

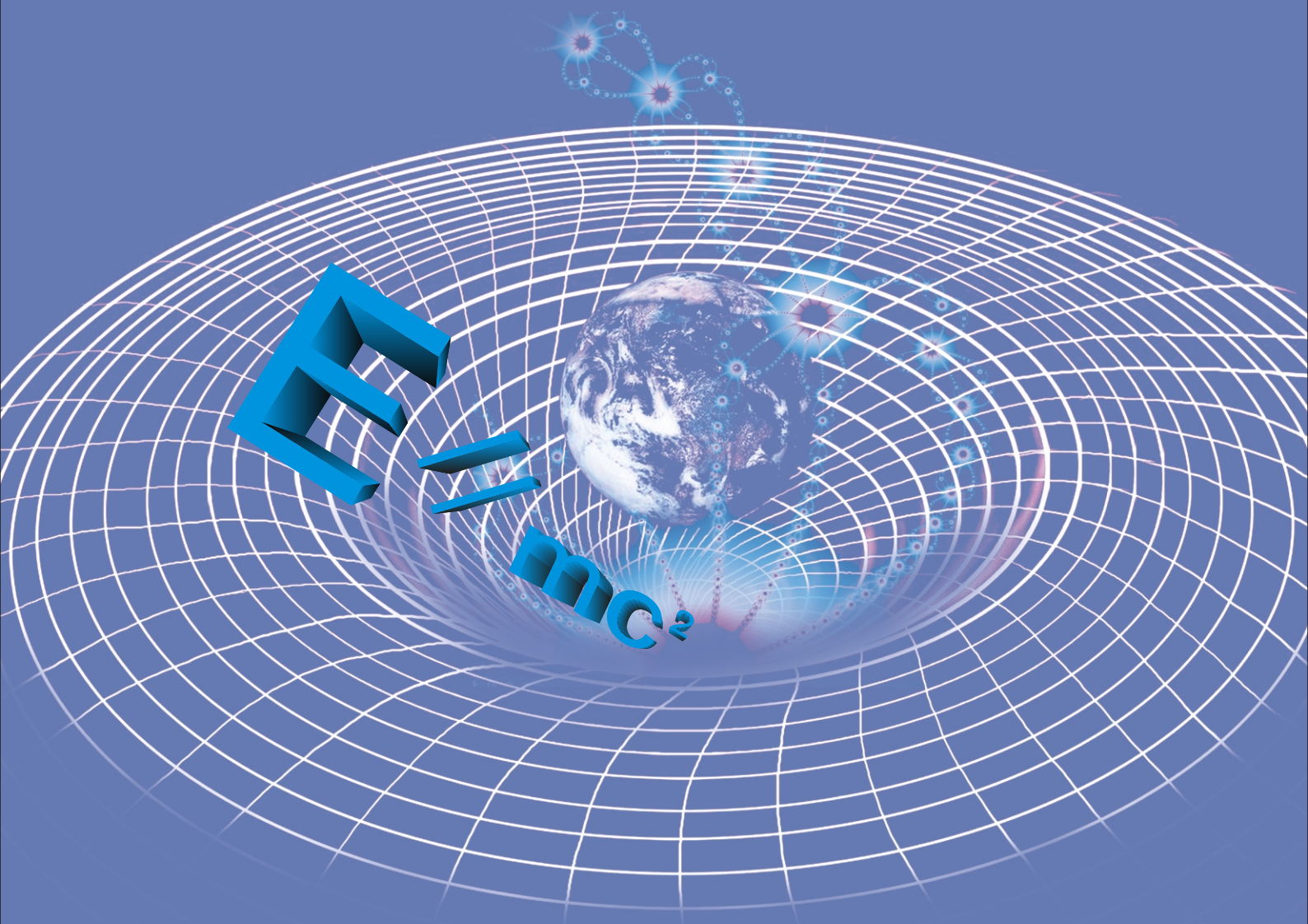
ISSN: 2153-1196 (Print), 2153-120X (Online)

Vol.1, No.1, April 2010



Scientific
Research

Journal of Modern Physics



www.scirp.org/journal/jmp

Journal Editorial Board

ISSN: 2153-1196 (Print) ISSN: 2153-120X (Online)

<http://www.scirp.org/journal/jmp>

Editor-in-Chief

Dr. Marko Markov

Erie Community College, USA

Editorial Board

Prof. Sadhan Kumar Adhikari

Institute of Theoretical Physics, Brazil

Prof. Sami M. AL-Jaber

AN-Najah National University, Palestine

Dr. Ksenofontov Alexandre

Moscow Engineering Physics Institute, Russia

Prof. Roberto Oscar Aquilano

Rosario Physics Institute, Argentina

Prof. Salvatore Capozziello

University of Naples “Federico II, Italy

Dr. Riccardo Cerulli

Gran Sasso National Laboratory, INFN, Italy

Prof. Papadopoulos Demetrios

Aristotle University of Thessaloniki, Greece

Prof. Constantin Fetecau

“Gh. Asachi” Technical University of Iasi, Romania

Prof. Roman Kezerashvili

The City University of New York, USA

Prof. Bouzid Menaa

Fluorotronics, Inc., CA, USA

Prof. Karo Michaelian

National Autonomous University of Mexico, Mexico

Prof. Zdzislaw E. Musielak

The University of Texas at Arlington, USA

Prof. Luciano Nunziante

University of Napoli Federico II, Italy

Prof. Sergey Dmitrievich Odintsov

Space Research Institute(ICE) of CSIC-IEEC, Spain

Prof. Jingli Ren

Zhengzhou University, China

Prof. Alexandre I. Rykov

The University of Tokyo, Japan

Prof. Mohindar Singh Seehra

West Virginia University, USA

Prof. Er-Chang Shang

Chinese Academy of Science, China

Prof. Gabriela Slavcheva

Imperial College London, UK

Dr. Raghvendra Singh Yadav

University of Allahabad, India

Prof. Victor Yashnikov

Russian Academy of Sciences, Russia

Prof. A. Zerarka

Academy of Science, New York., Algeria

Dr. S. Zerbini

University of Trento, Italy

Editorial Assistant

Xuefang Xiong

Scientific Research Publishing

Email: jmp@scirp.org

Xiaoxue Li

Scientific Research Publishing

Email: jmp@scirp.org

TABLE OF CONTENTS

Volume 1 Number 1

April 2010

Plastic Flow Macrolocalization: Autowave and Quasi-Particle

L. B. Zuev, S. A. Barannikova.....1

Translational Motion of a Free Large Polaron and Broadening of Absorption Spectra

V. Mukhomorov.....9

On the Nature of Dark Matter and Dark Energy

Baurov Yu. A., Malov I. F.17

Studying Magnetization Distribution in Magnetic Thin Films under Transversal Application of Magnetic Fields

A. A. Zakharenko.....33

Computation of the Schrödinger Equation via the Discrete Derivatives Representation Method: Improvement of Solutions Using Particle Swarm Optimization

A. Zerarka, H. Saidi, A. Attaf, N. Khelil.....44

Fracture and Damage Behaviors of Concrete in the Fractal Space

H. Zhang, D. M. Wei.....48

Supertranslations to All Orders

R. Dick.....59

Tilted Bianchi Type VI₀ Cosmological Model in Saez and Ballester Scalar Tensor Theory of Gravitation

S. K. Sahu.....67

The Possible Dimension, Additional to Space-Time, which Physicists Ignore

A. S. Issar, S. Sorek.....70

Instability of Thermally Conducting Self-Gravitating Systems

S. Shaikh, A. Khan.....77

Measurement of Relative Metastable Level Population of Gd Atoms in Hollow Cathode Lamp with LIF Method

S. H. Nabavi, A. Koohian.....83

Bohr Correspondence Principle and Multiphoton Nature Raleigh Light Scattering

V. E. Ogluzdin.....86

The Double-Slit Experiment and Particle-Wave Duality: Toward a Novel Quantum Interpretation

I. Orion, M. Laitman.....90

Journal of Modern Physics (JMP)

Journal Information

SUBSCRIPTIONS

The *Journal of Modern Physics* (Online at Scientific Research Publishing, www.SciRP.org) is published bimonthly by Scientific Research Publishing, Inc., USA.

Subscription rates:

Print: \$50 per issue.

To subscribe, please contact Journals Subscriptions Department, E-mail: sub@scirp.org

SERVICES

Advertisements

Advertisement Sales Department, E-mail: service@scirp.org

Reprints (minimum quantity 100 copies)

Reprints Co-ordinator, Scientific Research Publishing, Inc., USA.

E-mail: sub@scirp.org

COPYRIGHT

Copyright©2010 Scientific Research Publishing, Inc.

All Rights Reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except as described below, without the permission in writing of the Publisher.

Copying of articles is not permitted except for personal and internal use, to the extent permitted by national copyright law, or under the terms of a license issued by the national Reproduction Rights Organization.

Requests for permission for other kinds of copying, such as copying for general distribution, for advertising or promotional purposes, for creating new collective works or for resale, and other enquiries should be addressed to the Publisher.

Statements and opinions expressed in the articles and communications are those of the individual contributors and not the statements and opinion of Scientific Research Publishing, Inc. We assumes no responsibility or liability for any damage or injury to persons or property arising out of the use of any materials, instructions, methods or ideas contained herein. We expressly disclaim any implied warranties of merchantability or fitness for a particular purpose. If expert assistance is required, the services of a competent professional person should be sought.

PRODUCTION INFORMATION

For manuscripts that have been accepted for publication, please contact:

E-mail: jmp@scirp.org

Plastic Flow Macrolocalization: Autowave and Quasi-Particle

Lev B. Zuev, Svetlana A. Barannikova

Institute of Strength Physics and Materials Science, Tomsk, Russia

E-mail: lbz@ispms.tsc.ru

Received March 2, 2010; revised March 26, 2010; accepted April 7, 2010

Abstract

A new approach is proposed to describe the autowave processes responsible for plastic deformation localization in metals and alloys. The existence of a quasi-particle, which corresponds to a localized plastic flow autowave, is postulated and its characteristics are determined. The above postulate leads to a number of corollaries and quantitative assessments that are considered herein. The deformation processes occurring on the macro- and micro-scale levels are found to be directly related.

Keywords: Localization, Plastic Deformation, Autowave, Quasi-Particle

1. Introduction

The experimental studies of plastic flow in solids carried out during last decades [1,2] allow one to throw light upon the plasticity phenomenon and to detect the most important experimental fact that the plastic flow would exhibit a localization behavior from yield point to failure. Recently strong experimental evidences for the above viewpoint were presented independently by several workers [3-5]. In order to visualize the localization patterns observed by mechanical testing, a technique of double-exposure speckle-photography was developed [6].

Some of the observed localization patterns are demonstrated in **Figure 1(a)**. As is seen from **Figure 1(b)**, these have very complex structure characterized by a typical macroscopic scale of about 10^{-2} m. This suggests that the deforming medium becomes spontaneously stratified into macroscopic layers, with deforming (active) layers alternating with non-deforming (passive) ones. In a general case, the boundaries between such layers are mobile; therefore, the process of plastic flow is conventionally considered as evolution of localized plastic flow patterns.

The phenomenology and quantitative characteristics of the localization effect have been fully elucidated by now. Thus a detailed investigation of space-time periodic localization patterns [1,2] allows one to refer the localization phenomenon to self-organization processes. The above assumption is only valid provided the term ‘self-organization’ is taken to imply, according to Haken [7], that the system acquires spatial, temporal or functional

structure in the absence of any specific periodic external action. Of major importance is the finding that the localized plastic flow patterns have all the particular features of autowave (self-excited) process. This comes into particular prominence at the linear stage of deformation hardening as the plastic flow localization takes on the form of phase autowave, which has length $\lambda \approx 10^{-2}$ m and propagation rate $10^{-5} \leq V_{aw} \leq 10^{-4} \text{ m} \cdot \text{s}^{-1}$. The autowaves in question are distinct from the well-known plastic deformation waves that are generated in solids under shock loading which are described by Kolsky in [8].

However, the nature of localized plastic flow phenomena is poorly understood so that the challenge of interpreting rich experimental evidences on plastic flow macrolocalization can be daunting. To accomplish this demanding task, a new model of plastic flow localization is proposed herein.

2. On the Observation of Localization Phenomena

As noted above, the experimental observation of localized plastic flow autowaves was carried on with help of a specially developed speckle photography technique related to focused-image holography [6]. The method developed makes feasible the experimental determination of displacement vector fields and the calculation of plastic distortion tensor components for the deforming specimen. A vast array of wavelength and propagation rate data has been acquired and stored digitally in a computer.

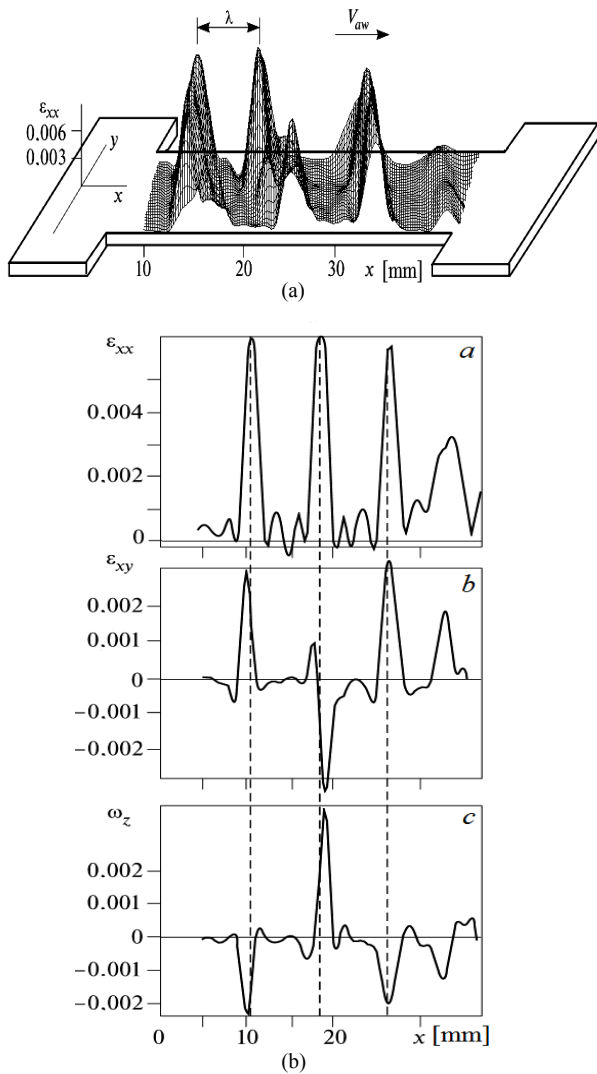


Figure 1. (a) A typical example of localized plastic flow autowave generated at the linear work hardening stage in the single crystal of alloyed γ -Fe; (b) ε_{xx} - local elongation; x and y - specimen length and width, respectively; λ - nucleus spacing (autowave length); V_{aw} - autowave propagation rate; the distributions of the plastic distortion tensor components, ε_{xx} , ε_{xy} and ω_z , within the localization zone in the single crystal of alloyed γ -Fe.

The spatial distributions of plastic distortion tensor components can be used to locate localized plastic flow nuclei; the kinetic characteristics of the nuclei can be determined from the temporal evolution thereof. The characteristics of autowaves are defined as follows. First the spatial period (length of autowave, λ) is determined from the variation in the space co-ordinates of localization nuclei with time (see Figure 2); then the time of variation, T , is defined. Hence the phase rate of autowave propagation is given as $V_{aw} = \lambda/T = \omega/k$ (here $\omega = 2\pi/T$ is the frequency and $k = 2\pi/\lambda$ is the wave number).

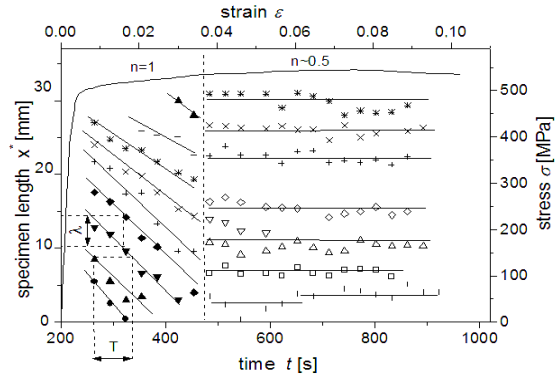


Figure 2. The spatial (λ) and temporal (T) periods of localized plastic deformation as determined for the single crystals of alloyed γ -Fe for $n = 1$ and $n \sim 0.5$ (linear and parabolic work hardening stages, respectively); $\sigma(\varepsilon)$ - stress-strain dependence; $X(t)$ - variation in the localization nucleus co-ordinates with time ($\bullet$; $\blacktriangle$; $\blacktriangledown$; $\blacksquare$; $+$; $\times$; $*$; $-$).

The quantitative characteristics of autowave processes involved in the plastic flow localization were determined experimentally in our investigations [1,2]. The test specimens were prepared from the single crystals of FCC BCC and HCP alloys (Al, Cu, Ni and γ -Fe; γ -Fe, V and Nb and Mg, Zn, Zr and Ti, respectively) and from polycrystalline metals and alloys. The alkali halide crystals KCl, NaCl and LiF and some rocks were also studied. The mechanical characteristics and the shape of plastic flow curve are found to be determined by chemical composition, grain size (in the case of polycrystals) and tension axis orientation (in the case of single crystals). The plastic localization patterns observed for all materials studied have many features in common which are discussed below.

3. Correspondence between the Localized Plastic Flow Patterns and the Work Hardening Stages

One of the striking results obtained in these investigations is the finding that the emergent localized plastic flow patterns strictly correspond to the well-known plastic strain stages [9].

Using the Ludwik equation [10], the flow curve $\sigma(\varepsilon)$ is readily separated into individual stages, *i.e.*

$$\sigma(\varepsilon) = \sigma_0 + \theta \varepsilon^n, \quad (1)$$

where σ_0 is the proof stress, θ is the work hardening coefficient and n is the hardening exponent. Especially convenient for this separation is the value n since it changes discretely with the plastic flow. Then the flow stages singled out on the curve $\sigma(\varepsilon)$ are matched against the respective specific patterns of plastic flow localization. In what follows the localized plastic flow patterns

are discussed in the order of their emergence.

The first to appear is a solitary front of localized plastic flow, which travels along the specimen elongation axis at the yield plateau in the polycrystals ($n = 0$) or at the easy glide in the single crystals ($n \approx 0$). In this case, an elastic-plastic transition takes place. The motion of such a front can be regarded as switching autowave [11]. The next pattern is a mobile set of parallel fronts separated by distance λ , which is observed at the linear work hardening stage in the single-crystal and polycrystalline specimens ($n = 1; \sigma \sim \varepsilon$). In accordance with [8], the latter pattern corresponds to a typical phase autowave having length λ and propagation rate V_{aw} . The third in the order of appearance is a group of immobile equidistant localized plasticity nuclei, which occurs at the parabolic work hardening (Taylor's) stage ($n = 1/2; \sigma \sim \varepsilon^{1/2}$) in most materials [9]. This pattern might be considered a stationary dissipative structure [11]. The autowave pattern emergent at the pre-failure stage ($0 < n < 1/2$) is specifically associated with 'collapse' of the autowave [12] which takes place concurrently with the onset of macro-necking. The final stage of the plastic flow process ($n \approx 0$) is ductile failure of material. The above phenomena would emerge spontaneously in the specimen under constant-rate tensile loading. Following Seeger and Frank [13], we regard these phenomena as processes of structure formation.

On the base of conclusive evidence obtained for a wide range of materials the following Rule of Correspondence is introduced: *in accordance with the acting work hardening law, $\sigma(\varepsilon)$, each plastic flow stage involves a special kind of autowave process*. The rule formulated above applies to all the plastic flow phenomena having a characteristic macro-scale of about 10^{-2} m. However, the work hardening law governs the mechanisms involved in the interaction of lattice defects [6]. Therefore, the said rule apparently applies to the defect subsystem of the deforming crystal [9]. For this reason, the plastic flow is expected to manifest certain microscopic features that are indications of microscopic effects. In our opinion, three such manifestations merit special note. These are considered below.

4. On the Manifestations of Microscopic Effects

4.1. Elastic and Plastic Deformation Invariant

On the base of experimental data a significant regularity is established for the autowave process of plastic flow localization in a range of metals. Thus *a close correlation is found to exist between the product of the macroscopic parameters of the autowave process, $\lambda \cdot V_{aw}$, and*

the product of the microscopic (lattice) parameters of material, $d \cdot V_{\perp}$. Here d is the spacing between the close-packed planes of the lattice and $V_{\perp}$ is the transverse elastic wave rate. The numerical data obtained for studied metals is listed in **Table 1**. Matching of this data suggests that the following equality is good within an acceptable range of accuracy

$$\lambda \cdot V_{aw} \approx 1/2 d \cdot V_{\perp} \approx 3/2 r_i \cdot V_{\perp} \quad (2)$$

Indeed, a numerical analysis shows that $d \approx 3r_i$ (here r_i is the Pauling ion radius [14]). Equation (2) is validated by the fact that the average ratio $\langle 2\lambda \cdot V_{aw} / d \cdot V_{\perp} \rangle \approx 1.04 \pm 0.52$ obtained for studied metals is about close to unity and the dependence $\lambda/d(V_{\perp}/V_{aw})$ is a linear one (see **Figure 3**).

Equation (2) is physically significant since it establishes a quantitative relationship between the micro-scale (lattice) characteristics (d and $V_{\perp}$) of elastic waves which govern elastic deformation processes on the one hand and the macro-scale characteristics (λ and V_{aw}) of localized plastic flow autowaves which are generated

Table 1. Matching of $\lambda \cdot V_{aw}$ and $d \cdot V_{\perp}$ values calculated from Equation (2).

Metal	$\lambda \cdot V_{aw} \cdot 10^7$ ($\text{m}^2 \text{s}^{-1}$)	$d \cdot 10^{10}$ (m)	$V_{\perp} \cdot 10^{-3}$ (ms^{-1})	$d \cdot V_{\perp} \cdot 10^7$ ($\text{m}^2 \text{s}^{-1}$)	$\frac{2\lambda \cdot V_{aw}}{d \cdot V_{\perp}}$
Cu	3.60	2.08	2.30	4.78	1.50
Al	7.92	2.33	3.23	7.52	2.10
Zr	1.92	2.46	2.25	5.53	0.70
Ti	3.50	2.24	2.96	6.63	1.06
V	2.80	2.14	2.83	6.06	0.92
γ -Fe	2.55	2.07	3.32	6.87	0.74
α -Fe	2.24	2.03	3.32	6.74	0.66
Ni	2.10	2.03	3.22	6.54	0.64

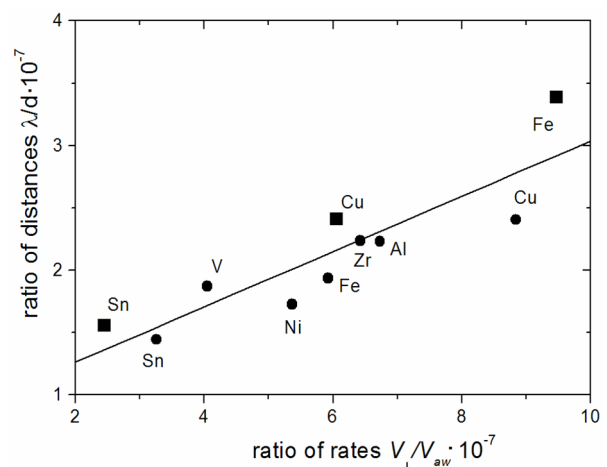


Figure 3. Verification of the validity of Equation (2) with help of a linear dependence between the ratios λ/d and $V_{\perp}/V_{aw}$; ■ – easy glide stage; ● – linear work hardening stage.

in deforming media on the other hand. In this case, it might be reasonable to regard the products $d \cdot V_{\perp}$ and $\lambda \cdot V_{aw}$ as invariants of elastic and plastic deformation processes, respectively. The above regularity suggests that the elastic and the plastic processes simultaneously involved in the deformation ($\varepsilon \ll 1$ and $\varepsilon \approx 1$, respectively) are closely related. The quantity $V_{\perp}$ is the rate of elastic stress redistribution in the deforming solid and the quantity V_{aw} is the rate of localized plasticity front rearrangement in the same solid. Thus, the macro-scale localization of plastic deformation can no longer be regarded as a mere disturbance of plastic flow homogeneity by necking. What is more, the localization phenomena are taken to be an attribute of the plastic deformation, with their characteristics being closely associated with and largely determined by the properties of crystal lattice.

4.2. Autowave Characteristics and the Planck Constant

A numerical analysis suggests that for all metals and alloys studied the following equality holds good:

$$\lambda \cdot V_{aw} \cdot \rho \cdot r_i^3 = h, \quad (3)$$

where ρ is material density. In other words, the quantum (Planck's) constant h can be calculated by multiplying the values V_{aw} and which are localized plastic flow characteristics measured experimentally by the values and r_i which are hand-book material constants. Indeed, the calculated values h listed in **Table 2** are close to the Planck constant $h = 6.626 \cdot 10^{-34}$ J·s, with the average value $\langle h \rangle$ being $(6.44 \pm 0.88) \cdot 10^{-34}$ J·s and the ratio $\langle h \rangle / h = 0.96 \pm 0.07$ being close to unity. Thus the Planck constant can be estimated directly from the macro-scale characteristics V_{aw} and λ , which appears striking in itself.

4.3. Form of Dispersion Relation

Let us consider the following quadratic dispersion law [15]

$$\omega = \omega_0 \pm \alpha(k - k_0)^2, \quad (4)$$

where α , ω_0 and k_0 are empirical constants. The values of these constants were derived for localized plastic flow autowaves from the experimental data in **Figure 4**. Note that the value α can be both negative and positive, *i.e.* for the easy glide stage, $\alpha < 0$ and for the linear work hardening stage, $\alpha > 0$. Equation (4) is readily reduced to the canonic form $\tilde{\omega} = 1 \pm \tilde{k}^2$ by substituting $\omega = \omega_0 \cdot \tilde{\omega}$ and $k = k_0 + \frac{\tilde{k}}{\sqrt{\text{sign} \alpha \cdot \alpha / \omega_0}}$ (here $\tilde{\omega}$ and $\tilde{k}$ are the di-

mensionless frequency and the wave number, respectively, and the signum function of α is $\text{sign} \alpha = \begin{cases} +1 \\ -1 \end{cases}$ for $\alpha > 0$ and $\alpha < 0$). Wave processes that have quadratic dispersion law of the latter form would generally satisfy a number of nonlinear equations, *e.g.* the Schrödinger nonlinear equation, the sine-Gordon equation, *etc.*, which are frequently employed to address self-organization processes occurring in nonlinear media [16]. Therefore, the dispersion relation (4) is taken to be an additional proof for plastic flow localization being involved in the self-organization of the deforming medium. Moreover, Equation (4) turned out to be formally equivalent to the de Broglie wave dispersion law for electrons in sharp-cornered potential well [17], which is significant in itself.

5. A Postulate of Plastic Flow Localization and the Ensuing Corollaries: Introduction of a New Quasi-particle

Taken as a whole the above localization patterns demonstrate that the phenomenon in question has not only macroscopic characteristics of autowave process (λ and V_{aw}) but also microscopic ones (h and d or r_i). In view of the ratio λ/r_i being about equal to 10^8 , the key problem in this case is how the macro- and micro-scales could be reconciled. To overcome this problem, we propose to use an approach based on the wave-particle dualism, which received wide application in the physics of solids [18]. We are led to postulate *the existence of aquasi-particle having effective mass, m_{ef} , quasimomentum, p , and energy, E , which corresponds to the autowave of localized plastic deformation having wavelength, λ and*

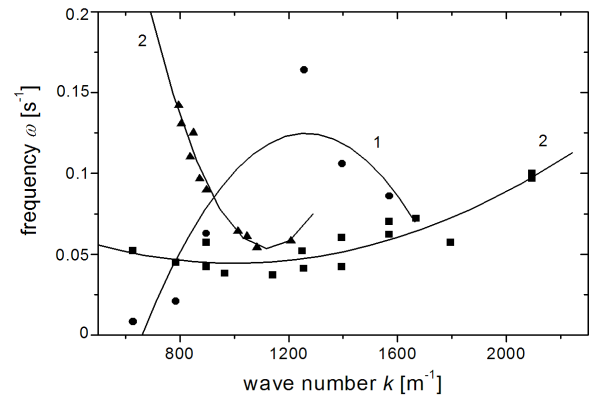


Figure 4. The dispersion law $\omega(k)$ established for localized plastic flow autowaves generated at the stages of easy glide (1) and linear work hardening (2); ● – single crystals of Cu, Sn and alloyed γ -Fe; ■ – single crystals of alloyed γ -Fe; ▲ – polycrystalline Al.

Table 2. Microscopic characteristics and the Planck constant values calculated from the data on localized plastic flow autowaves.

Metal	$\lambda \cdot 10^3$	$V_{aw} \cdot 10^5$	m_{ef}	$\mu \cdot 10^2$	n	$\Omega \cdot 10^{27}$	$d_\Omega \cdot 10^9$	$r_{ion} \cdot 10^9$	d_Ω/r_{ion}	$h \cdot 10^{34}$
	(m)	(ms ⁻¹)	(a.m.u)			(m ³)	(m)	(m)		(J·s)
Cu	4.5	8.0	1.1	1.74	1	0.21	0.059	0.072	0.82	8.14
Al	7.2	11	0.50	1.87	3	0.31	0.068	0.051	1.33	5.0
Zr	5.5	3.5	2.05	2.24	4	0.53	0.081	0.079	1.03	6.13
Ti	7.0	5.0	1.1	2.3	4	4.2	0.075	0.076	0.99	6.91
V	4.0	7.0	1.42	2.81	5	0.33	0.069	0.059	1.17	6.52
γ -Fe	5.0	5.1	1.76	2.81	8	0.33	0.069	0.064	1.08	6.32
α -Fe	4.3	5.2	1.77	3.0	8	3.75	0.072	0.064	1.13	6.32
Ni	3.5	6.0	0.89	3.24	10	0.32	0.068	0.069	0.99	6.17

propagation velocity, V_{aw} . Then it can be written that

$$m_{ef} = \hbar / \lambda V_{aw}. \quad (5)$$

Equality (5) is a mathematical expression of the above postulate to which we conventionally add the equations $p = m_{ef} V_{aw} = \hbar k$ and $E = \hbar \omega$ (here $\hbar = h/2\pi$) for momentum and energy, respectively [19,20]. It is common knowledge that to validate a postulate, one has to match the corollaries ensuing from the same against experimental evidence. It turns out that a set of corollaries ensues from the above postulate which give an insight into the nature of localized plastic flow processes. These corollaries are considered below.

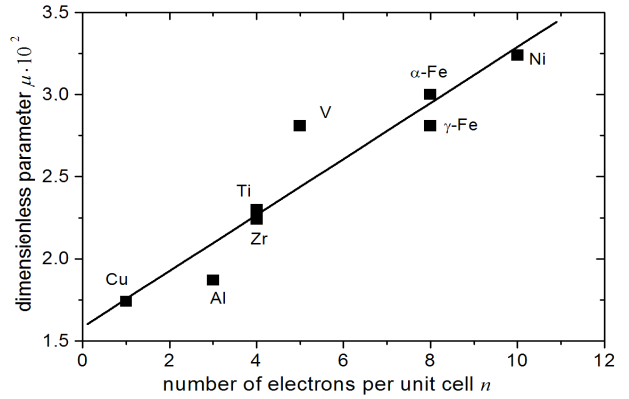
Corollary 1. First we will demonstrate that the effective mass, m_{ef} , calculated from Equation (5) has a physical meaning; this quantity depends on the characteristics of the deforming crystal. Indeed, the calculated values listed in **Table 2** for a number of metals are in the range $0.5 \leq m_{ef} \leq 2$ a.m.u. (here 1 a.m.u. = $1.66 \cdot 10^{-27}$ kg is atomic mass unit). Evidently, the volume is readily calculated as $\Omega = m_{ef} / \rho$; then the length is found as $d_\Omega = \sqrt[3]{\Omega}$. The latter value is close to the value r_i , i.e. $d_\Omega \approx r_i$, with the average ratio being $\langle d_\Omega / r_i \rangle = 1.07 \pm 0.09 \approx 1$. Thus, the effective mass, m_{ef} , turns out to be related to the lattice characteristics, ρ and r_i .

Apparently, the values m_{ef} calculated from Equation (5) differ slightly for each particular metal. The normalization of effective masses, m_{ef} , to the atomic masses, A_m , of respective metals yields normalized (dimensionless) mass $\mu = m_{ef} / A_m \ll 1$, which increases linearly with the number of electrons per unit cell, n (see **Figure 5**) [21], i.e.

$$\mu = \mu_0 + \kappa \cdot n. \quad (6)$$

The correlation coefficient for μ and n values is ~ 0.95 ; it has a high statistical significance.

Now we propose a physical interpretation of Equation (6) which is based on the consideration of jump-like dislocation motion. Thus a dislocation would become ar-

**Figure 5. The dimensionless parameter, as a function of the number of electrons per unit cell, n .**

rested at a local barrier until the thermal fluctuation causes its breakaway, which suggests that the plastic deformation results from the dislocation motion in a viscous medium [22]. For $V_{disl} = const$, the motion of dislocations is controlled by viscous drag force per unit length, $F_v \approx B V_{disl}$ (here B is the viscous drag factor for dislocations) [22]. For $V_{disl} \neq const$, an inertial term proportional to the dislocation acceleration, $\dot{V}_{disl}$, is added to the viscous drag force [23]. Then the total drag force, F_Σ , is given by

$$F_\Sigma = F_v + F_{in} \approx B V_{disl} + (B/\omega_a) \cdot \dot{V}_{disl}, \quad (7)$$

where ω_a is the frequency of an elementary deformation act and B/ω apparently has the meaning of added mass per unit length of dislocation.

In the case of metals, the factor B is determined by the interaction of dislocations with phonon and electron gases [22]; moreover, the contributions of phonon and electron gases, i.e. B_{ph} and B_e , respectively, are additive so that $B = B_{ph} + B_e$. In this case, the first and the second term in the right-hand side of Equation (6) are evidently connected with the contributions to the added mass, B/ω , of the viscous drag of both gases.

The term μ_0 from (6) is apparently independent of the kind of metal, which is reasonable since the properties of metal are only weakly dependent on the characteristics of its phonon spectrum at temperatures exceeding the Debye temperature [21]. However, the contribution of electron gas to the added mass should be proportional to n , i.e. $B_e \sim n$ [22]. Hence it can be written

$$F_{in} \sim (B_{ph} + B_e) / \omega \cdot \dot{V}_{disl} \sim (m_{ph} + m_e) \cdot \dot{V}_{disl} \sim (\mu_0 + \kappa \cdot n) \cdot \dot{V}_{disl} \quad (8)$$

Thus Equation (8) relates the normalized mass, μ , to the contributions of phonon and electron gases.

Corollary 2. Equation (5) can be rewritten as

$$\lambda V_{aw} = \frac{h}{m_{ef}} = \frac{h}{\rho \cdot r_i^3} = \varsigma \cdot \quad (9)$$

The quantity ς from (9) is calculated as $\varsigma = h / \rho r_i^3$. Then the calculated values, ς , are matched with the experimental λV_{aw} data. The average values obtained for the single γ -Fe crystals and polycrystalline Al are, respectively, $\langle \lambda V_{aw} \rangle = (2.86 \pm 0.44) \cdot 10^{-7} \text{ m}^2 \cdot \text{s}^{-1}$ ($\varsigma = 3.2 \cdot 10^{-7} \text{ m}^2 \cdot \text{s}^{-1}$) and $\langle \lambda V_{aw} \rangle = (7.75 \pm 1.36) \cdot 10^{-7} \text{ m}^2 \cdot \text{s}^{-1}$ ($\varsigma = 18.4 \cdot 10^{-7} \text{ m}^2 \cdot \text{s}^{-1}$).

Corollary 3. Equation (5) can be also rewritten as

$$V_{aw} = \frac{h}{m_{ef} \cdot \lambda} = \frac{h}{\rho \cdot r_i} \cdot \frac{1}{\lambda} = \frac{\varsigma}{2\pi} \cdot k \quad (10)$$

It is shown above that $V_{aw} = V_{gr} = d\omega/dk$ and $d\omega = (\varsigma/2\pi) \cdot k \cdot dk$. Hence we can write

$$\int_{\omega_0}^{\omega} d\omega = \frac{\varsigma}{2\pi} \int_0^{k-k_0} k \cdot dk \quad (11)$$

It follows from (11) that dispersion relation of quadratic form can be written for localized plastic flow autowaves, i.e. $\omega = \omega_0 + \frac{\varsigma}{4\pi} (k - k_0)^2 = \omega_0 + \alpha (k - k_0)^2$. Apparently, the latter relation corresponds to (4). The coefficient α from the dispersion relation of quadratic form can be found by matching the experimental $\omega(k)$ data against the calculated values $\alpha = \varsigma/4\pi$ obtained for Fe and Al. Thus the experimental values α obtained for Fe and Al are $5.4 \cdot 10^{-8} \text{ m}^2 \cdot \text{s}^{-1}$ and $7.9 \cdot 10^{-7} \text{ m}^2 \cdot \text{s}^{-1}$, respectively, and the calculated data are $2.5 \cdot 10^{-8} \text{ m}^2 \cdot \text{s}^{-1}$ and $1.46 \cdot 10^{-7} \text{ m}^2 \cdot \text{s}^{-1}$, respectively. Both sets of data have practically the same order of magnitude.

Corollary 4. It follows from Equation (5) that

$$\lambda V_{aw} \rho = h / r_i^3 \quad (12)$$

The terms in both sides of (12) evidently have the units of dynamic viscosity, i.e. $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1} \equiv \text{Pa} \cdot \text{s}$. The calculated value $\lambda \cdot V_{aw} \cdot \rho$ is about equal to $5 \cdot 10^{-4} \text{ Pa} \cdot \text{s}$

for all studied metals. Hence the latter quantity can be identified with the viscosity of phonon gas, B , which controls dislocation mobility by quasi-viscous motion [22]. This is good indirect evidence for the validity of the same quantity, in particular, by interpreting the physical meaning of dependence (6).

