

New Exact Solutions for Benjamin-Bona-Mahony-Burgers Equation

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Abstract

Obtaining the new solutions for the nonlinear evolution equation is a hot topic. Benjamin-Bona-Mahony-Burgers equation is this kind of equation, the solutions are very interest. Several new exact solutions for the nonlinear equation are obtained by using truncated expansion method in this paper. The numerical simulations with different parameters for the new exact solutions of Benjamin-Bona-Mahony-Burgers equation are given.

Keywords

Truncated Expansion Method, Benjamin-Bona-Mahony-Burgers Equation, Evolution, Nonlinear Equation

1. Introduction

Nonlinear partial differential equations are very important in the natural world; the solutions are the important way for us to know the nature. However, the different initial condition means different solutions exit. People had found out many powerful methods to obtain the solutions for nonlinear partial differential equations. Such as the useful methods-Soliton-Like Solutions, complex travelling wave, truncated expansion method, hyperbolic tangent method [1] [2] [3] [4] [5], high-order multi-symplectic schemes, simple fast method, Invariant-conserving finite difference algorithms, stable bound states [6]-[11], which help us deeply study the relation of the nature.

The Nonlinear Benjamin-Bona-Mahony-Burgers equation was proposed by Peregrine [12] and Benjamin [13], long waves on the surface of water in a channel with small-amplitude can be described by the nonlinear equation. The form of the Benjamin-Bona-Mahony-Burgers equation [14] is read as

$$u_t + u_x + uu_x - mu_{xx} - u_{xxt} = 0 \quad (1)$$

where the subscripts denote the partial derivatives of position x and time t . $u(x, t)$ is a real-valued function and u_{xx} is considered as dissipative term. The following structure of this work is organized as follows: Section 2 is a brief introduction to the truncated expansion method and its Properties. In Section 3, applying the truncated expansion method, some new exact wave solutions for Nonlinear Benjamin-Bona-Mahony-Burgers equation are given. The conclusion is summarized in the final.

2. The Truncated Expansion Method and Its Properties

The nonlinear partial differential equation with independent variables position x and time t is generally in the following form

$$Q(u, u_x, u_t, u_{xt}, u_{xx}, u_{tt}, \dots) = 0 \quad (2)$$

The above equation is a function about $u(x, t)$ variable with position x and time t , the subscripts denote the partial derivatives with x and t , respectively. The wave variable $\xi = ax + bt$ is applied to the Equation (2), which is changed into the following ordinary differential equation

$$Q(u, u_\xi, u_{\xi\xi}, u_{\xi\xi\xi}, \dots) = 0 \quad (3)$$

where $u_\xi, u_{\xi\xi}, \dots$ denotes the derivative with respect to the same variable ξ .

Generally, the function $u(x, t)$ in terms of truncated expansion method [15] [16] can be expressed as follow

$$u(\epsilon) = \sum_{n=0}^N A_n \varphi^n(\epsilon, t) \quad (4)$$

where A_n is constant parameter; N is determined by balancing the linear term of the highest order derivative with nonlinear term of Equation (3).

The function $\varphi(\epsilon, t)$ has the nature

$$\varphi_\epsilon = \lambda + \varphi^2 \quad (5)$$

where the λ is a constant. The second and third derivative of φ are read as the follow

$$\varphi_{\epsilon\epsilon} = 2\lambda\varphi + 2\varphi^3 \quad (6)$$

$$\varphi_{\epsilon\epsilon\epsilon} = 2\lambda^2 + 8\lambda\varphi^2 + 6\varphi^4 \quad (7)$$

The function $\varphi(\epsilon, t)$ with the form of Equation (5) has the follow form with different λ

$$\varphi = -(-\lambda)^{\frac{1}{2}} \tanh\left[(-\lambda)^{\frac{1}{2}} \epsilon\right], \quad \lambda < 0 \quad (8)$$

$$\varphi = -(-\lambda)^{\frac{1}{2}} \coth\left[(-\lambda)^{\frac{1}{2}} \epsilon\right], \quad \lambda < 0 \quad (9)$$

$$\varphi = -\frac{1}{\epsilon}, \quad \lambda = 0 \quad (10)$$

$$\varphi = -(\lambda)^{\frac{1}{2}} \tan \left[(\lambda)^{\frac{1}{2}} \epsilon \right], \quad \lambda > 0 \quad (11)$$

$$\varphi = -(\lambda)^{\frac{1}{2}} \cot \left[(\lambda)^{\frac{1}{2}} \epsilon \right], \quad \lambda > 0 \quad (12)$$

