

Quantum Fisher Information of Two-Level Atomic System under the Influence of Thermal Field, Intrinsic Decoherence, Stark Effect and Kerr-Like Medium

S. Jamal Anwar, M. Ramzan, M. Usman, M. Khalid Khan

Department of Physics, Quaid-i-Azam University, Islamabad, Pakistan

Email: sjamalanwar@gmail.com

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Abstract

In this paper, we have proposed the numerical calculations to study the quantum entanglement (QE) of moving two-level atom interacting with a coherent and the thermal field influenced by intrinsic decoherence (ID), Kerr medium (non-linear) and the Stark effect. The wave function of the complete system interacting with a coherent and the thermal field is calculated numerically affected by ID, Kerr (non-linear) and Stark effects. It has been seen that the Stark, Kerr, ID and the thermal environment have a significant effect during the time evolution of the quantum system. Quantum Fisher information (QFI) and QE decrease as the value of the ID parameter is increased in the thermal field without the atomic movement. It is seen that QFI and von Neumann entropy (VNE) show an opposite and periodic response in the presence of atomic motion. The non-linear Kerr medium has a more prominent and significant effect on the QE as the value of the Kerr parameter is decreased. At smaller values of the non-linear Kerr parameter, the VNE increases, however, QFI decreases, so QFI and VNE have a monotonic connection with one another. As the value of the Kerr parameter is increased, the effect of non-linear Kerr doesn't stay critical on both QFI and QE. However, a periodic response of QE is seen because of the atomic movement which becomes modest under natural impacts. Moreover, it has been seen that QFI and QE rot soon at the smaller values of the Stark parameter. However, as the value of the Stark parameter is increased, the QFI and QE show periodic response even when the atomic movement is absent.

Keywords

Quantum Fisher Information (QFI), Intrinsic Decoherence (ID), Thermal Environment, Two-Level Atomic System, Kerr and Stark Effect

1. Introduction

Quantum metrology (QM) is the most important and developing field of modern research in quantum information theory (QIT) [1] [2] [3] [4]. Quantum mechanics gives a new breakthrough to the field of parameter estimation [5] [6]. The main objective of quantum mechanics is to explore the restrictions of the parameter-estimation limited by the laws of quantum mechanics. Quantum mechanics has a large number of applications such as improving the time and frequency standards [7] [8], detection of gravitational waves [9], and synchronization of clock [10], etc. Quantum Fisher information (QFI) has a key part in quantum mechanics. Quantum Fisher Information (QFI) is a very fascinating and interesting idea thought for the examination of the situations that need phase sensitivity. Quantum Fisher information (QFI) has great importance for quantum estimation theory and QIT [11] [12] [13] [14]. In the quantum estimation theory, the main objective is to compute the value of an unknown parameter related to the quantum system, and the primary aim is to increase the precision and accuracy [15] [16] [17] [18]. The inverse of the QFI provides the lower bound of error of the estimation [19] [20]. Hence, how to improve the QFI turns into the basic problem to be resolved. Also, statistical distinguishability is measured with the help of QFI on the space of the density operators in the quantum information geometry [21] [22]. Recently, the QFI flow was suggested as a quantitative measure of the information flow and provided an innovative point of view on studying the non-Markovian behaviour in open quantum systems [23].

Different quantum entanglement (QE) measures for pure states have been suggested, like von Neumann entropy (VNE) [24] [25], concurrence [26] [27] and linear entropy [28], etc. The relationship between the QE and Fisher information (FI) has not been investigated extensively. In this context, the QE evaluation is calculated using atomic Fisher information (AFI) [29]. It has been observed that QE of a two-level atom can be determined by using AFI and its marginal distribution. In addition, the QFI gives an acceptable condition to observe multi-particle QE in an N qubit state [12]. Another use of the FI is to investigate polarization squeezing and multipartite QE of a quantum light field. Moreover, the FI determines the correlation between the quantized field and Kerr medium [30].

The interaction between the electromagnetic field and matter is a major quantum optical problem and it has a large number of applications in laser physics and quantum optics [31] [32]. The most simplified model in the atom optics is well-know Jaynes-Cummings model (JCM) [33] which describes the interaction between a single two-level atom and the quantized field in a rotating wave approximation (RWA). This model has been explored to a great extent before [34]. A number of generalized models have been studied [35]. Multiphoton and multimode processes can be discussed through these generalizations from a theoretical point of view [36]. However, in recent years, researches have strongly focused on the nonlinear interaction between a two-level atom and field in the

deformed JCM. The JCM has been investigated under the effect of Stark-shift and Kerr like medium [37] [38]. Abdel-Aty *et al.* [39] discussed the entropy evolution of the bimodal field interacting with a two-level atom in the presence of the Kerr medium. The interaction of a three-level atom with a single-mode cavity field having intensity-dependent coupling in a Kerr medium has been studied [40]. Abdalla *et al.* studied the interaction of a two-level atom with a single-mode multi-photon field in the medium including the Stark shift and Kerr medium effects with the coupling term which is supposed to be a function of time, but still linear with the intensity of light [41]. Besides, it is partially nonlinear only due to the effect of the Kerr medium. Anyhow, in the last few years, the effect of the atomic motion and field-mode structure on QE has become attractive for researchers [42].

Real quantum systems always have the effect of the surrounding environment. Decoherence is generated because of the interaction between the environment and quantum systems and hence it can also induce QE. Therefore, the QE always arises in the interaction of an arbitrary big system in any mixed state with a single qubit in the pure state. The effect of intrinsic decoherence in quantum systems is discussed in a large number of papers [43] [44]. One explanation of why quantum coherences are decreased and difficult to observe that the closed quantum systems don't evolve unitarily according to the Schrodinger equation, but are influenced by more generalized equations that include ID. In particular, a classic model of ID based on a simple change of a unitary Schrodinger evolution has been suggested. This model is established on the hypothesis that on adequately short time scales the closed system grows by an irregular arrangement of unitary phase changes developed by the system Hamiltonian.