Corollary 5. The dispersion relation obtained for localized plastic deformation autowaves can be rewritten in the form appropriate for quasi-particles, i.e. $E = E(p) = E_0 + \alpha' (p - p_0)^2$ (here E_0 , p_0 and α' are constants). Then the effective mass of the hypothetical quasi-particle is estimated [17] as

$$m_{ef} = (\partial^2 E / \partial p^2)^{-1} = \hbar \cdot (\partial^2 \omega / \partial k^2)^{-1} \quad (13)$$

The experimental and calculated effective masses obtained for iron and aluminum are 0.6 and 0.1 a.m.u., respectively. This lends credence to the existence of the hypothetical quasi-particle having $m_{ef} \approx 1 \text{ a.m.u.}$

Corollary 6. As is seen from **Figure 4**, the oscillation spectrum $\omega = \omega_0 + \alpha (k - k_0)^2$ has a narrow gap in the range $0 \leq \omega \leq \omega_0 \approx 10^{-2} \text{ Hz}$. Hence for any temperature, $\hbar \omega_0 \ll k_B T$ (here k_B is the Boltzmann constant). Localization phenomena are liable to occur spontaneously at any temperature provided geometric constraints place no restrictions, in particular, in the case of small-sized specimens [1].

Corollary 7. Finally, the jump-like plastic deformation in solids [24] can be explained with help of the above postulate. Let us rewrite Equation (5) as

$$\delta L \approx h (\rho r_i V_{aw})^{-1} \quad (14)$$

Now suppose that the specimen length L accommodates an integer $m = 1, 2, 3 \dots$ of autowaves having length λ , i.e. $\lambda = L/m$, which precludes the occurrence of deformation within the clamps of the testing machine. With growing total deformation, ε , the elongation of the specimen occurs as $L \approx L_0 (1 + \varepsilon) \approx L_0 + \delta L$ (here L_0 is the original specimen length). Hence from (14) follows that

$$\delta L \approx h (\rho r_i V_{aw})^{-1} \cdot m \quad (15)$$

For the linear work hardening stage, $V_{aw} = \text{const}$. From (15) apparently follows that the specimen length would vary discretely ($\delta L \sim m$) in accord with the jump-wise deformation behavior, i.e. the specimen length would be accommodated to the emergent autowave pattern. Deformation jumps may occur by different mechanisms depending on the kind of material so that Equation (15) only states that this kind of deformation behavior is a must.

From (15) follows that $\delta L \sim V_{aw}^{-1}$. The available experimental evidence [1] suggests that the autowave

propagation rate is proportional to the cross-head motion velocity, *i.e.* $V_{aw} \sim V_{mach}$. With increasing velocity of the movable clamp of the test machine, the amplitude of jumps is expected to grow less. The latter inference is supported by the experimental data obtained for Al at 1.4 K [24].

Corollary 8. Now it transpires that Equation (3) of the form $\lambda \cdot V_{aw} \cdot \rho \cdot r_i^3 = h$ is readily derived from the same postulate provided $m_{ef} = \rho \cdot r_i^3$.

6. Conclusions

It is pertinent to mention here the early efforts to apply quantum ideas to problems of strength and plasticity physics. Thus Steverding was the first to introduce the notion of elastic waves quantization by fracture [25]. By addressing the kinetics of brittle fracture, the existence of a quasi-particle in the deforming medium was postulated. The quasi-particle was identified with the tip of a growing crack; therefore, it got the name ‘crackon’ [26]. Quantum models were applied to explain the tunnel effect observed by dislocation motion [27,28]. On the other hand, recent theoretical and experimental studies indicate that the plastic flow involves wave processes [1-5,7,29,30].

The above findings justify the use of quantum concepts to address plastic flow development in solids. In this approach one has to reconcile micro- and macro-scale manifestations of the localization phenomenon; therefore, physicists dealing with plasticity problems might consider it invalid. When the micro-scale (dislocation) level is addressed, the quantization idea appears quite appropriate. In view of crystal lattice discreteness, the minimal possible shear is by the Burgers vector, b , which has microscopic scale d of about 10^{-10} m and hence might be regarded as a ‘quantum of shear deformation’. Hence the use of (2) allows the quantization concept to be extended to the macro-scale level, *i.e.* $\lambda \cdot V_{aw} \approx 1/2 d \cdot V_{\perp}$.

The fact that the Planck constant value calculated from the data derived in rather ‘rough’ macro-experiments is close to a handbook value might be due to the manifestation of so-called ‘universality concept’ [31], which postulates that the system’s measurables are virtually unaffected by the distribution of its major microscopic properties. In other words, both the qualitative and the quantitative characteristics of material substructure are only partly responsible for, *e.g.* plastic flow evolution, flow stress level, work hardening coefficient, *etc.*

The corollaries ensuing from the postulate proposed herein have proven to be very fruitful to study deformation localization processes, which validates the concept of wave-particle duality. By way of summary it should be emphasized that the macro-scale effects (characteris-

tic scale $L_{macro} \approx \lambda$) emerging in a plastically deforming multi-scale system are found to be directly related to the micro-scale effects (characteristic scale $L_{micro} \approx r_i$), with the scale ratio being $L_{macro}/L_{micro} \approx \lambda/r_i \geq 10^8$.

The above gives justification to the use of a common approach, *i.e.* postulation of a quasi-particle corresponding to a localized plastic flow autowave. This turns out to be a fruitful effort by explaining a number of relationships in plasticity physics, which have remained poorly understood in the frame of traditional models of crystal plasticity. The postulated quasi-particle would be named “*auto-localizon*”.

As far back as the 1960-ies Dzyaloshinski [32] was the first to discuss the possibility of measuring the Planck constant in mechanical experiments. In his paper this worker emphasizes the importance of studying the macroscopic manifestations of typical quantum effects, *e.g.* superfluidity, superconductivity and the quantum Hall effect discovered later on. In the light of the foregoing it is maintained that the plastic flow in solids is analogous to all these phenomena; therefore, this might also be regarded as a macroscopic quantum effect.

7. Acknowledgements

This work was partly supported by the grant of RFBR (09-08-00213-a).

8. References

- [1] L. B. Zuev, “Wave Phenomena in Low-Rate Plastic Flow of Solids,” *Annals of Physics*, Vol. 10, 2001, pp. 965-984.
- [2] L. B. Zuev, “On the Waves of Plastic Flow Localization in Pure Metals and Alloys,” *Annals of Physics*, Vol. 16, 2007, pp. 286-310.
- [3] R. J. McDonald, C. Efstathiou and P. Curath, “The Wave-Like Plastic Deformation of Single Crystals Copper,” *Journal of Engineering Materials and Technology*, Vol. 131, 2009, pp. 692-703.
- [4] A. Asharia, A. Beaudoin and R. Miller, “New Perspectives in Plasticity Theory: Dislocation Nucleation, Waves and Partial Continuity of Plastic Strain Rate,” *Mathematics and Mechanics of Solids*, Vol. 13, 2008, pp. 292-315.
- [5] C. Fressengeas, A. Beaudoin and D. Entemeyer, “Dislocation Transport and Intermittency in the Plasticity of Crystalline Solids,” *Physical Review B*, Vol. 79, 2009, pp. 014108-1-014108-9.
- [6] L. B. Zuev, V. V. Gorbatenko and S. N. Polyakov, “Instrumentation for Speckle Interferometry and Techniques for Investigating Deformation and Fracture,” *Proceedings of SPIE*, Vol. 4900, Part 2, 2002, pp. 1197-1208.
- [7] H. Haken, “Information and Self-Organization,” *A Macroscopic Approach to Complex Systems*, Springer Verlag, Berlin, 1989.
- [8] H. Kolsky, “Stress Waves in Solid,” Phoenix Education,

- Dover, 2003.
- [9] F. R. N. Nabarro, Z. S. Basinski and D. B. Holt, "The Plasticity of Single Crystals," Taylor and Francis Ltd, London, 1964.
 - [10] R. Hill, "The Mathematical Theory of Plasticity," University Press, Oxford, 1998.
 - [11] V. A. Davydov, V. S. Zykov and A. S. Michailov, "Kinematics of Autowave Structures in the Excited Media," *Physics-Uspekhi*, Vol. 161, 1999, pp. 45-85.
 - [12] B. B. Kadomtsev, "Dynamics and Information," *Physics-Uspekhi*, Vol. 164, 1994, pp. 449-530.
 - [13] A. Seeger and W. Frank, "Structure Formation by Dissipative Processes in Crystals with High Defect Densities," In L. P. Kubin and G. Martin Eds., "Non-Linear Phenomena in Material Science," Trans Tech Publications, New York, 1987, pp. 125-137.
 - [14] L. Pauling, "The Nature of the Chemical Bond," Cornell University Press, Ithaca, 1960.
 - [15] S. A. Barannikova, "Dispersion of the Plastic Strain Localization Waves," *Technical Physics Letters*, Vol. 30, 2004, pp. 338-340.
 - [16] A. Scott, "Nonlinear Science," *Emergence and Dynamics of Coherent Structures*, University Press, Oxford, 2003.
 - [17] L. D. Landau and E. M. Lifschitz, "Quantum Mechanics: Non-Relativistic Theory," *Course of Theoretical Physics*, Pergamon, London, 1977.
 - [18] J. M. Ziman, "Electrons and Phonons," University Press, Oxford, 2001.
 - [19] J. P. Billingsley, "The Possible Influence of the De Broglie Momentum-Wavelength Relation on Plastic Strain 'Autowave' Phenomena in 'Active Materials'," *International Journal of Solids and Structures*, Vol. 38, 2001, pp. 4221-4234.
 - [20] L. B. Zuev, "The Linear Work Hardening Stage and De Broglie Equation for Autowaves of Localized Plasticity," *International Journal of Solids and Structures*, Vol. 42, 2005, pp. 943-949.
 - [21] A. P. Cracknell and K. C. Wong, "The Fermi Surface. Its Concept, Determination and Use in the Physics of Metals," Clarendon Press, Oxford, 1973.
 - [22] V. I. Al'shits and V. L. Indenbom, "Mechanisms of Dislocations Drag," In F. R. N. Nabarro Ed., *Dislocations in Solids*, Amsterdam, 1986, pp. 43-111.
 - [23] L. D. Landau and E. M. Lifshits, "Hydrodynamics," Nauka Publications, Moscow, 1988.
 - [24] V. V. Pustovalov, "Serrated Deformation of Metals and Alloys at Low Temperatures," *Low Temperature Physics*, Vol. 34, 2008, pp. 683-723.
 - [25] B. Steverding, "Quantization of stress waves and Fracture," *Materials Science and Engineering*, Vol. 9, 1972, pp. 185-189.
 - [26] E. M. Morozov, L. S. Polack and Ya. B. Fridman, "On Variation Principles of Crack Development in Solids," *Soviet Physics-Doklady*, Vol. 156, 1964, pp. 537-540.
 - [27] T. Oku and J. M. Galligan, "Quantum Mechanical Tunneling of Dislocations," *Physical Review Letters*, Vol. 22, 1969, pp. 596-597.
 - [28] B. V. Petukhov and V. L. Pokrovsky, "Quantum and Classic Motion of Dislocations in the Potential Peierls Relief," *Journal of Experimental and Theoretical Physics*, Vol. 63, 1972, pp. 634-647.
 - [29] E.C. Aifantis, "Nonlinearity, Periodicity and Patterning in Plasticity and Fracture," *International Journal of Non-Linear Mechanics*, Vol. 31, 1996, pp. 797-809.
 - [30] E.C. Aifantis, "Gradient Plasticity," In J. Lemaître, Ed., *Handbook of Materials Behavior Models*, Academic Press, New York, 2001, pp. 291-307.
 - [31] Y. Imry, "Introduction to Mesoscopic Physics," University Press, Oxford, 2002.
 - [32] L. P. Dzyaloshinski, "Macroscopic Quantum Phenomena," *Soviet Physics-Uspekhi*, Vol. 90, 1966, pp. 623-629.

Translational Motion of a Free Large Polaron and Broadening of Absorption Spectra

Vladimir Mukhomorov

Agrophysical Institute, St.-Petersburg, Russia

E-mail: vmukhomorov@mail.ru

Received February 16, 2010; revised March 17, 2010; accepted March 20, 2010

Abstract

The translational motion of a large polaron as whole is analyzed in the context of its effect on the broadening of an absorption optical spectrum. It was open question how important the role of translational degrees of freedom and the corresponding velocities are on the broadening. The Bogolyubov method of canonical transformation of coordinates is formulated for a system of an electron and field, taking into account rigorous fulfillment of the conservation laws. Separation of variables is carried out for the coordinates describing the translational degrees of freedom and the electron oscillations in a polarization well. The equations obtained for the electronic states explicitly depend on the velocity of the free polaron as a whole. An estimate is made for free polaron in ammonia.

Keywords: Large Polaron, Translational Motion, Canonical Transformation, Broadening, Absorption Spectra, Ammonia

1. Introduction

ITS is well known that the absorption spectra of free large polaron consists of a broad featureless asymmetric band with a long tail extended to the short-wave length region. There is vast literature on the possible mechanisms of broadening the optical spectrum of free polaron. S. I. Pekar [1] have studied the broadening of the optical absorption spectra of large polaron in crystal as a function of phonon dispersion. In the work [2] authors have calculated the optical absorption coefficient for free polarons using the multiphonons mechanism. The main idea in ref. [3] consist that broadening of the absorption spectra depends on width of an electronic band. In this case broadening of the absorption spectra to proportionally electron effective mass at the bottom of a conductivity band [4]. However the question of a contribution to the broadening from the translational degrees of freedom and from the corresponding velocities is still unanswered. In this work the thermal motion of free quasiparticle as a whole is analyzed in the context of its effect on the broadening of the optical absorption spectra.

2. Mathematical Method

To analyze the effect of translational motion of a free large polaron on its absorption spectra, one must separate

in the Hamiltonian the translation-invariant degrees of freedom from the coordinates describing the motion of the polaron as whole and derive the velocity dependent equations for the electron transitions. If the electron and quantum field are strongly coupled then the collective localized state of the field and particle is formed. In such a formation the electron motion is rather intricate. On the one hand the electron oscillated within a rather deep polarization potential well and undergoes the optical transitions, and on the other, it moves together with the center of inertia of the system and participates in the translational random walk. The problem is to separate these motions correctly, rigorously taking into account the conservation laws. This can be conveniently done using Bogolyubov [5] method of canonical transformation to the collective coordinates. The Bogolyubov method is the most powerful analytic techniques available for discussing electron-phonon problems. This transformation removes the translational degeneracy and allows one to develop the successive approximation algorithm for the energy and wave function while simultaneously fulfilling the law of conservation of total momentum of the system. Some of the transformed variables are generalized coordinates whose canonically conjugated momenta are the integrals of motion, which are defined by the symmetry properties of the original Hamiltonian and hence ultimately ensure fulfillment of the conservation laws. Fol-

lowing the Bogolyubov method, we reformulate the adiabatic theory of the particle strongly interacting with the quantum field. The resulting equations determine the electron transitions and depend explicitly on the translational velocity of free polaron.

Within the effective mass one-electron continual approximation the Hamiltonian of the electron-phonon system has the form

$$H = \frac{p_r^2}{2m^*} + \gamma^2 \sum_f (V_f^{(0)} e^{i\mathbf{r}} b_f + V_f^{(0)*} e^{-i\mathbf{r}}) + \frac{1}{2} \sum_f \hbar \omega_f (b_f^+ b_f + b_f b_f^+) \quad (1)$$

where the interaction form-factor is defined as $V_f^{(0)} = i(\hbar \omega_f / f u^{1/2})(4\pi/V)^{1/2}$, $u = (2m^* \omega_f / \hbar)^{1/2}$ and the dimensionless coupling constant is $\gamma^2 = \alpha_c$, $\alpha_c = (1/\varepsilon_\infty - 1/\varepsilon_s) e^2 u / 2\hbar \omega_f$; m^* is the isotropic effective mass of electron, ω_f is the frequency of the long-wave length longitudinal optical phonons of the polar medium; ε_∞ and ε_s are the high-frequency and low-frequency dielectric constants of the isotropically polarizable dielectric continuum, and $\mathbf{r}$ is the electron coordinate. The quantum amplitudes b_f and b_f^+ of the polarization field, respectively, annihilate and create the field quantum $\hbar \omega_f$ and obey the Bose-Einstein commutation rule $[b_f, b_{f'}^+] = \delta_{ff'}$.

In order to develop the iterative procedure for calculating the eigenfunctions and energy eigenvalues of Hamiltonian (1) we modify the canonical transformation of coordinates. For this purpose the electron coordinate is written as the vector sum of two variables

$$\mathbf{r} = \boldsymbol{\lambda} / \gamma + \mathbf{q}, \quad (2)$$

where $\mathbf{q}$ is independent of $\mathbf{r}$ and means the coordinate of the center of gravity of the system, and $\boldsymbol{\lambda}$ describes the electron motion relative to the center. Before developing the perturbation theory, let us introduce in Hamiltonian (1) instead of the phonon creation and annihilation operators, the complex phonon coordinates q_f and the corresponding operators of conjugated momentum p_f :

$$q_f = (b_f + b_{-f}^+) / \gamma \sqrt{2}, \quad p_f = i\gamma(b_f^+ - b_{-f}) / \sqrt{2}, \quad (3)$$

which satisfy the commutation rule $[q_f, p_{f'}] = i\delta_{ff'}$. The $1/\gamma$ factor in (2) and (3) allows one to describe the electron motion relative to the center of inertia even within the lowest-order nonvanishing term of the expansion of the Hamiltonian in γ powers. Using Equations (2) and (3) and taking into account that $\partial / \partial \mathbf{r} = (\partial \boldsymbol{\lambda} / \partial \mathbf{r}) \partial / \partial \boldsymbol{\lambda} = \gamma \partial / \partial \boldsymbol{\lambda}$ one can transform Hamiltonian (1) as

$$H = \frac{\gamma^2 \mathbf{P}_\lambda^2}{2m^*} + \gamma^2 \sqrt{2} \sum_f V_f^{(0)} q_f e^{i\mathbf{f}(\mathbf{q} + \boldsymbol{\lambda} / \gamma)} + \frac{\gamma^2}{2} \sum_f \hbar \omega_f q_{-f} q_f +$$

$$\frac{1}{2\gamma^2} \sum_f \hbar \omega_f p_{-f} p_f. \quad (4)$$

One can see that, after changing the variables, the energy of the electron - field interaction and the field potential energy are indeed of the same order in the γ parameter. Hamiltonian (4) is translation-invariant. According to Equation (3) the operator of the total momentum of the system can be written as $-i\hbar \partial / \partial \mathbf{q} = -i\hbar \partial / \partial \mathbf{r} - i\hbar \sum_f \mathbf{f} q_f p_f$, so that it is a strict integral of motion. It then follows that the $\mathbf{q}$ vector indeed means the coordinate of the center of gravity of the system. Because of a smallness of the last term in Equation (4), the effect of interaction of the electron with the quantum field reduces mainly to the appearance of a potential well [second term in Equation (4)] whose depth depends on the magnitude of the dimensionless coupling constant. As a result of the strong interaction, the quasi-particle is characterized by its own internal structure. The appropriate internal states can be coupled to one another by the electronic transitions.

The interaction of phonons with the charged particles is known to shift the equilibrium positions of the field oscillators relative to their unperturbed values. We thus supplement transformation (2) by the transformation of the field coordinates q_f :

$$q_f = (u_f + Q_f / \gamma) \exp(-i\mathbf{f}\mathbf{r}), \quad u_f = u_{-f}^*, \quad Q_f^+ = Q_{-f} \quad (5)$$

The translation-invariant Q_f variables allow for the quantum fluctuations of the field near its new self-consistent classical value which is determined by the set of c -numbers u_f to be evaluated in the follows. Within the new variables (5) the interaction potential between the electron and the quantum polarization field retains its order of magnitude in γ . Note that the introduction of new coordinates (2) results in the appearance of three extra degrees of freedom in comparison to the original system. We therefore impose three additional conditions on the Q_f coordinates, which can be chosen in a linear form without loss of generality:

$$\sum_f \mathbf{f} v_f^* Q_f = 0 \quad (6)$$

This requirements, allow one to retain the number of independent variables after introducing the new electron and field coordinates. The v_f values can be chosen in such a way that the orthonormalization condition

$$\sum_f f_\alpha f_\beta v_f^* u_f = \delta_{\alpha\beta}, \quad \alpha, \beta = 1, 2, 3 \quad (7)$$

is fulfilled together with the requirement that $v_f^* = v_{-f}$. The coordinate transformations (2) and (5) provide fulfillment of the conservation law for the total momentum. Hamiltonian (4) can be further transformed after the operator of momentum p_f is expressed in the terms of the

new variables $\mathbf{q}$, λ and Q_f :

$$p_f = -i \frac{\partial}{\partial q_f} = -\sum_f \frac{\partial Q_k}{\partial q_f} \frac{\partial}{\partial Q_k} - i \frac{\partial \mathbf{q}}{\partial q_f} \frac{\partial}{\partial \mathbf{q}} - i \frac{\partial \lambda}{\partial q_f} \frac{\partial}{\partial \lambda}. \quad (8)$$

Putting u_f independent of q_k and differentiating (5) with respect to q_k we get

$$\begin{aligned} \frac{\partial Q_f}{\partial q_k} &= \gamma \frac{\partial q_f}{\partial q_k} e^{i f q} + i \gamma f q_f e^{i f q} \frac{\partial \mathbf{q}}{\partial q_k} = \\ &= \gamma e^{i f q} [\delta_{fk} + i f (u_f + \frac{1}{\gamma} Q_f) e^{-i f q} \frac{\partial \mathbf{q}}{\partial q_k}]. \end{aligned} \quad (9)$$

The derivative $\partial \mathbf{q} / \partial q_k$ is found by inserting (5) in the additional condition (6) and differentiating the identity obtained. The result is

$$\sum_f f v_f^* (\delta_{kf} e^{i f q} + i f q_f e^{i f q} \frac{\partial \mathbf{q}}{\partial q_f}) = 0. \quad (10)$$

The equation for the partial derivative of $\mathbf{q}$ with respect to q_k is obtained from Equation (10) taking into account transformation (5) and the condition (7)

$$\frac{\partial \mathbf{q}}{\partial q_k} = i k v_k^* e^{i k q} - \frac{1}{\gamma} \sum_f (f f) v_f^* Q_f \frac{\partial \mathbf{q}}{\partial q_k} \quad (11)$$

This equation can be solved by iteration, with $1/\gamma$ as a small parameter. The following solution is then obtained within an accuracy of the terms on the order of $1/\gamma^2$:

$$\begin{aligned} \frac{\partial \mathbf{q}}{\partial q_k} &= i e^{i k q} [k v_k^* - \frac{1}{\gamma} \sum_f f(f k) v_k^* v_f^* Q_f + \\ &+ \frac{1}{\gamma^2} \sum_{f, l} k(f l)(f l) v_k^* v_f^* v_l^* Q_f Q_l + \dots] \end{aligned} \quad (12)$$

Using the transformation of variables (2), one determines the partial derivative $\partial \lambda / \partial q_f = -\gamma \partial \mathbf{q} / \partial q_f$. Now, using this equality and Equation (12), one finally obtains the following expansion series for the $\partial / \partial q_f$ operator in terms of the λ , $\mathbf{q}$, and Q_f variables

$$\begin{aligned} \frac{\partial}{\partial q_f} &= e^{i q f} \{ i \gamma P_f - i \gamma f v_f^* \frac{\partial}{\partial \lambda} + \sum_k (f k) v_f^* Q_k \frac{\partial}{\partial Q_k} - \\ &- \sum_k k(f k) v_f^* v_k^* Q_k (i \sum_l l u_l \frac{\partial}{\partial Q_l} - \frac{\partial}{\partial \lambda}) + f v_f^* \frac{\partial}{\partial \mathbf{q}} + \\ &+ \frac{1}{\gamma} [\sum_{k, l} f(k l)(k l) v_f^* v_k^* v_l^* Q_k Q_l (i \sum_l l u_l \frac{\partial}{\partial Q_l} - \frac{\partial}{\partial \lambda}) - \\ &- \sum_k k(k f) v_f^* v_k^* Q_k \sum_l l Q_l \frac{\partial}{\partial Q_l} - \sum_k k(k f) v_f^* v_k^* Q_k \frac{\partial}{\partial \mathbf{q}}] + \\ &+ \frac{1}{\gamma^2} [\sum_{k, l} f(k l)(k l) v_f^* v_k^* v_l^* Q_k Q_l \sum_m m Q_m \frac{\partial}{\partial Q_m} + \end{aligned}$$

$$\sum_{k, l} f(k l)(k l) v_f^* v_k^* v_l^* Q_k Q_l \frac{\partial}{\partial \mathbf{q}}] + \dots \} \quad (13)$$

where P_f stands for the field generalized momentum; the latter is expressed as a linear combination of the $-i \partial / \partial Q_f$ momentum: $i P_f = \partial / \partial Q_f - f v_f^* \sum_k k u_k \partial / \partial Q_k$.

As the $\mathbf{q}$ coordinate is a cyclic variable, the corresponding canonically conjugated operator of momentum $-i \hbar \partial / \partial \mathbf{q}$ (which coincides with the total momentum of the system) commutes with Hamiltonian (4). Correspondingly, the $\partial / \partial \mathbf{q}$ operator will be further replaced throughout by the total momentum $\mathbf{P} = \mathbf{p}_r + \hbar \sum_f f b_f^+ b_f$. In order to allow for the momentum even in the first approximation, we introduce the $\mathbf{I}$ vector such that $\mathbf{P} = \gamma^2 \mathbf{I}$. As a result, translational effects appear even in the first order. The total eigenfunction of the system can then be written as

$$\Psi(\lambda, \mathbf{q}, Q_f) = \exp(i \gamma^2 \mathbf{I} \mathbf{q} / \hbar) \Phi(\lambda, Q_f). \quad (14)$$

This function realizes a certain representation of the translation group and corresponds to the state with a fixed total momentum $\mathbf{P}$ of the system. It is convenient to perform, according to [5], one more unitary transformation of the wave function with respect to the Q_f variable and rewrite the total wave function as

$$\Psi(\lambda, \mathbf{q}, Q_f) = \exp(i \gamma \sum_f s_f Q_f) \exp(i \gamma^2 \mathbf{I} \mathbf{q} / \hbar) \Phi(\lambda, Q_f), \quad (15)$$

The complex numbers $s_f^* = s_{-f}$ and can be chosen in a way to satisfy the condition

$$\sum_f f u_f s_f = 0. \quad (16)$$

Transformation (15) can be used to expand the collective coordinates Hamiltonian in descending powers of the γ parameter:

$$H = H_2 \gamma^2 + H_1 \gamma + H_0 + \dots \quad (17)$$

where the following notations are used:

$$\begin{aligned} H_2 &= \frac{p_z^2}{2m^*} + \sqrt{2} \sum_f V_f^{(0)} u_f e^{i f \lambda / \gamma} + \sum_f \frac{\hbar \omega_f}{2} u_{-f} u_f + \\ &+ \frac{1}{2} \sum_f \hbar \omega_f (s_{-f} - \frac{i f}{\hbar} v_{-f}^*) (s_f + \frac{i f}{\hbar} v_f^*), \quad (18) \\ H_1 &= \sum_f \hbar \omega_f (s_f + \frac{i f}{\hbar} v_f^*) (f v_f^* \frac{\partial}{\partial \lambda} + P_f) + \\ &+ \sum_f [\sqrt{2} V_f^{(0)} e^{i f \lambda / \gamma} + \hbar \omega_f u_{-f} + \\ &+ (s_f + \frac{i f}{\hbar} v_f^*) \sum_m \hbar \omega_m (s_m + \frac{i m}{\hbar} v_m^*) (f m) v_{-m}^*] Q_f, \quad (19) \end{aligned}$$

$$\begin{aligned}
H_0 = & \frac{1}{2} \sum_f \hbar \omega_f Q_f Q_f - \frac{1}{2} \sum_f \hbar \omega_f [2(s_f + \frac{i\mathbf{f}}{\hbar} v_f^*) \times \\
& (-\sum_k \mathbf{k}(\mathbf{f}\mathbf{k}) v_f^* v_k Q_k \sum_m m u_m \frac{\partial}{\partial Q_m} + f v_f^* \sum_k Q_k \mathbf{k} \frac{\partial}{\partial Q_k}) + \\
& (f v_f^* \frac{\partial}{\partial \lambda} + P_f + f v_f^* \sum_k k s_k Q_k + i \frac{I}{\hbar} \sum_k \mathbf{k}(\mathbf{f}\mathbf{k}) v_f^* v_k^* Q_k) \\
& (f v_f^* \frac{\partial}{\partial \lambda} - P_f + f v_f^* \sum_k k s_k Q_k + \\
& + i \frac{I}{\hbar} \sum_k \mathbf{k}(\mathbf{f}\mathbf{k}) v_f^* v_k^* Q_k)] . \quad (20)
\end{aligned}$$

We also require that

$$\omega_f(s_f + i v_f^* \mathbf{f} \mathbf{f} / \hbar) = -i u_f \mathbf{f} \mathbf{C} . \quad (21)$$

The physical meaning of the $\mathbf{C}$ vector will be given below. Let us expand the total wave function Φ and energy E in powers of γ :

$$\begin{aligned}
E = & \gamma^2 E_2 + \gamma E_1 + E_0 + \dots , \\
\Phi = & \Phi_0 + \frac{1}{\gamma} \Phi_1 + \frac{1}{\gamma^2} \Phi_2 + \dots \quad (22)
\end{aligned}$$

Upon substituting (22) in the equation $H\Phi = E\Phi$ with Hamiltonian (17) and collecting the terms with the same γ powers, we obtain the following set of equations:

$$\begin{aligned}
H_2 \Phi_0 &= E_2 \Phi_0 , \\
H_1 \Phi_0 + H_2 \Phi_1 &= E_1 \Phi_0 + E_2 \Phi_1 , \\
H_0 \Phi_2 + H_1 \Phi_1 + H_2 \Phi_0 &= E_0 \Phi_2 + E_1 \Phi_1 + E_2 \Phi_0 , \dots (23)
\end{aligned}$$

Because the H_2 operator acts only on the field variables Q_f the zero-order wave function can be written in a multiplicative form $\Phi_0(\lambda, Q_f) = \varphi_0(\lambda) \chi(Q_f)$, where $\chi(Q_f)$ is an arbitrary function of the Q_f coordinates. Taking into account that the functions Φ_0 and Φ_1 are orthonormal, one has from the second equation in (23): $\langle \varphi_0(\lambda) | H_2 - E_2 | \Phi_1 \rangle = 0$ so that the $\chi(Q_f)$ function obeys the following equation: $\langle \varphi_0(\lambda) | H_1 | \varphi_0(\lambda) \rangle \chi(Q_f) = E_1 \chi(Q_f)$. This equation has a regular solution $\chi(Q_f)$ only if $\langle \varphi_0(\lambda) | H_1 | \varphi_0(\lambda) \rangle$ is equal to zero, because the H_1 operator is linear in the Q_f variables. Taking into account the form of Hamiltonian (19) and the obvious requirement that $E_1 = 0$, one obtains from (23) the following relation for determining the u_f values for an arbitrarily chosen $\chi(Q_f)$:

$$\begin{aligned}
\sqrt{2} V_f^{(0)} \langle \varphi_0(\lambda) | e^{i\mathbf{f}\lambda/\gamma} | \varphi_0(\lambda) \rangle + \hbar \omega_f u_f - (s_f + i \mathbf{f} v_f^* / \hbar) \times \\
\sum_m \hbar \omega_m (s_m^* - i v_m \mathbf{m} I / \hbar) (\mathbf{f} \mathbf{m}) v_m^* = 0 . \quad (24)
\end{aligned}$$

The Equation (24) is derived under the assumptions that the operators P_f satisfy the condition $\sum_f \mathbf{f} u_f P_f = 0$ that directly follows from the P_f definition. Sub-

stituting the additional requirement (21) and the condition (7) in (24) and assuming that the ground electronic state is described by the wave function $\varphi_0(\lambda)$, one finds from Equation (24) the self-consistent classical field components

$$u_f = - \frac{\sqrt{2} V_f^{(0)*} \omega_f \langle \varphi_0(\lambda) | \exp(-i \mathbf{f} \lambda / \gamma) | \varphi_0(\lambda) \rangle}{\hbar(\omega_f^2 - (\mathbf{f} \mathbf{C})^2)} . \quad (25)$$

In the strong coupling limit, the H_2 term in the Hamiltonian expansion (17) dominates and bears nontrivial information about the system. Using transformations (21) and taking into account that Hamiltonian H_2 depends only on the λ variable, one can write the energy eigenvalue of the state as $H_2 \Phi_0(\lambda, Q_f) = E_2 \Phi_0(\lambda, Q_f)$, where the notation

$$E_2 = W_2 + \sum_f \frac{\hbar}{2} \omega_f |u_f|^2 + \sum_f \frac{\hbar}{2} \omega_f |u_f|^2 \left(\frac{\mathbf{f} \mathbf{C}}{\omega_f} \right)^2 \quad (26)$$

is introduced. The first two terms in Equation (26) define the internal energy of large polaron.

The equation for determining the lowest energy state wave function $\varphi_0(\lambda)$ has the form

$$\left(\frac{\mathbf{p}_\lambda^2}{2m^*} + \sqrt{2} \sum_f V_f^{(0)} u_f e^{i\mathbf{f}\lambda/\gamma} \right) \varphi_0(\lambda) = W_2 \varphi_0(\lambda) . \quad (27)$$

Using Equation (25), it can be recast as

$$\begin{aligned}
\left(\frac{\mathbf{p}_\lambda^2}{2m^*} - 2 \sum_f \frac{\omega_f |V_f^{(0)}|^2}{\hbar(\omega_f^2 - (\mathbf{f} \mathbf{C})^2)} \langle \varphi_0(\lambda) | e^{-i\mathbf{f}\lambda/\gamma} | \varphi_0(\lambda) \rangle e^{i\mathbf{f}\lambda/\gamma} \right) \varphi_0(\lambda) \\
= W_0 \varphi_0(\lambda) \quad (28)
\end{aligned}$$

which parametrically depends on the $\mathbf{C}$ vector. The integro-differential Equation (28) must be generally solved using the self-consistent method because the classical component of the field is influenced by the electronic state to the same extent as u_f influence the electronic state.

Let us now clarify the physical meaning of the $\mathbf{C}$ vector. For this purpose, we differentiate E_2 in (26) with respect to $\mathbf{C}$

$$\begin{aligned}
\frac{\partial E_2}{\partial C_\alpha} = & \frac{\partial W_2}{\partial C_\alpha} + \sum_f \frac{\hbar \omega_f}{2} \left(u_f \frac{\partial u_f^*}{\partial C_\alpha} + u_f^* \frac{\partial u_f}{\partial C_\alpha} \right) \times \\
& \left(1 + \left(\frac{\mathbf{f} \mathbf{C}}{\omega_f} \right)^2 \right) + \sum_f \hbar |u_f|^2 f_\alpha \frac{(\mathbf{f} \mathbf{C})^2}{\omega_f} , \quad \alpha = 1, 2, 3, \quad (29)
\end{aligned}$$

The $\partial W_2 / \partial \mathbf{C}$ derivative can be found using Equation (27). This can be done by differentiating (27) with respect to $\mathbf{C}$,

$$\left(\frac{\mathbf{p}_\lambda^2}{2m^*} + \sqrt{2} \sum_f V_f^{(0)} u_f e^{i\mathbf{f}\lambda/\gamma} \right) \frac{\partial \varphi_0}{\partial C_\alpha} + \sqrt{2} \sum_f V_f^{(0)} \frac{\partial u_f}{\partial C_\alpha} e^{i\mathbf{f}\lambda/\gamma} \varphi_0 -$$

$$-\frac{\partial W_2}{\partial C_\alpha} \varphi_0 - W_2 \frac{\partial \varphi_0}{\partial C_\alpha} = 0 \quad (30)$$

The average of Equation (30) for the state with the wave function $\varphi_0(\lambda)$ is

$$\frac{\partial W_2}{\partial C_\alpha} = \sqrt{2} \sum_f V_f^{(0)} \left\langle \varphi_0(\lambda) \left| \frac{\partial u_f}{\partial C_\alpha} e^{if\lambda/\gamma} \right| \varphi_0(\lambda) \right\rangle \quad (31)$$

Using the value obtained of the classical field component u_f (25), Equation (31) can be transformed to

$$\sqrt{2} V_f^{(0)*} \left\langle \varphi_0(\lambda) \left| e^{if\lambda/\gamma} \right| \varphi_0(\lambda) \right\rangle = -\frac{\hbar u_f}{\omega_f} (\omega_f^2 - (f\mathbf{C})^2), \quad (32)$$

Then, instead of (31), the required derivative can be represented as

$$\frac{\partial W_2}{\partial C_\alpha} = -\sum_f \frac{\hbar u_f^*}{\omega_f} (\omega_f^2 - (f\mathbf{C})^2) \frac{\partial u_f}{\partial C_\alpha}. \quad (33)$$

Substituting Equation (33) into (30), we get

$$\begin{aligned} \frac{\partial E_2}{\partial C_\alpha} = & \sum_f \frac{\hbar (f\mathbf{C})^2}{\omega_f} \left(u_f^* \frac{\partial u_f}{\partial C_\alpha} + u_f \frac{\partial u_f^*}{\partial C_\alpha} \right) + \\ & \sum_f \hbar \omega_f |u_f|^2 f_\alpha \left(\frac{f\mathbf{C}}{\omega_f} \right)^2 \end{aligned} \quad (34)$$

We now determine the $\mathbf{I}$ vector. For this purpose, let us multiply condition (21) by $f u_f$ and sum over the wave vector f . After applying the requirement (7) and condition (16), we obtain the following expression for the $\mathbf{I}$ vector:

$$\mathbf{I} = \hbar \sum_f \frac{f(f\mathbf{C}) |u_f|^2}{\omega_f}. \quad (35)$$

Let us differentiate (35) with respect to the $\mathbf{C}$ vector,

$$\begin{aligned} \frac{\partial \mathbf{I}}{\partial \mathbf{C}} = & \hbar \sum_f \frac{(ff)}{\omega_f} |u_f|^2 + \hbar \sum_f f \frac{(f\mathbf{C})}{\omega_f} u_f \frac{\partial u_f}{\partial \mathbf{C}} + \\ & \hbar \sum_f f \frac{(f\mathbf{C})}{\omega_f} u_f^* \frac{\partial u_f^*}{\partial \mathbf{C}} \end{aligned} \quad (36)$$

One can easily see from a comparison of Equation (34) with (36) that

$$\frac{\partial E_2}{\partial C_\beta} = \sum_\alpha C_\alpha \frac{\partial I_\alpha}{\partial C_\beta}, \quad \alpha, \beta = 1, 2, 3. \quad (37)$$

We finally obtain

$$\begin{aligned} \frac{\partial E_2}{\partial I_\alpha} = & \sum_\gamma \frac{\partial E_2}{\partial C_\gamma} \frac{\partial C_\gamma}{\partial I_\alpha} = \sum_\beta C_\beta \left(\sum_\gamma \frac{\partial I_\beta}{\partial C_\gamma} \frac{\partial C_\gamma}{\partial I_\alpha} \right) \\ = & \sum_\beta C_\beta \frac{\partial I_\beta}{\partial I_\alpha} = C_\alpha \end{aligned} \quad (38)$$

Consequently, the following result is obtained after

using the definition for the total momentum $\mathbf{P} = \gamma^2 \mathbf{I}$: $\mathbf{C} = \gamma^2 \partial E_2 / \partial \mathbf{P}$. However, by definition, $\partial E_2 / \partial \mathbf{P}$ is merely the velocity $\mathbf{v}$. Therefore, the $\mathbf{C}$ vector is related to the translational velocity of the polaron by expression:

$$\mathbf{C} = \gamma^2 \partial E_2 / \partial \mathbf{P} = \gamma^2 \mathbf{v}, \quad (39)$$

and determines to the γ^2 factor, the mean velocity of the center of inertia of system. Hence the energy eigenvalue (28) of the self-consistent ground electronic state W_2 explicitly depends on the translational velocity of the quasiparticle.

