

A Sufficient Clarification of “Super-Quantum Correlations: A Necessary Clarification” by Pierre Uzan

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Abstract

In the paper “Super-Quantum Correlations: A Necessary Clarification” by Uzan [1], it is suggested that stronger than quantum (or supra-quantum) correlations are not possible. The main point of Uzan’s argumentation is the belief that the intuitive definition of No-Signalling (NS) is different from the statistical definition of No-Signalling (NS_{stat}), and that situations exist where NS_{stat} is respected while NS isn’t. In this paper we show why these definitions are one and the same, and where the example from the original paper breaks down. We provide a broader context to help the reader understand intuitively the situation.

Keywords

Nonlocality, No-Signalling, Super-Quantum Correlations

1. Introduction

1.1. A Brief History of No-Signalling and Non-Local Correlations

In 1935 was published a paper [2] by Albert Einstein, Boris Podolsky, and Nathan Rosen on what they called “spooky action at a distance”. They were referring to the idea that if two entangled particles are separated far in space, then collapsing the wave function of one of the particles will instantly collapse the second particle to a new state. This idea, that information might travel from the first location of the first particle to that of the second particle, violates special relativity, as information would be transferred faster than the speed of light. As this would break a major part of the physics known at the time, they proposed the hypothesis that reality was governed by “local hidden variables” (LHV), which means that those variables would influence the seemingly random out-

comes of quantum states. This hypothesis was further studied by John Stewart Bell in his 1964 paper [3] entitled “On the Einstein Podolsky Rosen Paradox”. In this paper he formulated the so-called “Bell Inequalities”, which if violated would indicate that the LHV theory is insufficient and there exist greater-than-local correlations occurring instantaneously, but of course, without enabling a transfer of information. This gives rise to the concepts of nonlocality and nonlocal correlations (which are explained in great detail in [4]). In 1982 was devised an experiment by Alain Aspect, Philippe Grangier, and Gérard Roger [5], seemingly demonstrating violation of Bell inequalities, but it was pointed out that there were loopholes present in that experiment. Solid experimental results were achieved in 2015 (in [6], among others), giving persuasive evidence that the local hidden variables theory is insufficient to describe our observable reality.

1.2. The Rise of Non-Local Correlations

We define here the notion of correlation in the context of a correlation experiment that involves two parties which are asked questions by an experimenter. We observe the distribution of their answers in relation to their questions. The most restrictive nonlocal class is the LHV class. If two parties participating in a correlation experiment are limited to the set of local correlations, they are considered classical and they can only share hidden variables along with any kind of joint strategy. They can make any calculation involving their pre-agreed variables and their own individual question given by the experimenter, but they can have no further resource. Formally, the LHV class consists of all correlations resulting from such calculations. Now, if we have two parties that can also share entanglement, *i.e.* by each having a particle from an entangled pair, then they can reach greater correlation than in the classical case by having a strategy that involves quantum measurements on their particle. This is called quantum nonlocality and we denote the class of all correlations obtained this way by QNL . Recalling our earlier discussion, we mentioned that the locality of a correlation may be refuted by showing it violates the Bell inequalities. There is a way to quantify “how much” we violate locality and Bell’s inequalities (see [4]), and it was shown in [7] that there is a limit to how much quantum resources can be used to violate Bell’s inequalities. This leaves a gap between the quantum theory and what is forbidden by special relativity: we do not know if the maximum theoretical violation is a mathematical artifact or whether it represents a physical property of our universe.

This area where supra-quantum nonlocal correlation is possible is called No-Signalling. In fact, the LHV class is part of the QNL class, which is itself part of the NOSIG class: the class of all correlations that cannot be used to signal. The question is whether or not there exist supra-quantum correlations part of the NOSIG class that can be obtained instantaneously in our physical reality.