In the last few years, researchers focused on the QE and QFI of a two-level atomic system as it can be considered as one qubit system. A.-S. F. Obada studied the QE evolution with atomic Fisher information (AFI) [29]. S. Abdel-Khalek and his fellows studied the QFI of a single-qubit system [45]. K.Berrada and co-workers investigated the QFI of the qubit system placed inside the dissipative cavity [46]. Buzek studied the effect of Kerr-like medium on the dynamics of two-level atom [47]. H. R. Baghshahi and fellows described the effect of Kerr-like medium and Stark on the dynamics of the Jaynes-Cummings model [48].

In this present work, our main focus is to investigate the QFI and QE of moving two-level atom which is interacting with coherent field and thermal field in presence of ID, Kerr medium and Stark shift. We have calculated VNE and QFI of the two-level atomic system in the presence of atomic motion and without atomic motion. Here, we use the QFI, based on symmetric logarithmic derivative (SLD) operator, to quantify entanglement of moving two-level atom interacting with a single-mode thermal field under the influence of ID. This study mainly investigates the time evolution of QFI and VNE under ID for moving and non moving two-level atomic system interacting with a thermal field. The time evolution of the wave function of the complete system is numerically evaluated. Then, the dynamics of the two-level atom by considering "QFI" and VNE are

discussed, in detail. It is deduced from the numerical results that, both the ID and thermal environment play an important role during the time evolution of the quantum system. QFI and QE show an opposite response during its time evolution in the presence of the thermal environment. The time evolution of QFI is found to be highly sensitive to ID as compared to the QE. Furthermore, the degree of entanglement changes drastically when we increase the ID parameter in the absence of atomic motion. The increased damping behaviour of QE validates the basic fact that the system is more prone to intrinsic decoherence for larger time-scales. We study the dynamics of QE and QFI for the two-level atomic system under the influence of Stark shift and Kerr medium. The time evolution of QFI and QE for a two-level atomic system influenced by Stark effect and Kerr-like medium is investigated. It is seen that Stark and Stark and Kerr's medium plays a dominant during the time evolution of the quantum system. Dynamics of QFI are heavily influenced under the effect of linear Kerr medium as compared to the QE. Whereas the effect of non-linear Kerr medium is more prominent on the QE instead of QFI. Similarly, the Stark effect strongly influences the QE of the two-level atomic system. The QFI and QE evolve with time as we increase the Stark effect parameter. Finally, the quantum system is found highly sensitive to these environmental influences.

The organization of the paper is as follows: In Section 2, we describe the brief introduction of QFI and VNE and their formulas related to numerical calculations. The Hamiltonian of the system and dynamics are presented in Section 3. In Section 4, we present detailed results and numerical discussions. In Section 5, we present a brief conclusion.

2. Quantum Fisher Information (QFI) of Bipartite System

Von Neumann entropy (VNE) is the most important and basic QE measurement tool when the quantum system is in the pure state, therefore VNE is used to measure QE between the field and the two atoms. The VNE is presented in the form of eigenvalues of the atomic density matrix as [49] [50]

$$S_{AB} = -\text{Tr}(\rho_{AB} \ln \rho_{AB}) = -\sum_{i=1}^2 r_i \ln r_i, \quad (1)$$

where r_i are the eigenvalues of the atomic density matrix ρ_{AB} .

The QFI gives the maximum information about the estimated parameter and the QFI which is related to θ can be represented as [51] [52] [53]

$$I_\theta = \sum_k \frac{(\partial_\theta \lambda_k)^2}{\lambda_k} + 2 \sum_{k,k'} \frac{(\lambda_k - \lambda_{k'})^2}{\lambda_k + \lambda_{k'}} \left| \langle k | \partial_\theta k' \rangle \right|^2, \quad (2)$$

where $\lambda_k > 0$ and $\lambda_k + \lambda_{k'} > 0$. The first term in Equation (2) is classical Fisher information (CFI) and the second term represents its quantum counterpart. Therefore, we can define the QFI of a bipartite density operator ρ_{AB} in terms of θ as [54]

$$I_{QF}(t) = I_{AB}(\theta, t) = \text{Tr} \left[\rho_{AB}(\theta, t) \{L^2(\theta, t)\} \right], \quad (3)$$

where $L(\theta, t)$ is the quantum score [55] or the symmetric logarithmic derivative (SLD) which can be defined as

$$\frac{\partial \rho_{AB}(\theta, t)}{\partial \theta} = \frac{1}{2} [L(\theta, t) \rho_{AB}(\theta, t) + \rho_{AB}(\theta, t) L(\theta, t)]. \quad (4)$$

3. Intrinsic Decoherence Model

In real practical situations, any quantum system which is said to be open can never be completely isolated from the environment. Milburn [56] gives a simple model of ID by making a modification in standard quantum mechanics. He supposed that on a sufficiently small time scale τ , the system state takes the form

$$\rho(t + \tau) = \exp\left[-\frac{i}{\hbar} \theta(\tau) H\right] \rho(t) \exp\left[\frac{i}{\hbar} \theta(\tau) H\right], \quad (5)$$

with a probability of $\rho(t)$. The master equation presenting the ID under the Markovian approximations is given by

$$\frac{d\rho(t)}{dt} = -i[H, \rho(t)] - \frac{\gamma}{2} [H, [H, \rho(t)]], \quad (6)$$

where γ is the ID parameter. The above master equation can be expressed by the solution as given below

$$\rho(t) = \sum_{k=0}^{\infty} \frac{(\gamma t)^k}{k!} M^k \rho(0) M^{\dagger k}, \quad (7)$$

where $\rho(0)$ is the density operator of the initial state and M^k is defined as

$$M^k = H^k e^{-iHt} e^{-\frac{\gamma t}{2} H^2}. \quad (8)$$

4. The System Hamiltonian and Its Dynamics

We have considered a system of moving two-level atom interacting with a coherent and thermal field as shown in **Figure 1**.