3. The Truncated Expansion Method for Benjamin-Bona-Mahony-Burgers Equation

Consider the Nonlinear Benjamin-Bona-Mahony-Burgers equation in the form of Equation (1) with $\epsilon = ax + bt$, we obtain

$$bu_{\epsilon} + au_{\epsilon} + auu_{\epsilon} - ma^2u_{\epsilon\epsilon} - a^2bu_{\epsilon\epsilon\epsilon} = 0 \quad (13)$$

By using N to balance the highest order derivative term and the nonlinear term of Equation (13), that means uu_{ϵ} and $u_{\epsilon\epsilon\epsilon}$ are the nonlinear term and the highest order derivative, $2N+1 = N+3$, we have $N=2$. Then, Equation (4) reduced as

$$u(\epsilon) = A_0 + A_1\varphi + A_2\varphi^2 \quad (14)$$

Substituting Equation (14) into the equation Equation (13) and collecting all terms with the same power term together and equating each coefficient of φ to zero, a set of simultaneous algebraic equations are yielded as follows:

$$a\lambda A_1 + aA_0\lambda A_1 + \lambda A_1b - 2a^2\lambda^2 A_1b - 2ma^2 A_2\lambda^2 = 0 \quad (15)$$

$$a\lambda A_1^2 + 2a\lambda A_2 + 2a\lambda A_0A_2 + 2\lambda A_2b - 16ba^2 A_2\lambda^2 - 2ma^2 A_1\lambda = 0 \quad (16)$$

$$aA_1 + aA_0A_1 + 3a\lambda A_1A_2 + A_1b - 8ba^2 A_1\lambda - 8ma^2 A_2\lambda = 0 \quad (17)$$

$$aA_1^2 + 2aA_2 + 2aA_0A_2 + 2\lambda A_2^2a + 2bA_2 - 2ma^2 A_1 - 40ba^2 A_2\lambda = 0 \quad (18)$$

$$3aA_1A_2 - 6a^2 A_1b - 6ma^2 A_2 = 0 \quad (19)$$

$$2A_2^2 - 24abA_2 = 0 \quad (20)$$

Solving the above algebraic equations, we get the results:

$$m = \frac{\sqrt{24b^2 + 24ab - 100\lambda a^2 b^2}}{a}; \quad A_0 = \frac{200\lambda a^2 b^2 - a^2 m^2 - ab - b^2}{25ab}; \quad (21)$$

$$A_1 = \frac{12am}{5}; \quad A_2 = 12ab$$

By using Equations (8)-(10) and (14), we obtained the solution of Equation (1) as the follow:

$$u(x, t)_1 = \frac{200\lambda a^2 b^2 - a^2 m^2 - ab - b^2}{25ab} - \frac{12am}{5} (-\lambda)^{\frac{1}{2}} \tanh \left[(-\lambda)^{\frac{1}{2}} (ax + bt) \right] + 12ab \left((-\lambda)^{\frac{1}{2}} \tanh \left[(-\lambda)^{\frac{1}{2}} (ax + bt) \right] \right)^2, \quad \lambda < 0 \quad (22)$$

$$u(x, t)_2 = \frac{200\lambda a^2 b^2 - a^2 m^2 - ab - b^2}{25ab} - \frac{12am}{5} (-\lambda)^{\frac{1}{2}} \coth \left[(-\lambda)^{\frac{1}{2}} (ax + bt) \right] + 12ab \left((-\lambda)^{\frac{1}{2}} \coth \left[(-\lambda)^{\frac{1}{2}} (ax + bt) \right] \right)^2, \quad \lambda < 0 \quad (23)$$

$$u(x, t)_3 = \frac{200\lambda a^2 b^2 - a^2 m^2 - ab - b^2}{25ab} - \frac{12am}{5(ax + bt)} + 12ab \left(\frac{-1}{ax + bt} \right)^2, \quad \lambda = 0 \quad (24)$$

$$u(x, t)_4 = \frac{200\lambda a^2 b^2 - a^2 m^2 - ab - b^2}{25ab} + \frac{12am}{5} (\lambda)^{\frac{1}{2}} \tan \left[(\lambda)^{\frac{1}{2}} \epsilon \right] + 12ab \left((\lambda)^{\frac{1}{2}} \tan \left[(\lambda)^{\frac{1}{2}} \epsilon \right] \right)^2, \quad \lambda > 0 \quad (25)$$

$$u(x, t)_5 = \frac{200\lambda a^2 b^2 - a^2 m^2 - ab - b^2}{25ab} - \frac{12am}{5} (\lambda)^{\frac{1}{2}} \cot \left[(\lambda)^{\frac{1}{2}} \epsilon \right] + 12ab \left(-(\lambda)^{\frac{1}{2}} \cot \left[(\lambda)^{\frac{1}{2}} \epsilon \right] \right)^2, \quad \lambda > 0 \quad (26)$$