Let us now determine the translational effective mass of the polaron. Using (26) and assuming that the velocity of the center of inertia is small, which ordinary holds for thermal motion, we expand the energy eigenvalue of the system in series

$$E_2^{(0)} = W_2^{(0)} + \sum_f \frac{\hbar \omega_f}{2} |u_f^{(0)}|^2 + \sum_f \frac{\hbar}{2} |u_f^{(0)}|^2 \frac{(f\mathbf{C})^2}{\omega_f} + \dots \quad (40)$$

The quantities that correspond to the zero translational velocity of the polaron are labeled by superscripts in Formula (40). After substituting relation (39) in (40), the following expression is finally obtained for the ground state energy of the system:

$$E_2^{(0)} = W_2^{(0)} + \sum_f \frac{\hbar \omega_f}{2} |u_f^{(0)}|^2 + \sum_f \frac{\gamma^4 \hbar}{2} |u_f^{(0)}|^2 \frac{(f\mathbf{v})^2}{\omega_f} + \dots \quad (41)$$

The last term in (41) can be regarded as the kinetic energy of the translational motion of the free particle as a whole: $E_{kin} = m^{**} v^2 / 2$, where the notation m^{**} stands for the ground state translational mass of the large polaron:

$$\begin{aligned} m^{**} = & \frac{\gamma^4}{3} \sum_f \frac{\hbar (ff) |u_f^{(0)}|^2}{\omega_f} = \\ & \frac{\gamma^4}{3} \sum_f \frac{2 |V_f^{(0)}|^2 |\langle \varphi_0(\lambda) | \exp(i\lambda f / \gamma) | \varphi_0(\lambda) \rangle|^2 (ff)}{\hbar \omega_f^3} \end{aligned} \quad (42)$$

If the electron is trapped by the polarization field, the interaction of the particle with the field fully “consumes” the rest mass of an electron. Indeed, it follows from the order-of-magnitude analysis of the variables in Equation (42) that the effective mass $m^{**} \approx \gamma^8 m^* \gg m^*$ is dominated by the field inertia.

The translational mass m^{**} can be calculated using (42) if the wave function $\varphi_0(\lambda)$ is known. It can be found by solving the nonlinear integro-differential Equation (27). However, in practice, it is convenient to determine the ground state wave function using a direct variational method and varying the total energy functional

$$F[\varphi_0(\lambda)] = -\frac{\hbar^2}{2m^*} \langle \varphi_0(\lambda) | \nabla^2 | \varphi_0(\lambda) \rangle - \frac{1}{2} \sum_f \hbar \omega_f |u_f^{(0)}|^2 \quad (43)$$

with

$$u_f^{(0)} = -\frac{\sqrt{2}V_f^{(0)*} \langle \varphi_0(\lambda) | \exp(-if\lambda/\gamma) | \varphi_0(\lambda) \rangle}{\hbar\omega_f}.$$

The approximate analytic form of a trial variational wave function $\varphi_0(\lambda)$ of a nondegenerate ground state can be established by expanding the exponential in Equation (27). Upon restricting ourselves to the quadratic terms in the resultant series, we obtain the oscillator equation

$$\left\{ \frac{p_\lambda^2}{2m^*} + \sqrt{2} \sum_f V_f^{(0)} u_f^{(0)} \left[1 + i \frac{f\lambda}{\gamma} - \frac{1}{2} \left(\frac{f\lambda}{\gamma} \right)^2 + \dots \right] \right\} \varphi_0(\lambda) = W_2^{(0)} \varphi_0(\lambda), \quad (44)$$

whose solutions are the Hermite polynomials. These functions can be regarded as good approximations to the wave functions of the ground and low excited states of a system with large γ . Therefore, for slow translational motion of particle we choose the trial ground state wave function in the following analytic form: $\varphi_0(\lambda) = (\pi^3 \alpha^6 \gamma^6)^{-1/4} \exp(-\lambda/\sqrt{2}\alpha\gamma)^2$, where α is the variational parameter. Such an approximation for the trial function is consistent with the results of the shifted $-1/N$ -expansion numerical technique that was applied in [6] and technique solving of nonlinear integral Equation [7] to the analysis of Equation (43).

It was established earlier that the optical transition from the ground to the lowest lying electronic p -state is most probable (oscillator strength of 0.77 [1]). Because the transition time $\tau_0 = (\Delta E/\hbar)^{-1} \approx 10^{-15} s$ is much shorter than the relaxation time $\tau \approx 10^{-13} s$ of the polar lattice, the optical transition can be considered as vertical, *i.e.*, proceeding at a fixed value $u_f^{(0)}$ of the classical component of the polarization field in the lowest electronic state. According to this premise and based on Equation (28), the initial electronic state is described by the equation:

$$\left\{ \frac{p_\lambda^2}{2m^*} - 2 \sum_f \frac{|V_f^{(0)}|^2}{\hbar\omega_f} \langle \varphi_0(\lambda) | e^{-if\lambda/\gamma} | \varphi_0(\lambda) \rangle e^{if\lambda/\gamma} \right\} \varphi_0(\lambda) - 2 \sum_f \frac{|V_f^{(0)}|^2}{\hbar\omega_f} \langle \varphi_0(\lambda) | e^{-if\lambda/\gamma} | \varphi_0(\lambda) \rangle e^{if\lambda/\gamma} \left(\frac{f\mathbf{C}}{\omega_f} \right)^2 \varphi_0(\lambda) = (W_0^{(0)} + W_0^{(1)}) \varphi_0(\lambda), \quad (45)$$

whereas the final state (k) of electronic optical transition obeys the equation

$$\left\{ \frac{p_\lambda^2}{2m^*} - 2 \sum_f \frac{|V_f^{(0)}|^2}{\hbar\omega_f} \langle \varphi_0(\lambda) | e^{-if\lambda/\gamma} | \varphi_0(\lambda) \rangle e^{if\lambda/\gamma} \right\} \varphi_k(\lambda) - 2 \sum_f \frac{|V_f^{(0)}|^2}{\hbar\omega_f} \langle \varphi_0(\lambda) | e^{-if\lambda/\gamma} | \varphi_0(\lambda) \rangle e^{if\lambda/\gamma} \left(\frac{f\mathbf{C}}{\omega_f} \right)^2 \varphi_k(\lambda) =$$

$$(W_k^{(0)} + W_k^{(1)}) \varphi_k(\lambda). \quad (46)$$

In Equations (45) and (46), the translational velocity of large polaron is assumed to be small, *i.e.*, $(f\mathbf{v})^2 < \omega_f^2$ and only quadratic terms are retained in the expansion of the potential. The wave function of the excited electronic p -state is chosen in the form

$$\varphi_k(\lambda) = \left(\frac{2}{\pi^{3/2} \gamma^5 \beta^5} \right)^{1/2} \frac{\lambda}{\gamma} \cos \theta e^{-(\lambda/\sqrt{2}\beta\gamma)^2} \quad (47)$$

where β is the variational parameter.

It is convenient to transfer from Equation (45) to the equations

$$\begin{aligned} W_0^{(0)} + W_0^{(1)} &= \left\langle \varphi_0(\lambda) \left| \frac{p_\lambda^2}{2m^*} \right| \varphi_0(\lambda) \right\rangle - 2\gamma^2 \sum_f \frac{|V_f^{(0)}|^2}{\hbar\omega_f} \frac{\rho_f^{(0)} \rho_f^{(0)*}}{\hbar\omega_f} - \\ &\quad 2\gamma^2 \sum_f \frac{|V_f^{(0)}|^2}{\hbar\omega_f} \frac{\rho_f^{(0)} \rho_f^{(0)*}}{\hbar\omega_f} \left(\frac{f\mathbf{v}}{\omega_f} \right)^2, \\ W_k^{(0)} + W_k^{(1)} &= \left\langle \varphi_k(\lambda) \left| \frac{p_\lambda^2}{2m^*} \right| \varphi_k(\lambda) \right\rangle - 2\gamma^2 \sum_f \frac{|V_f^{(0)}|^2}{\hbar\omega_f} \frac{\rho_f^{(0)} \rho_f^{(k)*}}{\hbar\omega_f} - \\ &\quad 2\gamma^2 \sum_f \frac{|V_f^{(0)}|^2}{\hbar\omega_f} \frac{\rho_f^{(0)} \rho_f^{(k)*}}{\hbar\omega_f} \left(\frac{f\mathbf{v}}{\omega_f} \right)^2 \end{aligned} \quad (48)$$

Here $\rho_f^{(0)}$ and $\rho_f^{(k)}$ are the Fourier transforms of electron densities in the ground and excited electronic states. In the adopted approximation, the frequency of the most active electronic dipole transition can be written as

$$\hbar\Omega_{0k} = (W_k^{(0)} - W_0^{(0)}) + (W_k^{(1)} - W_0^{(1)}) = \hbar\Omega_{0k}^{(0)} + \hbar\Omega_{0k}^{(1)}, \quad (49)$$

where the second term depends on the polaron velocity, whereas the first term determines the optical transition frequency at the band maximum in the state with the zero center-of-mass velocity. The frequency $\hbar\Omega_{0k}^{(1)}$ can be found from Equation (48)

$$\hbar\Omega_{0k}^{(1)} = S_{0k} v^2. \quad (50)$$

The following notation is used in Equation (50)

$$S_{0k} = 2\gamma^4 \sum_f \frac{(ff) |V_f^{(0)}|^2}{\hbar\omega_f^3} \rho_f^{(0)} (\rho_f^{(0)} - \rho_f^{(k)}). \quad (51)$$

Such an approach in the phototransition calculation is justified if the impurity absorption spectrum lies between the IR-absorption region of the polar lattice oscillating and the absorption region of the strongly bound electrons of base material. We assume that the polarons are in thermal equilibrium and that the quasiparticle distribution over the velocities $\mathbf{v}$ is Maxwellian: $F(\mathbf{v}) = \pi^{-3/2} v_0^{-3} e^{-(v/v_0)^2}$, where $v_0^2 = 2kT/m^*$. Then, the full width at half maximum of the optical absorption spectra is related to the standard deviation D as $W_{1/2} = 2D\sqrt{2\ln 2}$. In

this approximation, the intensity is symmetrically distributed relative to the $\hbar\Omega_0^{(0)}$ frequency. The band becomes asymmetric in the presence of photo transitions to high-lying electron excited states. With the Maxwellian velocity distribution, the variance is $D^2 = \langle \Omega_{0k}^2 \rangle - \langle \Omega_{0k} \rangle^2 = 6S_{0k}^2 (kT/m^{**})^2$. Then, the band width $W_{1/2}$ is equal

$$W_{1/2} = 4 |S_{0k}| (kT/m^{**}) \sqrt{3 \ln 2}. \quad (52)$$

The approach presented to estimating the broadening of the absorption spectra is valid if the inequality $t \gg c/\Omega_{0k}^{(0)}v$ is fulfilled, where t is the mean free path time of the quasi-particle, and c is the light velocity. This inequality is fulfilled for the transition frequencies and temperatures of interest.

3. Discussions and Conclusions

The theory is applied to free polaron in ammonia. The electron is self-trapped owing to strong interaction with the quantum polarization field, which is generated by the dipole ammonia molecules librating around their equilibrium positions. Various investigations [8-12] have shown that many properties of electrons in ammonia may be described using the model of continual polarons. Within the framework of this model the possibility of existence coupled of two-electronic bipolaron formations in singlet state [13,14] has been established and magnetic and optical properties of metal-ammonia systems are explained [8,9]. The criteria for validity of the theory reduce to the following inequality: $\hbar\omega_f < \hbar\omega_e < \hbar\omega_m$. For an electron in ammonia, $\hbar\omega_e = 0.885 \text{ eV}$ is the energy of the most active optical transition of a self-trapped electron [11], $\hbar\omega_f \approx 0.4 \text{ eV}$ is the energy of the longitudinal polarization oscillations of the medium [15], and $\hbar\omega_m \approx 6 \text{ eV}$ is the excitation energy of electrons of the main substance [11]. The orientational oscillations of molecules about their equilibrium position in a polar liquid form elastic waves that may be treated as in crystal. As a result of the directionality and saturation of the intermolecular hydrogen bonds for ammonia, the "quasicrystallinity" of the structure is comparatively well defined. Far from the critical point, the thermal vibrations of the molecules may be reduced to a set of Debye waves, as in a polar crystal, where the spectrum of collective oscillations in the liquid has a cutoff at longer wavelengths than in crystals [16] on account of the translational motion of the particles. The elastic continuum approximation does not generally allow for anisotropy and is far better applicable to a liquid than to a crystal [17].

The width $W_{1/2}$ of the optical spectrum of free polaron in ammonia can be numerically estimated if the numerical parameters of the theory are given. At low concentrations of the polarons, the dielectric constants ϵ_∞ and ϵ_s

can be set equal to their values in pure ammonia; i.e., $\epsilon_\infty = 1.756$ and $\epsilon_s = 22.7$. The electron effective mass m^* is usually determined from a comparison of the experimental and theoretical positions of the absorption band maximum. At sufficiently low temperatures, the transition frequency is dominated by the first term in (49). Indeed, for the experimental measurements at temperature $T = 225 \text{ K}$ [11], we have the ratio

$$\frac{\hbar\Omega_{0k}^{(1)}}{\hbar\Omega_{0k}^{(0)}} = \frac{S_{0k}v^2}{\hbar\Omega_{0k}^{(0)}} = \frac{3S_{0k}kT}{m^{**}\hbar\Omega_{0k}^{(0)}} \approx \frac{1}{\gamma^4} \ll 1 \quad (53)$$

In this estimate, it is taken into account that, according to Formula (42), the effective mass of the solvated electron is $m^{**} = 0.02\gamma^8 m^*$. Therefore, the translational velocity contributes only insignificantly to the optical transition. It is mainly determined by the $\hbar\Omega_{0k}^{(0)} = W_0^{(0)} - W_k^{(0)}$ term. A comparison of the theoretical position of the band maximum with its experimental value 0.88 eV [11,18] yields the value of $m^* = 1.73m$ for the electron effective mass, where m is the mass of a free electron.

Let us estimate numerically the contribution from the translations of quasiparticle as whole to the full width at half maximum of the absorption spectra. For definiteness, we use the following parameter values: $\omega_0 = 5.8 \times 10^{13} \text{ s}^{-1}$ [8], $\gamma^2 = 13.5$. Then, Formulas (51) and (52) yield the value of $W_{1/2} = 0.23 \text{ eV}$ for the contribution from the thermal motion of the quasiparticle, which represents an appreciable part of the experimentally observed value 0.46 eV [11,18]. The remaining part in the broadening of the absorption spectra of the polaron is likely to be due to the fluctuations of the polarization field [4] or other mechanisms which short discussed in Introduction. Equations (51) and (52) can be also used to calculate the temperature band-width coefficient; it occurred to be equal to $dW_{1/2}/dT = 1.03 \times 10^{-3} \text{ eV/K}$. The experimentally measured [19,20] range $(0.6 - 1.6) \times 10^{-3} \text{ eV/K}$ of the temperature coefficient is in satisfactory agreement with the calculated value.

4. References

- [1] S. I. Pekar, "Research in Electron Theory of Crystals," *US AEC Report*, 1960.
- [2] M. J. Goovaerts, J. M. De Sitter and J. T. Devreese, "Numerical Study of Two-Phonon Sidebands in the Optical Absorption of Free Polarons in the Strong-Coupling Limit," *Physical Review B*, Vol. 7, No. 6, 1973, pp. 2639-2644.
- [3] D. Emin, "Optical Properties of Large and Small Polarons," *Physical Review B*, Vol. 48, 1993, pp. 13691-13696.
- [4] Y. T. Mazurenko and V. K. Mukhomorov, "On the Mechanism of Optical Spectra Broadening of Polarons," *Optics and Spectroscopy (USSR)*, Vol. 41, 1976, pp. 51-56.

- [5] N. N. Bogolyubov, "About One New Form of an Adiabatic Perturbation Theory in a Problem about Interaction Particles with a Quantum Field," *Ukrainskii Matematicheskii Zhurnal (Russian Edition)*, Vol. 2, 1950, pp. 3-24.
- [6] V. K. Mukhomorov, "On a Spectra of the Coupled States of Polaron in a Limit of Adiabatic and Strong Coupling," *Optics and Spectroscopy (USSR)*, Vol. 71, 1991, pp. 958-965.
- [7] V. K. Mukhomorov, "Singlet and Triplet States of a Continuum Bipolaron for Adiabatic and Strong Coupling Limit," *Optics and Spectroscopy (USSR)*, Vol. 74, 1993, pp. 544-655.
- [8] V. K. Mukhomorov, "Bipolaron States of Electrons and Magnetic Properties of Metal-Ammonia Systems," *Physica Status Solidi B*, Vol. 219, 2000, pp. 71-89.
- [9] V. K. Mukhomorov, "Ground and Excited States of a Three-Dimensional Continual Bipolaron," *Physica Status Solidi B*, Vol. 231, 2002, pp. 462-476.
- [10] J. Jortner, "Energy Levels of Bound Electrons in Liquid Ammonia," *Journal of Chemical Physics*, Vol. 30, 1959, pp. 839-846.
- [11] J. C. Thompson, "Electrons in Liquid Ammonia," Clarendon, Oxford, 1976.
- [12] N. F. Mott, "Metal-Insulator Transitions," Taylor and Francis, London, 1974.
- [13] V. K. Mukhomorov, "Stability of Bipolarons, Electron-Electron Correlations, the Variational Principle, and the Virial Theorem," *Physics of the Solid State*, Vol. 48, 2006, pp. 864-870.
- [14] V. K. Mukhomorov, "About the Problem of the Existence of a Three-Dimensional Bipolaron," *Physica Scripta*, Vol. 69, 2004, pp. 139-145.
- [15] V. K. Mukhomorov, "Bipolaron Formations and Interpolaron Interactions in Dielectric Layers," *Physica Scripta*, Vol. 79, No. 6, 2009, pp. 065704-065709.
- [16] I. Z. Fisher, "Statistical Theory of Liquids," University of Chicago Press, Chicago, 1965.
- [17] A. I. Gubanov, "Quantum Electron Theory of Amorphous Conductors," Consultants Bureau, New York, 1965.
- [18] E. J. Hart and M. Anbar, "The Hydrated Electron," Wiley, New York, 1970.
- [19] R. C. Dauthit and J. L. Dye, "Absorption Spectra of Sodium and Potassium in Liquid Ammonia," *Journal of the American Chemical Society*, Vol. 82, 1960, pp. 4472-4478.
- [20] S. Arai and M. G. Sayer, "Absorption Spectra of the Solvated Electron in Polar Liquids: Dependence on Temperature and Composition of Mixture," *Journal of Chemical Physics*, Vol. 44, 1966, pp. 2297-2301.

On the Nature of Dark Matter and Dark Energy

Baurov Yury Alexeevich¹, Malov Igor Fedorovich²

¹Central Research Institute of Machine Building, Korolyov, Russia

²P. N. Lebedev Physical Institute, Russian Academy of Sciences, Moscow, Russia

E-mail: baurov@mail.ru, malov@prao.ru

Received February 19, 2010; revised March 17, 2010; accepted March 20, 2010

Abstract

It is known that all candidates in dark matter (DM) particles (neutrinos, axions, supersymmetric particles *etc.*) can not explain the basic properties of DM. The same can be said on the proposed candidates in dark energy (DE) (for example, quintessence). In the paper it is shown that some problems connected with DM and DE can be solved in the framework of the byuon theory. Basic axioms and some conclusions of this theory are discussed. The existence of fundamental unobserved elements in nature, byuons is declared. Physical space in our Universe is the quantum medium of special objects 4b, formed in four-contact interactions of byuons ($m_{4b} c^2 \approx 33\text{eV}$). These objects determine the average density of substance (DM) in the Universe $\sim 10^{-29} \text{ g cm}^{-3}$. The byuon theory predicts a new interaction of natural objects with physical vacuum. This new force can cause the observed acceleration of our Universe. The estimations show that it is higher than the gravitational force at distances of order to 10^{26} - 10^{28} cm. Some other consequences of the byuon theory are considered.

Keywords: Dark Matter, Dark Energy, Byuon, New Force

1. Introduction

Some hard problems have appeared in astrophysics during the last dozens of years. Observations show that approximately 4% of the cosmological energy density is accounted for by baryons, 23% by “the dark matter” and the reminder by “the dark energy” (see, for example, [1,2]).

There are some evidences for the existence of dark matter (DM) and dark energy (DE). Here we enumerate basic ones only.

1) In 1937 F.Zwicky measured velocities of galaxies in the Coma cluster and concluded that the total mass of this cluster must be much more than observable one to prevent the escaping of investigated galaxies from the cluster.

2) The summarized mass of the observed gas and galaxies in the number of clusters is not enough to keep them inside of the cluster.

3) The gravitational lensing by clusters of galaxies gives the mass of such lens much more than calculated masses of clusters.

4) The rotation curves of galaxies [3] show that the total mass of the individual galaxy is approximately one order higher than the mass of gas and all stars observed in this galaxy.

5) The observations of supernovae in distant galaxies (see, for example, [4]) show that our Universe expands with an acceleration, and there is a source causing such type of expansion.

The nature of dark matter and dark energy is unknown up to now.

2. Dark Matter

DM is not observed as a shining matter and must be characterized by extremely weak electromagnetic interactions. It must be approximately collisionless and non-relativistic.

DM is not primarily baryonic. The calculated amount of deuterium should be much smaller than observed one if the average baryon density was an order of magnitude higher than the modern value (~ 0.3 baryons per cubic meter).

The mass interval for the possible candidates in DM is huge (from 10^{-22} eV to $10^6 M_\odot \approx 10^{72} \text{ eV}$).

Let us discuss the most probable candidates in DM.

1) Axions, light pseudo-scalar bosons [5,6] with mass $\mu\text{eV} \lesssim m \lesssim \text{meV}$. They could be detected by resonant axion-photon conversion in a magnetic field [7,8].

2) Neutrinos. Some laboratory experiments and cos-

mological restrictions give the mass interval for all kinds of neutrinos:

$50 \text{ meV} \lesssim \Sigma m_\nu \lesssim 0.7 \text{ eV}$, or $0.0005 < \Omega_\nu h^2 < 0.0076$, where $\Omega_\nu = \rho_\nu/\rho_c$, $h = H_0/100 \text{ km/sec/Mpc}$, $\rho_c = 3 H_0^2/(8\pi G)$ is the critical density of the Universe, H_0 is the Hubble constant.

Super-symmetric theories put bosons and fermions in common multiplets. They give some possible candidates in DM.

1) The super-partner of the graviton, gravitino with the spin $3/2$ [9].

2) Neutralinos. These are the four spin $1/2$ Majorana fermion super-partners of the neutral gauge and Higgs bosons (χ_{1-4}^0) [10]. There are also two charged Dirac fermion super-partners of charge gauge and Higgs bosons, charginos ($\chi_{1-2}^\pm$).

3) Axinos, a spin $1/2$ partner of the axion [11].

4) Non-topological solitons, Q-balls [12].

5) If our four-dimensional space-time is embedded in a higher dimensional space, the Kaluza – Klein excitations of Standard Model states along the orthogonal dimensions may be as DM candidates [13].

6) Objects of many dimensions (branes) are described in string theories. Their fluctuations have been considered as particles (branons) which could be DM candidates [14].

7) DM could be an ordinary matter in the mirror world where the only communication is gravitational. In this case our Universe and a mirror universe are two branes in a higher dimensional space [15].

8) At the last stages of inflation gravitational interactions can produce a lot of weakly interacting massive particles which for mass scales of 10^{13} GeV could account for DM [16].

9) Primordial black holes have been considered as candidates in DM as well [17].

So, as observations give, baryons provide approximately 4% of DM, neutrinos $\sim 0.3\text{-}3\%$ of it. The rest (20-25%) is a non-baryonic DM. The nature of this part of DM is unclear. There are many problems with theoretical foundations and experimental evidences of the existence of particles mentioned above and described in cited papers.

3. Dark Energy

The nature of DE is much more unclear than that of DM. It is necessary for it to have the equation of state of the following form (see, for example, [18]):

$$p = w \rho \quad (1)$$

where p is pressure and ρ is the energy density. The most probable value of the parameter w is approximately -1 , as follows from the known observations. This implies that the energy density of such substance is constant and corresponds to the flat universe, *i.e.* the curvature K of

the spatial sections (slices at constant cosmic time) is equal to zero. DE causes the acceleration of the expansion of our Universe. **Figure 1** shows the sum (solid line) of two potentials: The usual (negative) gravitational potential φ_1 (broken line) causing the attraction of two bodies and a positive constant potential φ_2 giving the repulsion at large distances ($r > r_*$).

One of the possible sources of DE is “quintessence” [19], a scalar field Φ rolling slowly in a potential. Most quintessence models give for such scalar fields

$$m_\Phi c^2 \sim 10^{-33} \text{ eV} \quad (2)$$

In quantum field theory light scalar fields are hard to understand. In any case these fields should give rise to long-range forces which must be observable. It is surprisingly why such quintessence field has not been detected up to now.

There are many problems with other models of DE (see, for example, [18]).

In this paper we shall try to explain DM and DE in the framework of the byuon theory. First of all we will describe briefly the foundations of this theory.

4. Basic Axioms and Hypotheses: Space, Time, and Physical Vacuum in the Light of the Byuon Theory

Any theory begins with axioms, that is, with basic postulates accepted without proofs.

Thus, let us assume that there are no space, no time, no world of elementary particles of which all physical bodies consist, but there is an unobservable object, a byuon $\Phi(i)$ [20-25], being unobservable in itself and characterized by discrete states (*i.e.* numbered by the series of natural numbers) having inherent “vectorial” property. The expression for $\Phi(i)$ is

$$\Phi(i) = [\mathbf{A}_g X(i)] \vee (-\sqrt{-1} [\mathbf{A}_g X(i)]) \quad (3)$$

where $X(i)$ is “length” of the byuon, a real (positive or negative) value depending on the index $i = 0, 1, 2, \dots, k, \dots$

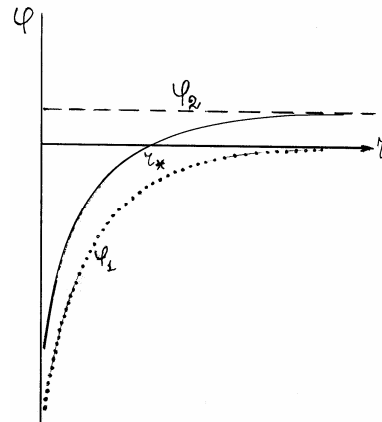


Figure 1. Scheme of two body interactions in our Universe.

Index i is a quantum number for $\Phi(i)$ ¹. The explanation of square brackets has been given further. The dimension of byuons is equal to the dimension of electric charge (in the CGSE-system) or of magnetic flux or of the Dirac's monopole. The quantity A_g is some internal potential being equal in magnitude to the cosmological vectorial potential A_g , a new fundamental vectorial constant introduced in [26,27] ($A_g \approx 1.95 \cdot 10^{11} \text{Gs}\cdot\text{cm}$).

Thus, $\bar{\Phi}(i)$ can take both real and pure imaginary values.

The whole set $\Phi(i)$ forms a one-dimensional space R_1 in index i .

According to this conception, by the discrete time is meant, for the byuon, a discrete change in the index i (its increase or decrease) is possible. In connection with the discrete time, a quantum of time τ_0 and quantum of space $\tilde{x}_0$ are introduced in the one-dimensional discrete space R_1 formed by byuons ($\tau_0 \approx 0.9 \times 10^{-43} \text{s}$, $\tilde{x}_0 \approx 2.8 \times 10^{-33} \text{cm}$). The distance between byuons is defined therewith as a difference in their lengths $x(i)$. The space R_1 is discrete by definition.

Since the space R_1 is discrete, one of methods of parametrization of $X(i)$ is $X(i) = \tilde{x}_0 \cdot i$, or $X(i) = -\tilde{x}_0 \cdot i$.

Statics. In the set $\{\Phi(i)\}$, there are meant no static states with time $t > \tau_0$.

Kinematics. Depending on whether the length $X(i)$ positive or negative, decreases or increases in magnitude, free byuons (*i.e.* not interacting one with another) can be only in one of the four so called vacuum states (VS) II^+ , I^+ , I^- , II^- . Further we will omit sometimes VS in the expressions like VS II^+ .

Introduce the following definitions.

1) A free byuon is in the state II^+ if its positive length discretely, in a quantum of time τ_0 , increases by a quantum of distance $\tilde{x}_0$ with the speed of propagation (increase in length) $c = \frac{\tilde{x}_0 - 0}{\tau_0} = c_0$ (c_0 is the light speed).

Hence the speed of byuons is the ratio of their lengths to the postulated quantum of time.

2) A free byuon is in the state I^+ if its positive length discretely, in a quantum of time τ_0 , decreases by $\tilde{x}_0$. In this case $c = \frac{0 - \tilde{x}_0}{\tau_0} = -c_0$.

3) A free byuon is in II^- if the modulus of its negative length increases by $\tilde{x}_0$ in time τ_0 with $c = \frac{-\tilde{x}_0 - 0}{\tau_0} = -c_0$.

4) A free byuon is in I^- if the modulus of its negative

length discretely, in time τ_0 , decreases by $\tilde{x}_0$. In this case $c = \frac{0 - (-\tilde{x}_0)}{\tau_0} = c_0$.

From the definition of byuons it is seen that they are in perpetual dynamics of generation and annihilation, extension and contraction. The collection of free (not interacting) byuons in VSs II^+ , I^+ , I^- , II^- forms physical vacuum of the one-dimensional space R_1 of index i (about properties of R_1 will be said below). Recall however that in this model of physical vacuum, *time is a sequence of events of byuon generation* (extension) and "collapse" (contraction). These correspond to each byuon its own count of time measured by the natural number series. One of the two directions of the one-dimensional space R_1 , coincident with that of a byuon with the maximum $x(i)$ in VS II^+ , is taken for the positive direction of the vector $\bar{A}_G$ and $\bar{\Phi}(i)$.

The average magnitudes for byuons being in the above described VSs at maximum $i=k$, are determined from the following expressions (see 1, and [20-22]):

$$\begin{aligned} [\bar{A} \cdot X]_{II^+}^{i+1} &= \bar{A}_G \left[\frac{2i+1}{2} \cdot \frac{\text{const}_1}{k \cdot A_G} \right] & X > 0, \\ [\bar{A} \cdot X]_{I^+}^i &= -\sqrt{-1} \cdot \bar{A}_G \left[\frac{2i+1}{2} \cdot \frac{\text{const}_1}{k \cdot A_G} \right] & X > 0, \\ [\bar{A} \cdot X]_{II^-}^{k-i} &= \sqrt{-1} \cdot \bar{A}_G \left[\frac{2(k-i)-1}{2} \cdot \frac{\text{const}_1}{k \cdot A_G} \right] & X < 0, \\ [\bar{A} \cdot X]_{I^-}^{k-i-1} &= -\bar{A}_G \left[\frac{2(k-i)-1}{2} \cdot \frac{\text{const}_1}{k \cdot A_G} \right] & X < 0, \end{aligned} \quad (4)$$

where $\text{const}_1 = \frac{\sqrt{hc_0}}{4\sqrt{3}} \cdot \frac{hc_0}{e_0^2}$, is some constant. As we will see later h is equal to the Plank constant and e_0 is the electron charge. (See Appendix 1). The square brackets mean the average value for byuons between previous and subsequent magnitudes because any observations are possible during time intervals much more than a time quantum only.

Assume that for the byuons with the length greater than $\tilde{x}_0$, only contact interactions are realized, by which we will mean existence of at least two byuon VSs at a quantum of space R_1 .

Hypothesis 1. Assume the observable three-Dimensional space R_3 to appear as a result of minimization of the potential energy (PE) of VSs byuon interactions in the one-dimensional space R_1 formed by them. We construct PE from the taking into consideration of dimensions. More precisely, the space R_3 is fixed by us as the result of this byuon dynamics. In the space R_3 therewith the dynamical processes for objects with the residual positive potential energy of byuon interactions originate, and in consequence, the wave properties of elementary

¹It should be explained that the vector $\Phi(i)$ is not an ordinary vector in some space but an object with "inner" vectorial properties that are manifesting themselves when the value $x(i)$ changes in the process of physical space formation.

particles arise.

PE means the extreme value of the expression with the dimension of energy. This expression is formed using all possible vacuum states of byuons and the distance in the R_1 space. This distance is taken positive values only.

The proposed hypothesis requires to develop a mathematical model based on a new algebra of probabilistic events since the elementary events (a discrete decrease ($\bar{A}$) or increase (D) in the length of byuon) are assumed to be probabilistic in character. Hence for the byuons of the minimum length we may say about the existence, with certain probability, of the events $\bar{A} \cdot D \neq 0$. Note that in [9] an algebra of events is given, being a development of the Boolean algebra with the proviso that $\bar{A} \cdot D = 1$. For the deterministic approach used in [28], the event $\bar{A} \cdot D \neq 0$ is illogical by von Neumann, but in the probabilistic space of events the existence of $\bar{A} \cdot D \neq 0$ is possible.

In this paper only the physical statement of the problem will be considered, and results of evaluations made in support of the hypothesis advanced, will be given.

The space R_1 is formed from the set of byuons in such a manner that at its i -th point there exist all the byuons with the lengths smaller than $X(i)$ or equal to $X(i)$ for $X(i) > 0$, and those with the absolute values smaller than $X(k-i)$ for $X(k-i) < 0$, where k is some period in i .

The assumption that two neighboring byuons (the i -th and $(i+1)$ -th; $(i+1)$ -th and $(i+2)$ -th etc.) being in vacuum states II^+ will interact, is unreasonable since in this case the definition of byuons for this VS would be violated at the point of interaction. Such interaction is possible only between the i -th and $(i-k)$ -th byuons in the state II^+ if they form a "loop" in the space R_1 (by the "loop", the periodicity of the process in i is implied), i.e. the two byuons II_i^+ and II_{i-k}^+ will be observed simultaneously at one point of the space R_1 . The least possible value of k is $k = 3$. In **Figure 2** shown is the interaction of byuons in the vacuum states II_1^+ and II_4^+ (the smallest loop). The byuons in the state II^- interact likely.

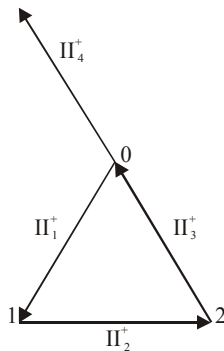


Figure 2. Interaction of byuons in vacuum states II_1^+ and II_4^+ (the smallest loop).

The byuon states I^+ and I^- can occur only if the byuons have already been in VS II^+ and II^- , respectively. At maximum positive potential energy of byuon interaction there exists a single variant of "occupancy" (**Figure 3**).

The probability of the minimum four-contact interaction of the neighboring in i byuons $I^+II^+ \times I^-II^-$ ("×" symbolizes interaction) with randomly appearing states I^+ and I^- , is equal to $1/16$ [20-22]. That is quite understandable when analyzing possible four-contact interactions (see **Figure 4**). All other possible variants of the four-contact interaction are unobservable either because one cannot introduce them without violating the definition of byuons or in view of imaginary energy of such interaction.

Note once more that there exist only two directions in the one-dimensional world, the first of which corresponds to increasing index i for byuons with $X > 0$ (vacuum II^+), and the second corresponds to decrease in i for

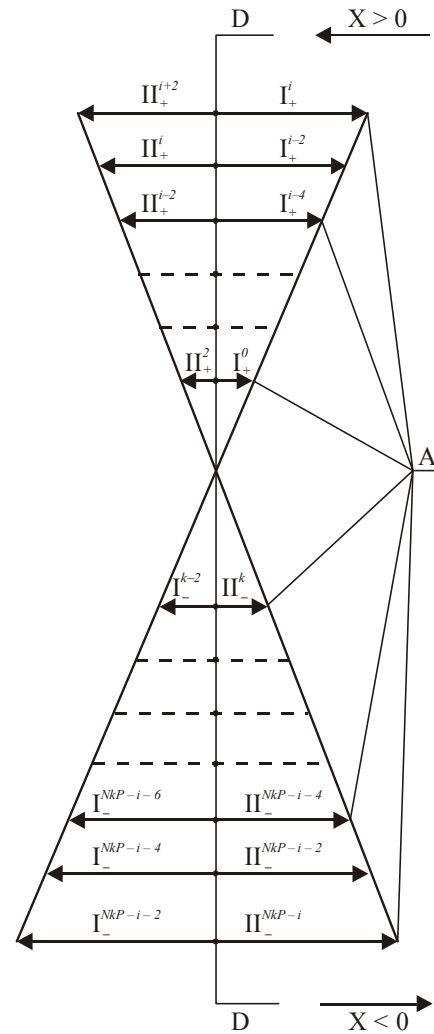


Figure 3. Completion of vacuum states II^+ and II^- by vacuum states I^+ and I^- , respectively, at the maximum potential energy of interaction.

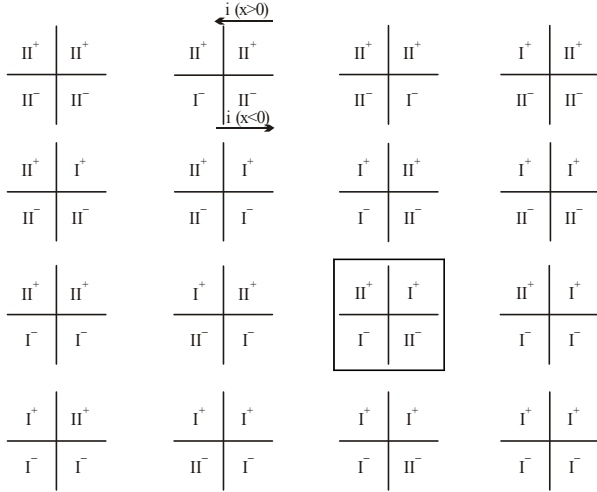


Figure 4. The possible variants of four-contact interaction of byuons. Square means that this interaction can realize in nature.

such byuons (vacuum I^+). These directions are coincident with those for byuons with $X < 0$: II^- with I^+ , and I^- with II^+ . It is clear from above definitions that the byuon with the maximum length $X(i)$ in VS II^+ determines the positive direction, and directions of other byuons are correspondent with it.