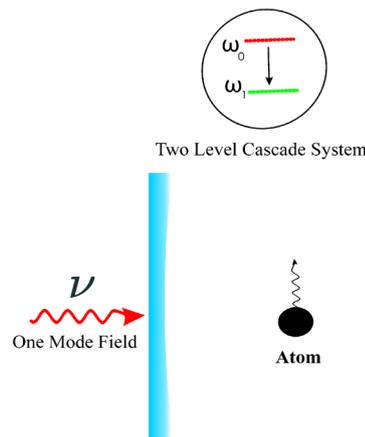


Figure 1. (Systematic diagram) Interaction of two-level atom with the single-mode coherent field.

The total Hamiltonian of the system \hat{H}_T under the RWA for a particular system can be described as [57]

$$\hat{H}_T = \hat{H}_{\text{Atom-Field}} + \hat{H}_I, \tag{9}$$

where $\hat{H}_{\text{Atom-Field}}$ is representing the Hamiltonian for the non-coupling atom and field, and the interaction part is given by \hat{H}_I . We will write $\hat{H}_{\text{Atom-Field}}$ as

$$\hat{H}_{\text{Atom-Field}} = \sum_j \omega_j \hat{\sigma}_{j,j} + \nu \hat{a}^\dagger \hat{a}, \tag{10}$$

where $\hat{\sigma}_{j,j} = |j\rangle\langle j|$ are describing as population operators for the j th level and ν is the frequency of oscillation.

The interaction Hamiltonian \hat{H}_I of two-level atomic system for the case which is not resonant is written as [58]

$$\hat{H}_I = \sum_{s=1}^1 \Omega(t) \left[\hat{a} e^{-i\Delta_s t} (\hat{\sigma}_{s,s+1}) + (\hat{a} e^{-i\Delta_s t} (\hat{\sigma}_{s,s+1}))^\dagger \right]. \tag{11}$$

We can define the detuning parameter as

$$\Delta_s = \nu - (\omega_s - \omega_{s+1}). \tag{12}$$

We have also considered the case in which the moving two-level atom interacting with a coherent field in the presence of non-linear Kerr-Like medium as shown in **Figure 2**.

$$\hat{H}_I = \sum_{s=1}^1 \Omega(t) \left[\hat{a} e^{-i\Delta_s t} (\hat{\sigma}_{s,s+1}) + (\hat{a} e^{-i\Delta_s t} (\hat{\sigma}_{s,s+1}))^\dagger \right] + \chi (\hat{a}^\dagger \hat{a})^2, \tag{13}$$

and when the Stark effect is included in the interaction Hamiltonian, it can be written as

$$\hat{H}_I = \sum_{s=1}^1 \Omega(t) \left[\hat{a} e^{-i\Delta_s t} (\hat{\sigma}_{s,s+1}) + (\hat{a} e^{-i\Delta_s t} (\hat{\sigma}_{s,s+1}))^\dagger \right] + \beta \hat{a}^\dagger \hat{a} |g\rangle\langle g|, \tag{14}$$

where $|g\rangle$ is representing the ground state of two-level atom and the coupling constant for atom and field is G , $\Omega(t)$ represents the shape function of the cavity-field mode [59] and the atomic motion is along z -axis. A realization of particular interest is

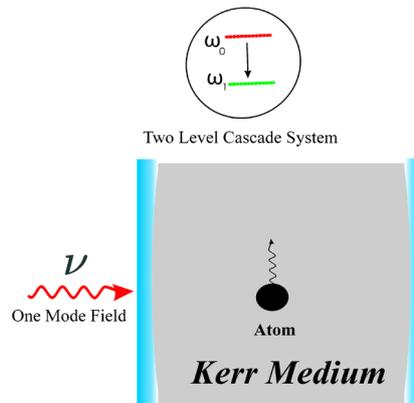


Figure 2. (Systematic diagram) Interaction of two-level atom with the single-mode coherent field in the presence of Kerr-like medium.

$$\begin{aligned}\Omega(t) &= G \sin(\eta\pi vt/L) \text{ in the presence of atomic motion, } \eta \neq 0, \\ \Omega(t) &= G \text{ in the absence of atomic motion } \eta = 0,\end{aligned}\quad (15)$$

where v denotes the atomic motion velocity and η is equal to the number of half wavelengths of the mode in the cavity and L represents the cavity length in the z -direction. The atomic motion velocity is given as $v = GL/\pi$ which leads to

$$\Omega_1(t) = \int_0^t \Omega(\tau) d\tau = \frac{1}{\eta} (1 - \cos(\eta\pi vt/L)) \text{ for } \eta \neq 0, \quad (16)$$

$$= Gt \text{ for } \eta = 0. \quad (17)$$

The optimal input state after the phase gate operation for a two-level atomic system interacting with a single-mode cavity field can be written as

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}} (|1\rangle + e^{i\phi} |0\rangle) \otimes |\alpha\rangle, \quad (18)$$

where $|1\rangle$ is the ground state and $|0\rangle$ is the excited state of an atom and α is the coherent state of the input field given as

$$|\alpha\rangle = \sum_{n=0}^{\infty} \alpha^n \sqrt{e^{-|\alpha|^2}/n!} |n\rangle. \quad (19)$$