1.3. Signalling and No-Signalling

Given a channel that is intended for communication, we say that the channel can be used to signal if it is possible to transmit information using it. This is quite vague, so we would rather say that a channel is signalling if it is not No-Signalling. What is No-Signalling then? We say that a channel is No-Signalling if for whatever we feed into one of the inputs of the channel, the probability distribution of the other output does not change. That means that, if we imagine our fictitious channel with a friend on the other side, then whatever output our friend receives, he has no way of knowing what input we fed to the channel, and therefore has no way of knowing what message we were actually trying to send him. Yes, they do get “something”, but that something is, seemingly, random, so we cannot communicate him any information.

2. To Signal or Not to Signal, That Is the Question

We will now prove to the reader that the statistical definition of No-Signalling is in fact equivalent to its “intuitive” definition. Let us represent any channel by a black box that takes in a set of inputs and returns a set of outputs. In what follows, we assume the box has two inputs and two outputs. We say that the box is No-Signalling if for a fixed input on one side of the box, the distribution of outputs on that side is independent of the input on the other side. Formally, for all inputs $\{x_0, x_1\}$ and outputs $\{a_0, a_1\}$ where the indices 0 and 1 indicate the sides of the box:

$$\forall i \in \{0, 1\}, a_i, x_i, x_{\bar{i}}, \Pr(a_i | x_i) = \Pr(a_i | x_i, x_{\bar{i}}).$$

Why does this imply that it is impossible to send a signal using this box? Because whatever outputs the box returns, since it always respects the same probability distribution, there is no way of knowing which input it was given. When absolutely no such restriction is applied, the most general SIG class results (consult [Figure 1](#)): the class of all correlations achieved by signalling parties.

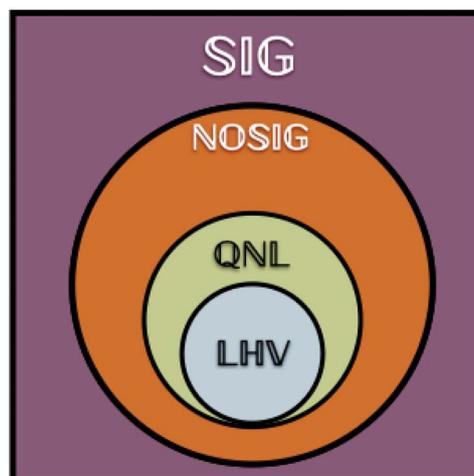


Figure 1. Nonlocality hierarchy.

As soon as this definition is broken, then there are at least two inputs for which the probability distribution changes, and by repeatedly using the box with those two flawed inputs (thus amplifying the signal) it is possible to exploit the box to send a signal that can be detected. We can see, by taking the contrapositive

$$\exists i \in \{0,1\}, a_i, x_i, x_{\bar{i}}, \Pr(a_i | x_i) \neq \Pr(a_i | x_i, x_{\bar{i}}),$$

which is equivalent to

$$\exists i \in \{0,1\}, a_i, x_i, x_{\bar{i}}, x'_{\bar{i}}, \Pr(a_i | x_i, x'_{\bar{i}}) \neq \Pr(a_i | x_i, x_{\bar{i}}),$$

we obtain a pair of inputs $x_{\bar{i}}, x'_{\bar{i}}$ for which the distributions are different. By having party i fix his input so that with party \bar{i} they jointly input either $(x_i, x_{\bar{i}})$ or $(x_i, x'_{\bar{i}})$ repeatedly, say ℓ times, party i should receive a_i roughly $\mu \stackrel{\text{def}}{=} \Pr(a_i | x_i, x_{\bar{i}})\ell$ times in the former case or $\mu' \stackrel{\text{def}}{=} \Pr(a_i | x_i, x'_{\bar{i}})\ell$ times out of the total ℓ in the latter case. For ℓ large enough, determining whether the number of occurrences of a_i is smaller or larger than $\frac{\mu + \mu'}{2}$ will distinguish $x_{\bar{i}}$ from $x'_{\bar{i}}$ with probability converging to 1. By selecting which input is sent repeatedly, we can choose one of two values, and hence transmit one bit of information.