The four-contact interaction of byuons is realized within a time $\tau = \tilde{\tau}_0$ only at points D of the R_1 -space (**Figure 3**), i.e. at the points where introducing an interaction with $PE > 0$ is possible. In **Figure 3** the arrows corresponding to byuons show directions of a decrease or an increase in their lengths relative to the origin of the coordinates introduced, for example, where $i \rightarrow 0$ (in its direction the absolute value of the byuon length decreases (states I^+ and I^-), and it increases in the opposite directions (states II^+ and II^-)). At the points A in **Figure 3**, the coordinate denoting place (time) of byuon interaction cannot be fixed because of violating, in such a case, the definitions of the byuon states (in one quantum of the R_1 -space within a time τ_0 , the byuons with $II^+ I^+ I^- II^-$ should not be present). It is assumed that before the origin of VS II^+ with the minimum length ($i = 1$), the byuon vacuum states II^- and I^- with any possible lengths are already in existence.

The propagation of byuons in VSs $II^+ I^+$ and $I^- II^-$, the interaction between which occurs with imaginary energy (see below), presents two wave-like processes (see below) directed towards each other at $X(i) > 0$ and $X(i) < 0$, respectively. These processes are unobservable. A really observable signal can be transmitted by means of such processes only in the four-contact byuon interaction $II^+ I^+ \times I^- II^-$.

Let us obtain an equation characterizing the propagation of the four-contact interaction of byuons in R_1 . Introduce functions of index i , characterizing the origin of

such or another VS by byuons: $\Psi_{II^+}^{i+2}$, $\Psi_{II^-}^{k-i}$, determining the processes of byuon length magnitude origin and increase at positive and negative $X(i)$, respectively; $\Psi_{I^+}^i$, $\Psi_{I^-}^{k-i-2}$, determining the processes of byuon length magnitude cancellation and decrease at positive and negative $X(i)$, respectively.

The physical sense of the introduced functions consists in that their product determines the probability of two-contact interaction of byuons (for example, $\Psi_{II^+}^{i+2} \cdot \Psi_{II^+}^{i+2-k}$ determines the probability of interaction of byuons $[A \cdot X]_{II^+}^{i+2}$ и $[A \cdot X]_{II^+}^{i+2-k}$), the product of four functions determines the probability of four-contact interaction, the product of eight functions gives the probability of eight-contact interaction. These products should be positive, and in this case only they can describe an observed event.

The probability of a single event is no greater than 1.

Depending on which range is i in ($0 \leq i < k$, $k < i < Nk$, $Nk < i < NkP$ where k, N, P are the assumed periods in i) various types of contact interactions between byuons may be introduced. Hence the normalization of the introduced functions should be dependent on i .

Let us normalize the introduced functions for the case $0 \leq i < k$ in the following manner

$$\sum_{\xi=0}^{(NkP-k)/2} \sum_{j=0}^{j=i} \Psi_{II^+}^{j+2} \cdot \Psi_{I^+}^j \cdot \Psi_{II^-}^{NkP-j-2-2\xi} \cdot \Psi_{I^-}^{NkP-j-2-2\xi} = \frac{NP}{2} \quad (5)$$

$$\sum_{\xi=1}^{NP-1} \sum_{j=0}^{j=i} \Psi_{II^-}^{NkP-j} \cdot \Psi_{II^-}^{NkP-j-2-\xi k} = P, \quad (6)$$

$$\sum_{\xi=0}^{(NkP-k)/2} \sum_{j=0}^{j=i} \Psi_{II^+}^{j+2} \cdot \Psi_{I^-}^{NkP-j-2-2\xi} = \frac{NP}{2}, \quad (7)$$

$$\sum_{\xi=0}^{(NkP-k)/2} \sum_{j=0}^{j=i} \Psi_{I^+}^j \cdot \Psi_{II^-}^{NkP-j-2-2\xi} = \frac{NP}{2}. \quad (8)$$

When normalizing, it is taken into account that within a period in $i = k$, one four-contact interaction occurs with probability 1.

Let us obtain an equation in terms of Ψ -functions, describing the propagation of a four-contact interaction of byuons. For that we may write the following relationships as to the origin of VSs $II^+(f_{II^+})$, $I^+(f_{I^+})$, $II^-(f_{II^-})$, $I^-(f_{I^-})$ depending on certain VSs of the byuons neighbouring in the index i :

$$\begin{aligned} \Psi_{II^+}^{i+2} &= f_{II^+} [\Psi_{II^+}^{i+1}, \Psi_{I^+}^{i+2}, \Psi_{II^-}^{NkP-i}, \Psi_{I^-}^{NkP-i-2}], \\ \Psi_{I^+}^i &= f_{I^+} [\Psi_{I^+}^{i+1}, \Psi_{II^+}^i, \Psi_{II^-}^{NkP-i}, \Psi_{I^-}^{NkP-i-2}], \\ \Psi_{II^-}^{NkP-i} &= f_{II^-} [\Psi_{II^-}^{NkP-i-1}, \Psi_{I^-}^{NkP-i-1}, \Psi_{II^+}^{i+2}, \Psi_{I^+}^i], \\ \Psi_{I^-}^{NkP-i-2} &= f_{I^-} [\Psi_{I^-}^{NkP-i-1}, \Psi_{II^-}^{NkP-i-2}, \Psi_{II^+}^{i+2}, \Psi_{I^+}^i]. \end{aligned} \quad (9)$$

Assuming only linear dependences in Expressions (9) as well as equiprobability of VSs of byuons neighbouring in i , we obtain the following equations for Ψ -functions of four-contact interactions of byuons:

$$\begin{aligned}\Psi_{II^+}^{i+2} &= \Psi_{II^+}^{i+1} - \Psi_{I^+}^{i+2} - \Psi_{II^-}^{NkP-i} + \Psi_{I^-}^{NkP-i-2}, \\ \Psi_{I^+}^i &= \Psi_{I^+}^{i+1} - \Psi_{II^+}^i + \Psi_{II^-}^{NkP-i} - \Psi_{I^-}^{NkP-i-2}, \\ \Psi_{II^-}^{NkP-i} &= \Psi_{II^-}^{NkP-i-1} - \Psi_{I^-}^{NkP-i-1} - \Psi_{II^+}^{i+2} + \Psi_{I^+}^i, \\ \Psi_{I^-}^{NkP-i-2} &= \Psi_{I^-}^{NkP-i-1} - \Psi_{II^-}^{NkP-i-2} + \Psi_{II^+}^{i+2} - \Psi_{I^+}^i.\end{aligned}\quad (10)$$

From the first and second pairs of Equations (10) we obtain, respectively, the following equations:

$$\Delta[\Psi_{II^+}^{i+1} + \Psi_{I^+}^{i+1}] + \Psi_{II^+}^{i+1} + \Psi_{I^+}^{i+1} = 0, \quad (11)$$

$$\begin{aligned}\Delta E(i) &= \Psi_{II^-}^{NkP-i-k} \Psi_{II^-}^{NkP-i} E_{II^-II^-}^{NkP-i-k, NkP-i} \cos_{II^-II^-} + \Psi_{II^-}^{NkP-i-2k} \Psi_{II^-}^{NkP-i} E_{II^-II^-}^{NkP-i-2k, NkP-i} \cos_{II^-II^-} + \\ &+ \Psi_{II^-}^{NkP-i-3k} \Psi_{II^-}^{NkP-i} E_{II^-II^-}^{NkP-i-3k, NkP-i} \cos_{II^-II^-} + \dots + \Psi_{II^+}^{i+2} \Psi_{I^-}^{NkP-i-2} E_{II^+I^-}^{i+2, NkP-i-2} \cos_{II^+I^-} + \\ &+ \Psi_{II^+}^{i+2} \Psi_{I^-}^{NkP-i-2-2} E_{II^+I^-}^{i+2, NkP-i-2-2} \cos_{II^+I^-} + \Psi_{II^+}^{i+2} \Psi_{I^-}^{NkP-i-2-2 \times 2} E_{II^+I^-}^{i+2, NkP-i-2-2 \times 2} \cos_{II^+I^-} + \\ &+ \Psi_{II^+}^{i+2} \Psi_{I^-}^{NkP-i-2-2 \times 3} E_{II^+I^-}^{i+2, NkP-i-2-2 \times 3} \cos_{II^+I^-} + \dots + \Psi_{II^-}^{NkP-i} \Psi_{I^+}^i E_{II^-I^+}^{NkP-i, i} \cos_{II^-I^+} + \Psi_{II^-}^{NkP-i-2} \Psi_{I^+}^i E_{II^-I^+}^{NkP-i-2, i} \cos_{II^-I^+} + \\ &+ \Psi_{II^-}^{NkP-i-2 \times 2} \Psi_{I^+}^i E_{II^-I^+}^{NkP-i-2 \times 2, i} \cos_{II^-I^+} + \Psi_{II^-}^{NkP-i-2 \times 3} \Psi_{I^+}^i E_{II^-I^+}^{NkP-i-2 \times 3, i} \cos_{II^-I^+} + \dots + \\ &+ \Psi_{I^+}^i \Psi_{II^+}^{i+2} \Psi_{I^-}^{NkP-i-2} \Psi_{II^-}^{NkP-i} \sqrt{E_{I^+II^+}^{i, i+2} \cos_{I^+II^+} E_{I^-II^-}^{NkP-i-2, NkP-i} \cos_{I^-II^-}} + \\ &+ \Psi_{I^+}^i \Psi_{II^+}^{i+2} \Psi_{I^-}^{NkP-i-2-2} \Psi_{II^-}^{NkP-i-2} \sqrt{E_{I^+II^+}^{i, i+2} \cos_{I^+II^+} E_{I^-II^-}^{NkP-i-2-2, NkP-i-2} \cos_{I^-II^-}} + \\ &+ \Psi_{I^+}^i \Psi_{II^+}^{i+2} \Psi_{I^-}^{NkP-i-2-2 \times 2} \Psi_{II^-}^{NkP-i-2 \times 2} \sqrt{E_{I^+II^+}^{i, i+2} \cos_{I^+II^+} E_{I^-II^-}^{NkP-i-2-2 \times 2, NkP-i-2 \times 2} \cos_{I^-II^-}} + \\ &+ \Psi_{I^+}^i \Psi_{II^+}^{i+2} \Psi_{I^-}^{NkP-i-2-2 \times 3} \Psi_{II^-}^{NkP-i-2 \times 3} \sqrt{E_{I^+II^+}^{i, i+2} \cos_{I^+II^+} E_{I^-II^-}^{NkP-i-2-2 \times 3, NkP-i-2 \times 3} \cos_{I^-II^-}} + \dots +\end{aligned}\quad (13)$$

where $E_{II^-II^-}^{NkP-i-k, NkP-i}, \dots, E_{I^+II^+}^{i, i+2}$, etc. are maximum values of potential energy of interactions of byuons with the lengths $X(NkP-i-k) < 0$ and $X(NkP-i) < 0$ in VS II^- , as well as byuons with the lengths $X(i)$ and $X(i+2)$ in VSs I^+ and II^+ , respectively; $\cos_{I^+II^+}$, $\cos_{I^-II^-}$ etc. are functions minimizing the potential energy of interactions of byuons entering into the expressions for $E_{I^+II^+}^{i, i+2}, \dots, E_{I^-II^-}^{NkP-i-2, NkP-i}$, etc.; these functions are “responsible” for the appearance of a minimum plane object and the introduction of the concept of spin (see below). Functions $\cos_{I^+II^+}$ are not the usual Cos-functions, because we work up to now in one-dimensional space of i -indices. These Cos/s can have values from 0 to 1. Upper and lower indices in (13) correspond to interactions of byuon VSs,

The difference in the average values of byuon lengths calculated basing upon the definition of byuons with the use of the rule of circular arrow (see **Figure 5**), is taken as a distance between the interacting byuons to find $\Delta E(i) > 0$. The meaning of this rule is that the said distance is

$$\Delta[\Psi_{II^-}^{NkP-i-1} + \Psi_{I^-}^{NkP-i-1}] + \Psi_{II^-}^{NkP-i-1} + \Psi_{I^-}^{NkP-i-1} = 0 \quad (12)$$

where Δ denote the second finite differences in index i .

It is seen from Equations (11) and (12) that the process in i is of oscillatory character for the functions $(\Psi_{II^+}^{i+1} + \Psi_{I^+}^{i+1})$ and $(\Psi_{II^-}^{NkP-i-1} + \Psi_{I^-}^{NkP-i-1})$. These functions determine the A -type points in the space of index i shown in **Figure 3**, i.e. the points at which we cannot introduce interaction of byuons. The A -type points in the space R_1 determine as if “ruptures” in i , between which there exist objects with energy $E > 0$.

For the case $i \leq k$, write an equation for an increment in potential energy of a byuon interaction $\Delta E(i)$, corresponding to the occurrence of VSs II_+^{i+2} and I_+^i . Minimization of $E(i)$ is assumed to be going at each step in i .

calculated as the difference in the length values of the subsequent and preceding byuons in the direction pointed by an arrow.

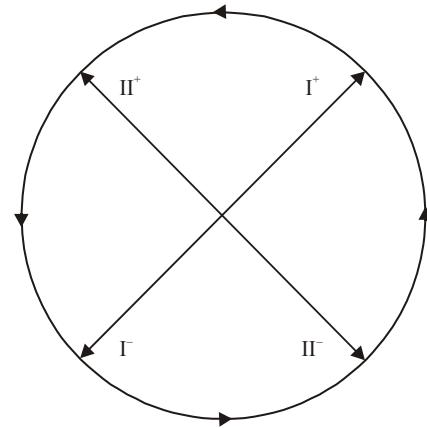


Figure 5. The rule of circular arrow for determining the distance between byuons.

The lengths of byuons are:

$$X_{II_{cp}^+}^{i+1} = \frac{2i+1}{2} \cdot \frac{const_1}{k \cdot A_G}; \quad X_{II_{cp}^+}^{k-i} = -\frac{2(k-i)-1}{2} \cdot \frac{const_1}{k \cdot A_G};$$

$$X_{I_{cp}^+}^i = \frac{2i+1}{2} \cdot \frac{const_1}{k \cdot A_G}; \quad X_{I_{cp}^+}^{k-i-1} = -\frac{2(k-i)-1}{2} \cdot \frac{const_1}{k \cdot A_G}$$

etc.

The distance between byuons does not depend on i and may take by magnitude only two values:

$$X_{II^+I^+}^{k-i,i} = \frac{const_1}{A_G}; \quad X_{II^+I^-}^{i+1,k-i-1} = -\frac{const_1}{A_G};$$

$$X_{I^+II^+}^{i,i+2} = \frac{const_1}{k \cdot A_G}; \quad X_{I^-II^-}^{k-i-2,k-i} = -\frac{const_1}{k \cdot A_G};$$

or multiples of them, for example,

$$X_{II^+I^+}^{Nk-i,i} = \frac{const_1 N}{A_G}.$$

The expressions determining the maximum energy of byuon interactions are written as

$$E_{II^+I^+}^{k-i,i} = \frac{[Ax]_{II^+I^+}^{k-i,i} [Ax]_{I^+II^+}^i}{X_{II^+I^+}^{k-i,i}} = \frac{const_1}{4k^2} A_G [2(k-i)-1] (2i+1);$$

$$E_{II^+I^-}^{i+2,k-i-2} = \frac{const_1}{4k^2} A_G [2(k-i)-1] (2i+1);$$

$$E_{I^+II^+}^{i,i+2} = -\sqrt{-1} \frac{const_1}{4k^2} A_G (2i+3)(2i+1);$$

$$E_{I^-II^-}^{k-i-2,k-i} = \sqrt{-1} \frac{const_1}{4k^2} A_G [2(k-i)-1] [2(k-i)-3];$$

$$E_{II^+II^-}^{NkP-i-k,NkP-i} = \frac{const_1}{4k^2} A_G [2(NkP-i-k)-1] [2(NkP-i)-1].$$

(14)

The minimization of $\Delta E(i)$ is achieved in the functional space of the following variables:

$$\Psi_{I^+}^0, \Psi_{II^+}^2, \Psi_{II^-}^{NkP}, \Psi_{I^-}^{NkP-2}, \cos_{I^+II^+}^{i,i+2}, \cos_{II^+II^-}^{NkP-k-i,NkP-i}, k, NP$$

(15)

It is assumed therewith that the conditions of symmetry during the "closure of the loop" in i are fulfilled as well as symmetry of the world and antiworld, which conditions can be represented as:

$$\cos_{I^+II^+}^{i,i+2} = \cos_{I^-II^-}^{NkP-i-2,NkP-i} = \cos_{I^-II^-}^{NkP-i-2-2,NkP-i-2}$$

$$= \cos_{I^-II^-}^{NkP-i-2-2 \times 2, NkP-i-2 \times 2} = \dots; \quad (16)$$

$$\cos_{II^+II^-}^{NkP-i-k,NkP-i} = \cos_{II^+II^-}^{NkP-i-2k,NkP-i} = \dots;$$

The functions $\cos_{II^+I^-}$, $\cos_{II^-I^+}$ are considered as equal to 1.

Initial conditions for Ψ function are preset to be

$$\Psi_{II^+}^0 \approx 0, \Psi_{I^+}^0 \approx 1, \Psi_{II^-}^{NkP} \approx 1, \Psi_{I^-}^{NkP} \approx 0, \Psi_{I^+}^1 \approx \Psi_{I^-}^{NkP-2}, \Psi_{II^-}^{NkP-1} = \Psi_{II^+}^2, \Psi_{I^-}^{i+2} + \Psi_{I^+}^{i+1} = 0$$

(17)

Using the solutions of the Equations (11) and (12):

$$\Psi_{II^+}^{i+1} + \Psi_{I^+}^{i+1} = A \cos\left(\frac{2\pi i}{k}\right) + B \sin\left(\frac{2\pi i}{k}\right), \quad \text{we contract}$$

the space of variables down to four: $\cos_{I^+II^+}^{i,i+2}$, $\cos_{II^+II^-}^{NkP-i-2,NkP-i}$, k , NP .

Now, taking into account the normalizing Expressions (5-8), seek for $\min \Delta E(i)$ by the steepest descent method. When retaining only 14 terms of the series in (16) and (5), $\min \Delta E(i)$ will correspond to the following values:

$$\Psi_{I^+}^0 = 0.999(6), \quad \Psi_{II^+}^2 = 1.00136 \times 10^{-4}, \quad \Psi_{II^-}^{NkP} = 0.999(8),$$

$$\Psi_{I^-}^{NkP-2} = 1.100043 \times 10^{-4}, \quad \cos_{I^+II^+}^{i,i+2} = 1.01887 \times 10^{-5},$$

$$\cos_{II^+II^-}^{NkP-i-k,NkP-i} = 1.20013 \times 10^{-5}, \quad k = 6.2 \times 10^{15}, \quad NkP = 3 \times 10^{60}$$

(15)

With increasing n , as is seen from the solutions given and **Figure 6**, k (the first period in i) approaches its value obtained in [25,26] on the base of physical considerations as an integer part of the ratio $x_0/\tilde{x}_0 = 3.2 \times 10^{15}$. Thus, we can now obtain, with the aid of the calculated k , one of the fundamental dimensions in physics of elementary particles, $x_0 \approx 10^{-17}$ cm, with the only quantum of space $\tilde{x}_0$ given. This mathematical result raises prospects that the advanced hypothesis is true. It reflects the nature of physical space and vacuum.

The minimum $\Delta_1 E(i)$ was sought for a case when $Nk < i \leq NkP$. In this case the normalizing expressions for the arising interactions of byuons have the form:

$$\sum_{j=Nk}^j \sum_{\xi=0}^{\xi=\frac{NkP-i-2}{2}} \Psi_{I^+}^j \Psi_{II^+}^{j+2} \Psi_{I^-}^{NkP-j-2-2\xi} \Psi_{II^-}^{NkP-j-2\xi} = \frac{iNP}{2k};$$

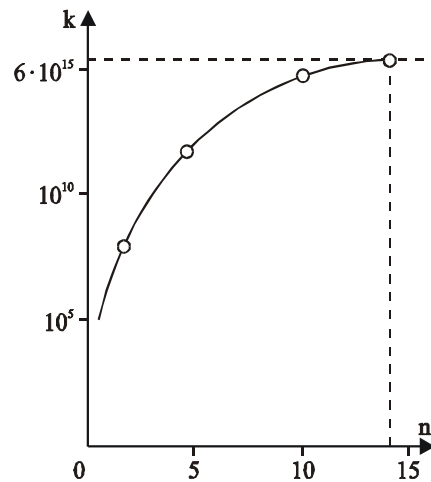


Figure 6. k as a function of the number n of the terms of the series.

$$\sum_{\xi=1}^{\frac{NkP-i-2}{2}} \Psi_{II^-}^{NkP-i} \Psi_{II^-}^{NkP-i-\xi k} = \frac{NkP-i}{Nk}$$

$$\sum_{\eta=1}^{i/k} \Psi_{II^+}^{i+2} \Psi_{II^+}^{i+2-\eta k} = \frac{i}{Nk};$$

$$\sum_{\xi=0}^{\frac{NkP-i-2}{2}} \sum_{j=Nk}^{j=i} \Psi_{II^+}^{j+2} \Psi_{II^-}^{NkP-j-2-2\xi} = \frac{iNP}{2k};$$

$$\sum_{\xi=0}^{\frac{NkP-i-2}{2}} \sum_{j=Nk}^{j=i} \Psi_{II^-}^{NkP-j-2\xi} \Psi_{II^+}^j = \frac{iNP}{2k}. \quad (19)$$

The expression for $\Delta_1 E(i)$ becomes more complicated:

$$\begin{aligned} \Delta_1 E(i) = & \Psi_{II^+}^{i+2} \cdot \Psi_{II^+}^{i+2-k} \cdot E_{II^+II^+}^{i+2, i+2-k} \cdot \cos_{II^+II^+} + \Psi_{II^+}^{i+2} \cdot \Psi_{II^+}^{i+2-2k} \cdot E_{II^+II^+}^{i+2, i+2-2k} \cdot \cos_{II^+II^+} + \dots + \\ & + \Psi_{II^+}^{i+2} \cdot \Psi_{II^-}^{NkP-i-2} \cdot E_{II^+II^-}^{i+2, NkP-i-2} \cdot \cos_{II^+II^-} + \Psi_{II^+}^{i+2} \cdot \Psi_{II^-}^{NkP-i-2-2} \cdot E_{II^+II^-}^{i+2, NkP-i-2-2} \cdot \cos_{II^+II^-} + \\ & + \Psi_{II^+}^{i+2} \cdot \Psi_{II^-}^{NkP-i-2-2 \times 2} \cdot E_{II^+II^-}^{i+2, NkP-i-2-2 \times 2} \cos_{II^+II^-} + \dots + \Psi_{II^+}^{i+2-k} \cdot \Psi_{II^-}^{NkP-i-2+k} \cdot E_{II^+II^-}^{i+2-k, NkP-i-2+k} \cos_{II^+II^-} + \\ & + \Psi_{II^+}^{i+2-k} \cdot \Psi_{II^-}^{NkP-i-2-2+k} \cdot E_{II^+II^-}^{i+2-k, NkP-i-2-2+k} \cos_{II^+II^-} + \\ & + \Psi_{II^+}^{i+2-k} \cdot \Psi_{II^-}^{NkP-i-2-2 \times 2+k} \cdot E_{II^+II^-}^{i+2-k, NkP-i-2-2 \times 2+k} \cos_{II^+II^-} + \dots + \\ & + \Psi_{II^+}^{i+2-2k} \cdot \Psi_{II^-}^{NkP-i-2+2k} \cdot E_{II^+II^-}^{i+2-2k, NkP-i-2+2k} \cos_{II^+II^-} + \dots + \Delta E(i) \end{aligned}$$

($\Delta E(i)$ is taken from Equation (10), taking into account for multiplication over index i as was shown above ($i+2$, $I+2-k$, $i+2-2k$ etc.).

The search for $\min \Delta E(i)$ with the use of the chain of Equations (7) in the space of similar variables with similar initial conditions (where $i=0$ corresponds now to $i=Nk$), leads to practically the same results:

$$\begin{aligned} \Psi_{II^+}^{Nk} &= 0.9998, \quad \Psi_{II^+}^{Nk+2} = 1.006 \times 10^{-4}, \quad \Psi_{II^-}^{NkP} \\ &= 0.9999, \\ \Psi_{II^-}^{NkP-2} &= 1.101 \times 10^{-4}, \quad \cos_{II^+II^+}^{i, i+2} = \cos_{II^+II^+}^{i+2, i+2-k} \\ &= 1.1019 \times 10^{-5}, \\ \cos_{II^-II^-}^{NkP-i-k, NkP-i} &= 1.203 \times 10^{-5}, \quad k = 4.475 \times 10^{15}, \quad NkP \\ &= 3.169 \times 10^{60} \end{aligned}$$

It is interesting that at $i < k$ and even at $i \approx Nk$ (Table 1), a significant part in the magnitude of $\Delta E(i)$ is potential energy of the byuon interaction in VS II^+II^- , and at $i \rightarrow NkP$ the four-contact interaction of byuons in VS $II^+II^-II^-$ becomes determining.

Let us consider a simplified case when only the period in i , equal to k , is present in the antiworld. Then we have from the necessary condition of minimum $\Delta E(i)$ with respect to the function $\Psi_{II^+}^{i+2}$ (i.e. from the equation $\frac{\partial \Delta E(i)}{\partial \Psi_{II^+}^{i+2}} = 0$):

Table 1. The values of potential energy E in vacuum states (II^+II^+), ($II^+II^-II^-$), (II^-II^-) depending on the i index.

i	Nk	NkP
$E_{II^+II^+}$	10^{13}	10^{95}
$E_{II^+II^-II^-}$	10^{70}	10^{111}
$E_{II^-II^-}$	10^{95}	10^{13}

$$\cos_{II^+II^+} = -\frac{1}{\Psi_{II^+}^i \cdot \Psi_{II^-}^{k-i}} \cdot \frac{1}{k} \cdot \frac{\sqrt{[(k-i)-1](2i+1)}}{(2i+3)[(2k-i)+3]}. \quad (20)$$

If $i < k$, $N=P=1$, and hence, according to Equation (5) $\sum_{j=0}^{j=i} \Psi_{II^+}^j \cdot \Psi_{II^-}^{k-j} \rightarrow 1$, we obtain from Equation (20) $\cos_{II^+II^+} \approx 1/k$.

Thus, as show our numerical calculations and analytical estimations, the minimization of $\Delta E(i)$ leads to values of functions $\cos_{II^+II^+}$ etc. if not zero but extremely small.

Note that the functions $\cos_{II^+II^+}$ for the case of NP “loops” in VS II^- (see (18)) and for the case when $N=P=1$, differ by ten orders of magnitude. The physical meaning of these functions will be shown further.

Put a question, where disappears and into what kind of energy the potential energy of byuon interaction transforms? An answer seems to be simple, of course, - into the kinetic energy of rotation (since $\cos_{II^+II^+} < 1$, see (18, 20)). But the rotation of what and around what? And why do we assume the law of conservation of energy to be fulfilled here? After all, it makes no sense to say about uniformity in time for this statement in which the time is discrete! Let us answer these questions.

As it was shown above, the optimum values of the functions $\cos_{II^+II^+}^{i+2, i}$, $\cos_{II^-II^-}^{NkP-i, NkP-i-2}$, etc., are much less than 1 but non-zero. The smallest values of $\cos_{II^+II^+} \approx 1/k$ correspond to residual (finite) potential energies E_k , from which, as we will further assume, the smallest part (associated with the formation of the own space of elementary particles) of the potential self-energy of elementary particles corresponding to the known Einstein’s relationship $E_k = mc^2$, is added together. Determine the minimum value of E_k . It is seen from Equation (13) that for the

simplest objects with N and P approximately equal to I , the minimum E_k is equal to the potential energy of the four-contact interaction of byuons with the minimum values of $\cos_{II^+I^+}$, $\cos_{II^-I^-}$.

In view of the normalization (5) we have for this case

$$E_{k_{\min}}^{(0)} = \sqrt{E_{I^+II^+}^{0,2} \cos_{I^+II^+} E_{I^-II^-}^{k-2,k} \cos_{I^-II^-}} \quad (21)$$

From that, with the condition (16), and using Equation (14) and the equality $\cos_{II^+I^+} \approx 1/k$, we obtain $E_{k_{\min}}^{(0)} \approx 33 \text{ eV}$.

Consider the process of energy transformation for the four-contact byuon interaction occurring within a time quantum τ_0 (Hence, the transformation must be discrete).

Any value of i can be redenoted with an another index, for example, with j , γ , ξ , etc., which may be set equal to zero. At each point then, where $j = \gamma = \xi = \dots = 0$, there will be always present an own system of account of the indices j , γ , ξ , etc., as well as the minimum energy of four-contact byuon interaction $E_{k_{\min}}^{(0)} \approx 33 \text{ eV}$. Note that this minimum energy is limited in index i by values $i = 0$ and $i = 2$ (point A **Figure 3**).

Thus, we have in R_1 two sets of points $A(\{A\})$ and $D(\{D\})$, between which the dynamic process of renunciation goes in connection with the properties of vacuum states of the discrete objects, the byuons II^+ , I^+ , I^- , II^- :

$$\text{the time } \tau_{i+1} \begin{cases} \{D\} = i+1, i-1, \dots \\ \{A\} = i+2, i, \dots \end{cases}$$

$$\text{the time } \tau_{i+1} + \tau_0 \begin{cases} \{D\} = i+2, i, \dots \\ \{A\} = i+3, i+1, \dots \end{cases}$$

The set R_1 may be represented, at some i -th point of time as the join of $\{A\}$ and $\{D\}$, i.e. $R_1 = \{A\} \cup \{D\}$ (**Figure 7**).

Hence the space R_1 segregates into the subspaces R_D of D -points and R_A of A -points. Thus, we may say about the motion of D -points relative to A -points. A new, second coordinate appears, symbolize it by Y_{AD} (**Figure 7**).

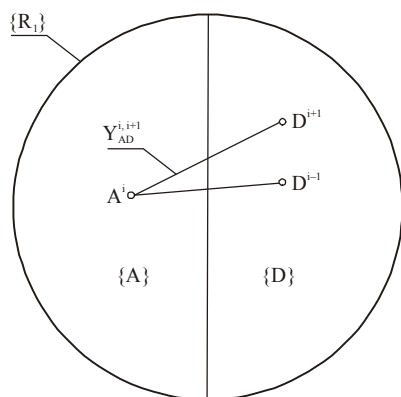


Figure 7. Representation of the set $\{R_1\}$ as a union of sets of the points $\{A\}$ and $\{D\}$.

The minimum value of Y_{AD} : $Y_{AD_{\min}} = C_0 \tau_0 |i+1|_A$

$-i_D| = \tilde{X}_0$. The minimum object appears. Assume that its appearance (new coordinate) corresponds to the minimum action h (see [1-3], $h = ([A_r x_0]_{II^+} [A_r x_0]_{I^-}) / c_0 x_0 / ct^*$ and elementary electric charge $e_0^2 = (1/(4\sqrt{3})) A_g^2 x_0^2 (x_0 / ct^*)^{3/2}$). We may then introduce the concept of momentum for objects with the residual PE of a byuon interaction by writing the relationship $P_D^{i+1} \times Y_{AD}^{i,i+1} = h$ where P_D^{i+1} is the momentum of the point D numbered $i+1$ relative to the point A with the number i . Similar relationship can be written in any point from $\{A\}$. The direction of the momentum vector $\vec{P}_D^{i+1}$ corresponds to that towards the point D^{i+1} of the subspace R_D . The direction of the coordinate $Y_{AD}^{i,i+1}$ corresponds to the vector directed from the point A^i to the point D^{i+1} .

The appearance of the minimum plane object and realization of the minimum action are connected with the origin of the quantum spin number $\vec{S} = [\vec{P}_D^{i+1} \times \vec{Y}_{AD}^{i,i+1}]$, expressed numerically in minimum actions h .

The function $\cos_{II^+I^+}$ etc. minimizing $\Delta E(i)$, will be further considered by us as cosines of the angles between the vector $\vec{P}_D^{i+1}$ and $\vec{Y}_{AD}^{i,i+1}$ i.e. before the byuon interaction, the space R_1 represents, at some time point τ , a certain discrete straight line of points $\{D\}$ and $\{A\}$, and at the time point $\tau + \tau_0$ forms a line, broken at points $\{D\}$. That is the minimum interaction of byuons has occurred.

If an "observer" was able to perceive objects with $E > 0$ only every $\tau = k\tau_0$, he would simultaneously (within a time quantum τ_0) fix all the planes arrived to the point of "observation", and "see" already the three-dimensional world formed from the plane world in the result of its dynamics within the time $\tau = \tau_0$. Why the three-dimensional and not N -dimensional one? Because the set of two-contact interactions of byuons is divided, depending on reference point (**Figure 2**, $i = 0, 1, 2$), into three subsets M_0, M_1, M_2 (the lower index denotes reference point), corresponded by three one-dimensional subspaces $R_{1,0}, R_{1,1}, R_{1,2}$ while introducing metric properties. Explain the above said.

As was indicated above, any value of index i can be always redenoted by j and then $j = 0, 1, 2$ corresponds to reference points. Redenoting $i+1$ by ξ , $i+2$ by γ etc. leads, depending on reference points, to formation of three families of subspaces embedded in each other:

$$\text{reference point "0"} \Rightarrow R_{1,0}^i \supset R_{1,0}^j \supset R_{1,0}^\xi \supset R_{1,0}^\gamma \text{ etc.};$$

$$\text{reference point "1"} \Rightarrow R_{1,1}^i \supset R_{1,1}^j \supset R_{1,1}^\xi \supset R_{1,1}^\gamma \text{ etc.};$$

$$\text{reference point "2"} \Rightarrow R_{1,2}^i \supset R_{1,2}^j \supset R_{1,2}^\xi \supset R_{1,2}^\gamma \text{ etc., if}$$

$$i > j > \xi > \gamma \text{ etc.}$$

Thus, in connection with the existence of three independent reference points for the new pair interactions,

the three independent coordinates should be given to fix a pair interaction with respect of the three reference points, *i.e.* R_3 can be represented as $R_3 = R_{1,0} \times R_{1,1} \times R_{1,2}$. Note that $R_{1,0}$, $R_{1,1}$, $R_{1,2}$ consist of the sets of points $\{A\}$ and $\{D\}$, *i.e.* at each subsequent point in time, reenumeration of points A and D and “spinning” of objects with $E > 0$ in the subspaces $R_{1,0}$, $R_{1,1}$, $R_{1,2}$, occur. In this manner the concept of spin is introduced for objects with $E > 0$ in R_3 . For objects of a big size (as a result of described minimization of PE), the rotation will be always take place since the byuons are not closed in a volume of R_3 due to $\cos_{II+II+} \neq 0$. That is why, planets, stars and so on rotate, with the main part of potential energy of byuons being transformed into energy of rotation.

Advance (without proving) the following theorem:

If a system is closed, the amount of information in it is constant.

That is, transformation of one information image into another is possible in the system, but the total amount of information does remain invariable.

Note that by information we mean here not informativity as in theory of information developed by Hartly and Shannon [29] on the basis of entropic approach, but the numbers of information bits (the values “0” and “1”) in one or another information subsystems of the system of considered objects (the combinatoric approach [30]). By “1” we imply here accomplishing the minimal act (minimum action $\hbar/2$) in the system with formation of an object with $E > 0$ from byuons, and by “0” disappearing of the object with $E > 0$ is meant.

On the base of the theorem, write the following equality for informational units (bits) in the subspaces R_1 and R_3 :

$$2^{\frac{\tau_0 E_1 S_1 k \cdot 2}{\hbar}} = 2^{S_3 N} \quad (22)$$

where N is the number of information images in R_3 ($N = ct^*/x_0$ - the second period in i); S_3 the complex of the information image in R_3 (number of “loops” of length N ; $S_3 = 1, 2, \dots$); E_1 potential energy of minimum four-contact interaction of byuons in R_1 if $\cos_{II+I+} = \cos_{II-I-} = 1$;

$\frac{\tau_0 E_1 S_1 \cdot 2}{\hbar}$ is a transformation factor of recounting the number of information images (k) in R_1 into that in R_3 for $i \geq kN$; S_1 is complex of the information image in R_1 (number of “loops” of length k , $S_1 = 1, 2, \dots$).

By an information image in R_3 , one means one or another quantum number of an elementary particle.

With Equation (22), the expressions for lepton masses obtained in [20-22] (see Appendix 1) become more understandable (for example, $m_e c^2 = E_{k_{\min}}^0 \cdot N = 33eV \cdot ct^*/x_0 = 2m_{\nu_e} c^2 \cdot ct^*/x_0$).

Taking k from the solution of the problem of searching the minimum ΔE and substituting $S_3 = 1$, $S_1 = 1$ into

Equation (22), we find $N = 1.544 \times 10^4 = ct^*/x_0$, and knowing N and NP from this solution, determine $P \approx 10^{42}$. The value of $\tilde{x}_0 kN \approx 10^{-13}$ cm.

Thus, we find all the periods of byuon motion in i , corresponding to the following scales of our World: 10^{-17} cm is a characteristic scale of weak interactions (for larger lengths our World is three-dimensional and almost orthogonal, for an empty space with the 10^{-15} precision); 10^{-13} cm is a characteristic size of proton and atomic nuclei, 10^{28} cm is the radius of our Metagalaxy or the observable part of our Universe.

(See: Appendix 1 – The expressions for masses of ultimate particles; Appendix 2 – A qualitative distinction between the theory of byuons and previous physical theories. See in [21] “Force-free physics. A qualitative pattern of a common approach to unifying all interactions. A novel principle of relativity.”)

It should be also noted that to calculate the fundamental constants $\hbar$, e_0 , c ; constants of known interactions, masses of main baryons, leptons, and mesons, only three numbers $\tilde{x}_0$, τ_0 , $|Ag|$ should be given since the characteristic dimensions $\tilde{x}_0 \approx 10^{-17}$ cm, $ct^* \approx 10^{-13}$ cm and 10^{28} cm are found from the minimum PE of byuons and from the information theorem.