The optimal input state for a two-level atomic system interacting with a thermal field can be written as

$$\rho = \rho(0) \otimes \rho_f(0),$$

where $\rho(0) = |\Psi(0)\rangle\langle\Psi(0)|$ and $\rho_f(0)$ is the state of input thermal field. We have used thermal state of the field as input field state in the atom field interaction of the two-level atomic system which is given as

$$\rho_f(0) = \sum_{n=0}^{\infty} P(n) |n\rangle\langle n|. \quad (20)$$

where $|n\rangle$ is the fock state.

$$P(n) = \frac{\bar{n}^n}{(\bar{n} + 1)^{(n+1)}}, \quad (21)$$

where \bar{n} is the mean photon number and is given as

$$\bar{n} = \frac{1}{e^{\hbar\omega_f/k_B T} - 1},$$

where k_B is the Boltzmann constant, ω_f is the frequency of the cavity mode and T is the temperature.

The wave function $|\Psi(t)\rangle$ in terms of unitary time evolution operator $\hat{U}(t)$ can be written as

$$|\Psi(t)\rangle = \hat{U}(t) |\Psi(0)\rangle, \quad (22)$$

where $\hat{U}(t)$ is given by

$$U(t) = \sum_{z=1}^2 \exp(E_z t) |\varphi_z(t)\rangle\langle\varphi_z(t)|, \quad (23)$$

where $|\varphi_z(t)\rangle$ and $E_z(t)$ are eigenvectors and eigenvalues of the Hamiltonian H_I , respectively and γ is the ID parameter. The QFI is calculated numerically for the atom-field density matrix as given below

$$\hat{\rho}(t) = \hat{U}(t)\rho(0)\hat{U}^\dagger(t). \quad (24)$$

One can write the explicit expression of the density matrix as

$$\hat{\rho}(t) = \sum_{m,n}^2 |\Psi_n(t)\rangle \langle \Psi_n(t)| \hat{\rho}(t) |\Psi_m(t)\rangle \langle \Psi_m(t)|.$$

The density matrix can also be written as

$$\hat{\rho}(t) = \sum_{mn} \exp\left(-\frac{\gamma t}{2}(E_m - E_n)^2 - i(E_m - E_n)t\right) \times \langle \Psi_m | \hat{\rho}(0) | \Psi_n \rangle |\Psi_m\rangle \langle \Psi_n|,$$

where $E_{m,n}$ and $\Psi_{m,n}$ are the eigenvalues and the corresponding eigenvectors of H_I respectively. Now, the effect of different environmental parameters γ, θ and η on the dynamics of QFI and VNE is discussed in detail in the next section.

5. Numerical Results and Discussions

In this section, we will present the results of the time evolution of QFI and VNE of a system of the two-level atom interacting with a coherent field and thermal field under the influence of Stark, Kerr (non-linear), and ID effects. We scaled out the time t , for example, one unit of time is described by the inverse of the coupling constant G . Initially, we investigate the time evolution of QFI and VNE for two-level atomic system interacting with coherent and thermal fields under the influence of ID, a (non-linear) Kerr medium, and a Stark effect with and without atomic motion.

In **Figure 3** and **Figure 4**, we plot the QFI and VNE as a function of time for the two-level atomic system interacting with a thermal field under the influence of ID for $|\alpha|^2 = 6$, $\gamma = 0, 0.0001$ and 0.001 for phase shift $\phi = 0, \pi/4$, and atomic motion parameter $\eta = 0, 1$. The gradual decay of QFI is observed in the thermal environment in the presence of ID. Particularly at value $\gamma = 0.001$, both QFI and VNE decrease rapidly as compared to the value at $\gamma = 0.0001$. So it means that as ID parameter increased there is a prominent decay of QE as compared to QFI. In the case of the thermal field in the presence of ID, We find that the QFI and QE show an opposite response during the time evolution when the atomic motion parameter is $\eta = 1$. Periodic and opposite behaviour of QFI and VNE is observed in the presence of atomic motion. It is shown that the atomic motion destroys the monotonic correlation between QFI and QE. It is found that the presence of a thermal environment leads to the suppression of QFI during its time evolution. Furthermore, the effect of the thermal environment dominates the ID effect. Paying attention to the effect of the atomic motion parameter, it is observed that an increase of atomic motion parameter and ID results in the loss of the purity of the state of the system. The damping behaviour of QE is seen under ID for larger time-scales in the absence of atomic