The rest section is the numerical simulation for the above sol are determined randomly in the corresponding interval. The contour plot of numerical simulation for Equation (22) with $a = 1$; $b = 0.5$; $\lambda = -2$; $x \in [-5, 5]$ and $t \in [-5, 5]$ is shown in **Figure 1**.

The contour plot of numerical simulation for Equation (23) with $a = 1$; $b = 0.5$; $\lambda = -1$; $x \in [3, 4]$ and $t \in [3, 4]$ is shown in **Figure 2**.

The contour plot of numerical simulation for Equation (24) with $a = 1$; $b = 0.5$; $x \in [1, 2]$ and $t \in [1, 2]$ is shown in **Figure 3**.

The contour plot of numerical simulation for Equation (25) with $a = 2$; $b = 6$; $\lambda = 0.05$; $x \in [-1, 1]$ and $t \in [-1, 1]$ is shown in **Figure 4**.

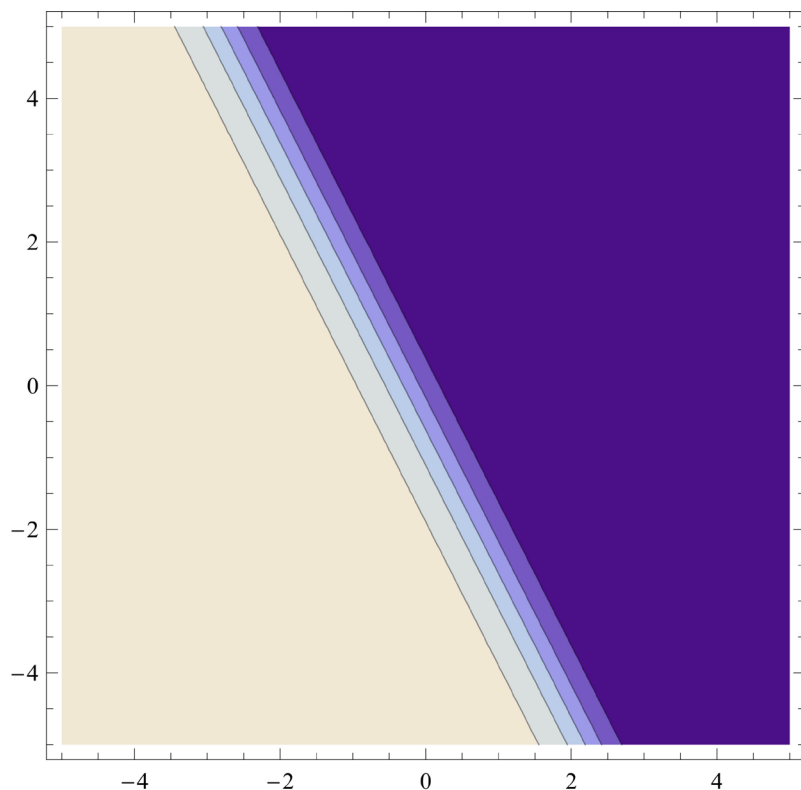


Figure 1. The numerical simulation for Equation (22) with $x \in [-5, 5]$ and $t \in [-5, 5]$.