5. Dark Matter and Dark Energy in the Byuon Theory

Determine the average density of substance in the Universe while taking $i = NkP$ and, hence, its characteristic dimension $\tilde{x}_0 NkP \approx 10^{28}$ cm (it coincides with the assumed radius of the Universe). The total energy in the Universe can be represented as $\frac{\hbar}{\tau_0} NkP$. Its value is $5.4 \times$

10^{77} erg, and the corresponding equivalent mass $\approx 6 \times 10^{56}$ g. The uniformity of distribution of substance over the sphere with the radius $\tilde{x}_0 NkP$ gives the density of substance in the Universe $\approx 10^{-29}$ g cm⁻³, which is measured in the known observations.

Through the set $\{A\}$, information exchange occurs between points of the set $\{D\}$, which is the main mechanism determining physical essence of the Heisenberg uncertainty interval in the conception of physical space and physical vacuum. Without introducing the points $\{A\}$, the connection between D -points is realized in one direction only, in that of increasing index i with the speed no greater than c_0 .

According to the developed conception of physical vacuum structure, we can determine a momentum and a coordinate of such a complex object as an elementary particle only with an accuracy of the momentum and the coordinate entering into the relationship $P_D^{i+1} Y_{AD}^{i+1} = \hbar$ governing the momentum of D -points mapped into R_3

according to (22). Here $Y_{AD}^{i,i+1}$ is any distance in one-dimensional space between the points A and D (cf. **Figure 7**). That is, writing the uncertainty relation in R_3 for some elementary object γ as $\Delta P \cdot \Delta X \geq h/2$, we mean ΔP and ΔX to be caused by the process of R_3 formation from byuons, *i.e.* determined by quantities of P_D and Y_{AD} type.

Its momentum corresponding to the minimum momentum for elementary particles, can be given in general form as [20-22]

$$P = \Phi \cdot E_{k_{\min}}^0 / c,$$

where Φ is probability of observing the object 4b formed in the process of the four-contact interaction in some region of space R_3 .

If the objects 4b are free (that is, they create not an elementary particle but space free of them), then

$$\Phi = \frac{1}{16} \cdot \frac{\tilde{x}_0^3}{4\pi x_0^2 \tilde{x}_0}, \text{ where } \tilde{x}_0 \approx 10^{-33} \text{ cm, and } x_0 \approx 10^{-17} \text{ cm.}$$

In this case, if the scatter in values of the momentum is P for an elementary object Δp , then the uncertainty in the coordinate in R_3 for the object 4b will be equal to 10^{28} cm. This value ΔX has given us earlier the possibility to obtain the density of matter in the Universe, observed in experiment, by way of averaging it over the sphere 10^{28} cm in radius. From the modern point of view [cf. Arxiv: 0710.3018v1 [physics.gen-ph] 16 Oct.2007] the 4b-objects with $m_{4b}c^2 = 33$ eV and $\Delta X = 10^{28}$ cm, form the so-called the cold dark matter – the quantum medium corresponding to the observed physical R_3 – space.

If the object 4b is not free (that is, it forms the internal geometry of an electron, for instance), then

$$\Phi = \frac{1}{16} \cdot \frac{x_0^3}{4\pi(Nx_0)^2 x_0}, \quad (23)$$

and we can write the following expression for an assemblage of objects 4b forming an electron (for which $m_e c^2 = N \cdot E_{k_{\min}}$):

$$\Delta p = \frac{1}{16} \cdot \frac{x_0^3}{4\pi(Nx_0)^2 x_0} \cdot \frac{NE_{k_{\min}}^0}{c} = \frac{1}{64\pi} \cdot \frac{E_{k_{\min}}^0}{Nc} \quad (24)$$

Using Equation (24) we obtain the uncertainty in coordinate Δx in R_3 of the order to 10 cm for the assemblage of N objects 4b, that is, the electron, due to wave properties of N objects 4b, carries information on its properties not over distances of 10^{-8} cm (characteristical dimension of the de Broglie wave for electron at the temperature of 300 K) as would be in the case of a point-wise particle but over distances of the order to 10 cm.

When considering not N objects but one object 4b in the electron (that is, when Formula (23) is valid) then $\Delta x \approx 10^5$ cm. Hence the less is an information on conditions of internal spatial characteristics of electron, the more is the scatter in coordinate.

In the modern terminology [cf. Arxiv: 0710.3018v1 [physics.gen-ph] 16 Oct.2007] 4b-objects forming elementary particles (their charges, masses and so on) and having ΔX from 10 to 10^5 cm create the hot or warm dark matter.

In [21] a qualitative pattern of a common approach to unifying all interactions is shown

The byuon theory predicts a new anisotropic interaction of natural objects with physical vacuum.

Peculiar “taps” to gain new energy are elementary particles because their masses are proportional to the modulus of some summary potential A_Σ that contains potentials of all known fields (Appendix 1). The value of A_Σ cannot be larger than the modulus of A_g [20-22]. In accordance with the experimental results shown in [22,31], this force ejects any substance from the area of the weakened A_Σ potential along conal surfaces at angles of $100^\circ \pm 10^\circ$ around the vector A_Σ direction. This vector has the following coordinates in the second equatorial system of coordinates: right ascension $\alpha \approx 293^\circ \pm 10^\circ$ ($19^h 20^m$), declination $\delta \approx 36^\circ \pm 10^\circ$ [22,31]. The vector A_Σ is parallel to the vector A_g practically.

The new force is of nonlinear and non-local character as to variation of some summary potential A_Σ and may be represented by some series in ΔA_Σ [20-22,32].

The expression for the new force takes the form :

$$|\vec{F}| = -2Nm_0 |\vec{A}_G| \cdot c^2 \lambda(\Delta A) \cdot \frac{\partial \lambda(\Delta A)}{\partial \Delta A} \cdot \frac{\partial \Delta A}{\partial X_1} \quad (25)$$

Here N is the number of stable elementary particles in the body (electrons, protons and neutrons).

Note that expression for the new force (25) is local (we cannot deal with the nonlocal ones as yet), therefore, to account for the nonlocality of the phenomenon, we will take ΔA equal to the difference in changes of the summary potential $|\vec{A}_\Sigma|$ at the location points of a test body and a sensor element [20-22].

These changes being equal, the force will be absent. Depending on the relative position of the sensor and the test body, ΔA can take as a positive, so a negative value.

To estimate a role of gravitational field in a change of $|\vec{A}_\Sigma|$ we put forward the maximal gravitational potential $\varphi_{\max}$, determined for proton by the following relationship:

$$m_p \varphi_{\max} = e |A_g|, \quad (26)$$

where m_p is the proton mass. Then the contribution of $\varphi_{\max}$ in the change of $|\vec{A}_\Sigma|$ is described by the following equality:

$$\varphi_{\max} \cos m_p = eAv/c, \quad (27)$$

where v is in this case the velocity of our Galaxy relative to the neighbouring galaxies, $\cos = \cos_{II-II}^{NKP-i-2, NKP-i}$ (15).

It characterizes nonorthogonality of our World at the moment of the formation of the space of elementary par-

ticles.. It is worth noting that potentials of physical fields have the physical meaning only for interacting byuons when elementary particles are generated with their masses and charge numbers. As for the vector potential of magnetic field it is gauged so that its value on the axis for example of the solenoid equals zero; x_l is the coordinate, directed from the point of the most decreased $\vec{A}_\Sigma$ on a winding to the vector $\vec{A}_G$.

The analysis of the specific experimental results with high field magnets (see [20-22,32-35]) has led to the following expression for $\lambda(\Delta A)$:

$$\lambda(\Delta A) = \sum_{k=1}^{\infty} \lambda_k \exp \left\{ - \left[\frac{\Delta A}{A_G} \cdot \frac{r}{\Delta y} \cdot \left(\frac{ct^*}{x_0} \right)^{3/2} \right]^k \right\} \cdot \Delta A^k \quad (28)$$

Here r is the radius of the circle where the test body is located on; Δy is the difference in coordinates y of the sensor and the body [20-22]; $(x_0/ct^*)^{3/2}$ is the part of energy $2m_{v_e}c_0^2 = 2m_0 |\vec{A}_G| c_0^2$, which can be acted upon by the electromagnetic field potentials.

Using the linear term only in the expansion of (25) by ΔA , we obtain the following formula for the modulus of the new force:

$$F = 2Nm_v c^2 \lambda_l^2 \Delta A_\Sigma (\Delta A_\Sigma / \Delta X) \quad (29)$$

It is worth noting that the experiments for the scanning the celestial sphere by the pulsed plasma generator [22,31] to detect some directions in space, where energy is more than the average value, are the final stage in the determination of the direction of the new force. This direction was determined before by the using the high field magnets [20-22,32-35], by the investigations of the rate of β -decay for a number of radioactive elements [20-22,24,25,36], by the investigations with high precision gravimeters [20-22,37], and by plasma generators of other types [20-22,38].

Experiments with high field magnets [1,2] showed that the new interaction had the most probably an isotropic component as well.

Let us discuss the nature of the dark energy in the framework of the byuon theory on the base of potentials of physical fields.

It is known that the gravitational potential ϕ is negative, and therefore for any summation of potentials it decreases the modulus of $\mathbf{A}_\Sigma$. Masses of elementary particles are proportional to this modulus. Hence the new force will push out any material body from the region of the decreased modulus of $\mathbf{A}_\Sigma$, because a defect of energy $\Delta E = \Delta mc^2$ will appear and the corresponding force will act to the region with undisturbed value of $\mathbf{A}_\Sigma$. Any material body decreases in its own region the modulus $\mathbf{A}_\Sigma$ due to potentials of physical fields of all its elementary components, i.e. creates the gradient $\Delta A_\Sigma / \Delta X$. Gravitationally acting mass, for example, our Galaxy, creates

around itself the gravitational potential ϕ . To estimate the action of one galaxy to another we put in the Formula (29) the potential $\phi_{\max}$ from (26) and ΔA_Σ from (27) ($\Delta A_\Sigma = A$). Let us estimate the distance R_{GG} where the new force F from (29) will be higher than the gravitational force F_g (Figure 1):

$$R_{GG} \geq GM_g^2 / (2Nm_v c^2 \lambda_l^2 \cos^2 \phi_{\max}^2 (m_p c / v_e)^2) \quad (30)$$

where G is the gravitational constant.

Here M_g is the mass of the one of interacting galaxies. We consider an interaction of two galaxies with 10^{10} stars, assume that the mass of each star is of order to the solar mass ($\sim 10^{33}$ g) and a relative velocity of each galaxy $v = 100$ km/sec and 1000 km/sec. From our experiments $\lambda_l = 10^{-12}$ [20-22,51]. As the result we obtain from (30) $R_{GG} \geq 10^{26}$ cm for $v = 100$ km/sec and $R_{GG} \geq 10^{28}$ cm for $v = 1000$ km/sec.

Thus we have estimated the magnitude of the distance between galaxies above which they scatter under the action of the new force. The estimate obtained seems as reasonable and indicates that the physics of byuons is perspective to explicate the nature of dark energy and dark matter.

6. References

- [1] M. J. Rees, "Dark Matter: Introduction," *Astro-Physics*, Vol. 361, 2003, pp. 2427-2434.
- [2] E. A. Baltz, "Dark Matter Candidates," *Astro-Physics*, Vol. 1, 8 December 2004.
- [3] V. Rubin, W. K. Ford and N. Thonnard, *Astrophysical Journal*, Vol. 238, 1980, p. 471.
- [4] J. Garcia-Boldino, "Cosmology and Astrophysics," *Astro-Physics*, Vol. 2, 2005, p. 139.
- [5] R. D. Peccei and H. R. Quinn, "CP Conservation in the Presence of Pseudoparticles," *Physical Review Letters*, Vol. 38, 1977, p. 1440.
- [6] M. S. Turner, "Windows on the Axion," *Physics Reports*, Vol. 197, 1990, pp. 67.
- [7] P. Sikivie, "Experimental Tests of the 'Invisible' Axion," *Physical Review Letters*, Vol. 51, 1983, p. 1415.
- [8] S. J. Asztalos, et al. "Experimental Constraints on the Axion Dark Matter Halo Density," *Astrophysical Journal*, Vol. 571, 2002, p. L27.
- [9] H. Pagels and J. R. Primack, "Supersymmetry, Cosmology, and New Physics at Teraelectronvolt Energies," *Physical Review Letters*, Vol. 48, 1982, p. 223.
- [10] G. Jungman, M. Kamionkowski and K. Griest, "Supersymmetric Dark Matter," *Physics Reports*, Vol. 267, 1996, p. 195.
- [11] L. Covi, J. E. Kim and L. Roszkowski, "Axinos as Cold Dark Matter," *Physical Review Letters*, Vol. 82, 1999, p. 4180.
- [12] A. Kusenko and P. J. Steinhardt, "Q-Ball Candidates for Self-Interacting Dark Matter," *Physical Review Letters*,

Vol. 87, 2001, p. 141301.

- [13] K. R. Dienes, E. Dudas and T. Gheghetta, "Grand Unification at Intermediate Mass Scales through Extra Dimensions," *Nuclear Physics B*, Vol. 537, 1999, p. 47.
- [14] J. A. R. Cembranos, A. Dobado and A. L. Maroto, "Branon Dark Matter," *High Energy Physics-Phenomenology*, p. 0406076.
- [15] Z. Berezhiani, D. Comelli and F. L. Villante, "The Early Mirror Universe: Inflation, Baryogenesis, Nucleosynthesis and Dark Matter," *Physics Letters B*, Vol. 503, 2001, p. 362.
- [16] D. J. H. Chang, E. W. Kolb and A. Riotto, "Superheavy Dark Matter," *Physical Review D*, Vol. 59, 1999, p. b3501.
- [17] K. Jedamzik, "Primordial Black Hole Formation during the QCD Epoch," *Physical Review D*, Vol. 55, 1997, p. 5871.
- [18] M. Trodden and S. M. Carroll, "TASI Lectures: Introduction to Cosmology," *Astro-Physics/0401547*.
- [19] P. J. E. Peebles and B. Ratra, "The Cosmological Constant and Dark Energy," *Reviews of Modern Physics*, Vol. 75, 2003, p. 559.
- [20] Y. A. Baurov, "Structure of Physical Space and New Method of Obtaining Energy (Theory, Experiment, Applications)," Moscow, Krechet, in Russian, 1998.
- [21] Y. A. Baurov, "On the Structure of Physical Vacuum and a New Interaction in Nature (Theory, Experiment and Applications)," *Nova Science*, NY, 2000.
- [22] Y. A. Baurov, "Global Anisotropy of Physical Space (Experimental and Theoretical Basis)," *Nova Science*, 2004.
- [23] Y. A. Baurov, "Structure of Physical Space and Nature of Electromagnetic Field," in Coll. Work. PHOTON: Old Problems in Light of New Ideas, *Nova Science*, New York, 2000, pp. 259-269.
- [24] Y. A. Baurov, "Structure of Physical Space and New Interaction in Nature (Theory and Experiment)," *Proceedings of conference Lorentz group, CPT and Neutrinos*, World Scientific, 2000, pp. 342-352.
- [25] Y. A. Baurov, "Structure of Physical Space and Nature of de Broglie Waves (Theory and Experiment)," *Journal Annales de Fondation de Broglie, Contemporary Electrodynamics*, Vol. 27, No. 3, 2002, pp. 443-461.
- [26] Y. A. Baurov, Y. N. Babaev and V. K. Ablekov, *Doklady Akademii Nauk*, Vol. 259, 1981, p. 1080.
- [27] Y. N. Babaev and Y. A. Baurov, "Preprint P-0362 of Institute for Nuclear Research of Russian Academy of Sciences (INR RAS)," Moscow, 1984 (in Russian).
- [28] V. V. Kolpachev and C. A. Nikolaev, "Preprint N 1695 of Institute for Space Researches of Russian Academy of Sciences (ISR RAS)," Moscow, in Russian, 1990.
- [29] C. E. Shannon, *Bell Labs Technical Journal*, Vol. 27, 1948, pp. 379,623.
- [30] A. I. Kholmogorov, "Information Theory and Theory of Algorithms," Moscow, Nauka, in Russian, 1987.
- [31] Y. A. Baurov, I. B. Timofeev, V. A. Chernikov, S. F. Chalkin and A. A. Konradov, "Experimental Investigation of the Distribution of Pulsed-Plasma-Generator at its Various Spatial Orientation and Global Anisotropy of Space," *Physics Letters A*, Vol. 311, 2003, p. 512.
- [32] Y. A. Baurov, "Space Magnetic Anisotropy and a New Interaction in Nature," *Physics Letters A*, Vol. 181, 1993, p. 283.
- [33] Y. A. Baurov, E. Y. Klimenko and S. I. Novikov, "Experimental Observation of Space Magnetic Anisotropy," *Doklady Akademii Nauk SSSR (DAN)*, Vol. 315, 1990, p. 1116.
- [34] Y. A. Baurov, E. Y. Klimenko and S. I. Novikov, "Experimental Observation of Space Magnetic Anisotropy," *Physics Letters A*, Vol. 162, 1992, p. 32.
- [35] Y. A. Baurov and P. M. Ryabov, "Experimental Investigations of Magnetic Anisotropy Using Quartz Piezoresonance Balances," *Doklady Akademii Nauk SSSR (DAN)*, Vol. 326, 1992, p. 73.
- [36] Y. A. Baurov, A. A. Konradov, E. A. Kuznetsov, V. F. Kushniruk, Y. B. Ryabov, A. P. Senkevich, Y. G. Sobolev and S. Zadorozhny, "Experimental Investigations of Changes in β -Decay Rate of ^{60}Co and ^{137}Cs ," *Modern Physics Letters A*, Vol. 16, No. 32, 2001, p. 2089.
- [37] Y. A. Baurov and A. V. Kopaev, "Experimental Investigation of Signals of a New Nature with the Aid of Two High Precision Stationary Quartz Gravimeters," *Hadronic Journal*, Vol. 25, 2002, p. 697.
- [38] Y. A. Baurov, G. A. Beda, I. P. Danilenko and V. P. Ignatko, "Experimental Investigation of New Method of Energy Generation in Plasma Devices Caused by Existence of Physical Space Global Anisotropy," *Hadronic Journal Supplement*, Vol. 15, 2000, pp. 195-210.
- [39] R. A. Alpher, M. Bethe and G. Gamov, *Physical Review*, Vol. 70, 1946, p. 572; Vol. 73, 1948, p. 803.
- [40] A. D. Linde, *Uspehi Fizicheskikh Nauk*, Vol. 144, 1984, p. 177.
- [41] G. Y. Bogoslovsky, "Theory of Locally Anisotropic Space-Time," edited by Moscow State University, in Russian, 1992.
- [42] K. Rund, "Differential Geometry of Finsler's Spaces," Moscow, Nauka, 1981.
- [43] "Time Structures in Natural Sciences: On the Road to Right Understanding of Time Phenomenon," Part 1, Interdisciplinary Research, edited By Moscow State University, in Russian, 1996.
- [44] N. N. Bogolyubov and D. V. Shirkov, "Introduction to the Theory of Quantized Fields," Moscow, Nauka, in Russian, 1976.
- [45] L. B. Okun', "Leptons and Quarks," Nauka, Moscow, in Russian, 1990.
- [46] J. H. Schwartz, "Superstrings," *World Scientific*, Singapore, 1985.
- [47] L. D. Landau and E. M. Lifshitz, "Theory of the Field," Nauka, Moscow, in Russian, 1967.
- [48] H. Weyl, "Raum, Zeit Material Vierte Erweiterte Auflage," Springer, Berlin, 1921.

- [49] H. Weyl, "Raum, Zeit, Materie, Fünfte, Umgearbeitete Auflage," Springer, Berlin 1923.
- [50] P. A. M. Dirac, *Proceedings of the Royal Society*, London, 1973, Vol. A333, p. 419.
- [51] Y. A. Baurov, A. A. Shpitalnaya and I. F. Malov, "Global Anisotropy of Physical Space and Velocities of Pulsars," *International Journal of Pure & applied Physics*, Vol. 1, No. 1, 2005, pp. 71-82.

Appendix 1:

The Planck constant and electric charge is determined by the following Expressions [20-22]:

$$h = (([A_r \cdot x_0]_{II} + [A_r \cdot x_0]_{I}) / c_0) \cdot x_0 / ct^*$$

$$e_0^2 = (1/(4\sqrt{3})) \cdot A_r^2 x_0^2 (x_0 / ct^*)^{3/2}$$

The masses of ultimate particles can be described by the following Expressions [20-22]:

$$m_p c^2 = \frac{\sqrt{hc}}{2} \left(\frac{4\sqrt{3}e_0^2}{hc} \right)^3 \left(1 - \frac{1}{2} \sqrt{\frac{3x_0}{ct^*}} \right) \times$$

$$\left[\sqrt{1 + \frac{1}{6 \left(1 - \frac{1}{2} \sqrt{\frac{3x_0}{ct^*}} \right)^2}} + 1 \right] |\vec{A}| = 924 \text{ meV},$$

(A1)

$$m_{\pi^0} c^2 = \frac{\sqrt{hc}}{2} \left(\frac{4\sqrt{3}e_0^2}{hc} \right)^3 \left(1 - 2 \sqrt{\frac{3x_0}{ct^*}} \right) \times$$

$$\left[\sqrt{1 + \frac{1}{6 \left(1 - 2 \sqrt{\frac{3x_0}{ct^*}} \right)^2}} - 1 \right] |\vec{A}| = 132 \text{ meV}.$$

Masses of all leptons:

$$2 m_{\nu_0} c^2 = \frac{\sqrt{hc}}{2k} \cdot \left(\frac{hc}{4\sqrt{3}e_0^2} \right) \cdot |\vec{A}|,$$

$$m_e c^2 = \frac{\sqrt{hc}}{2k} \cdot \left(\frac{hc}{4\sqrt{3}e_0^2} \right)^3 \cdot |\vec{A}|,$$

$$2 m_{\nu_\mu} c^2 = 2 \cdot 3^{1/4} \sqrt{hc} \cdot \left(\frac{4\sqrt{3}e_0^2}{hc} \right)^{5.5} \cdot |\vec{A}|$$

$$m_\mu c^2 = 3^{1/4} \sqrt{hc} \cdot \left(\frac{4\sqrt{3}e_0^2}{hc} \right)^{3.5} \cdot |\vec{A}| \quad (A2),$$

$$2 m_{\nu_\tau} c^2 = 3^{1/4} \cdot 96 \cdot \sqrt{2} \cdot \sqrt{hc} \cdot \left(\frac{4\sqrt{3}e_0^2}{hc} \right)^{5.5} \cdot |\vec{A}|,$$

$$m_\tau c^2 = 3^{1/4} \cdot 12 \cdot \sqrt{2} \cdot \sqrt{hc} \cdot \left(\frac{4\sqrt{3}e_0^2}{hc} \right)^{3.5} \cdot |\vec{A}|.$$

Appendix 2:

It is known that any novel physical model of the Universe must meet the following criteria. First, all the dis-

covered laws of nature as well as sufficiently well established models of one or another physical phenomena must follow from the new model as asymptotical approximations. Second, the new theory should have the capability for predictions. That is, it should guide an experimental way to the gain of new knowledge, as the theory itself gives nothing but only points such a way. Criterion of truth is an accurately performed experiment independently confirmed by various authors. The theory of byuons [20-22] seems to meet the above criteria. That is a theory of "life" of special discrete objects from which the surrounding space and the world of elementary particles form. The intrinsic dynamics of byuons determines such fundamental phenomena as the course of time, rotation of planets and stars, spins of elementary particles, asf.

What is a qualitative distinction between the theory of byuons and previous physical theories?

First, the physical space was always given, in one way or another, and motion equations for a system of objects under study were written in that space. Space could be uniform continuum (Newton, Minkovsky) or discrete, one-dimensional or multidimensional, asf. In present-day cosmological models of the Universe origin (the Gamov's Big Bang [39], the Linde's model of bulging Universe [40], and so on), space is always given, too. But in the theory of byuons, the physical space (necessarily three-dimensional one, not ten-or-more-dimensional as in some modern physical models) is a special quantized medium arising as the result of interaction of byuon's vacuum states (VSs). That is, space is not given but arises. Therewith the appearing three-dimensional space must have an insignificant global anisotropy, as distinct from all basic isotropic models with the same properties in various directions. The said anisotropy denotes the existence of some chosen direction caused by the existence, in nature, of a new fundamental vectorial constant, the cosmological vector-potential A_g entering into the definition of the byuon. That new constant is associated with the prediction of a novel anisotropic interaction of natural objects between themselves and with the physical vacuum, a lowest energy state of physical fields.

It should be noted that in the literature spaces with local rather than global anisotropy are considered [41], for example, the Finsler's space-time [42], but the local anisotropy is given therein "by hand". That is, an author himself directly introduces it into his model instead of obtaining from some general principle. For example, there are domain models of the Universe.

Secondly, the physical sense of time notion is not yet revealed in science in the present state of the art [43]. The general philosophic concept of time as a form of matter existence, which expresses the order of change of objects and phenomena as a sequence of events, does not indicate a common nature of those events. As a rule,

people tie their time to a particular periodic process: rotation of the Earth around its axis, Earth's orbiting around the Sun, oscillations of a quartz system, asf, without becoming aware of inner, profound sense of time. Standard physical time references, for example, quantum or, what is the same, atomic clock with instrument error on the level of 10^{-11} per year and moderate resolution of the order to 10^{-13} seconds, give us no possibility of approaching the knowledge of time essence. The byuon theory reveals physical essence of time as a discrete sequence of changes in the byuon's "length", its quantum number. A possibility therewith arises, to synchronize clocks at great distances comparable with dimensions of our Metagalaxy, due to the quantum process of physical space formation from the byuon's vacuum states (VSs). That possibility distinguishes substantially the theory of byuon's from A. Einstein's special theory of relativity (STR), in which clocks can be synchronized only when a signal has passed between them with speed of light c_0 . It should be noted at once that in the byuon theory, material objects cannot move with a speed faster than the light speed (that is similar to the STR's postulate on finite propagation velocity of interactions), but synchronization of clocks occurs by a quantum way without introducing the concept of speed. That is, some object originated in the course of interaction between byuon vacuum states and forming the physical space, is at a time in two spatial regions being very distant from each other in the three-dimensional space arising.

Third, an essential distinction of the byuon theory from modern models in the classical and quantum field theories [44] is that the potentials of physical fields (gravitational, electromagnetic, asf.) become, in the theory of byuons, exactly fixable, measurable values. Recall to the reader that ordinary methods of measurement are capable to measure solely a difference of potentials. Therefore, in the existing field theory, potentials are defined only with a precision of an arbitrary constant or the rate of change of the potentials in space or time (gauge models). But in the theory of byuons, field potentials become single-valued since there are formed, on the set of byuon VSs, field charge numbers which generate the fields themselves, as, for example, the electric charge of an electron generates an electric field. The physical sense of field as a special form of matter, loses its basic meaning because all the observable events can be described on the basis of the byuon theory without introducing the

concept of force, and hence of field.

An important methodic distinction between the byuon model and all those existent in the theoretical physics of today, is that the latter use images with properties of real objects, - for example, strings in the physics of elementary particles [45], superstrings, membranes when creating a unified field theory [46], asf. But the byuons are unobservable objects having no analogues in the nature though all the natural objects appear in the result of interaction of byuon VSs.

The proposed pattern of formation of the observed space R_3 on the basis of dynamics of the finite set of byuons animates, fills with a sense, and supplements the physical results on properties of elementary particles, described in [34,35]. For example, if some elementary object appearing in a byuon interaction has, with the probability near 1, the vacuum state I^+ of a byuon completing formation of its quantum numbers (the greatest period of byuon interaction of the order of kN), such an elementary object will be stable as well as its properties will, since quite a definite amount of information will be locked up by VS I^+ . This relates, for example, to the electron.

Thus, as opposed to gauge models in which the level of symmetry constantly grows for more complete, all-embracing, and unified description of the surrounding world [23-28], and to obtain massive particles it is necessary to use "by hands" the Higgs mechanism (spontaneous violation of symmetry), in the present model there exists first a one-dimensional world (its direction, *i.e.* that of $\mathbf{A}_g$, is determined by the byuon with the maximum $x(i)$), then its symmetrization takes place, and the space R_3 , with the world of elementary particles originates. At that some insignificant ($\sim 1/k$ ($\sim 10^{-15}$)) asymmetry of "empty" R_3 remain as well as that of the 10^{-5} order inside the elementary particles.

It should be also noted that to calculate the fundamental constants h , e_0 , c ; constants of known interactions, masses of main baryons, leptons, and mesons according to formulae ($1 \div 1e$), only three numbers $\tilde{x}_0$, τ_0 , $|\mathbf{A}_g|$ should be given since the characteristic dimensions $\tilde{x}_0 \approx 10^{-17}$ cm, $ct^* \approx 10^{-13}$ cm and 10^{28} cm are found from the minimum PE of byuons and from the information theorem.

Notice that in [20-22,24,25,31-38], results of some fundamental experiments in support of the basic theoretical statements have been described

Studying Magnetization Distribution in Magnetic Thin Films under Transversal Application of Magnetic Fields

Aleksey Anatolievich Zakharenko

International Institute of Zakharenko Waves, Krasnoyarsk, Russia

E-mail: aazaaz@inbox.ru

Received January 25, 2010; revised February 26, 2010; accepted March 20, 2010

Abstract

The problem of magnetization change across the direction of magnetic field for a magnetic layer with non-symmetric boundary conditions was treated. The exact solution of the problem for the magnetization components m_x and m_y was written in the form of complex combination of Jacobian elliptic functions and elliptic integrals. This allows one to demonstrate both the static mode and all dynamic modes for the magnetization distribution across the layer thickness. The static mode and several dynamic modes, as well as the first and second derivatives of the magnetization components, were calculated. Also, average values of the magnetization components $\langle m_x \rangle$ and $\langle m_y \rangle$ for the static mode and three dynamic modes were calculated in dependence on the magnetic field. The obtained results can represent an interest in the large amount of applications of magnetic devices such as recording media, memory chips, and computer disks. The results are also useful for checking different numerical methods recently applied to study the problem, because it is thought that any numerical method cannot demonstrate solutions for the dynamic modes.

Keywords: Landau-Lifshits Equations, Thin Films, Magnetization, Static and Dynamic Modes.

1. Introduction

The Landau-Lifshits equations first derived by Landau and Lifshits on a phenomenological ground in [1,2] are fundamental equations in the theory of ferromagnetism. The study of the equations can demonstrate the magnetization distribution inside a ferromagnetic material and is a very challenging problem in physics and mathematics. Indeed, the study of them can be useful for a set of applications of magnetic devices [3,4] such as recording media and computer sensors, disks, and memory chips. The numerical and theoretical studies of the equations can be found in many works carried out in the last several decades (for example see [3-10]). However, it is thought that any numerical treatment can not demonstrate existence of a set of additional solutions. These solutions can improve understanding of magnetization distributions in a ferromagnetic layer when different regimes of application of magnetic fields can be realized for a domain in the layer.

It is assumed that in a ferromagnetic layer on an anti-ferromagnetic substrate, the vector of magnetic momentum $\mathbf{M}$ can be clamped at the boundary between the layer and the substrate. In this two-layer system, some magnetic structures [11,12] can appear when an external magnetic field is applied in the layer plane.

These magnetic structures are characterized by inhomogeneous turn of the $\mathbf{M}$ across the layer thickness. In 1980, Zakharov and Khlebopros [13] reported that solutions written in [12] for the magnetization distribution in a magnetically-soft layer on a magnetically-hard substrate under axial application of a magnetic field can be written in a novel way. Also, some solutions of the problem were originally found by Aharoni [11] in 1959, using the Jacobian elliptic functions [14,15]. Also, the excellent and classical book [16] by Collatz provides solutions of eigenvalue problems with different boundary conditions.

In 1995, the further theoretical investigations by Zakharov [17] considered the magnetic reversal of a magnetic system in a layer across anisotropy. Studying one of the magnetization components of the first dynamic mode, it was shown that magnetization in the layer after the dynamic threshold can turn from the equilibrium position and becomes opposite to the field over the entire thickness of the layer. It was also assumed in [17] that this occurs in a similar manner as a rod loaded by a transverse force is bent oppositely to the force direction as soon as the first dynamic threshold is achieved. Note that an infinite number of all higher-order modes followed the single static mode can be called the dynamic modes. In 1949, this definition of dynamic loss of stability was first given by Lavrent'ev and Ishlinskii [18]

when they studied a shock loading applied to rods. Indeed, the problem of magnetization change along the direction of magnetic anisotropy of a magnetic layer with non-symmetric boundary conditions is similar to the Euler problem of stability of an elastic rod. Note that exact solutions for the problem of transversal loading of the rods were recently given in the work [19]. It is thought that magnetic systems are more convenient for studying the dynamic buckling because of many experiments on these systems which can be performed.

It is also thought that these magnetic layered structures can represent an interest in the development of idea of creation of the metallic transistor [20-22] because an applied magnetic field can easily create an inhomogeneous distribution of the magnetization. Indeed, it is possible to study some effects resulting from the problem. Also, it is thought that the theoretical studies can be important for grasping some processes when liquid crystals and seignette-electrics are switched by super-strong fields. Some interesting experimental data can be found in the review paper [23] for the novel evaluation of the problem.

This theoretical study of the magnetization distribution provides exact solutions leading to existence possibility of infinite number of dynamic modes in addition to the single static mode. In the studied case, the magnetic field is transversely applied. The following section describes the theory. The third section investigates the magnetization components concerning extreme and inflexion points. In addition, the fourth section provides the magnetization distribution in the case when the magnetic anisotropy is accounted.

2. Theory

The magnetic layered system is shown in **Figure 1** when a ferromagnetic layer with the thickness d is situated on an anti-ferromagnetic substrate. The z -axis is directed parallel to the normal to both the layer surface and the interface between the layers; $z = 0$ at the interface. The x -axis and y -axis lie in the plane of the interface. It is

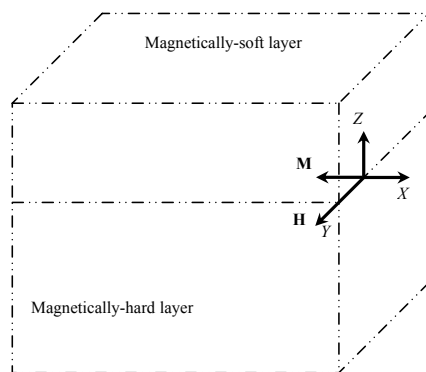


Figure 1. The configuration of a magnetically-soft layer on a magnetically-hard layer.

possible to treat domains with equal width and with negligibly thin walls such that the walls' energy can be omitted. The applied magnetic field $\mathbf{H}$ is directed along the y -axis, and the initial direction of the magnetization vector $\mathbf{M}$ is towards the x -axis negative values as shown in the figure. The applied $\mathbf{H}$ can turn the magnetization vector $\mathbf{M}$.

The Landau-Lifshitz Equation (1) can be written in the following form [13]:

$$\dot{\mathbf{M}}_j = g\mathbf{M}_j \times \mathbf{H}_j^{(e)} \quad (1)$$

where $\times$ is the vector cross product, g is the exchange coupling constant (gyromagnetic ratio); $j = 1, 2$. In Equation (1), the term on the left represents the first derivative of $\mathbf{M}_j$ with respect to time. The boundary conditions for Equation (1) are chosen as follows:

$$M_{ix} = -M, M_{iy} = M_{iz} = 0 \text{ at } z = 0 \text{ and } \partial \mathbf{M}_i / \partial n = 0 \text{ at } z = d \quad (2)$$

when the n is directed along the surface normal. In Equation (1), the effective magnetic fields $\mathbf{H}_i^{(e)}$ can be written in the following form for this case:

$$\mathbf{H}_i^{(e)} = \alpha \nabla^2 \mathbf{M}_i + \mathbf{H} - 2 \frac{\partial E_{dm}}{\partial \mathbf{M}_i} \quad (3)$$

where α is the constant of exchange for a ferromagnetics, $\mathbf{H}$ represents external constant and altering magnetic fields. The energy E_{dm} related to demagnetization fields existing at the domain boundaries can be written in the form of [13]:

$$E_{dm} = \frac{1}{8} \left[4\pi(M_{1z} + M_{2z})^2 + \eta_x(M_{1x} - M_{2x})^2 + \eta_y(M_{1y} - M_{2y})^2 + \eta_z(M_{1z} - M_{2z})^2 \right] \quad (4)$$

where the demagnetization factors η_j are as follows: $\eta_x = 0$, $\eta_y = 4\pi d/(d + D)$, $\eta_z = 4\pi D/(d + D)$ [13] with d and D representing the layer thickness and domain width, respectively. It is possible to use normalized field $\mathbf{h} = \mathbf{H}/M$ and normalized magnetization $\mathbf{m}^*_i = \mathbf{M}_i/M$. The $\mathbf{m}^*_i$ depend on the coordinate z and time t , and can be written as corresponding static and dynamic terms: $\mathbf{m}^*_i(z, t) = \mathbf{m}_i(z) + \boldsymbol{\mu}_i(z, t)$. The dynamic $\boldsymbol{\mu}_i(z, t)$ were treated in [13] and do not represent a studying subject of this work. Note that in the treated case, the static magnetization components of $\mathbf{m}_i(z)$ satisfy the following relationships:

$$m_{1x} = m_{2x} = m_x, m_{1y} = -m_{2y} = m_y, m_{1z} = m_{2z} = 0, m_x^2 + m_y^2 = 1 \quad (5)$$

After several complicated mathematical transformations described in [13] and accounting Equations (2)-(5), Equation (1) can be represented as:

$$\alpha \left(m_x \frac{\partial^2 m_y}{\partial z^2} - m_y \frac{\partial^2 m_x}{\partial z^2} \right) - h m_y - (\eta_y + \beta) m_x m_y = 0 \quad (6)$$

with boundary conditions

$$m_x = -1 \text{ at } z = 0 \text{ and } \partial m_x / \partial z = 0 \text{ at } z = d \quad (7)$$

In Equation (6), the material parameter β represents the constant of single-axis anisotropy; $h = H/M$. Using the coupling between the magnetization components $m_x = \cos(\chi)$ and $m_y = \sin(\chi)$, the Equation (6) can be rewritten in the following form:

$$\frac{d^2 \chi}{dz^2} + \frac{h}{\alpha} \sin \chi (1 + b \cos \chi) = 0 \quad (8)$$

with the boundary conditions for the angle χ in the case of transversal loading of a magnetic field (see **Figure 1**)

$$\chi = \pi/2 \text{ at } z = 0 \text{ and } \partial \chi / \partial z = 0 \text{ at } z = d \quad (9)$$

That leads to the equation of oscillations of the mathematical pendulum in the simplified case of $b = (\eta_y + \beta)/h = 0$ ($\eta_y \rightarrow 0$ and $\beta \rightarrow 0$):

$$\frac{d^2 \chi}{df^2} + q^2 \sin \chi = 0 \quad (10)$$

In Equation (10), the f represents the normalized layer thickness, $f = z/d$, and $q^2 = d^2 h / \alpha$.