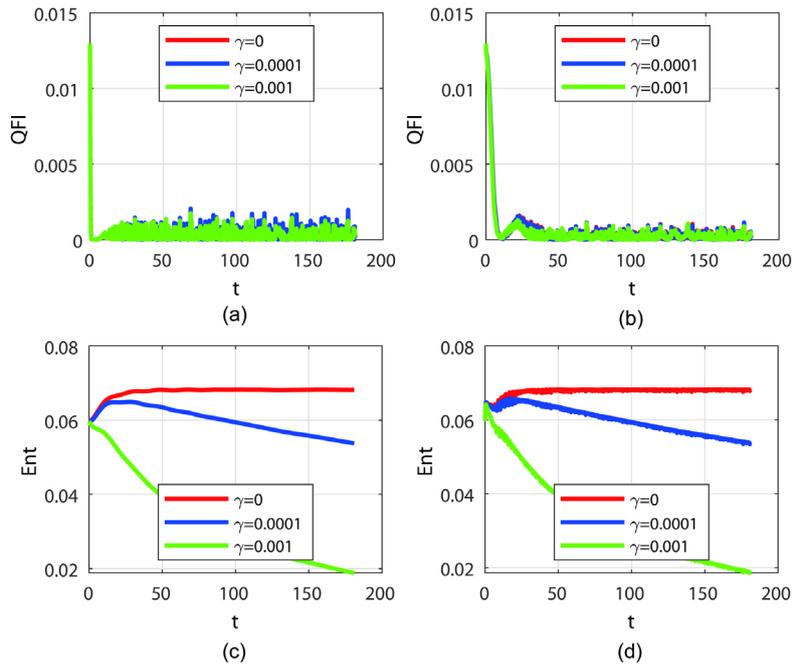


Figure 3. (Color online) The QFI (upper panel) and VNE (lower panel) as a function of time for two-level atom interacting with a thermal field for $\alpha=6$ in the presence ID parameter $\gamma=0,0.0001,0.001$ of and the phase shift estimator parameter $\phi=0$ (left panel) and $\pi/4$ (right panel). The atomic motion parameter η is neglected. (a) $\phi = 0, 2$ -level; (b) $\phi = \pi/4, 2$ -level; (c) $\phi = 0, 2$ -level; (d) $\phi = \pi/4, 2$ -level.

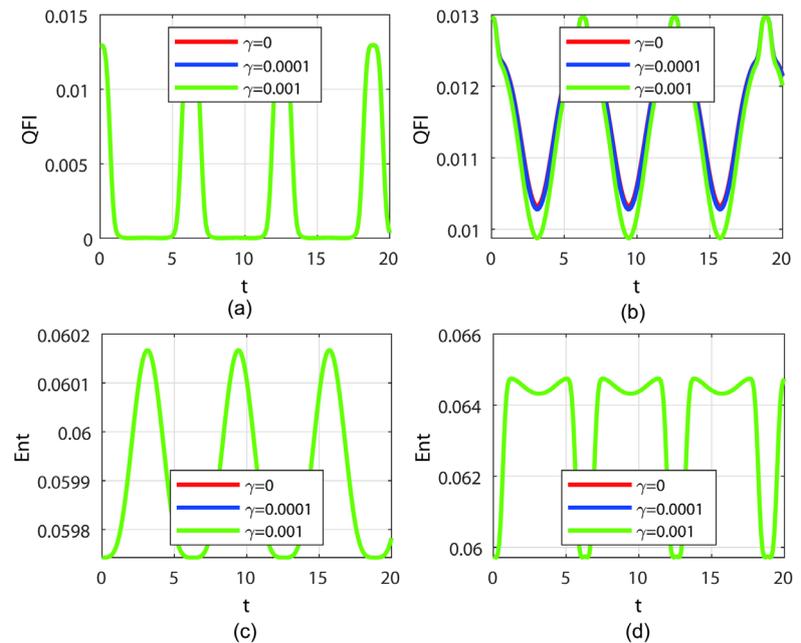


Figure 4. (Color online) The QFI (upper panel) and VNE (lower panel) as a function of time for two-level atom interacting with thermal for $\alpha=6$ in presence ID parameter $\gamma=0,0.0001,0.001$ and the phase shift estimator parameter $\phi=0$ (left panel) and $\pi/4$ (right panel), The atomic motion parameter is $\eta=1$. (a) $\phi = 0, 2$ -level; (b) $\phi = \pi/4, 2$ -level; (c) $\phi = 0, 2$ -level; (d) $\phi = \pi/4, 2$ -level.

motion. The periodic behaviour of QE in the presence of atomic motion becomes modest under environmental effects. The decay of the QE is suppressed in the presence of the thermal environment as compared to the ID. Therefore, the ID and thermal environment are found to suppress the nonclassical effects of the quantum system. However, QE and QFI saturate to a lower level for longer time-scales under the environmental influences. Finally, according to our numerical results, the decaying of the QE becomes faster in a thermal environment as we increased the value of ID parameter. Moreover, these results can be useful to perform long-distance quantum communication, especially when long-living QE is needed and the effect of environments cannot be neglected.

In **Figure 5** and **Figure 6**, we plot the QFI and VNE as a function of time for two-level system interacting with coherent field for $|\alpha|^2 = 6$ and the phase shift parameter $\phi = 0$ (left panel) and $\pi/4$ (right panel) for different values of χ (Non-linear Kerr medium) with and without atomic motion, *i.e.* $\eta = 0, 1$. The effect of non-linear Kerr medium is found to be more prominent on the QE and QFI at $\chi = 0.3$, QFI decreases as time evolves but VNE increases, so QFI and VNE are showing opposite behaviour. In the case of atomic motion, both QFI and VNE exhibit opposite and periodic behaviour in the presence of non-linear Kerr medium. At values $\chi = 1, 3$, the effect of non-linear Kerr is not very significant in the case of both QFI and VNE in the absence of atomic motion. The