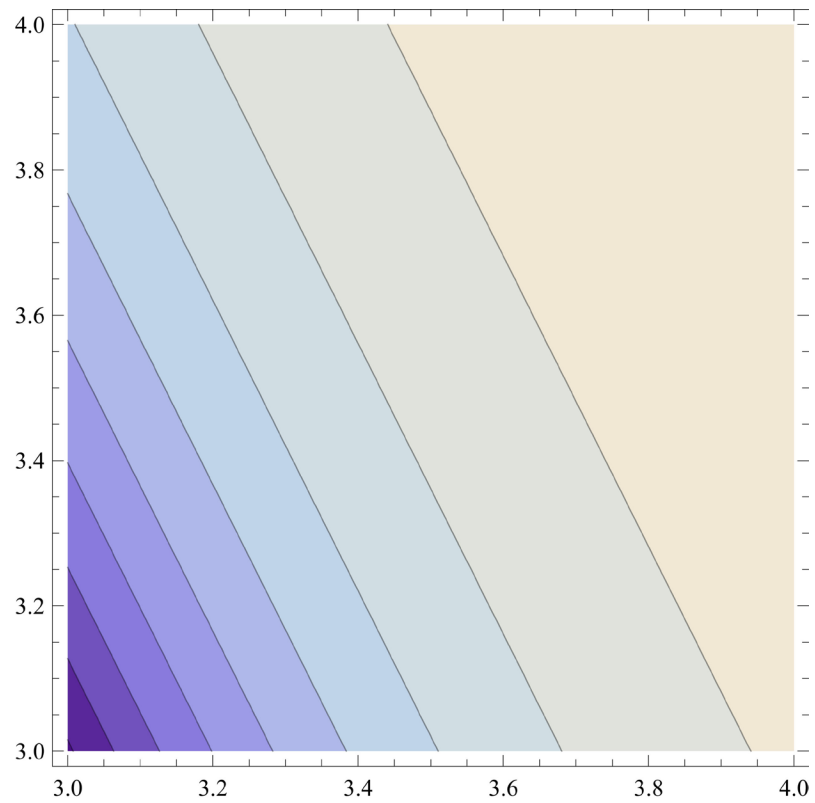


Figure 2. The numerical simulation for Equation (23) with $x \in [3, 4]$ and $t \in [3, 4]$.

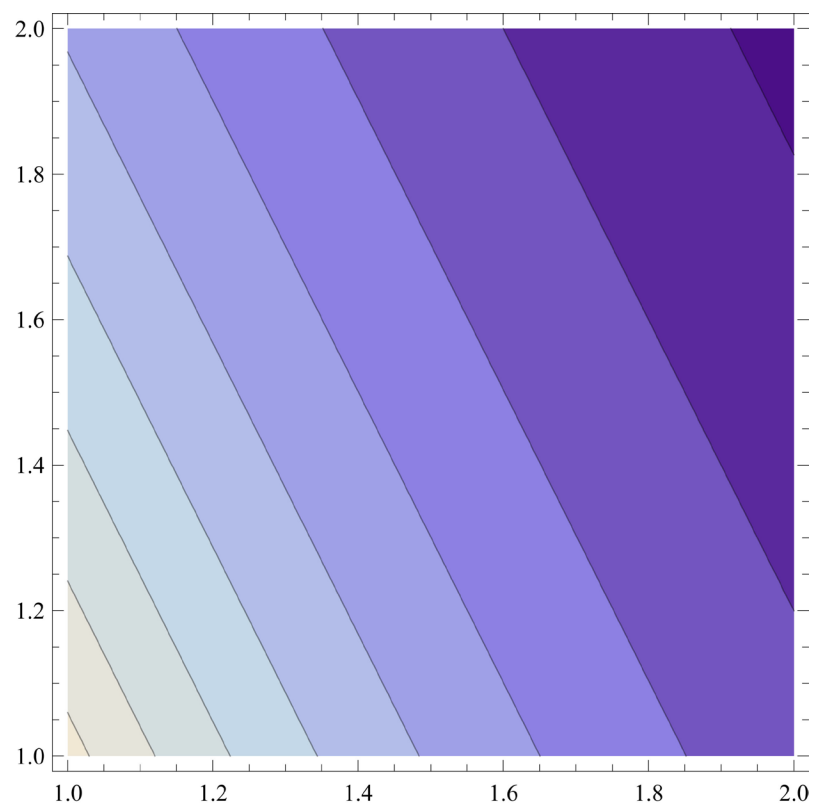


Figure 3. The numerical simulation for Equation (24) with $x \in [1, 2]$ and $t \in [1, 2]$.