It is now necessary to separate variables such as χ and f . Setting intermediate variable $T = d\chi/df$, it is possible to get the following equation $TdT/df = -q^2 \sin(\chi)[1 + b \cos(\chi)]$ resulting in $T^2 = 2q^2 \cos(\chi)[1 + 0.5b \cos(\chi)] + \text{const}$, and hence $d\chi/df = \sqrt{\text{const} + 2q^2 \cos(\chi)[1 + 0.5b \cos(\chi)]}$. Using the well-known trigonometric formula $\cos(\chi) = 1 - 2\sin^2(\chi/2)$, it is possible to write an intermediate result:

$$df = \frac{2d\psi}{C_1 \sqrt{1 - k^{*2} (1 + b \cos^2 \psi) \sin^2 \psi}} \quad (11)$$

with $C_1^2 = (2 + b)q^2 + \text{const}$, $\psi = \chi/2$ and the parameter $k^{*2} \equiv 1/k^2$. The complicated case of Equation (11) is discussed below. In the simplified case of $b = 0$, the constant $C_1^2 \rightarrow C^2 = 4q^2 k^2 = 2q^2 + \text{const}$ and

$$kqdf = \frac{d\psi}{\sqrt{1 - k^{*2} \sin^2 \psi}} \quad (12)$$

In order to obtain the function $\psi(f)$, the right side of equality (12) can be written in the form of the elliptic integral of the first kind:

$$kqf = F(\psi, k^*) + C^* \quad (13)$$

where C^* is a constant which should be determined from the boundary conditions. The elliptic integral of the first kind can be calculated with the descending Landen transformation [15]. Applying the boundary condition (9) at $z = 0$ ($f = 0$) and using the transformation formulas $F(\psi, k^*) = kF(\psi^*, k)$ and $\sin(\psi) = k \sin(\psi^*)$ for $k^{*2} = 1/k^2$ [14,15], the constant C^* can be found as a function of k : $C^* = kF_k(k)$ with

$$F_k(k) = F(\arcsin(\sqrt{2}/2k), k) \quad (14)$$

Equation (13) can then be written as

$$F\left(\arcsin\left(\frac{\sin \psi}{k}\right), k\right) = qf + F_k(k) \quad (15)$$

and applying the function such as sine to both sides of Equation (15), one can get

$$\sin \psi = k \text{sn}(u, k) \quad (16)$$

where $u = qf + F_k(k)$ and $\text{sn}(u, k)$ is the elliptic sine representing one of the twelve Jacobian elliptic functions [15]. The function $\text{sn}(u, k)$ can be also calculated with the descending Landen transformation [15]. Note that $F(\varphi, k) = u$ and $\varphi = \text{am}(u)$ from [14,15]. Hence

$$\chi = 2 \arcsin[k \text{sn}(u, k)] \quad (17)$$

Applying the boundary condition (9) at $z = d$ ($f = 1$) the parameter q can be also obtained as a function of k from the following equality:

$$\frac{d\chi}{df} = 2kq \text{cn}(u, k) \quad (18)$$

It is apparent that $d\chi/df = 0$ if the elliptic cosine $\text{cn}(u, k) = 0$ that is satisfied for $u = \rho K(k)$ with $\rho = 2\tau - 1$ and $\tau = 1, 2, 3, \dots$, where $K(k) = F(\pi/2, k)$ is called the complete elliptic integral of the first kind. Therefore

$$q^2 = d^2 h / \alpha = (\rho K(k) - F_k(k))^2 \quad (19)$$

and

$$u = (\rho K(k) - F_k(k))f + F_k(k) \quad (20)$$

Note that the parameter k^2 should be confined in the following range:

$$1/2 \leq k^2 \leq 1 \quad (21)$$

The exact solutions for the magnetization components m_x and m_y can be written as functions of the Jacobian elliptic functions $\text{sn}(u, k)$ and $\text{dn}(u, k)$ in the following complicated form:

$$m_x = -2k[\text{dn}(u, k)\text{sn}(u, k)] \quad (22)$$

$$m_y = -1 + 2k^2 \text{sn}^2(u, k) \quad (23)$$

where $\text{dn}(u, k)$ is called the elliptic delta-amplitude.

Using the effective magnetic field H_a of anisotropy [13] defined as $h_a = \alpha(\pi/2d)^2$, Equation (19) can be written as

$$h/h_a = (2/\pi)^2 (\rho K(k) - F_k(k))^2 \quad (24)$$

The average values, $\langle m_x \rangle$ and $\langle m_y \rangle$, of the magnetization components (22) and (23) can then be written in the following form:

$$\langle m_x \rangle = \frac{1}{d} \int_0^d m_x(z) dz = -2k \frac{\text{cn}(u, k) - \text{cn}(F_k(k), k)}{\rho K(k) - F_k(k)} \quad (25)$$

$$\begin{aligned} \langle m_y \rangle &= \frac{1}{d} \int_0^d m_y(z) dz = -f + \\ &+ 2 \frac{E(\text{am} u, k) - E(\arcsin(\sqrt{2}/2k), k)}{\rho K(k) - F_k(k)} \end{aligned} \quad (26)$$

where $E(\text{amu}, k)$ represents the elliptic integral of the second kind.

The exact solutions (25) and (26) were first introduced in [19] describing the dynamic instability in the nonlinear problem of a cantilever. **Figure 2** shows the dependence of the values of $\langle m_x \rangle$ and $\langle m_y \rangle$ on the normalized values of h/h_a for the static mode and several dynamic modes. The exact values of h/h_a calculated with formula (24) are listed in **Table 1**. It is thought that these tabulated values of h/h_a can be useful for researchers to check accuracy of different numerical methods.

The behaviors of the magnetization components m_x and m_y as functions of the $f = z/d$ are shown in **Figure 3** (static mode) and **Figures 4, 5, 6, and 7** (several corresponding dynamic modes) for several values of the parameter k^2 . It is clearly seen in all the figures that the satisfaction of the boundary condition of $m_x(f = 0) = -1$ occurs. It is also seen in **Figures 5 and 6** that the minimum values of the component $m_y = -1$ are significantly closer to each other for the different values of k^2 and the larger values of the f . This can mean that the magnetization vector $\mathbf{M}$ can be aligned along the y -axis just below the layer surface for the large values of ρ and $f \rightarrow 1$, and any values of k^2 in the range: $\frac{1}{2} \leq k^2 \leq 1$. Note that **Figure 7** shows the limit case of $k^2 = 1$ for the fourth

dynamic mode with $\rho = 9$. It is noted for comparison that in [17] homogeneous magnetization distributions were shown in the case of the axial loading of magnetic field. The following section theoretically investigates the magnetization components m_x and m_y written in Formulas

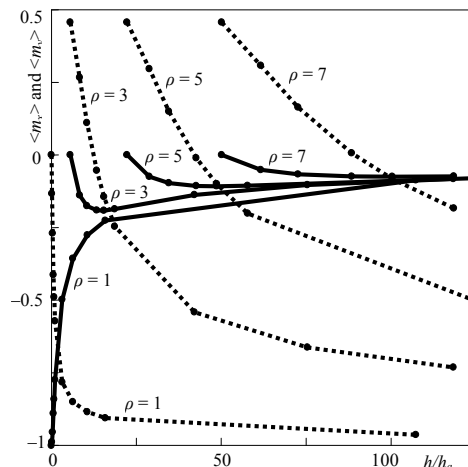


Figure 2. The dependence of the average magnetization components $\langle m_x \rangle$ and $\langle m_y \rangle$ on the values of h/h_a for the static mode ($\rho = 1$) and three dynamic modes ($\rho = 3, 5, 7$).

Table 1. The values of h/h_a with a high accuracy calculated with formula (24) for the static mode ($\rho = 1$) and four dynamic modes ($\rho = 3, 5, 7, 9$).

k^2	h/h_a				
	$\tau = 1, \rho = 1$	$\tau = 2, \rho = 3$	$\tau = 3, \rho = 5$	$\tau = 4, \rho = 7$	$\tau = 5, \rho = 9$
0.5	0.0	5.57281572	22.29126287	50.15534147	89.16505150
0.6	0.16567298	8.34803259	28.85368867	61.68264124	106.83489028
0.7	0.35546612	10.48881628	34.58708571	72.65027440	124.67838235
0.8	0.61298524	13.37289437	42.65213110	88.45069543	150.76858736
0.85	0.80045211	15.49579728	48.69611846	100.40141566	170.61168887
0.9	1.08028952	18.67880751	57.82731652	118.52581656	200.77430761
0.99	3.19667416	42.16367438	125.41284831	252.94419597	424.75771734
0.999	6.35289569	75.41909073	220.47314024	441.51504424	738.54480272
0.9999	10.58349145	118.41697367	342.64545246	683.26892783	1140.28739976
0.99999	15.88920579	171.10175649	491.73272506	977.78211149	1629.24991578

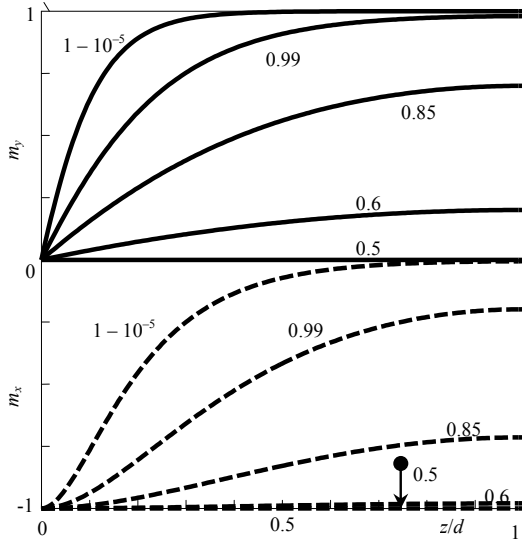


Figure 3. The magnetization distribution for the configuration of a magnetically-soft layer on a magnetically-hard layer versus the value of z/d ; the static mode with $\rho = 1$. Several values of the parameter k^2 are shown: $k^2 = 0.5, 0.6, 0.85, 0.99, 1 - 10^{-5}$.

(22) and (23) concerning study of their derivatives with respect to $f = z/d$.

3. Derivatives of the Magnetization Components

The first derivatives of the magnetization components m_x and m_y with respect to $f = z/d$ read:

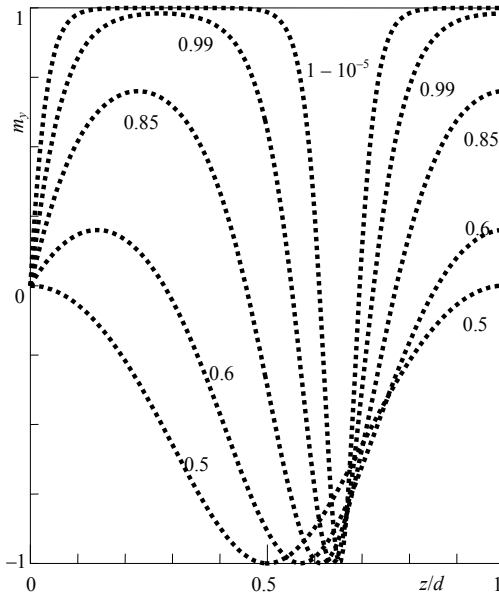
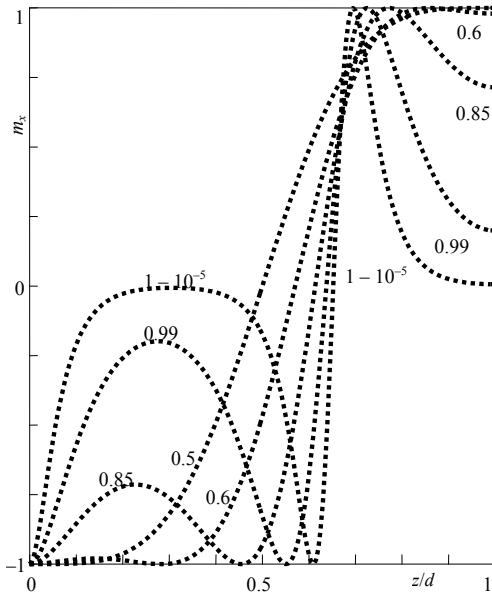


Figure 4. The magnetization components m_x and m_y versus the value of z/d ; the first dynamic mode with $\rho = 3$. The values of parameter k^2 are the same to those for the static mode shown in Figure 3.

$$\frac{dm_x}{df} = \frac{dm_x}{du} \frac{du}{df} \quad (27)$$

$$\frac{dm_y}{df} = \frac{dm_y}{du} \frac{du}{df} \quad (28)$$

where

$$du/df = \rho K(k) - F_k(k) \quad (29)$$

Note that the derivative du/df in Equation (29) is equal to a constant for each ρ and k . In Equations (27) and (28), the first derivatives of the m_x and m_y with respect to the function u can be written as follows:

$$\frac{dm_x}{du} = -2k \left[\text{dn}(u, k) \frac{\text{dsn}(u, k)}{du} + \text{sn}(u, k) \frac{\text{ddn}(u, k)}{du} \right] = - (2k) \text{cn}(u, k) [2\text{dn}^2(u, k) - 1] \quad (30)$$

$$\frac{dm_y}{du} = 4k^2 \text{sn}(u, k) \frac{\text{dsn}(u, k)}{du} = 4k^2 \text{sn}(u, k) \text{cn}(u, k) \text{dn}(u, k) \quad (31)$$

where

$$\frac{\text{dsn}(u, k)}{du} = \text{cn}(u, k) \text{dn}(u, k) \quad (32)$$

$$\frac{\text{ddn}(u, k)}{du} = -k^2 \text{cn}(u, k) \text{sn}(u, k) \quad (33)$$

Using Equations (27) and (28), the second derivatives of the m_x and m_y with respect to the f can be written as follows:

$$\frac{d^2 m_x}{df^2} = \frac{du}{df} \frac{d}{df} \left(\frac{dm_x}{du} \right) + \frac{dm_x}{du} \frac{d^2 u}{df^2} = \frac{d^2 m_x}{du^2} \left(\frac{du}{df} \right)^2 \quad (34)$$

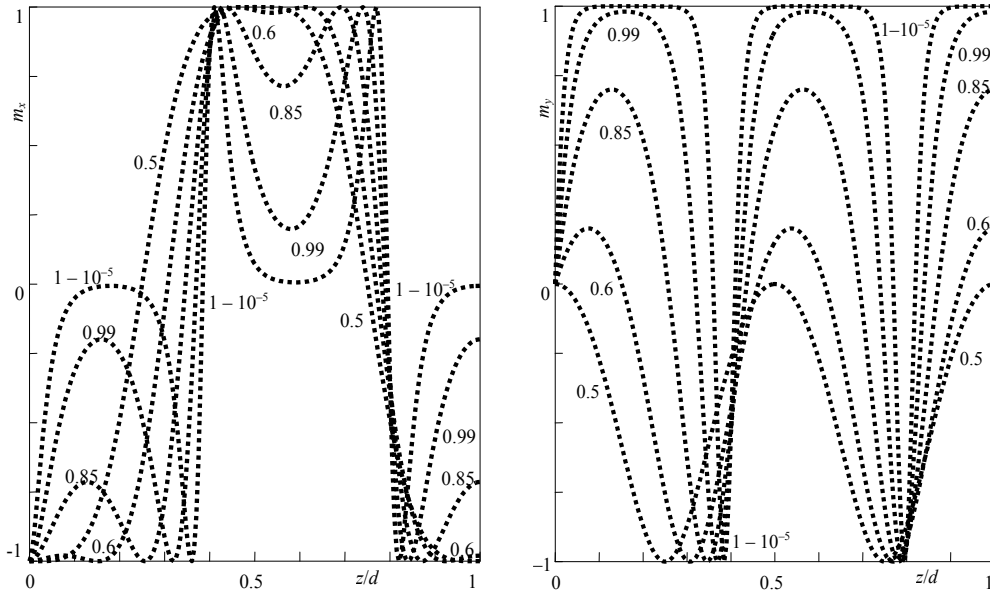


Figure 5. The magnetization components m_x and m_y versus the value of z/d ; the second dynamic mode with $\rho = 5$. The values of parameter k^2 are the same to those for the static mode shown in Figure 3.

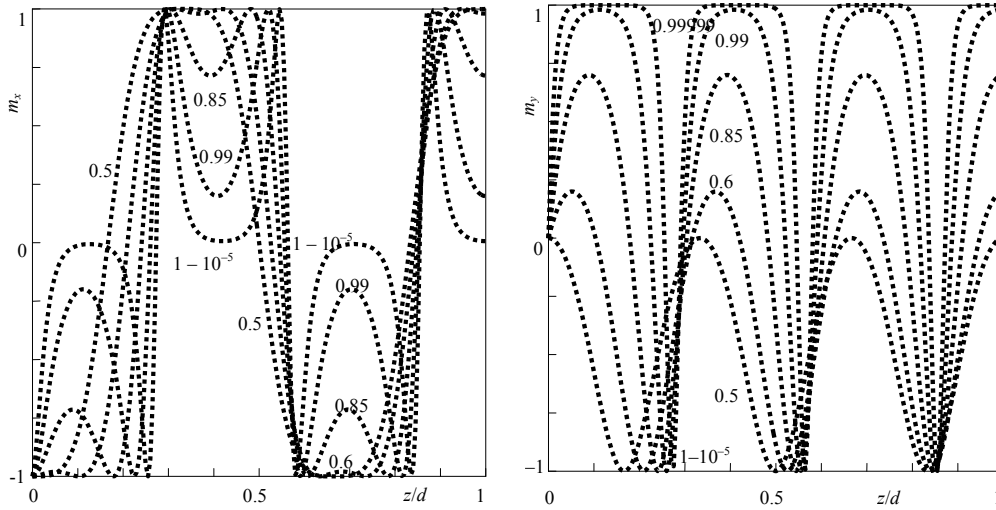


Figure 6. The magnetization components m_x and m_y versus the value of z/d ; the third dynamic mode with $\rho = 7$. The values of parameter k^2 are the same to those for the static mode shown in Figure 3.

$$\frac{d^2 m_y}{df^2} = \frac{du}{df} \frac{d}{df} \left(\frac{dm_y}{du} \right) + \frac{dm_y}{du} \frac{d^2 u}{df^2} = \frac{d^2 m_y}{du^2} \left(\frac{du}{df} \right)^2 \quad (35)$$

It is stressed that the second derivative of the function u with respect to f in Equations (34) and (35) equals zero due to the linear dependence $u(f)$ because $K(k)$ and $F_k(k)$ are constants. In Equations (34) and (35), the second derivatives of the m_x and m_y with respect to the function u can be written in the following form:

$$\frac{d^2 m_x}{du^2} = -2k \left[\operatorname{dn}(u, k) \frac{d^2 \operatorname{sn}(u, k)}{du^2} + \right.$$

$$\left. \operatorname{sn}(u, k) \frac{d^2 \operatorname{dn}(u, k)}{du^2} + 2 \frac{d \operatorname{sn}(u, k)}{du} \frac{d \operatorname{dn}(u, k)}{du} \right] \quad (36)$$

$$\frac{d^2 m_y}{du^2} = 4k^2 \left[\left(\frac{d \operatorname{sn}(u, k)}{du} \right)^2 + \operatorname{sn}(u, k) \frac{d^2 \operatorname{sn}(u, k)}{du^2} \right] \quad (37)$$

where

$$\frac{d^2 \operatorname{sn}(u, k)}{du^2} = -\operatorname{sn}(u, k) [\operatorname{dn}^2(u, k) + k^2 \operatorname{cn}^2(u, k)] \quad (38)$$

$$\frac{d^2 \operatorname{dn}(u, k)}{du^2} = -k^2 \operatorname{dn}(u, k) [\operatorname{cn}^2(u, k) - \operatorname{sn}^2(u, k)] \quad (39)$$

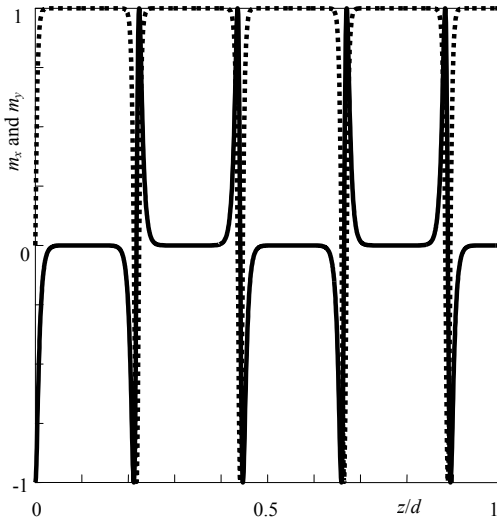


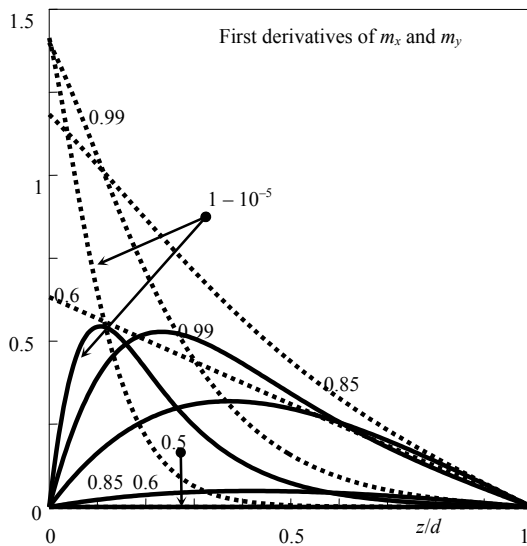
Figure 7. The magnetization components m_x (solid line) and m_y (dotted line) versus the value of z/d ; the fourth dynamic mode with $\rho = 9$. The value of parameter k^2 equals to 1.

In the same manner, it is possible here to write all derivatives of the m_x and m_y with respect to the f :

$$\frac{d^n m_x}{df^n} = \frac{d^n m_x}{du^n} \left(\frac{du}{df} \right)^n \quad (40)$$

$$\frac{d^n m_y}{df^n} = \frac{d^n m_y}{du^n} \left(\frac{du}{df} \right)^n \quad (41)$$

where the index n is an integer and $n > 0$.



The first and second derivatives of the m_x and m_y are shown in Figure 8 for the static mode with $\rho = 1$. It is clearly seen in Figure 8 that the first derivatives of the m_x commence with zero values at $f = 0$ and they together with the first derivatives of the m_y become equal to zero at $f = 1$. The second derivatives of the m_x and m_y for the static mode shown in Figure 8 were calculated with formulae (36)-(39). For the first dynamic mode, the first and second derivatives of the m_x and m_y are shown in Figures 9 and 10, respectively, as the functions of the values of z/d . It is seen in Figure 9 that the first derivatives of the m_x commence with zero at $f = 0$ and the derivatives of the m_x and m_y become equal to zero at $f = 1$ that is similar to the case of the static mode.

4. Non-Zero Value of the Parameter B and Discussions

In the case of non-zero parameter b , the magnetization components m_x and m_y were recently written as functions of the Jacobian elliptic functions $\text{sn}(u, k)$ and $\text{dn}(u, k)$ in the following complicated form [13]:

$$m_x = -2\sqrt{k^2 - \zeta^2} \text{dn}(u, k) \text{sn}(u, k) / (1 - \zeta^2 \text{sn}^2(u, k)) \quad (42)$$

$$m_y = 1 - 2\text{dn}^2(u, k) / (1 - \zeta^2 \text{sn}^2(u, k)) \quad (43)$$

According to [13], the parameter ζ represents a function of both parameters k and b , which are independent of each other. The parameter ζ can have any value from 0 to 1, and the parameter b can be written as the following function of the k and ζ [13]: $b(k, \zeta) = \zeta^2 / (k_0^2 - 2k_0^2 \zeta^2 + \zeta^2)$ with $k_0^2 = (k^2 - \zeta^2) / (1 - \zeta^2)$.

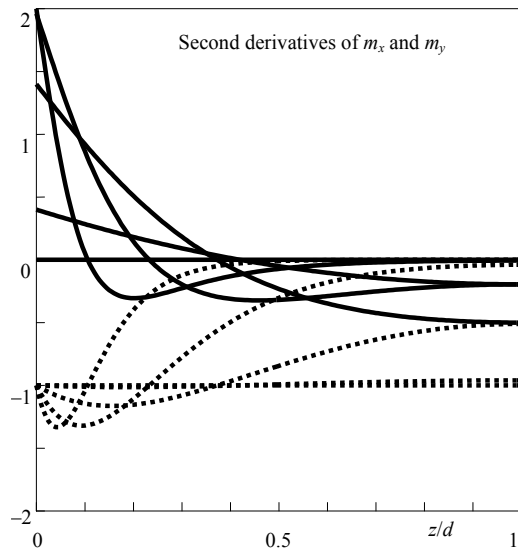


Figure 8. The first and second derivatives of m_x (solid line) and m_y (dotted line) with respect to the u versus the value of z/d ; the static mode with $\rho = 1$. The values of parameter k^2 are the same as those for the static mode shown in Figure 3. The factors $du/df = \rho K(k) - F_k(k)$ for the first derivatives of m_x and m_y are as follows: 0, 0.64, 1.41, 2.81, 6.26 for $k^2 = 0.5, 0.6, 0.85, 0.99, 1 - 10^{-5}$, respectively. The factors $(du/df)^2$ for the second derivatives of m_x and m_y are as follows: 0, 0.41, 1.98, 7.89, 39.2 for the corresponding k^2 .

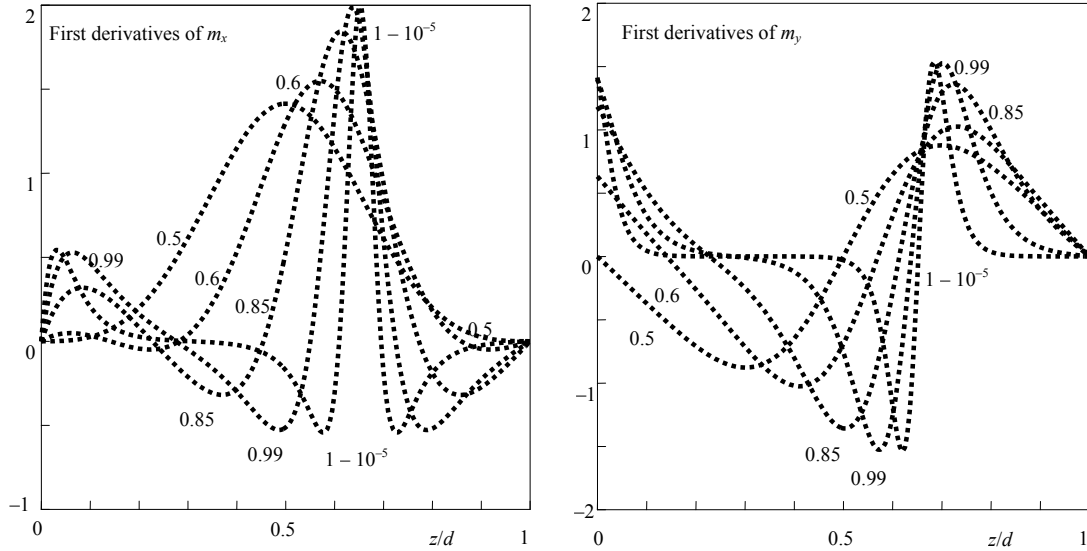


Figure 9. The first derivatives of m_x and m_y with respect to the u versus the value of z/d ; the first dynamic mode with $\rho = 3$. The values of parameter k^2 are the same to those for the static mode shown in Figure 3. The factors $du/df = \rho K(k) - F_k(k)$ for the first derivatives of m_x and m_y are as follows: 3.71, 4.54, 6.18, 10.2, 20.5 for $k^2 = 0.5, 0.6, 0.85, 0.99, 1 - 10^{-5}$, respectively.

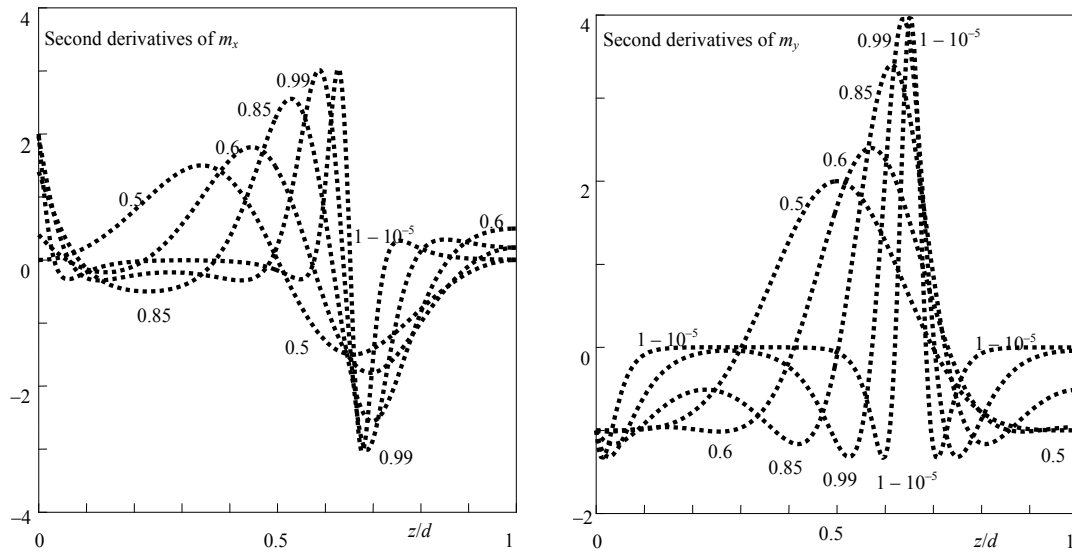


Figure 10. The second derivatives of m_x and m_y with respect to the u versus the value of z/d ; the first dynamic mode with $\rho = 3$. The values of parameter k^2 are the same to those for the static mode shown in Figure 3. The factors $(du/df)^2$ for the second derivatives of m_x and m_y are as follows: 13.8, 20.6, 38.2, 104, 422 for $k^2 = 0.5, 0.6, 0.85, 0.99, 1 - 10^{-5}$, respectively.

Zakharov and Khlebopros [13] represented solutions (42) and (43) as the exact solutions for the magnetization components m_x and m_y in the case of the axial loading of the magnetic field $\mathbf{H}$. Indeed, in the case of the axial loading, they satisfy the boundary condition $m_y(b \neq 0, f = 0) = m_y(b = 0, f = 0) = -1$ and the relationship between the components: $m_x^2 + m_y^2 = 1$. It is thought that the following must be fulfilled: $m_y(b \neq 0, f = 1) = m_y(b = 0, f = 1)$ because $\cos(\psi = \pi/2) = 0$ at $f = 1$ in Equation (11). However, that is also not fulfilled using solutions (42) and (43). Note that in the case of the axial loading of the

$\mathbf{H}$, the magnetization vector $\mathbf{M}$ should be directed towards negative values of the y -axis that is anti-parallel to the vector $\mathbf{H}$. It was also found that in the case of transversal loading of the magnetic field, solutions (42) and (43) can satisfy only the relationship $m_x^2 + m_y^2 = 1$, because it should be true for any u and k . In this case of transversal loading according to the boundary condition at $f = 0$, the magnetization component m_x should equal to -1 . However, that does not occur for any non-zero parameter ζ , using Equations (42) and (43). It is thought that any solution of the problem in both cases of the

transversal and axial loading for $b \neq 0$ should satisfy the boundary conditions similar to what occurs in the case of $b = 0$, using Equations (22) and (33) in both cases of the transversal and axial loading of the magnetic field $\mathbf{H}$. Therefore, one method is offered in this paper below to numerically obtain solutions for the case of $b \neq 0$, which entirely satisfy the boundary conditions at $f = 0$ and $f = 1$ for the cases of the transversal and axial loading such as in the problem of $b = 0$.

Equation (11) can be written as follows:

$$C_1 df = \frac{2d\psi}{\sqrt{1 - k^{*2} A^2 \sin^2 \psi}} \quad (44)$$

with $C_1^2 = (2 + b)q^2 + \text{const}$, $\psi = \chi/2$ and the function $A^2 = 1 + b \cos^2(\psi)$. Here, it is assumed that in Equation (44), the function A results in the parameter k^* that is convenient in order to cope with an integral in the form of the elliptic integral of the first kind. Hence, Equation (44) can be written as:

$$k_b q A df = \frac{d\psi}{\sqrt{1 - k_b^{*2} \sin^2 \psi}} \quad (45)$$

with the parameter $k_b^{*2} = k^{*2} A^2$ and $k_b^{*2} \equiv 1/k_b^2$, hence $k_b^2 = k^2/A^2$. Note that in this case in Equations (44) and (45), the constant $C_1^2 = 4q^2 k^2 = 4q^2 k_b^2 A^2$ represents a function of the parameter b and angle ψ , but it should also remain a constant.

It is thought that the following mathematical transformations can be written in the same manner as Formulas (13) to (23): the right side of equality (45) can be also written in the form of the elliptic integral of the first kind:

$$k_b q A f = F(\psi, k_b^*) + C^* \quad (46)$$

where C^* is a constant which is also determined from the boundary conditions. Applying the boundary condition (9) at $f = 0$ and using the transformations $F(\psi, k_b^*) = k_b F(\psi^*, k_b)$ and $\sin(\psi) = k_b \sin(\psi^*)$ for $k_b^{*2} = 1/k_b^2$, the constant C^* is analogically found as follows: $C^* = k_b F_{kb}(k_b)$ with

$$F_{kb}(k_b) = F(\arcsin(\sqrt{2}/2k_b), k_b) \quad (47)$$

giving

$$F\left(\arcsin\left(\frac{\sin \psi}{k_b}\right), k_b\right) = q A f + F_{kb}(k_b) \quad (48)$$

Note that in Equation (48), the parameter k_b is absent, and hence qA represents a function, but not a constant. Indeed, it is also possible to apply a harmonic function such as sine to both sides of Equation (48) that results in the following:

$$\sin \psi = k_b \sin(u_b, k_b) \quad (49)$$

with $u_b = qA f + F_{kb}(k_b)$. It is also noted that $F(\varphi, k_b) = u_b$ and $\varphi = \text{am}(u_b)$. Hence

$$\chi = 2 \arcsin[k_b \sin(u_b, k_b)] \quad (50)$$

Utilizing boundary condition (9) at $f = 1$, the parameter qA is also obtained as a function of k_b .

It is apparent that $d\chi/df = 0$ if the elliptic cosine $\text{cn}(u_b, k_b) = 0$ that is satisfied in the case of $b \neq 0$ already for $u_b = \rho K(k_b)$ with $\rho = 2\tau - 1$ and $\tau = 1, 2, 3, \dots$. Therefore, it is possible to write the following result: $q^2 A^2 = (\rho K(k_b) - F_{kb}(k_b))^2$. Hence

$$u_b = (\rho K(k_b) - F_{kb}(k_b))f + F_{kb}(k_b) \quad (51)$$

The solutions for the m_x and m_y can be also written in the form of the Jacobian elliptic functions, namely $\text{sn}(u_b, k_b)$ and $\text{dn}(u_b, k_b)$:

$$m_x = -2k_b [\text{dn}(u_b, k_b) \text{sn}(u_b, k_b)] \quad (52)$$

$$m_y = -1 + 2k_b^2 \text{sn}^2(u_b, k_b) \quad (53)$$

Using the effective magnetic field h_a of anisotropy, it is possible to write as follows:

$$h/h_a = (2/\pi)^2 (\rho K(k_b) - F_{kb}(k_b))^2 / A^2 \quad (54)$$

Note that solutions (52) and (53) for the case of $b \neq 0$ look like the exact solutions in Equations (22) and (23) for the case of $b = 0$. Therefore, they should satisfy the boundary conditions at both $f = 0$ and $f = 1$. It is obvious that solutions (52) and (53) are formed from the exact solutions in Equations (22) and (23) by the following substitutions: $k \rightarrow k_b$ and $u \rightarrow u_b$. Note that in the case of $b \neq 0$, the parameter k_b depends on both the parameter b and the angle $\psi = \chi/2$. Therefore, the angle ψ depends on the $k_b(\psi)$ in Equation (49), so that as soon as the angle ψ is changed, the $k_b(\psi)$ is also correspondingly changed. Indeed, it is necessary to apply the following recursive procedure using Equation (50): $\chi_{N+1} = 2 \arcsin[k_b(\chi_N) \text{sn}(u_b(\chi_N), k_b(\chi_N))]$ ($N = 0, 1, 2, \dots$). The right angle χ is found when $\chi_{N+1} = \chi_N$. Fortunately, this numerical problem can be resolved. It is thought that for the numerical procedure to compute magnetization components (52) and (53), the exactly determined angle χ in the case of $b = 0$ can be used as an initial guess χ_0 to numerically find the right angle χ_N in Equation (50) for the case of $b \neq 0$. It was set in the numerical procedure to interrupt the calculation process when $\text{abs}(\chi_{N+1} - \chi_N) < 10^{-7}$. Note that such numerical calculations can be readily completed with a modern computer, for instance, a laptop with a 20-inch monitor and a four-core processor. It is also thought that this numerical method can be useful for finding solutions when the function A represents more complicated function of the angle ψ , depending on several parameters b_i .

To compare the solutions for the cases of $b = 0$ and $b \neq 0$, **Figures 11** and **12** show the magnetization components m_x and m_y (transversal loading of a magnetic field) for the static mode ($\rho = 1$) and the first dynamic mode ($\rho = 3$) respectively. It is possible to notice in the figures that in the case of the dynamic mode in **Figure 12**, the

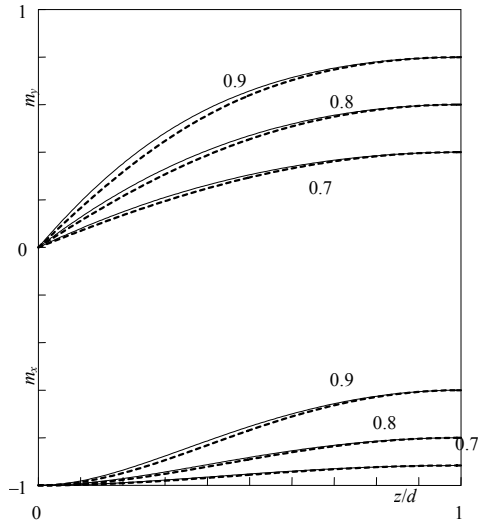


Figure 11. The magnetization components m_x and m_y versus the value of z/d for the static mode ($\rho = 1$) for $k^2 = 0.7, 0.8$, and 0.9 . Solid lines are for $b = 0$ and the dashed lines are for $b \sim 0.158, 0.139$, and 0.123 , respectively.

