cdot \boldsymbol{S}^{matter} = 2 \frac{\sigma^e}{\varepsilon_0 c} \left\| \boldsymbol{S}^{\boldsymbol{EB}} \right\| = 2 \frac{\sigma^m \mu_0}{c} \left\| \boldsymbol{S}^{\boldsymbol{EB}} \right\|. \tag{55}$$

5. Conclusions

The main aim of this work is to provide further discussion on the Maxwell-Lorentz equations in Dirac's symmetrisation. In particular, the symmetrisation permits to postulate a magnetic displacement current and a magnetomotive force; on the other hand, in a symmetrisation approach, I propose a magnetic monopole composed of two separated acts of electric current in the form of two juxtaposed circular semi-loops, that is, an alternative to the not-gauge-invariant, not-observable Dirac string stretching between two hypothetical Dirac monopoles.

Another aim of this work is to propose a cosmological model where I try to unify Electromagnetism, Gravitation, and Space-Time in terms of microscopic Maxwell-Lorentz-like equations in Dirac's symmetrisation, where the unifying trait is *c*. This cosmological model is, in my opinion, compatible with the concept of curved Space-Time [10] and also of flat Space-Time [11] [12].

In particular, if time is the source of space, I could imagine the Big Bang as an explosion (or a very quick extrusion) that distributed the time, so that space exists where time is.

The concept of space generated by time is in agreement with the discretization of Space-Time [10] and the hypothesised spatiotemporal waves could explain that "Space-Time cannot but fluctuate, and, moreover, its fluctuations are bounded from below, so that all processes become chaotic, and the observables become averaged over this chaos" [10].

On the other hand, according to the here proposed gravitational sources and gravitational light, I can cite a work of Shurtleff [11]: "… a flat spacetime exists whose symmetries can be the basis for motion in gravitational and electromagnetic fields… As indirect evidence, one can point to the observed paths followed by matter in electromagnetic and gravitational fields. After all, the observed paths are much like the geodesics that are determined by 1) the symmetries of that flat spacetime and 2) the assumption of a metric-based arc length".

In conclusion, in analogy to the positive and pushing radiation pressure of the electromagnetic light, I can suppose a negative and pulling pressure given by the gravitational light, whereas the spatiotemporal light should not exert any pressure; moreover, I can suppose that the gravitational light and the spatiotemporal light do not transfer energy as the electromagnetic light does.

In the future work, it would be interesting to speculate about this question: are gravitational light and spatiotemporal light just transverse waves without special characteristics? This is, in my opinion, a non-trivial question because Lorenz supposed "rotating vibrations" [13] of the light and Tamburini successfully used light's orbital angular momentum and vorticity in radio communication [14].

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Thidé, B. (2011) Electromagnetic Field Theory. 2nd Edition, Swedish Institute of Space Physics, Uppsala. https://web.archive.org/web/20160601102148/http://www.physics.irfu.se/CED/Book/EMFT_Book.pdf

[2] Finzi, B. (1959) Meccanica Razionale. 3rd Edition, Zanichelli, Bologna.

[3] Bialynicki-Birula, I. (1996) Photon Wave Function. *Progress in Optics*, **36**, 245-294. https://doi.org/10.1016/S0079-6638(08)70316-0

[4] Aste, A. (2012) Complex Representation Theory of the Electromagnetic Field. *Journal of Geometry and Symmetry in Physics*, **28**, 47-58. https://doi.org/10.7546/jgsp-28-2012-47-58

[5] Maxwell, J.C. (1865) A Dynamical Theory of the Electromagnetic Field. *Philosophical Transactions of the Royal Society of London*, **155**, 459-512. https://doi.org/10.1098/rstl.1865.0008

[6] Rosi, G., Sorrentino, F., Cacciapuoti, L., Prevedelli, M. and Tino, G.M. (2014) Precision Measurement of the Newtonian Gravitational Constant Using Cold Atoms. *Nature*, **510**, 518-521. https://doi.org/10.1038/nature13433

[7] Richter, F., Florian, M. and Henneberger, K. (2008) Poynting's Theorem and Energy Conservation in the Propagation of Light in Bounded Media. *EPL* (*Europhysics Letters*), **81**, Article ID: 67005. https://doi.org/10.1209/0295-5075/81/67005

[8] Ibragimov, N., Khamitova, R. and Thidé, B. (2007) Conservation Laws for the Maxwell-Dirac Equations with Dual Ohm's Law. *Journal of Mathematical Physics*, **48**, Article ID: 053523. https://doi.org/10.1063/1.2735822

[9] Mognaschi, M.E., Di Barba, P., Magenes, G., Lenzi, A., Naro, F. and Fassina, L.

(2014) Field Models and Numerical Dosimetry Inside an Extremely-Low-Frequency Electromagnetic Bioreactor: The Theoretical Link between the Electromagnetically Induced Mechanical Forces and the Biological Mechanisms of the Cell Tensegrity. *SpringerPlus*, **3**, Article No. 473. https://doi.org/10.1186/2193-1801-3-473

[10] Khlestkov, Y., Khlestkov, A., Lukashin, P., Lukashin, M. and Lukashina, N. (2020) Gravitational Space-Time Quantization for Charged Wormholes and the Diophantine Uncertainty Relation. *Journal of Modern Physics*, **11**, 1761-1778. https://doi.org/10.4236/jmp.2020.1111109

[11] Shurtleff, R. (2005) Poincaré Connections in Flat Spacetime. https://doi.org/10.48550/arXiv.gr-qc/0502021

[12] Petry, W. (2020) The Hubble Constant Problem and the Solution by Gravitation in Flat Space-Time. *Journal of Modern Physics*, **11**, 214-220. https://doi.org/10.4236/jmp.2020.112013

[13] Lorenz, L. (1867) On the Identity of the Vibrations of Light with Electrical Currents. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, **34**, 287-301. https://doi.org/10.1080/14786446708639882

[14] Tamburini, F., Mari, E., Sponselli, A., Thidé, B., Bianchini, A. and Romanato, F. (2012) Encoding Many Channels on the Same Frequency through Radio Vorticity: First Experimental Test. *New Journal of Physics*, **14**, Article ID: 033001. https://doi.org/10.1088/1367-2630/14/3/033001