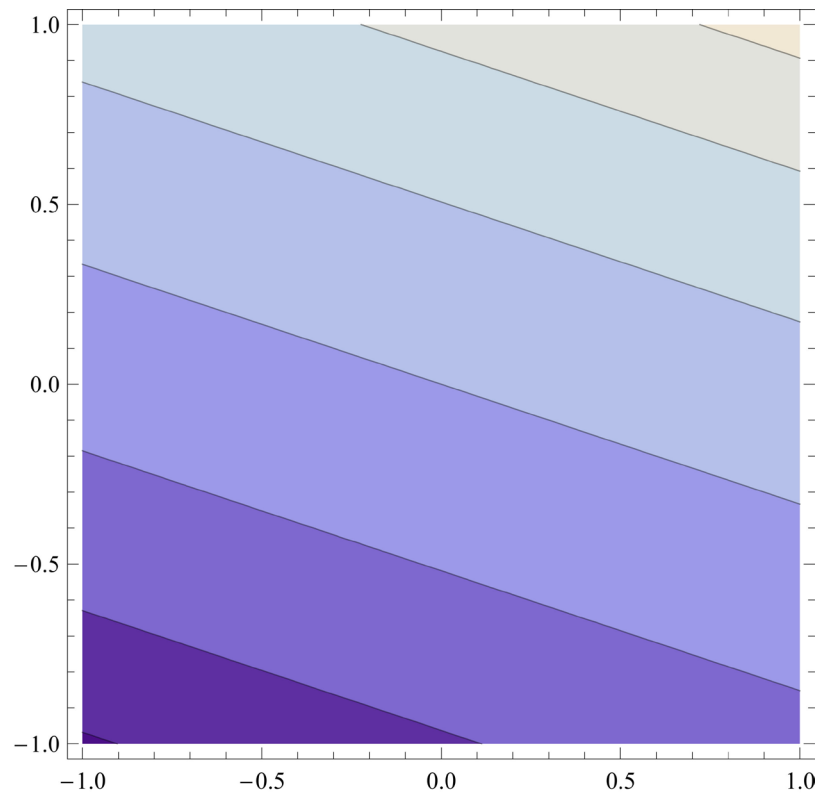


Figure 4. The numerical simulation for Equation (25) with $x \in [-1, 1]$ and $t \in [-1, 1]$.

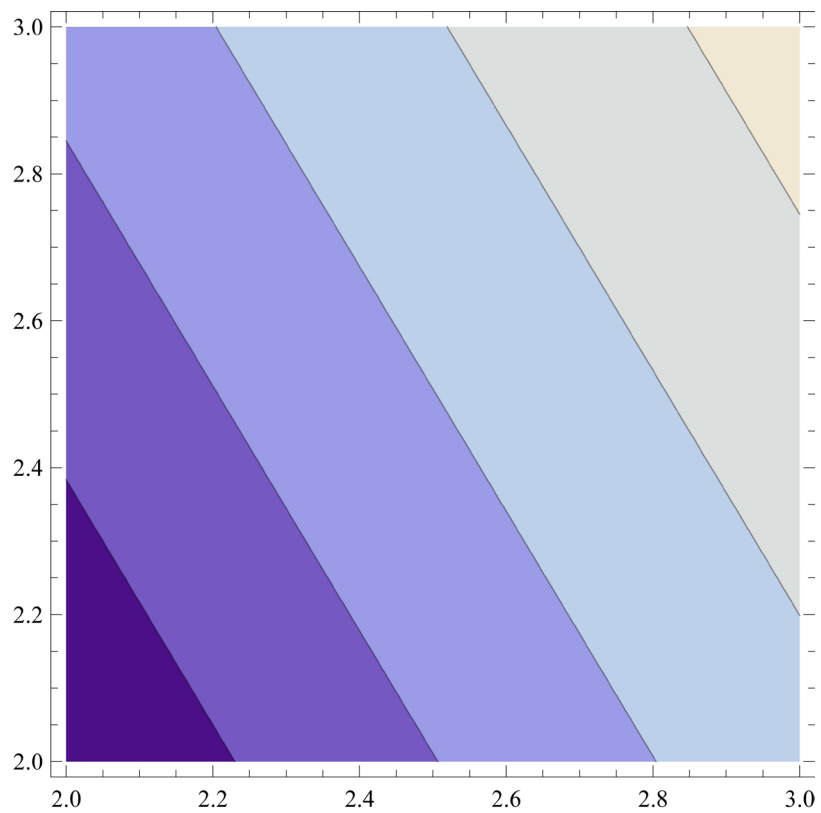


Figure 5. The numerical simulation for Equation (26) $x \in [2, 3]$ and $t \in [2, 3]$.

The contour plot of numerical simulation for Equation (26) with $a = 10$; $b = 6$; $\lambda = 0.01$; $x \in [2, 3]$ and $t \in [2, 3]$ is shown in **Figure 5**.

4. Conclusion

The Nonlinear Benjamin-Bona-Mahony-Burgers equation in the given form Equation (1) have been further studied, some new exact solutions for the equation are obtained by means of the truncated expansion method, which are shown in Equations (22)-(26). The numerical simulation results with contour plot are appended with **Figures 1-5**. The evolutions of the travelling wave are clear shown.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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