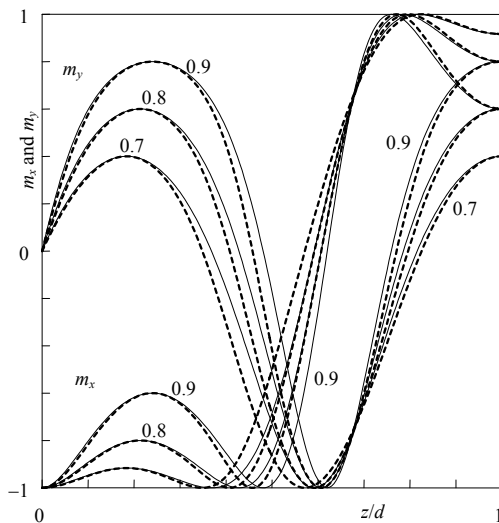


Figure 12. The magnetization components m_x and m_y versus the value of z/d for the first dynamic mode ($\rho = 3$) for $k^2 = 0.7, 0.8$, and 0.9 . Solid lines are for $b = 0$ and the dashed lines are for $b \sim 0.158, 0.139$, and 0.123 , respectively.

difference between the cases of $b = 0$ (solid line) and $b \neq 0$ (dashed line) is more significant than that for the case of the static mode in **Figure 11**. The figures show the magnetization behaviors for relatively small values of $b < 0.2$. Also, it is clearly seen that for the dynamic mode, the values of the components m_x and m_y reach -1 at the smaller values of z/d for the case of $b \neq 0$.

5. Conclusions

This paper demonstrated the magnetization distribution

in a magnetically-soft layer (ferromagnetics) on a magnetically-hard substrate (anti-ferromagnetics) when the applied magnetic field is perpendicular to the initial magnetization. Solutions were written in the form of combination of the Jacobian elliptic functions and elliptic integrals. The average values of magnetization components, $\langle m_x \rangle$ and $\langle m_y \rangle$, were calculated in dependence on the applied magnetic field. The static mode and several dynamic modes of magnetization components m_x and m_y were also calculated in order to illuminate their distributions across the layer thickness. The first and second derivatives of the magnetization components were also calculated. Also, it was found that the inclusion of magnetic anisotropy ($b \neq 0$) in calculations can complicate the finding of the magnetization components and show a significant difference. Note that the utilized solutions for the problem completely satisfy the boundary conditions applied to the magnetically-soft layer with inhomogeneous boundaries.

6. References

- [1] L. Landau and E. Lifshits, "On the Theory of the Dispersion of Magnetic Permeability in Ferromagnetic Bodies," *Physikalische Zeitschrift der Sowjetunion*, Vol. 8, 1935, pp. 153-169.
- [2] L. Landau and E. Lifshits, "On the Theory of the Dispersion of Magnetic Permeability in Ferromagnetic Bodies," *Ukrainian Journal of Physics*, Special Issue, Vol. 53, 2008, pp. 14-22.
- [3] J. Li, "A Two-Dimensional Landau-Lifshitz Model in Studying Thin Film Micromagnetics," *Abstract and Applied Analysis*, Vol. 13, 2009.
- [4] J.-N. Li, X.-F. Wang and Z.-A. Yao, "An Extension Landau-Lifshitz Model in Studying Soft Ferromagnetic Films," *Acta Mathematicae Applicatae Sinica, English Series*, Vol. 23, No. 3, 2007, pp. 421-432.
- [5] L. G. Korzunin, B. N. Filippov and F. A. Kassan-Ogly, "Dynamics of Vortex-like Domain Walls in Triaxial Magnetic Films with the Goss Orientation of the Surface," *Technical Physics*, Vol. 52, No. 11, 2007, pp. 1453-1457.
- [6] B. N. Filippov and L. G. Korzunin, "Non-linear Dynamics of Domain Walls with Vortex-like Inner Structure in Magnetically-single-axis Films with Plane Anisotropy," *Journal of Experimental and Technical Physics*, Moscow, Vol. 121, No. 2, 2002, pp. 372-387.
- [7] P. L. Sulem, C. Sulem and C. Bardos, "On the Continuous Limit for a System of Classical Spins," *Communications in Mathematical Physics*, Vol. 107, No. 3, 1986, pp. 431-454.
- [8] W. E and X.-P. Wang, "Numerical Methods for the Landau-Lifshitz Equation" *SIAM Journal on Numerical Analysis*, Vol. 38, No. 5, 2000, pp. 1647-1665.
- [9] A. DeSimone, R. V. Kohn, S. Müller and F. Otto, "A Reduced Theory for Thin-Film Micromagnetics," *Communications on Pure and Applied Mathematics*, Vol. 55,

- No. 11, 2002, pp. 1408-1460.
- [10] A. DeSimone, R. V. Kohn, S. Müller, F. Otto and R. Schäfer, "Two-Dimensional Modelling of Soft Ferromagnetic Films," *Proceedings of the Royal Society of London, Series A*, Vol. 457, No. 2016, 2001, pp. 2983-2991.
 - [11] A. Aharoni, E. H. Frei and S. Shtrikman, "Theoretical Approach to the Asymmetrical Magnetization Curve," *Journal of Applied Physics*, Vol. 30, No. 12, 1959, pp. 1956-1961.
 - [12] N. M. Salansky and M. S. Eruchimov, "The Peculiarities of Spin-Wave Resonance in Films with Ferro-Antiferromagnetic Interaction," *Thin Solid Films*, Vol. 6, No. 2, 1970, pp. 129-140.
 - [13] Y. V. Zakharov and E. A. Khlebopros, "Magnetization Curves and Frequencies of Magnetic Resonance in Films with Domain Structures on Anti-Ferromagnetic Substrate," *Solid State Physics*, Moscow, Vol. 22, No. 12, 1980, pp. 3651-3657.
 - [14] H. Bateman and A. Erdélyi, "Higher Transcendental Functions," McGraw-Hill, New York, Vol. 3, 1955.
 - [15] M. Abramowitz and I. A. Stegun, Eds., "Handbook of Mathematical Functions with Formulas," *Graphs and Mathematical Tables*, National Bureau of Standards in Applied Mathematics, Washington, 1964, p. 1058.
 - [16] L. Collatz, "Eigenvalue Problems with Engineering Applications," *Fizmatgiz*, Moscow, 1968.
 - [17] Y. V. Zakharov, "Static and Dynamic Instability of a Ferromagnetic Layer under Magnetic Reversal," *Doklady of Russian Academy of Science*, Vol. 344, No. 3, 1995, pp. 328-332.
 - [18] M. A. Lavrent'ev and A. Y. Ishlinskii, "Dynamic Buckling Modes of Elastic Systems," *Doklady Akademii Nauk USSR (Moscow)*, Vol. 64, No. 6, 1949, pp. 779-782.
 - [19] Y. V. Zakharov and A. A. Zakharenko, "Dynamic Instability in the Nonlinear Problem of a Cantilever," *Computational Technologies*, Vol. 4, No. 1, 1999, pp. 48-54.
 - [20] M. Johnson, "Bipolar Spin Switch," *Science*, Vol. 260 No. 5106, 1993, pp. 320-323.
 - [21] D. J. Monsma, J. C. Lodder, T. J. A. Popma and B. Dieny, "Perpendicular Hot Electron Spin-Valve Effect in a New Magnetic Field Sensor: The Spin-Valve Transistor," *Physical Review Letters*, Vol. 74, No. 26, 1995, pp. 5260-5263.
 - [22] M. Johnson, "Spin Injector in Metal Films: The Bipolar Spin Transistor," *Materials Science and Engineering B*, Vol. 31, 1995, pp. 199-205.
 - [23] V. Y. Shur and E. L. Rumyantsev, "Kinetics of Ferroelectric Domain Structure during Switching: Theory and Experiment," *Ferroelectrics*, Vol. 151, 1994, pp. 171-189.

Computation of the Schrödinger Equation via the Discrete Derivatives Representation Method: Improvement of Solutions Using Particle Swarm Optimization

Abdelwahab Zerarka, H. Saidi, A. Attaf, N. Khelil

Laboratory of Theoretical Physics, University Med Khider, Biskra, Algeria

E-mail: azerarka@hotmail.fr

Received January 16, 2010; revised February 3, 2010; accepted March 18, 2010

Abstract

We develop the discrete derivatives representation method (DDR) to find the physical structures of the Schrödinger equation in which the interpolation polynomial of Bernstein has been used. In this paper the particle swarm optimization (PSO for short) has been suggested as a means to improve qualitatively the solutions. This approach is carefully handled and tested with a numerical example.

Keywords: Discrete Derivatives, Spectra, Wave Function, Particle Swarm

1. Introduction

Several different methods, analytical and numerical have also been formulated and modeled during the past decades for the study of the solutions of the wave equation with different structures. It is known also that for very limited potentials, Schrödinger equation is exactly solvable [1-6].

The latest numerical approach to date is the differential quadrature method [1] introduced for energy spectra estimate. It was first applied to Schrödinger equation in the linear case, where the solution is not correctly reproduced in the domain in which strong oscillations can arise, or simply for instance in the case of highly excited states.

Further calculations are pursued for the construction of the solution by a suitable choice of the interpolating points using the particle swarm optimization (PSO) [7] together with the discrete derivatives representation method. The aim of the present work is to develop a general numerical procedure for the wave equations that is universally applicable.

2. Formulation of the Discrete Derivatives Representation Method (DDR)

In this section, the description of the discrete derivatives representation method can be summarized as follows: the radial Schrödinger equation in the framework of the spherically symmetric potential $V\left(\left|\vec{r}\right|=r\right)$ is written as

$$\left[-\frac{d^2}{dr^2} + w(r)\right]S_{n,l}(r) = \varepsilon S_{n,l}(r) \quad (1)$$

where $\varepsilon = \frac{2m}{\hbar^2} E$. We treat the case where the potential is central, and the Equation (1) is identified as the reduced Schrödinger equation, $w(r) = \frac{2m}{\hbar^2} V(r) + \frac{K}{r^2}$ is the effective potential where K is expressed in terms of the angular momentum quantum number l by $l(l+1)$, and the radial function $R_{n,l}(r)$ is related to $S_{n,l}(r)$ by the relation $S_{n,l}(r) = rR_{n,l}(r)$. The radial variable r runs from a to b with $a > 0$ and b can be infinite. In general, in some problem, the Schrödinger operator requires a change of variables. At this point, we need to make a universal transformation on the variable r .

Let $z = \varphi(r)$ be the new variable, where $\varphi(r)$ is a smooth invertible function ($r = \varphi^{-1}(z)$), and it is also easy to see that this definition preserves always the eigenvalue equation.

We can express the solution $S_{n,l}(z)$ by making the substitution

$$S(\varphi^{-1}(z)) = S_{as}(z)z^\rho\Psi(z) \quad (2)$$

where we have now dropped the n,l subscript for simplicity. $\Psi(z)$ is a polynomial function in which will be defined in the following, $S_{as}(z)$ is a asymptotic solution to be determined, and ρ is arbitrary quantity, and can be

expressed in terms of the parameters of the potential.

After substitution in (1) it can be verified that the function $\Psi(z)$ must be solution of the equation

$$h\Psi(z) = 0 \quad (3)$$

where the differential operator h is defined by

$$h = -F(z) \frac{d^2}{dz^2} - 2A(z) \frac{d}{dz} + D(z) \quad (4)$$

for simplicity, we abbreviate as follows

$$\begin{cases} F(z) = (\varphi')^2 \\ A(z) = (\varphi')^2 \left[\frac{S'_{as}}{S_{as}} + \frac{\rho}{z} + \frac{\varphi''}{2(\varphi')^2} \right] \\ D(z) = w(\varphi^-(z)) - \varepsilon - (\varphi')^2 \left[C(z) + \frac{2\rho}{z} \frac{\varphi''}{(\varphi')^2} + \frac{\rho(\rho-1)}{z^2} \right] \\ \text{where } C(z) = S_{as}^{-1} \left[S'_{as} \left(\frac{2\rho}{z} + \frac{\varphi''}{(\varphi')^2} \right) + S''_{as} \right] \end{cases} \quad (5)$$

Now, we introduce the discrete derivatives representation method in which any derivative discretized at any grid point can be expressed by a linear combination of functional values at all discrete points over the interval $[\varphi(a), \varphi(b)]$ of the variable z .

The term $[h\Psi]$ involves the different derivatives and can be expressed as a constant coefficient eigenfunction combination at all discrete points over the interval $[\varphi(a), \varphi(b)]$ as

$$[h\Psi]_i = \sum_{j=0}^N \beta_{ij} \Psi(z_j) \text{ for } i = 0, \dots, N \quad (6)$$

$\Phi(z_i)$ represents the eigenfunction value at grid point z_i . The weighting coefficients β_{ij} are established with the choice of the test function and specifically taken as the Bernstein interpolated polynomial of N th degree as

$$B_{N,j}(x) = \binom{N}{j} x^j (1-x)^{N-j}, \quad j = 0, \dots, N, \text{ and } x \in [0, 1] \quad (7)$$

and the associated sequences $\{z_j\}$, $1 \leq j \leq N$ of the z -variable linked to Bernstein points $x_j = \frac{j}{N}$ by the relation $z_j = (\varphi(b) - \varphi(a))x_j + \varphi(a)$. The term $\binom{N}{j}$ in (7) denotes the binomial coefficient. A given function

$g(x)$ can then be approached using (7) by

$$g(x) \approx g_N(x) = \sum_{j=0}^N g(x_j) B_{N,j}(x) \quad (8)$$

It follows that from (6,7,8), we can establish the unknown weighting coefficients β_{ij} for the total Hamiltonian h as

$$\beta_{ij} = D(z_i) \alpha_{ij}^{(0)} - 2A(z_i) \alpha_{ij}^{(1)} - F(z_i) \alpha_{ij}^{(2)}, \quad (9)$$

the superscripts 0, 1 and 2 in parentheses do not indicate powers, but merely identify the derivatives of the Bernstein's polynomial with which the quantities α_{ij} are associated.

$$\alpha_{ij}^{(k)} = \frac{1}{(\varphi(b) - \varphi(a))^k} \frac{d^k B_{N,j}(x_i)}{dx^k}, \quad k = 0, 1, 2 \quad (10)$$

Having found the weighting coefficients β_{ij} in terms of the energy, one can accurately solve the following matrix equation and therefore the original problem (1)

$$[\beta] \Psi = 0 \quad (11)$$

In the above expression, $[\beta]$ is a $(N+1) \times (N+1)$ matrix with elements β_{ij} , and Ψ is a column vector with components $(\Psi(z_0), \Psi(z_1), \dots, \Psi(z_N))$. more complete description will be given later on with two specific examples.

3. Strategy of Particle Swarm Optimization

A new stochastic algorithm has recently appeared, namely "particle swarm optimization" PSO. The term 'particle' means any natural agent that describes the swarms behavior. The PSO model is an appropriate particle simulation concept, and was first proposed by Eberhart and Kennedy [11-13].

In what follows, we present the main steps of the strategy of the PSO algorithm. We assume that each agent (particle) i can be represented in a multidimensional search space N by its current position $X_i = (x_{i1}, x_{i2}, \dots, x_{iN})$ and its corresponding specific velocity $V_i = (v_{i1}, v_{i2}, \dots, v_{iN})$. Also a memory of its personal (previous) best position is represented by $P_i = (p_{i1}, p_{i2}, \dots, p_{iN})$, called (pbest), the subscript i range from 1 to s , where s indicates the size of the swarm. Commonly, each particle localizes its best value so far (pbest) and its position, and consequently identifies its best value in the group (swarm), called also (sbest) among the set of values (pbest).

Now each particle i moves according to the following system as

$$v_{ij}^{k+1} = w_j v_{ij}^k + c_1 r_1^k [(pbest)_{ij}^k - x_{ij}^k] + c_2 r_2^k [(sbest)_j^k - x_{ij}^k] \quad (12)$$

$$x_{ij}^{k+1} = v_{ij}^{k+1} + x_{ij}^k \quad (13)$$

where x_i^{k+1} , v_i^{k+1} are the position and the velocity vector of particle i respectively at iteration sequence $k+1$, c_1 and c_2 are acceleration coefficients for each term exclusively situated in the range of 2 to 4, w_j is the inertia weight with its value that ranges from 0.9 to 1.2, whereas r_1^k , r_2^k are uniform random numbers between zero and one. For more detail, the double subscript in the relations (12) and (13) means that, the first subscript for the particle i and the second one for the dimension j . The good choice of the inertia weight w_j is crucial in the PSO success. In the general case, it can be initially set equal to its maximum value, and progressively we decrease it if the better solution is not reached. In the relation (12), v_{ij}^{k+1} is often replaced by v_{ij}^{k+1} / σ , where σ denotes the constriction factor that controls the velocity of the particles.

The features of this algorithm can be summarized with the following steps:

Step 1: Set the values of the dimension space N , and the size s of the swarm (s can be taken randomly).

Step 2: Initialize the iteration number k (in the general case is set equal to zero).

Step 3: Evaluate for each agent, the velocity vector using its memory and Equation (12), where pbests and sbest can be modified.

Step 4: Each agent must be updated by applying its velocity vector and its previous position using Equation (13).

Step 5: The steps 3, 4 and 5 can be repeated, successively until a convergence condition is satisfied.

The practical part of using PSO procedure is examined in the following example.

4. Example

It is interesting to take the same case as in [1] of the quasi-exact solutions for the singular even-power anharmonic potential to cast a light on the previous and present results.

$$V(r) = ar^2 + br^{-4} + cr^{-6}; \quad a, c > 0 \quad (14)$$

a , b and c are free parameters, whose bound states can, of course, be found in closed form [5,6,8]. This type of potential has been handled by Varshni [9]. The details of the solutions can be found in [9]. The discrete points generated with the algorithm examined above, have been applied successfully on this examples are listed in **Table 1**.

With this potential, the results for the first three energy levels obtained by the present method, the [1], the numerical integration of the Schrödinger equation, and the introduction of an ansatz for the state-function [9] are listed in **Table 2**, where the error tolerance: $TOL = 10^{-8}$. With this tolerance and the number of the interpolation points $N = 17$, the PSO results under consideration are very satisfactory. The wavefunction $R(r)$ is displayed in **Figure 1**. This illustration corresponds to following pairs of parameter (c , b) of **Table 2**: (10, -30.6637974), and (1, -14.2653094) for the first excited state, and the second excited state respectively.

Table 1. The best interpolating points x_i generated by PSO algorithm for this example.

i	x_i	i	x_i
1	0.2910	12	2.5975
2	0.7778	13	2.8191
3	0.8263	14	3.0029
4	0.8359	15	3.6242
5	0.9794	16	3.7235
6	1.0779	17	3.8154
7	1.2836		
8	1.6411		
9	1.6526		
10	1.9432		
11	2.3253		

Table 2. Values of the energies E_0 , E_1 , and E_2 in a.u. obtained for the ground state, the first excited state, and the second excited state respectively, (with $a = 1$, $l = 0$), where the superscripts a, b, c, and d denote the results obtained by numerical integration of Schrödinger equation: [9], by the ansatz for the first three bound states: [9], by the [1], and by the present work respectively.

c	1	10	100
b	-14.2653094	-30.6637974	7.8573936
E_0			8.7857393 ^a
			8.7857394 ^b
			8.7857394 ^c
			8.7857393 ^d
E_1		2.3032559 ^a	
		2.3032559 ^b	
		2.3032559 ^c	
		2.3032559 ^d	
E_2	-2.2653095 ^a		
	-2.2653094 ^b		
	-2.2653094 ^c		
	-2.2653095 ^d		

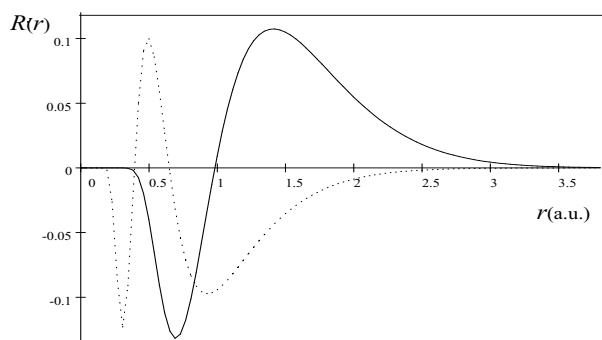


Figure 1. wavefunction $R(r)$ obtained with the present method, where r is in a.u. for the singular even-power anharmonic potential. Full curve ($c = 10$, $b = -30.6637974$), first excited state; dash curve ($c = 1$, $b = -14.2653094$), second excited state.

5. Comments and Concluding Remarks

In this work we have presented a new formulation that uses the PSO algorithm together with the DDR method for the computation of the bound-state eigenvalues and the associated eigenfunctions of linear differential operators such as the Schrödinger-like equation resulting from a quantum system, with which one can receive results that are not available with the interpolating points of Tchebychev type, especially when the wavefunction is not smooth.

Although the previous DDR method with the PSO procedure provides substantially better accuracy than the conventional Tchebychev's interpolating points used in [1] which are always known to be the only best points which permits a good approach of the interpolating function. The preliminary results, obtained through the use of the PSO method, show a good improvement of solutions for the example which has been selected here as a testbed. For instance, **Figure 1** shows graphically the wavefunction obtained by PSO procedure. Furthermore, the shape of the wave functions is preserved for all configurations and the error tolerance is 10^{-8} .

6. Acknowledgments

The authors are grateful to Prof. Hans for interesting discussions and for a support of this work. One of the authors (A. Z) wishes to express its sincere thanks to Dr. Dyn Keit for bringing his attention to the PSO method. This work was sponsored in part by the M.E.R.S (Ministère de l'Enseignement et de la Recherche Scientifique): Under contract No. D0701/01/08.

7. References

- [1] A. Zerarka, S. Hassouni, H. Saidi and Y. Boumedjane, "Energy Spectra of the Schrödinger Equation and the Differential Quadrature Method," *Communication in Nonlinear Science and Numerical Simulation*, Vol. 10, 2005, pp. 737-745.
- [2] A. Zerarka and A. Soukeur, "A Generalized Integral Quadratic Method: I. An Efficient Solution for One-Dimensional Volterra Integral Equation," *Communication in Nonlinear Science and Numerical Simulation*, Vol. 10, 2005, pp. 653-663.
- [3] F. Iachello, "Algebraic Methods in Quantum Mechanics with Applications to Nuclear and Molecular Structure," *Nuclear Physics A*, Vol. 560, 1993, pp. 23-34.
- [4] M. Selg, "Numerically Complemented Analytic Method for Solving the Time-Independent One-Dimensional Schrödinger Equation," *Physical Review E*, Vol. 64, No. 5, 2001, 056701.
- [5] S. H. Dong, "Exact Solutions of the Two-Dimensional Schrödinger Equation with Certain Central Potentials," *International Journal of Theoretical Physics*, Vol. 39, 2000, p. 1119.
- [6] S. K. Bose and N. Gupta, "Exact Solution of Nonrelativistic Schrödinger Equation for Certain Central Physical Potentials," *Il Nuovo Cimento B*, Vol. 113, 1998, p. 299.
- [7] J. Kennedy and R. C. Eberhart, "Particle Swarm Optimization," *Proceedings IEEE International Conferences Neural Networks*, Piscataway, 1995, pp. 1942-1948.
- [8] Maitland, *et al.*, "Intermolecular Forces," University Press, Oxford, 1987.
- [9] Y. P. Varshni, "The First Three Bound States for the Potential $V(r) = ar^2 + br^{-4} + cr^{-6}$," *Physics Letters A*, Vol. 183, 1993, pp. 9-13.
- [10] S. H. Dong, "Quantum Monodromy of Schrödinger Equation with the Decatic Potential," *International Journal of Theoretical Physics*, Vol. 41, No. 1, January 2002, pp. 89-99.
- [11] R. C. Eberhart and J. Kennedy, "A New Optimizer Using Particles Swarm Theory," *Sixth International Symposium on Micro Machine and Human Science*, Nagoya, Japan, 1995, pp. 39-43.
- [12] R. C. Eberhart and Y. Shi, "Parameter Selection in Particle Swarm Optimization," *Lecture Notes in Computer Science-Evolutionary Programming VII*, Porto, V. W., Saravanan, N. Waagen, D., Eiben, A. E., Springer, Vol. 1447, 1998, pp. 591-600.
- [13] T. I. Cristian "The Particle Swarm Optimization Algorithm: Convergence Analysis and Parameter Selection," *Information Processing Letters*, Vol. 85, No. 6, 2003, pp. 317-325.

Fracture and Damage Behaviors of Concrete in the Fractal Space

Heng Zhang¹, Demin Wei^{1,2}

¹Department of Civil Engineering, South China University of Technology, Guangzhou, China

²State Key Laboratory of Subtropical Building Science, South China University of Technology, Guangzhou, China

E-mail: heng.zh@mail.scut.edu.cn

Received January 22, 2010; revised March 17, 2010; accepted March 20, 2010

Abstract

The fracture toughness, the driving force and the fracture energy for an infinite plate with a fractal crack are investigated in the fractal space in this work. The perimeter-area relation is adopted to derive the transformation rule between damage variables in the fractal space and Euclidean space. A plasticity yield criterion is introduced and a damage variable tensor is decomposed into tensile and compressive components to describe the distinct behaviors in tension and compression. A plastic damage constitutive model for concrete in the Euclidean space is developed and generalized to fractal case according to the transformation rule of damage variables. Numerical calculations of the present model with and without fractal are conducted and compared with experimental data to verify the efficiency of this model and show the necessity of considering the fractal effect in the constitutive model of concrete. The structural response and mesh sensitivity of a notched unreinforced concrete beam under 3-point bending test are theoretical studied and show good agreement with the experimental data.

Keywords: Fracture Mechanics, Damage Variable, Fractal Space, Constitutive Model

1. Introduction

Concrete has been widely used in civil engineering for its good in-situ casting and molding abilities. As a quasi-brittle material, the fracture behavior of concrete receives much of researchers' concerns. Though classic fracture mechanics which is based on the assumption of smooth cracking in materials can analyze concrete properties and meet the need of structure design in a certain extent, no advance is made to explain the failure mechanism from the change of the concrete internal structure. The difficulty mentioned above has a hindering effect for researchers to improve the mechanical property of concrete.

Fractal geometry is established by Mandelbrot in 1970s [1], which plays an important role in the development of fracture mechanics theory. Researches show that the fracture zone of metal, rock and concrete has fractal characteristics [2-4]. This leads a widely use of fractal geometry in many fields of material science, for instances, the Sierpinski carpet was adopted by Carpinteri *et al.* [5] to simulate the composition of concrete cross section, and the fractal effect was also introduced into the cohesive crack model. Another remarkable application of fractal geometry is to describe the roughness of

cracks quantitatively. Saouma *et al.* [6] and Issa *et al.* [7] investigated the crack profiles of concrete through tests and pointed out that cracks in concrete have an average fractal dimension of 1.1. Meanwhile, Issa *et al.* [7] analyzed the fracture surface of concrete and found its fractal dimension is about 2.1 to 2.3.

Studies on the damage of concrete point out that the propagation micro defects, *i.e.* microvoids and micro-cracks, etc, is the mainly cause of the macro fracture of materials. Based on this fact, a new kind of constitutive model for concrete, called the damage constitutive model, is developed within the framework of continuum damage mechanics. In this model, the choice of damage variable is a key to control the effectiveness and performance. Because of the heterogeneity of concrete, definition of the damage variable still remains at the state that the change of material macro property, such as elastic modulus and stress, are used to reflect the development of damage indirectly, and no direct relation is set with the intrinsic defects. As the progress in studying fractal phenomena, some researchers try to explore the damage growth by means of fractal geometry. Zhao [8] defined a damage variable as a function of the area of fracture surface, and proposed a new damage constitutive model for

rock in which the fractal effect was taken into account; Guarracino [9] gave out a damage variable as the ratio of the porous and REV (or the representative elementary volume) volume fraction of materials and presented a fractal constitutive model for rock.

In this work, the fracture behaviors of a material with fractal cracks are investigated by using fractal geometry. Theoretical expressions of the fracture toughness, the driving force and the fracture energy is derived consequently. The transformation rule of a fractal damage variable in the fractal space and a apparent damage variable in the Euclidean space is obtained by adopting the perimeter-area relation. This rule is introduced into a new plastic damage constitutive model of concrete presented in this research. A notched plain concrete beam under 3-point bending test is simulated to verify the efficiency of the model.

2. Fracture Parameters in the Fractal Space

2.1. Simplification of Fracture Zone

Figure 1 illustrates an infinite plate with a fractal cut in uniaxial tension. The cut releases the stress in a fracture domain, whose shape can be approximated as an ellipse [10]. A standard Koch fractal curve is employed to construct the boundary of the crack, see **Figure 2**. n denotes the construction step. Keep the area of fracture zone as a constant of ηa_0^2 , then the fractal dimension D of the crack is independent on the yardstick $\delta = 3^{-n} \cdot a_0$, where η is a shape parameter and $\eta = 2\pi$ when smooth cracking.

2.2. Critical Cracking Stress

According to the fractal theory [1], the real length $2a$ and the apparent length (projected to the axial) $2a_0$ of the crack has the following relation:

$$a = a_0^D \cdot \delta^{1-D} \quad (1)$$

The surface energy of the fracture surface is:

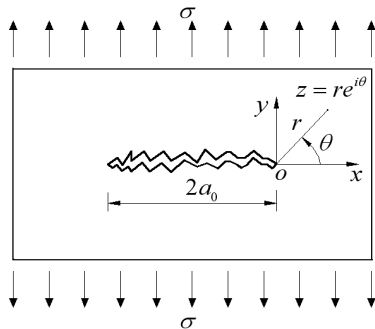


Figure 1. A fractal crack in the infinite plate.

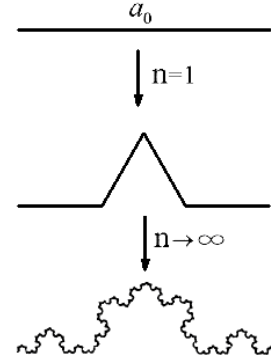


Figure 2. Construction of the crack boundary with a standard Koch fractal curve.

$$\Pi(a) = 4\gamma t a = 4\gamma t \cdot a_0^D \cdot \delta^{1-D} \quad (2)$$

where t is the plate thickness. The perimeter C of the fracture zone is:

$$C = 4a = 4a_0^D \cdot \delta^{1-D} \quad (3)$$

Thus, one gets the zone area A as follows by adopting the perimeter-area relation

$$A(\delta) = \eta a_0^2 = m \cdot C(\delta)^{\frac{2}{D}} \cdot \delta^{\frac{2D-2}{D}} = m \cdot 2^{\frac{4}{D}} \cdot a_0^2 \quad (4)$$

where the proportional coefficient m is:

$$m = \eta \cdot 2^{-4/D} \quad (5)$$

Therefore, the strain energy released during the crack-ing process can be written as:

$$\Delta U = -\frac{t\sigma^2}{2E_0} \cdot A(\delta) = -\frac{t\sigma^2}{2E_0} \cdot \eta \cdot a_0^2 \quad (6)$$

where σ denotes the tensile stress of the plate; E_0 is the elastic modulus. For a plate strain state, one needs to replace E_0 in Equation (6) with $E_0/(1-\nu_0^2)$, where ν_0 is the Poisson's ratio.

The Griffith fracture criterion state that: materials cracks when the released elastic energy ΔU equals to the surface energy Π concentrated in the fracture zone for an infinitesimally small increment of the crack length da_0 , i.e.,

$$\frac{d(\Pi + \Delta U)}{da_0} = 0 \quad (7)$$

Substituting Equations (2) and (6) into Equation (7), one obtains the critical cracking stress σ_c for materials as:

$$\sigma_c = \sqrt{4\gamma E_0 \eta^{-1} \cdot D \cdot a_0^{D-2} \cdot \delta^{1-D}} \quad (8)$$

For a smooth cut case, $D=1.0$, $\eta=2\pi$, and we have:

$$\sigma_c = \sqrt{\frac{2\gamma E_0}{\pi a_0}} \quad (9)$$

2.3. Fracture Toughness

Wunk and Yavari [11] studied the stress field at the fractal crack tip, and presented the component σ_y in y direction as shown in **Figure 1**.

$$\sigma_y = \frac{K_I^f}{(2\pi r)^\alpha} \left\{ \cos(\alpha\theta) + \alpha \sin\theta \sin[(\alpha+1)\theta] \right\} \quad (10)$$

where K_I^f is the fractal stress intensity factor; α represents the singularity order of the stress field, and can be expressed as:

$$\alpha = \frac{2-D}{2}, \quad 1 \leq D \leq 2 \quad (11)$$

in terms of D for a self-similar fractal crack.

For $D=1.0$, we have $\alpha=1/2$ and $K_I^f = K_I$, and Equation (10) degenerates to the smooth cracking case as:

$$\sigma_y = \frac{K_I}{\sqrt{(2\pi r)}} \left[\cos \frac{\theta}{2} + \frac{1}{2} \sin \theta \sin \left(\frac{3\theta}{2} \right) \right] \quad (12)$$

When $\sigma_y = \sigma_c$, $K_I^f = K_{IC}^f$, the crack begins to grow. Referring to Equations (8) and (10), one obtains the fractal fracture toughness for materials:

$$K_{IC}^f = \frac{(2\pi r)^\alpha \sqrt{4\gamma \cdot E_0 \cdot \eta^{-1} \cdot D \cdot a_0^{D-2} \cdot \delta^{1-D}}}{\cos(\alpha\theta) + \alpha \sin\theta \sin[(\alpha+1)\theta]} \quad (13)$$

Figure 3 illustrates the relation among K_{IC}^f , D and a_0 . It can be noted from **Figure 3** that K_{IC}^f decreases with the increasing of the crack length; If $a_0=0$,

K_{IC}^f tends to be infinite, and no crack exists in materials, which verifies the concept that the growth of initial defects leads to the final failure of materials. The greater the D value, the more bifurcated the cracks are, and the larger K_{IC}^f is, which indicates that roughness has a hin-

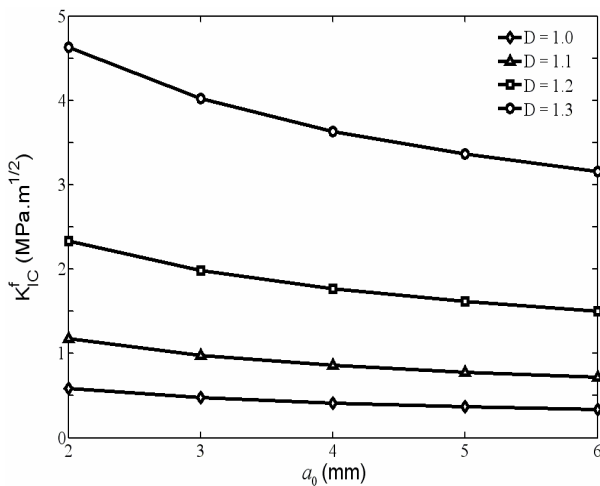


Figure 3. Influences of D and a_0 on K_{IC}^f .

dering effect on the cracking of materials.

2.4. Driving Force

Classic driving force is defined as the strain energy dissipated to form a unit fracture area [11], and can be expressed as:

$$G = -\frac{1}{2t} \cdot \frac{\partial(\Delta U)}{\partial a} = \frac{(K_I)^2}{E_0} \quad (14)$$

Submitting Equations (1) and (6) into Equation (14), one gets:

$$G^f = \frac{\eta\sigma^2}{2E_0D} \cdot a_0^{2-D} \cdot \delta^{D-1} \quad (15)$$

For the mode I cracking case, G^f reaches its maximum value $G_{\max}^f$ when σ increases to the tensile strength σ_t of materials:

$$G_{\max}^f = \frac{\eta\sigma_t^2}{2E_0D} \cdot a_0^{2-D} \cdot \delta^{D-1} \quad (16)$$

We also have the cracking resistance G_{IC} of materials as:

$$G_{IC} = \frac{1}{2t} \cdot \frac{\partial \Pi}{\partial a} = 2\gamma \quad (17)$$

If $G_{\max}^f > G_{IC}$, cracking occurs, and cracks does not grow when $G_{\max}^f < G_{IC}$. This is the G-fracture criterion for fractal cracks.

2.5. Fracture Energy

The fracture energy G_F of materials is defined as the area under the stress vs. the crack open displacement curve in the cohesive law, and represents the energy dissipated on the unitary crack surface [12]. G_F is usually determined by tests. For a large size specimen, the fracture energy G_F^* equals to the max driving force $G_{\max}^f$ expressed in Equation (16), approximately. Therefore, fracture energy G_F for a normal size specimen has relation with $G_{\max}^f$ as:

$$G_F \cdot A_N = G_F^* \cdot A^* = G_{\max}^f \cdot A^* \quad (18)$$

where A_N and A^* are the real areas of the fracture surfaces corresponding to the normal size and large size specimens, respectively, with a transformation as:

$$A^* = \left(\frac{l_{ch}^*}{l_{ch,N}} \right)^{2-2D} \cdot A_N \quad (19)$$

where l_{ch}^* and $l_{ch,N}$ are the characteristic lengths corresponding to A^* and A_N , respectively. Usually, l_{ch}^* is taken to be the minimum threshold of characteristic lengths. For concrete, $l_{ch}^* = 0.15$ mm when $f_0^- = 100$ MPa

and $l_{ch}^* = 0.15$ mm when $f_0^- = 200$ MPa [13], and l_{ch}^* can be obtained by linear interpolation for other strengths. $l_{ch,N}$ is given by [14]:

$$l_{ch,N} = 2a_0/b \quad (20)$$

where the constant $b < 1$ represents the initially cracked portion of the interface prior to load application, $2a_0$ is the initial crack diameter and is assumed to be equal to the maximum aggregate size in concrete.

Substituting Equations (16) and (19) into Equation (18), one obtains

$$G_F = \left(\frac{l_{ch,N}}{l_{ch}^*} \right)^{2D-2} \cdot \frac{\eta \sigma_t^2}{2E_0 D} \cdot a_0^{2-D} \cdot \delta^{D-1} \quad (21)$$

Here $2a_0$ needs to be considered to be the final length of the main crack for concrete, and represents a specimen size. **Figure 4** shows the behaviors of G_F as the changes of a_0 and D . We can find that G_F increases with the increasing of these two factors.