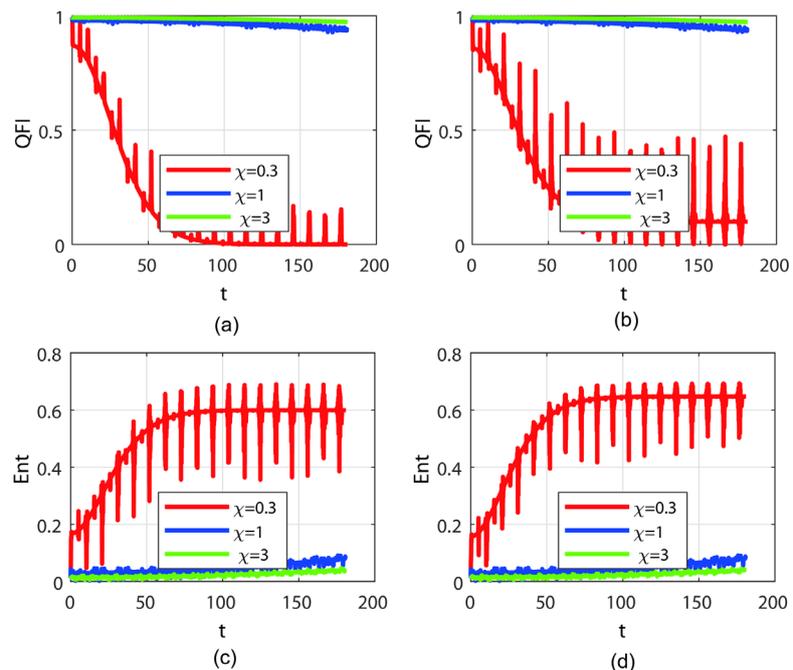


Figure 5. (Color online) The QFI (upper panel) and VNE (lower panel) as a function of time for a system of a two-level atom having interaction with a coherent field for $|\alpha|^2 = 6$ and the phase shift estimator parameter $\phi = 0$ (left panel) and $\pi/4$ (right panel). The parameter η of atomic motion is ignored and the value of $\chi = 0.3, 1, 3$ (Non-linear Kerr). (a) $\phi = 0$, 2-level; (b) $\phi = \pi/4$, 2-level; (c) $\phi = 0$, 2-level; (d) $\phi = \pi/4$, 2-level.

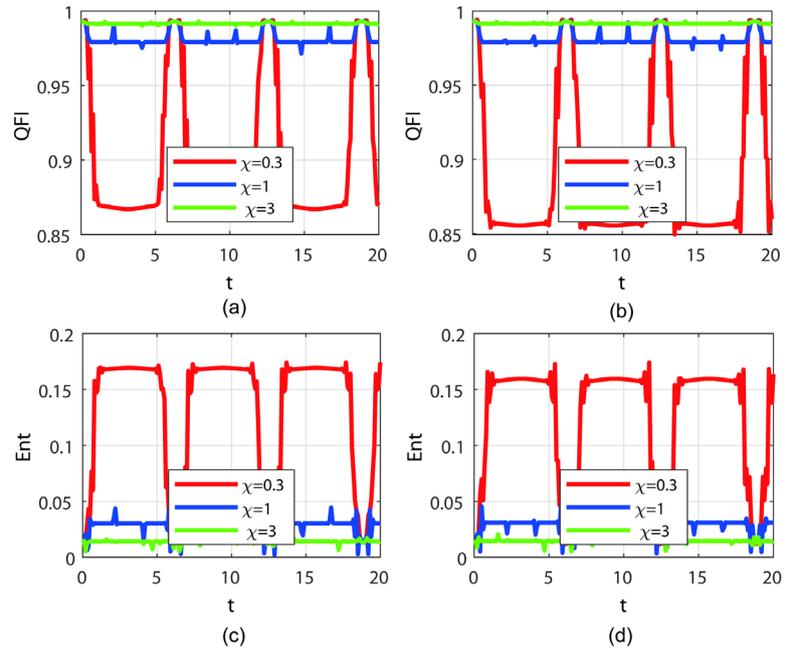


Figure 6. (Color online) The QFI (upper panel) and VNE (lower panel) as a function of time for a system of a two-level atom having interaction with a coherent field for $|\alpha|^2 = 6$ and the phase shift estimator parameter $\phi = 0$ (left panel) and $\pi/4$ (right panel). The parameter η of atomic motion is $\eta = 1$ and the value of $\chi = 0, 1, 3$ (Non-linear Kerr). (a) $\phi = 0$, 2-level; (b) $\phi = \pi/4$, 2-level; (c) $\phi = 0$, 2-level; (d) $\phi = \pi/4$, 2-level.

effect of the non-linear Kerr medium is found to be more prominent on the QE as compared to the QFI in the absence of atomic motion. It is seen that the non-linear Kerr medium plays a dominant role during the time evolution of the quantum system. The periodic behavior of QFI and QE is further suppressed under the effect of the non-linear Kerr medium. These results show the strong dependence of QFI and QE on the non-linear Kerr medium. It is seen that at $\chi = 0.3$ QFI decreases and VNE increases but this increasing and decreasing response becomes saturated as time evolves. At $\chi = 1, 3$, VNE decreases, but this decrease becomes saturated as time evolves. Hence, it is concluded that at higher values of the Kerr parameter, the QE decreases when compared with the smaller values. However, the non-linear Kerr medium has no prominent effect on QFI at higher and lower values of the Kerr parameter. In the presence of atomic motion, both QFI and VNE show periodic behavior.

In **Figures 7-10** we plot QFI and VNE as a function of time for the two-level atomic system interacting with coherent field for $|\alpha|^2 = 6$ and the phase shift parameter $\phi = 0$ (left panel) and $\pi/4$ (right panel) for different values of $\beta = 0.3, 1, 3$ (Stark shift) with and without atomic motion, *i.e.* $\eta = 0, 1$ respectively. The Stark effect is found to influence the QE of the two-level atomic system very strongly. Moreover, it has been seen that QFI and QE decay at the smaller values of the Stark parameter ($\beta = 0.3$) but as the value of the stark parameter is increased ($\beta = 1, 3$), the QFI and QE show periodic behaviour even