As we will see in a moment, the problem in Uzan’s paper is a simple mix-up of communication channels, as well as a confusion between how a correlation is implemented (what the box is made of) and what can be achieved from such a correlation (what the box is used for). Uzan thinks No-Signalling correlations are not allowed to be made from signalling ones while the rest of the community defined No-Signalling as the correlations that cannot lead to signalling ones (see [4] and references within). The bigger problem with this argument is that the box used in Uzan’s example to show that supra-quantum correlations are impossible is not supra-quantum: it is local deterministic. While it is true that supra-quantum correlations are, at this point in time, only possible in theory, local correlations, though, are very real. The mix-up in channels results in equating the two, which means that a signalling channel would be equivalent to a local one, which we know is not true. For example, if we have two normal, less-than-quantum humans being interrogated for some crime, it is intuitively understandable that they will answer their interrogation differently if they are able to communicate or if they are separated by walls. Note that while it is possible to add resources to a No-Signalling channel in order to make it signalling, the inverse is not true, as adding resources to a channel can only make it more signalling.

3. Corrections to the Paper

Uzan’s paper presents a deterministic box “for which x_i determines the outcome a_i for Alice and x_j determines the outcome b_j for Bob” and it is also mentioned that “Alice systematically informs Bob of her choice of action x ” after

each round. The problem in this situation stems from the confusion of two channels. We recall that signalling implies conveying information from one party to another, in this situation, from Alice (A) to Bob (B). The box presented in section 3.2 of the author's paper clearly cannot be used to signal between A and B. Indeed, since

$$\forall i, a_i, x_i, y, \Pr(a_i | x_i) = \Pr(a_i | x_i, y)$$

and

$$\forall j, b_j, y_j, x, \Pr(b_j | y_j) = \Pr(b_j | x, y_j)$$

by construction of the box, it is impossible to signal using this box. We can say that this channel is non-signalling, where both the author's NS and NS_{stat} definitions are respected. Moreover, not only is this channel No-Signalling but it is also *local* (in the \mathbb{LHV} class), which means that the correlations achieved by its use do not require quantum nor supra-quantum resources. So if the argument of [1] was correct, it would actually prove that local is signalling, which is clearly not true. Now, obviously, if we also add the possibility for A and B to have access to a second channel it may certainly be signalling. The combined channel result of the box together with the second channel is clearly signalling but the box is still No-Signalling. Looking at the very definition of No-Signalling, we see that it is not respected neither for the second channel nor for the combined channel: if A can simply tell B of her input into the box, then in this second channel the output (what B hears) is completely dependent of the input (what A says). Both the author's definitions (NS and NS_{stat}) of the No-Signalling principle are then violated, *only for this combined channel where the players can speak freely*, not the box in itself. The status of the previous box has not been changed in any way at all and still satisfies both NS and NS_{stat} .

An important misconception that may easily lead to incorrect conclusions is the confusion between *how one implements a particular box* and *what one can do with such a box*. If a box is local or No-Signalling, it may certainly be implemented in a signalling fashion but doing so will not affect at all the (local or No-Signalling) character of the box. We believe this is what misled the author of [1] to this fallacious claim.

As for section 3.1 of Uzan's paper, a complete detailed contradiction of those claims can be found in [8], with an explicit derivation in the Supplementary Material of that paper that *Relativistic Causality* and *No-Signalling* are exactly equivalent conditions for two-party correlations.

4. Conclusion

We have shown that the statistical and intuitive definitions of No-Signalling are indeed one and the same, and that the arguments in Uzan's paper leading to the contrary arise from a confusion between two different channels. The conclusion that supra-quantum, No-Signalling correlations are impossible is therefore unjustified. However, it remains an open question whether supra-quantum No-

Signalling correlations are observable simultaneously by distant observers in our world.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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