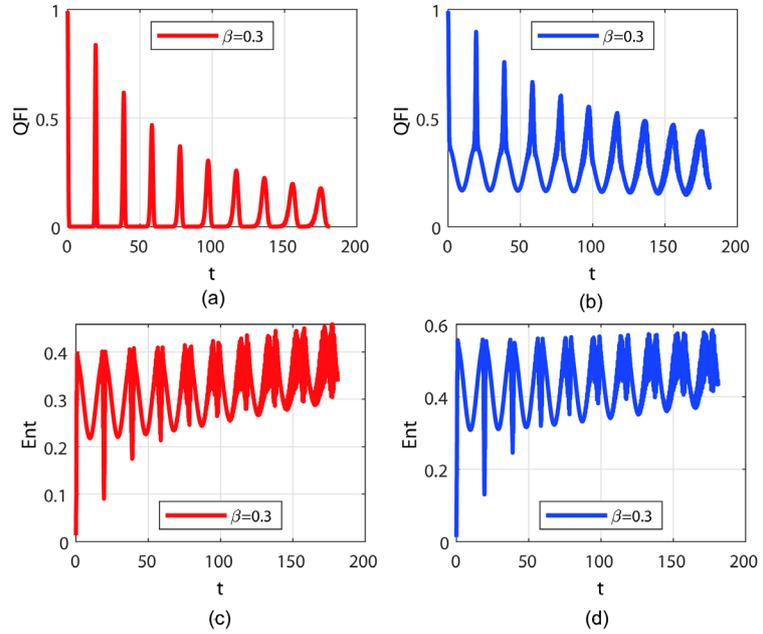


Figure 7. (Color online) The QFI (upper panel) and VNE (lower panel) as a function of time for a system of a two-level atom having interaction with a coherent field for $|\alpha|^2 = 6$ and the phase shift estimator parameter $\phi = 0$ (left panel) and $\pi/4$ (right panel). The parameter η of atomic motion is ignored and the value of $\beta = 0.3$ (Stark effect). (a) $\phi = 0$, 2-level; (b) $\phi = \pi/4$, 2-level; (c) $\phi = 0$, 2-level; (d) $\phi = \pi/4$, 2-level.

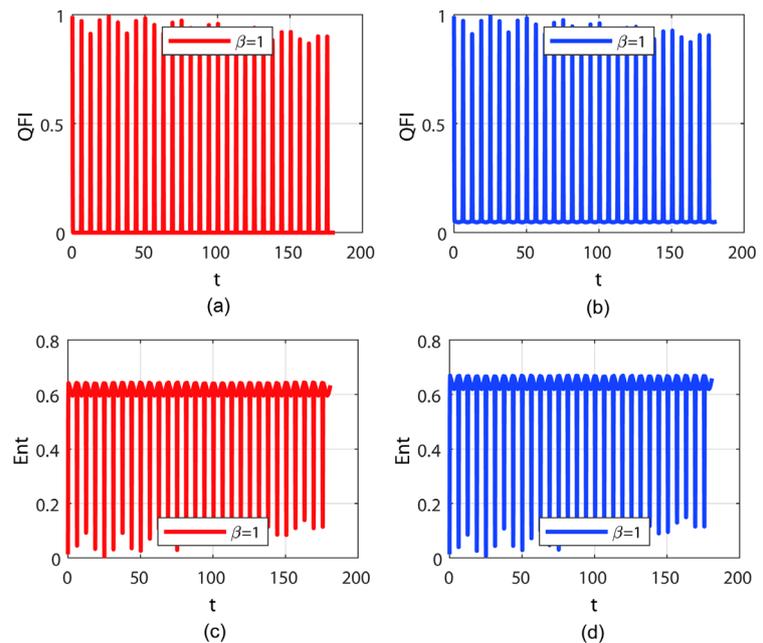


Figure 8. (Color online) The QFI (upper panel) and VNE (lower panel) as a function of time for a system of a two-level atom having interaction with a coherent field for $|\alpha|^2 = 6$ and the phase shift estimator parameter $\phi = 0$ (left panel) and $\pi/4$ (right panel). The parameter η of atomic motion is ignored and the value of $\beta = 1$ (Stark effect). (a) $\phi = 0$, 2-level; (b) $\phi = \pi/4$, 2-level; (c) $\phi = 0$, 2-level; (d) $\phi = \pi/4$, 2-level.

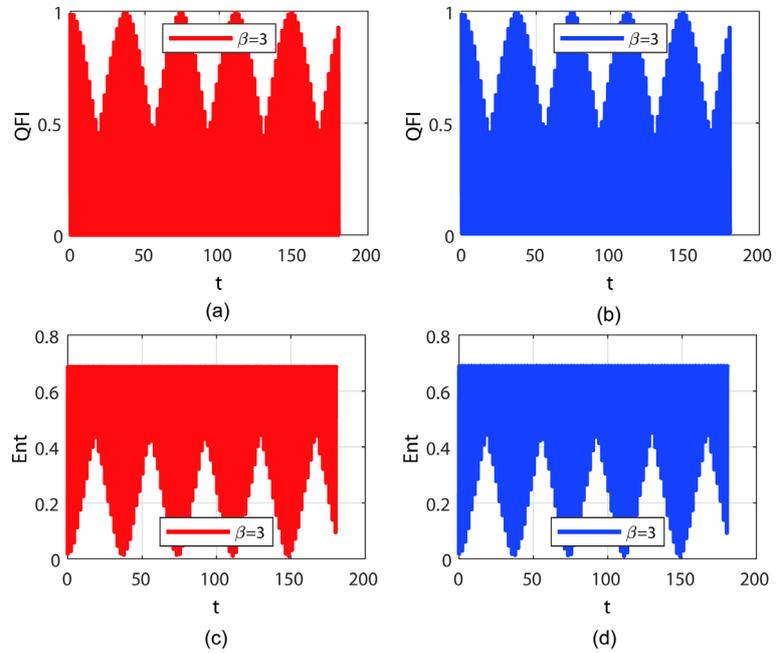


Figure 9. (Color online) The QFI (upper panel) and VNE (lower panel) as a function of time for a system of a two-level atom having interaction with a coherent field for $|\alpha|^2 = 6$ and the phase shift estimator parameter $\phi = 0$ (left panel) and $\pi/4$ (right panel). The parameter η of atomic motion is ignored and the value of $\beta = 3$ (Stark effect). (a) $\phi = 0$, 2-level; (b) $\phi = \pi/4$, 2-level; (c) $\phi = 0$, 2-level; (d) $\phi = \pi/4$, 2-level.

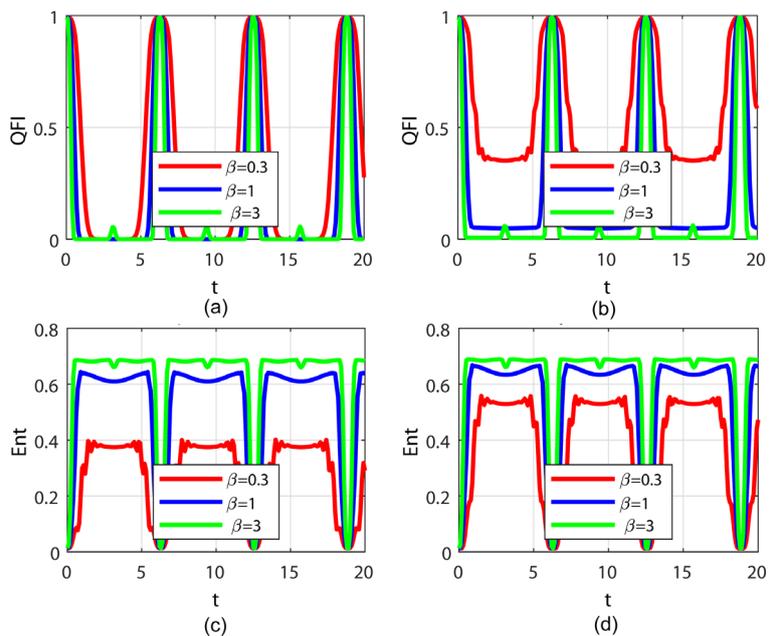


Figure 10. (Color online) The QFI (upper panel) and VNE (lower panel) as a function of time for a system of a two-level atom having interaction with a coherent field for $|\alpha|^2 = 6$ and the phase shift estimator parameter $\phi = 0$ (left panel) and $\pi/4$ (right panel). The parameter η of atomic motion is $\eta = 1$ value of $\beta = 0.3, 1, 3$ (Stark effect). (a) $\phi = 0$, 2-level; (b) $\phi = \pi/4$, 2-level; (c) $\phi = 0$, 2-level; (d) $\phi = \pi/4$, 2-level.

when the atomic motion is not present. Therefore, at higher values of β , the QE is sustained but at lower values of β , it decreases. In the presence of atomic motion, QFI and VNE show a periodic response at different β values, so increasing the value of β in the presence of atomic motion does not affect the QE. Finally, the quantum system is found highly sensitive to these environmental influences.

6. Conclusions

In summary, we have studied the dynamical evolution of QE and QFI for moving two-level atom interacting with the thermal field in the presence of ID. The time evolution of the entire system interacting with a thermal environment in the presence of ID is investigated numerically. It is seen that ID and thermal environment play an important and critical role in the time evolution of the quantum system. QFI and QE exhibit an opposite behaviour in a thermal environment in the presence of ID. It is observed that both QFI and QE are sensitive to ID in the presence of the thermal environment. It has been observed that QE drastically decreases when we increase the value of ID in the thermal field in the absence of atomic motion. The damping behaviour of QE is observed at larger time-scales at larger values of the ID parameter in the thermal field. The periodic behaviour of QE due to atomic motion becomes modest under the environmental effects. The ID and thermal environment are found to suppress the nonclassical effects of the quantum system. However, QE and QFI saturate to a lower level for longer time-scales under these environments. It ought to be mentioned that in the thermal environment decaying of the QE is faster at the larger values of ID. The results of our numerical calculations show that the two-level atomic system is more robust against the variations in the chosen parameters. Finally, it is revealed that the presented system can be useful for generating and maintaining QE in the presence of these environments in the presence of atomic motion. But in the case without atomic motion, the decay of QE is investigated under the influence of the thermal field and ID.

We have investigated the dynamics of QE and QFI for a two-level atomic system under the influence of the Stark shift and Kerr medium. The time evolution of QFI and QE for a two-level atomic system influenced by the Stark effect and Kerr-like medium is investigated. It is seen that Stark and Kerr's medium play a dominant role during the time evolution of the quantum system. The effect of non-linear Kerr medium is more prominent on the QFI and VNE at the smaller value of the Kerr parameter. As the value of the Kerr parameter is increased, the effect of non-linear Kerr does not remain significant on both QFI and QE. Similarly, the Stark effect strongly influences the QE of a two-level atomic system. Furthermore, it has been observed that QFI and QE decay soon at the smaller values of the Stark parameter but as the value of the Stark parameter is increased, the QFI and QE show periodic behaviour even when the atomic motion is not present. So it is concluded that at larger values of the Stark parameter, the QE is maintained and sustained. QE and QFI exhibit opposite and periodic

behaviour in the presence of atomic motion in case of linear, non-linear Kerr medium and stark effect. Finally, the quantum system is found highly sensitive to these environmental influences.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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