3. Damage Variable

An apparent damage variable is defined as the ratio of the effective bearing area A_k and the cross section area A_c of materials, and has the following form:

$$\phi = \frac{A_k}{A_c} \quad (22)$$

For a Euclidean shape, the area A_0 and the perimeter C_0 have the following relation:

$$A_0 = m_0 C_0^2 \quad (23)$$

where m_0 is a shape constant. Substituting Equation (23) into Equation (22), one obtains the expression for the apparent damage variable ϕ as follows:

$$\text{and } \phi = \left(\frac{C_k}{C_c} \right)^2 \quad (24)$$

where C_k and C_c are the perimeters corresponding to A_k and A_c , respectively. It is reasonable for us to take C_k

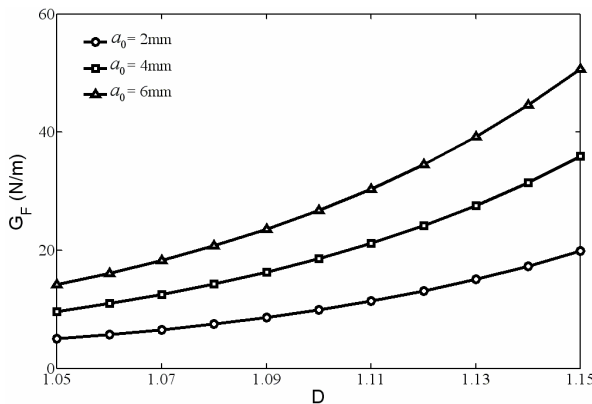


Figure 4. Influences of D and a_0 on G_F .

C_c as the crack length in one direction and the total length of all cracks in all directions in a unit cell at failure, respectively.

By referring to Equation (1), the perimeter C of a fractal damaged surface and the counterpart C_0 in the Euclidean space has the following relation:

$$C = C_0^D l_{ch}^{1-D} \quad (25)$$

Substituting Equation (4) into Equation (22), concerning Equation (25), and based on the fact that cracks in concrete is statistically self-similar fractal when the yardstick δ ranges from 0.263 to 1 [15], one obtains the following expression for a fractal damage variable in the fractal space:

$$\hat{\phi} = 16^{\left(\frac{1}{D_c} - \frac{1}{D_k} \right)} \cdot \phi \quad (26)$$

where $D_k = 1.0 \sim 2.0$. From Equation (29), we find that only the fractal dimension is different between the fractal and apparent damage variables, and δ has no influence on this relation. Since A_c is the cross section area of materials, which indicates $D_c = 1.0$, thus Equation (26) can be rewritten as:

$$\hat{\phi}_{ij} = 16^{\left(1 - \frac{1}{D_k} \right)} \cdot \phi_{ij} \quad (27)$$

For a smooth crack, $D_k = 1.0$, and no fractal effect exists, and we have:

$$\hat{\phi}_{ij} = \phi_{ij} \quad (28)$$

From the above discussion, we notice that the apparent damage variable in the Euclidean space is a special case of the fractal damage variable in the fractal space, and the fractal damage variable is the generalization of the apparent damage variable. **Figure 5** illustrates the differences between the two kind damage variables in uniaxial compression with an assumption that D_k takes the average value of 1.1, and the original evolution data for apparent damage variables is referred from reference [16].

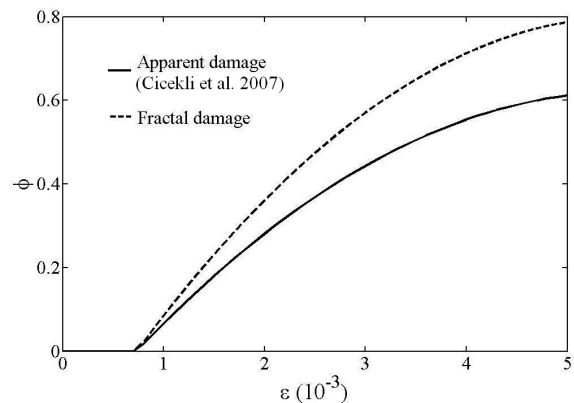


Figure 5. Evolutions of damage variables for concrete in uniaxial compression cases.

4. Plastic Damage Constitutive Model for Concrete

Among the various existed constitutive models for concrete, the plastic damage model has a better effect to characterize the stiffness degeneration, the strain softening and the unilateral effect of concrete under various loading conditions.

4.1. Decomposition of Effective Stress Tensor

In view of the fact that the typical failure modes of concrete are cracking in tension and crushing in compression, we decompose the effective stress tensor into tensile and compressive parts (denoted by $\bar{\sigma}^+$ and $\bar{\sigma}^-$, respectively) by utilizing spectral decomposition technique [17,18]:

$$\bar{\sigma}^+ = \mathbf{P}^+ : \bar{\sigma} \quad (29)$$

$$\bar{\sigma}^- = \bar{\sigma} - \bar{\sigma}^+ = \mathbf{P}^- : \bar{\sigma} \quad (30)$$

where $\mathbf{P}^+$ and $\mathbf{P}^-$ are the fourth-order projection tensors expressed as [19]:

$$\mathbf{P}^+ = \sum_i H(\bar{\sigma}_i) (\mathbf{p}_{ii} \otimes \mathbf{p}_{ii}) \quad (31)$$

$$\mathbf{P}^- = \mathbf{I} - \mathbf{P}^+ \quad (32)$$

where $\mathbf{I}$ is the fourth-order identity tensor; $H(\bar{\sigma}_i)$ represents the Heaviside function calculated for the i th eigenvalue $\bar{\sigma}_i$ of $\bar{\sigma}$; $\mathbf{p}_{ij}$ is the second-order tensor and is defined as:

$$\mathbf{p}_{ij} = \mathbf{p}_{ji} = \frac{1}{2} (\mathbf{n}_i \otimes \mathbf{n}_j + \mathbf{n}_j \otimes \mathbf{n}_i) \quad (33)$$

where $\mathbf{n}_i$ is the i th normalized eigenvector corresponding to $\bar{\sigma}_i$.

4.2. Plasticity

We adopt a plasticity yield function f and a plastic potential function F^p as:

$$f(\bar{\sigma}, \kappa) = \left(\alpha \bar{I}_1 + \sqrt{3\bar{J}_2} + \beta(\kappa) \langle \hat{\bar{\sigma}}_{\max} \rangle \right) - (1 - \alpha) c^-(\kappa) \leq 0 \quad (34)$$

$$F^p = \sqrt{3\bar{J}_2} + \alpha^p \bar{I}_1 \quad (35)$$

where $\langle x \rangle = (|x| + x)/2$ denotes the Macaulay bracket function, $\hat{\bar{\sigma}}_{\max}$ is the algebraically maximum effective principal stress. α , β and c are parameters with the following forms [20]:

$$\alpha = \frac{f_{b0}^- - f_0^-}{2f_{b0}^- - f_0^-}; \quad \beta = \frac{c^-(\kappa)}{c^+(\kappa)} (1 - \alpha) - (1 + \alpha)$$

$$c^+(\kappa) = \bar{f}^+(\kappa); \quad c^-(\kappa) = \bar{f}^-(\kappa) \quad (36)$$

where f_{b0}^- and f_0^- are the initial equibiaxial and uniaxial compressive yield stresses, respectively. f_{b0}^-/f_0^- lies between 1.10 and 1.20 from experiments, therefore, α varies from 0.08 to 0.12. $c^\pm(\kappa)$ represent the inner cohesion, and $\bar{f}^\pm(\kappa)$ are the evolution stresses (positive values are used here in compression) in the effective stress space due to plastic hardening or softening under uniaxial tension and compression, respectively. $\bar{I}_1$ is the first invariant of the effective stress tensor, $\bar{J}_2$ is the second invariant of the effective deviatoric stress tensor. $\alpha^p \geq 0$ is a dilation parameter with $0.2 \leq \alpha^p \leq 0.3$ for concrete.

According to the flow rule, the rate of the effective plastic strain $\dot{\bar{\epsilon}}^p$ can be written as:

$$\dot{\bar{\epsilon}}_{ij}^p = \dot{\lambda}^p \frac{\partial F^p}{\partial \bar{\sigma}_{ij}} = \dot{\lambda}^p \left(\frac{3}{2} \frac{\bar{s}_{ij}}{\sqrt{3\bar{J}_2}} + \alpha^p \delta_{ij} \right) \quad (37)$$

where $\dot{\lambda}^p$ is a plastic consistency factor, and can be determined by the consistency condition for the yield surface f , which can be expressed in the Kuhn-Tucker form as:

$$f \leq 0, \quad \dot{\lambda}^p \geq 0, \quad \dot{\lambda}^p f = 0, \quad \dot{\lambda}^p \dot{f} = 0 \quad (38)$$

In this study, the linear isotropic hardening rules are introduced to describe the change of the yield surfaces in the effective stress space, and can be expressed in simple forms as [21]:

$$\bar{f}^\pm(\kappa) = \bar{f}_y^\pm + E^{p\pm} \kappa^\pm \quad (39)$$

where $\bar{f}_y^\pm$ are the effective yield strengths in uniaxial tension and compression, and have approximate values as $\bar{f}_y^+ = f_0^+$ and $\bar{f}_y^- = f_0^-$, respectively. $f_0^\pm$ are the uniaxial yield strengths of concrete corresponding to tension and compression, respectively. $E^{p\pm}$ are the effective plastic hardening modulus in uniaxial case, and have relation with the elastoplastic tangent modulus $E^{ep\pm}$ as [22]:

$$E^{p\pm} = \frac{E_0 E^{ep\pm}}{E_0 - E^{ep\pm}} \quad (40)$$

According to the plasticity consistency condition, we have:

$$\dot{f} = \frac{\partial f}{\partial \bar{\sigma}} \dot{\bar{\sigma}} + \frac{\partial f}{\partial \kappa} \dot{\kappa} = 0 \quad (41)$$

and obtain the rate form of the constitutive equation as follows:

$$\dot{\bar{\sigma}}_{ij} = C_{0,ijkl} \left(\dot{\bar{\epsilon}}_{kl} - \dot{\lambda}^p \frac{\partial F^p}{\partial \bar{\sigma}_{kl}} \right) \quad (42)$$

where $C_{0,ijkl}$ is the fourth-order undamaged elastic stiff-

ness tensor.

Accounting for the coupling of tension and compression, $\dot{\mathbf{\kappa}}$ can be written as [22]:

$$\dot{\mathbf{\kappa}} = \left\{ w \dot{\hat{\sigma}}_{\max}^p, -(1-w) \dot{\hat{\sigma}}_{\min}^p \right\}^T \quad (43)$$

where $\dot{\hat{\sigma}}_{\max}^p$ and $\dot{\hat{\sigma}}_{\min}^p$ are the maximum and minimum values of the equivalent plastic strains $\dot{\hat{\sigma}}_{ij}^p$; w is the weight factor and has the following form:

$$w = \frac{\sum_{i=1}^3 \langle \hat{\sigma}_i \rangle}{\sum_{i=1}^3 |\hat{\sigma}_i|} \quad (44)$$

where $\hat{\sigma}_i$ are the principle stresses.

Substituting Equations (42) and (43) into Equation (41), we obtain:

$$\dot{\lambda}^{p\pm} = \frac{1}{h^{p\pm}} \frac{\partial f}{\partial \bar{\sigma}_{ij}} C_{0,ijkl} \bar{\epsilon}_{kl} \quad (45)$$

where $h^{p\pm}$ can be expressed in the following forms:

$$h^{p+} = \frac{\partial f}{\partial \bar{\sigma}_{ij}} C_{0,ijkl} \frac{\partial F^p}{\partial \bar{\sigma}_{kl}} - w \frac{\partial f}{\partial \kappa^+} \frac{\partial F^p}{\partial \hat{\sigma}_{\max}^p} \quad (46)$$

$$h^{p-} = \frac{\partial f}{\partial \bar{\sigma}_{ij}} C_{0,ijkl} \frac{\partial F^p}{\partial \bar{\sigma}_{kl}} + (1-w) \frac{\partial f}{\partial \kappa^-} \frac{\partial F^p}{\partial \hat{\sigma}_{\min}^p} \quad (47)$$

where $\hat{\sigma}_{\max}$ and $\hat{\sigma}_{\min}$ are the maximum and minimum effective principle stresses, respectively. Therefore, we can rewrite the rate form of the relation Equation (42) for the effective stress and the strain as:

$$\dot{\bar{\sigma}}_{ij} = C_{ijkl} \dot{\bar{\epsilon}}_{kl} \quad (48)$$

where C_{ijkl} is the elasto-plastic tangent stiffness tensor and has the following form:

$$C_{ijkl} = C_{0,ijkl} - \frac{1}{h^p} C_{0,ijrs} \frac{\partial F^p}{\partial \bar{\sigma}_{rs}} \frac{\partial f}{\partial \bar{\sigma}_{mn}} C_{0,mnkl} \quad (49)$$

4.3. Helmholtz Free Energy

A damage constitutive model of a material is based on the second law of thermodynamics which states that all the selected internal variables must satisfy the Clausius-Duhem inequality for any irreversible process under an isothermal condition, and has a simple form as:

$$\boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}} - \dot{\psi} \geq 0 \quad (50)$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\epsilon}$ are the stress and strain tensors, ψ is the total Helmholtz free energy (HFE) which can be considered as the sum of the elastic part ψ^e and the plastic part ψ^p , that is:

$$\psi(\boldsymbol{\epsilon}^e, \boldsymbol{\kappa}, \boldsymbol{\Phi}) = \psi^e(\boldsymbol{\epsilon}^e, \boldsymbol{\Phi}) + \psi^p(\boldsymbol{\kappa}, \boldsymbol{\Phi}) \quad (51)$$

where $\boldsymbol{\Phi}$ is the damage variable tensor.

We decompose ψ^e into tensile and compressive parts as:

$$\psi^e(\boldsymbol{\epsilon}^e, \boldsymbol{\Phi}) = \psi^{e+}(\boldsymbol{\epsilon}^e, \phi^+) + \psi^{e-}(\boldsymbol{\epsilon}^e, \phi^-) \quad (52)$$

where

$$\psi^{e+}(\boldsymbol{\epsilon}^e, \phi^+) = (1 - \phi^+) \psi_0^{e+}(\boldsymbol{\epsilon}^e) = \frac{1}{2} (1 - \phi^+) \bar{\boldsymbol{\sigma}}^+ : \boldsymbol{\epsilon}^e \quad (53)$$

$$\psi^{e-}(\boldsymbol{\epsilon}^e, \phi^-) = (1 - \phi^-) \psi_0^{e-}(\boldsymbol{\epsilon}^e) = \frac{1}{2} (1 - \phi^-) \bar{\boldsymbol{\sigma}}^- : \boldsymbol{\epsilon}^e \quad (54)$$

where $\psi_0^{e\pm}(\boldsymbol{\epsilon}^e) = (\bar{\boldsymbol{\sigma}}^\pm : \boldsymbol{\epsilon}^e)/2$ represent the initial elastic strain energy of materials; $\phi^\pm$ are the tensile and compressive components of $\boldsymbol{\Phi}$. Therefore, we derive:

$$\begin{aligned} \boldsymbol{\sigma} &= (1 - \phi^+) \frac{\partial \psi_0^{e+}(\boldsymbol{\epsilon}^e)}{\partial \boldsymbol{\epsilon}^e} + (1 - \phi^-) \frac{\partial \psi_0^{e-}(\boldsymbol{\epsilon}^e)}{\partial \boldsymbol{\epsilon}^e} \\ &= (1 - \phi^+) \bar{\boldsymbol{\sigma}}^+ + (1 - \phi^-) \bar{\boldsymbol{\sigma}}^- \end{aligned} \quad (55)$$

The incremental form of Equation (55) can be expressed as:

$$\dot{\boldsymbol{\sigma}} = (1 - \phi^+) \dot{\bar{\boldsymbol{\sigma}}}^+ + (1 - \phi^-) \dot{\bar{\boldsymbol{\sigma}}}^- - \bar{\boldsymbol{\sigma}}^+ \dot{\phi}^+ - \bar{\boldsymbol{\sigma}}^- \dot{\phi}^- \quad (56)$$

Equations (56) and (48) form the final plastic damage constitutive equations for the plain concrete.

Similarly, we can rewrite ψ^p as:

$$\psi^p(\boldsymbol{\kappa}, \phi) = \psi^{p+}(\boldsymbol{\kappa}, \phi^+) + \psi^{p-}(\boldsymbol{\kappa}, \phi^-) \quad (57)$$

Referring to the fact that the contribution to the plastic HFE from plastic strains of concrete in tension is much smaller comparing to the one in compression, we assume that $\psi^{p+} = 0$. Thus, we have:

$$\psi^p(\boldsymbol{\kappa}, \phi) = \psi^{p-}(\boldsymbol{\kappa}) = (1 - \phi^-) \psi_0^{p-}(\boldsymbol{\kappa}) \quad (58)$$

Substituting Equations (51), (52) and (58) into Equation (50), we get:

$$\left(\boldsymbol{\sigma} - \frac{\partial \psi^e}{\partial \boldsymbol{\epsilon}^e} \right) : \dot{\boldsymbol{\epsilon}}^e + \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^p - \left(\frac{\partial \psi^p}{\partial \boldsymbol{\kappa}} \dot{\boldsymbol{\kappa}} + \frac{\partial \psi}{\partial \boldsymbol{\Phi}} \dot{\boldsymbol{\Phi}} \right) \geq 0 \quad (59)$$

Since the above inequality must be satisfied for any elastic strain $\boldsymbol{\epsilon}^e$, we have:

$$\boldsymbol{\sigma} = \frac{\partial \psi^e}{\partial \boldsymbol{\epsilon}^e} \quad (60)$$

$$\mathbf{Y} \dot{\boldsymbol{\Phi}} \geq 0 \quad (61)$$

$$\boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^p - \frac{\partial \psi^p}{\partial \boldsymbol{\kappa}} \dot{\boldsymbol{\kappa}} \geq 0 \quad (62)$$

where $\mathbf{Y}$ is the damage energy release rate and can be expressed as:

$$\mathbf{Y} = - \frac{\partial \psi}{\partial \boldsymbol{\Phi}} \quad (63)$$

According to the above discussions about the total HFE, we can rewrite $\mathbf{Y}$ as:

$$Y^\pm = \psi_0^\pm \quad (64)$$

We define the damage criteria in tension and compression for concrete, respectively, as:

$$g^{\pm}(Y^{\pm}, r^{\pm}) = Y^{\pm} - r^{\pm} \leq 0 \quad (65)$$

where $r^{\pm}$ are the current damage thresholds whose initial values is denoted by $r_0^{\pm}$. Equation (65) indicates that damage is initiated when $Y^{\pm}$ exceed the corresponding damage thresholds $r^{\pm}$.

Initial strain energy of materials can be written as:

$$\psi_0^+(\boldsymbol{\varepsilon}^e) = \psi_0^{e+}(\boldsymbol{\varepsilon}^e) = \frac{1}{2} \boldsymbol{\varepsilon}^e : \mathbf{C}_0 : \boldsymbol{\varepsilon}^e \geq 0 \quad (66)$$

$$\psi_0^- = \psi_0^{e-}(\boldsymbol{\varepsilon}^e) + \psi_0^{p-}(\boldsymbol{\kappa}) \quad (67)$$

where ψ_0^{e-} is the initial strain energy of materials in compression, and can be expressed as:

$$\psi_0^{e-} = \frac{1}{2E_0} \left[2(1+\nu_0)\bar{J}_2^- + \frac{1-2\nu_0}{3}(\bar{I}_1^-)^2 - \nu_0\bar{I}_1^+\bar{I}_1^- \right] \geq 0 \quad (68)$$

$$\begin{aligned} \psi_0^{p-}(\boldsymbol{\kappa}) &= \frac{\lambda^{p-}}{\|\bar{\mathbf{s}}\|} \left(\sqrt{6}\bar{J}_2^- + \alpha^p\bar{I}_1^- \cdot \sqrt{2\bar{J}_2^-} - \frac{1}{2\sqrt{3}}\bar{I}_1^+\bar{I}_1^- \right) \\ &= \frac{\Omega}{2E_0} \left(\sqrt{6}\bar{J}_2^- + \alpha^p\bar{I}_1^- \cdot \sqrt{2\bar{J}_2^-} - \frac{1}{\sqrt{6}}\bar{I}_1^+\bar{I}_1^- \right) \end{aligned} \quad (69)$$

where $\|\bar{\mathbf{s}}\| = \sqrt{\bar{\mathbf{s}} : \bar{\mathbf{s}}} = \sqrt{2\bar{J}_2^-}$ is the norm of $\bar{\mathbf{s}}$; $\bar{J}_2^- = \frac{1}{2} \bar{\mathbf{s}} : \bar{\mathbf{s}}$ is the second invariant of the compressive effective deviatoric stress tensor $\bar{\mathbf{s}}^-$; $\bar{I}_1^+$ and $\bar{I}_1^-$ are the first invariant of $\bar{\boldsymbol{\sigma}}^+$ and $\bar{\boldsymbol{\sigma}}^-$, respectively. $\Omega = 2E_0\lambda^{p-}/\|\bar{\mathbf{s}}\|$ is a material parameter.

Therefore, ψ_0^- has the following form in compression case:

$$\psi_0^- = \Omega_0 \left[3\bar{J}_2^- + \Omega_1\bar{I}_1^- \sqrt{2\bar{J}_2^-} + \Omega_2(\bar{I}_1^-)^2 - \Omega_3\bar{I}_1^+\bar{I}_1^- \right] \quad (70)$$

where parameters $\Omega_0, \Omega_1, \Omega_2, \Omega_3$ are

$$\begin{aligned} \Omega_0 &= \frac{\sqrt{6}\Omega + 2(1+\nu_0)}{6E_0}; \quad \Omega_1 = \frac{3\Omega\alpha^p}{\sqrt{6}\Omega + 2(1+\nu_0)} \\ \Omega_2 &= \frac{1-2\nu_0}{\sqrt{6}\Omega + 2(1+\nu_0)}; \quad \Omega_3 = \frac{0.5\Omega + 3\nu_0}{\sqrt{6}\Omega + 2(1+\nu_0)} \end{aligned} \quad (71)$$

Assume that concrete is in biaxial compression, which indicates $\sigma_3 \equiv 0$, and we have:

$$\begin{aligned} Y^- = \psi_0^- &= \frac{\sqrt{6}\Omega + 3}{6E_0}(\sigma_1^2 + \sigma_2^2) - \frac{\sqrt{6}\Omega + 6\nu_0}{6E_0}\sigma_1\sigma_2 \\ &+ \frac{\sqrt{6}\Omega\alpha^p}{6E_0}(\sigma_1 + \sigma_2)\sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \end{aligned} \quad (72)$$

For an uniaxial compression case, we denote the uniaxial compressive ultimate strength as f_0^- , and have $\sigma_1 = f_0^-, \sigma_2 = 0$. We can derive the initial damage threshold r_0^- as:

$$r_0^- = \frac{(f_0^-)}{6E_0} \left[3 + \sqrt{6}\Omega(1-\alpha^p) \right] \quad (73)$$

And for an equibiaxial compression case, we use f_{b0}^- to denote the biaxial compressive ultimate strength. One gets $\sigma_1 = \sigma_2 = f_{b0}^-$, and derives r_0^- as:

$$r_0^- = \frac{(f_{b0}^-)}{6E_0} \left[6(1-\nu_0) + \sqrt{6}\Omega(1-2\alpha^p) \right] \quad (74)$$

Noticing that the initial damage threshold is unique for a material, we obtain the expression for Ω from Equations (73) and (74):

$$\Omega = \frac{6(1-\nu_0)(f_{b0}^-/f_0^-)^2 - 3}{\sqrt{6}(1-\alpha^p) - \sqrt{6}(f_{b0}^-/f_0^-)^2(1-2\alpha^p)} \quad (75)$$

For concrete, one has $f_{b0}^-/f_0^- = 1.10 \sim 1.16$ and $\alpha^p = 0.2 \sim 0.3$. Assuming $\nu_0 = 0.2$, and substituting the above three parameters into Equation (74), we find that $\Omega \geq 0$ is always hold. Thus we have $\psi_0^{p-} \geq 0$ (see Equation (69)), and $Y^- = \psi_0^- = \psi_0^{e-} + \psi_0^{p-} \geq 0$, and finally have $Y^{\pm} \geq 0$. Accounting for the fact that damage is irreversible, we get $\phi^+, \phi^- \geq 0$. Therefore, the total HFE defined in Equation (51) satisfies the thermodynamic consistency, or satisfies the inequality of Equation (61).

Equation (75) reaches the limit state $\Omega \rightarrow +\infty$ when

$$\alpha^p = \frac{(f_{b0}^-/f_0^-)^2 - 1}{2(f_{b0}^-/f_0^-)^2 - 1}, \text{ and the undamaged area of a material in compression can be characterized by the following inequality:}$$

$$\begin{aligned} \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 + \alpha^p(\sigma_1 + \sigma_2)\sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \\ \leq (1-\alpha^p)f_0^- \end{aligned} \quad (76)$$

Figure 6 illustrates the changes of the undamaged areas with α^p . We find that the increase of α^p will lead an expansion of the undamaged area.

4.4. Evolution Laws

The evolution laws proposed by Faria *et al.* [18] are adopted in this research. Damage variables (in the Euclidean space) and their rate forms are expressed as:

$$\phi^+ = 1 - \frac{r_0^+}{r^+} \exp \left[B^+ \left(1 - \frac{r^+}{r_0^+} \right) \right] \quad (77)$$

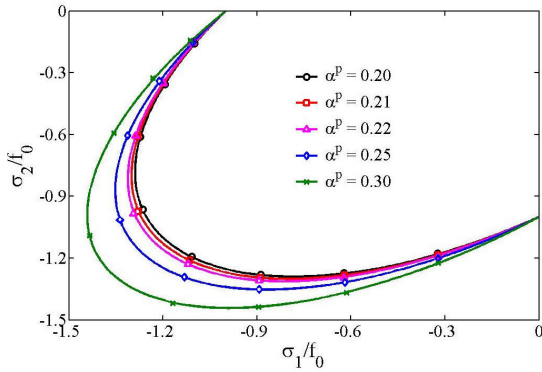


Figure 6. Influences of α^p on undamaged domain.

$$\phi^- = 1 - \left\{ \frac{r_0^-}{r^-} (1 - A^-) + A^- \exp \left[B^- \left(1 - \frac{r^-}{r_0^-} \right) \right] \right\} \quad (78)$$

$$\dot{\phi}^\pm = \dot{r}^\pm h^{d^\pm} \geq 0 \quad (79)$$

where

$$h^{d^+} = \frac{\partial G^+(r^+)}{\partial r^+} = \frac{B^+ r^+ + r_0^+}{(r^+)^2} \exp \left[B^+ \left(1 - \frac{r^+}{r_0^+} \right) \right] \quad (80)$$

$$h^{d^-} = \frac{\partial G^-(r^-)}{\partial r^-} = (1 - A^-) \frac{r_0^-}{(r^-)^2} + \frac{A^- B^-}{r_0^-} \exp \left[B^- \left(1 - \frac{r^-}{r_0^-} \right) \right] \quad (81)$$

where A^- is a material parameter, and can be determined by the uniaxial compression test; $B^\pm$ can be expressed as:

$$B^\pm = \left[\frac{G_F^\pm E_0}{l_{ch} (f_0^\pm)^2} - \frac{1}{2} \right]^{-1} \geq 0 \quad (82)$$

where $G_F^\pm$ is the fracture energy of a material and can be determined by tests or from Equation (21).

Replacing $\phi^\pm$ in the above model by $\hat{\phi}^\pm$ in Equation (27), we can generalize the Euclidean constitutive model for concrete to the fractal space.

5. Example Analysis

5.1 Comparison of Constitutive Models

In this section, numerical simulations of the present model considering fractal effect are performed for concrete under different loading conditions, and comparisons of the results are done with some experimental data, *i.e.* the uniaxial loading test by Karsan and Jirsa [23] and the biaxial loading one by Kupfer *et al.* [24]. Material parameters for concrete are listed in Table 1. The values of E_0 , ν_0 , α^p and α are obtained from the study of Lee and

Fenves [18]; Average values of $l_{ch,N}$, l_{ch}^* , η , a_0 , b and D_k are used here because of their narrow range intervals.

Therefore, we can obtain the fracture energy for concrete from Equation (21) as: $G_F^+ = 45.3 \text{ N/m}$ in tension and $G_F^- = 1497 \text{ N/m}$ in compression.

5.1.1. Uniaxial Tension

Both the predictions of the present model with and without the fractal effect and the test data obtained by Karsan and Jirsa [23] for concrete under uniaxial tension are illustrated in Figure 7(a). We can find that the two kind results of the present model agree well with the test data in the stress hardening stage. Comparing with the case of no fractal, the prediction considering the fractal is more coincident with the test. In the last large deformation stage, the three behaviors are close with each other.

5.1.2. Uniaxial Compression

Figure 7(b) shows the comparison of the calculation results of the present model and the test data for concrete under uniaxial compression obtained by Karsan and Jirsa

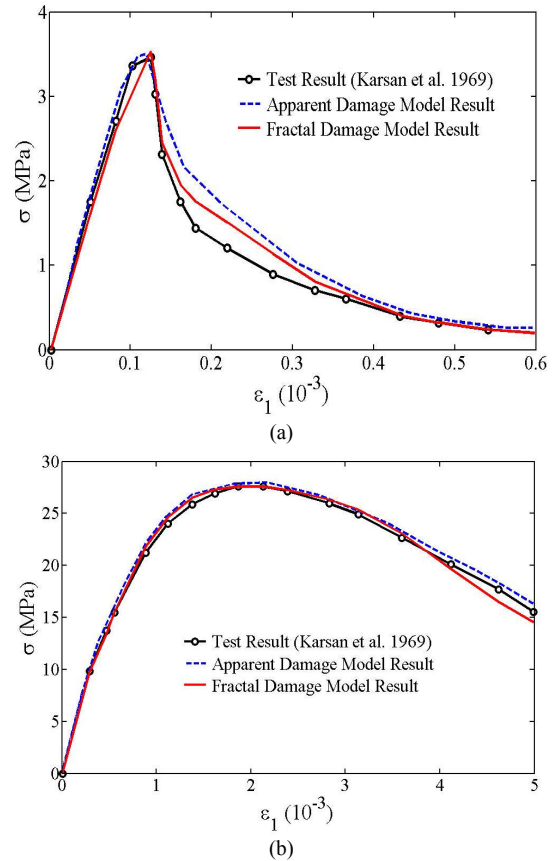


Figure 7. Comparison of the constitutive curves of the present model with test results (Karsan and Jirsa 1969) in the uniaxial loading conditions. (a) Uniaxial tension; (b) Uniaxial compression.

[23]. Comparing with the uniaxial tension case, both of the two kind results of the model are more efficient to simulate the concrete compressive behaviors. The predictions considering fractal is slightly lower than the two other data.

5.1.3. Biaxial Tension

In this simulation, equibiaxial tensile condition is adopted, and material parameters are taken as the same as those listed in Table 1. Figure 8(a) gives the behaviors obtained from the present model and the test in biaxial tension [24]. We find that the two kind theoretical results are coincident with the test data; in the initial strain softening stage, the result concerning no fractal agrees with the test better, but there is obviously a difference between them comparing with the data considering the fractal effect. All these show the superiority of the proposed model.

Table 1. Material parameters for concrete.

$E_0=31.7\text{GPa}$	$\nu_0=0.2$	$=3.48\text{MPa } f_0^+$	$=20\text{MPa } f_0^-$
$\alpha=0.12$	$\alpha^p=0.2$	$\delta=0.286$	$=0.156\text{mm } l_{ch}^*$
$\eta=2\pi$	$a_0=6.0\text{mm}$	$b=0.79$	$D_k=1.16$

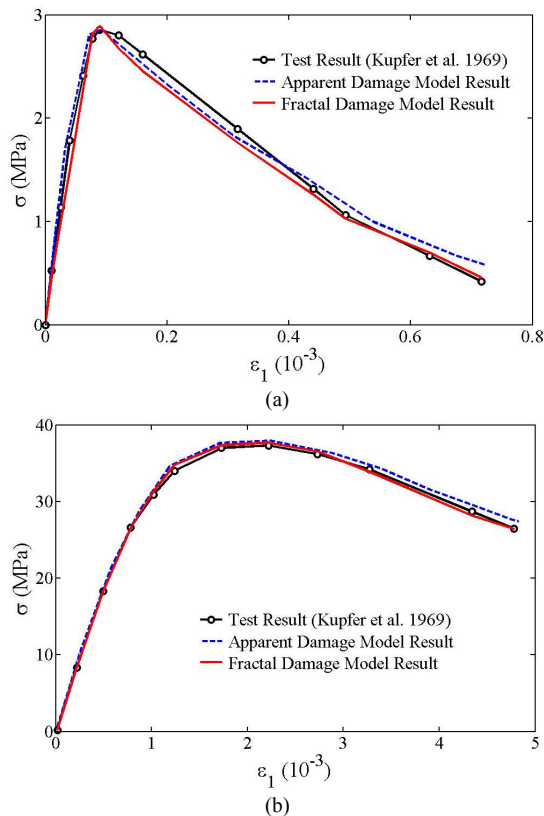


Figure 8. Comparisons of the constitutive curves of the present model with test results (Kupfer *et al.* 1969) in the biaxial loading conditions. (a) Biaxial tension ($\sigma_1: \sigma_2=1:1$); (b). Biaxial compression ($\sigma_1: \sigma_2=-1:-1$).

5.1.4. Biaxial Compression

In equibiaxial compression, concrete shows a good plastic deformation ability which can be found both in the proposed model and the test [24]; see Figure 8(b). The three curves are well close with each other. Comparing with the three other loading cases discussed above, the peak stress increases obviously, accompanied by the slowest decreasing softening stage in biaxial compression, which indicates a good compressive capacity of concrete under the confining pressure condition. Specially, model with fractal damage variables is more accurately than the one with apparent damage variables.

5.2. Structural Analysis

An unreinforced notched concrete beam under 3-point bending is simulated to verify the efficiency of the present concrete damaged plasticity model. This problem has been studied extensively both experimentally by Petersson [25] and analytically by Meyer *et al.* [26], among others. This beam is simply supported at both ends with concentrated force acting at the center. Its sketch is illustrated in Figure 9 (unit: m).

The measured parameters of concrete are: elastic modulus $E_0=30\text{GPa}$, Poisson's ratio $\nu_0=0.2$, density $\rho_0=2400\text{ kg/m}^3$ and uniaxial tensile strength $f_t=3.33\text{ MPa}$.

The present constitutive model considering the fractal effect of concrete is adopted for the theoretical analysis. The model parameters are taken as: $G_F^+ = 138\text{ N/m}$, $f_0^+ = f_t = 3.33\text{ MPa}$, $f_0^- = 30\text{ MPa}$, and other properties are same as that listed in Table 1. The beam is under the plane stress condition. Accounting for the symmetry of both the structure and the load, only one half of the beam is modeled. The model is meshed with 280 4-node bilinear, reduced integration plane elements. The beam is loaded by prescribing the vertical displacement at the center of the beam until it reaches a value of 0.0015 m. The Riks method is used to solve this problem.

Figure 10 illustrates the variation curves of the concentrated force and the center displacement of the beam calculated from the theoretical analysis and the Petersson's test [25]. We can note that the theoretical result is coincidence with the test at the loading stage and slightly higher than the test at the unloading branch. Figure 11 shows the distribution of the principal tensile stress of

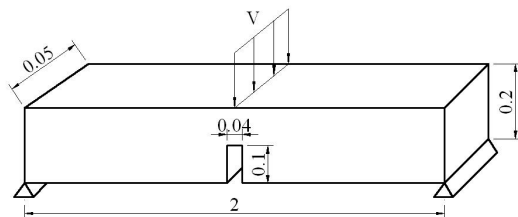


Figure 9. Notched beam: geometry and dimensions (unit: m).

the structure as well.

Mesh sensitivity is investigated in this study by meshing the structure into coarse and fine grids, respectively, with 70 and 1120 same type elements with the medium mesh case. Resolving the above problem at the same condition, we get the relation between the load and displacement in center. **Figure 12** represents the relation curves corresponding to the three kind meshes. We find that: comparing with the coarse mesh case, the structural responses agree well for the other two meshes. The structure is not sensitive to the mesh size in general.

6. Conclusions

In this research, the fracture toughness, the driving force

and the fracture energy of a material with fractal cracks are investigated and their theoretical expresses in the fractal space are derived based on fracture mechanics and fractal geometry. The surface energy and the strain energy in the fractal fracture zone are theoretical expressed in the fractal space. The transformation rule of damage variables in the fractal space and the Euclidean space is obtained which indicates that the apparent damage variable in the Euclidean space is a special case of the fractal one in the fractal space with the fractal dimension of cracks equals to 1. We introduce a plastic yield function and decompose the damage variable tensor into tensile and compressive parts to establish a plastic damage constitutive model for concrete in the Euclidean space. Generalization of this model to the fractal space is done by utilizing the damage variable transformation rule.

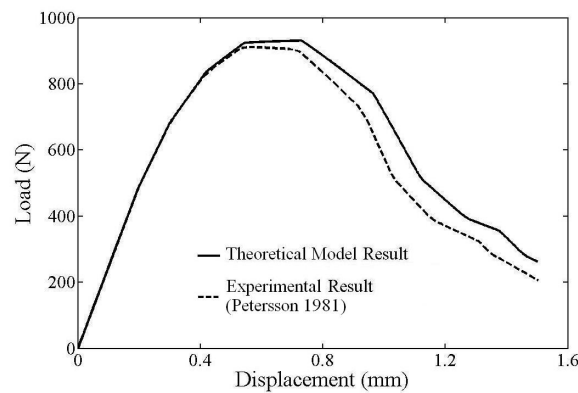


Figure 10. Comparison of the theoretical and experimental data for the load vs. displacement curve.

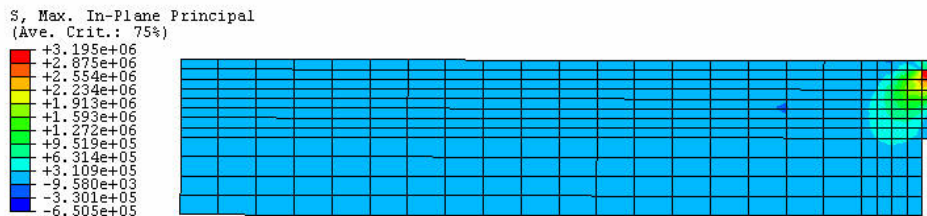


Figure 11. Distribution of the principle tensile stress.

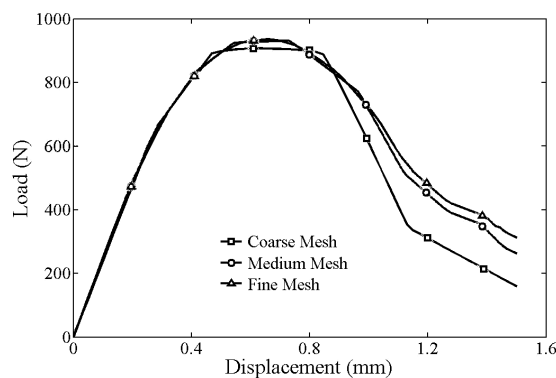


Figure 12. Influences of mesh size on structural response.

Comparisons of the results obtained from the present model and tests for concrete under different loading conditions are done to verify the efficiency of this model and show the necessity of considering the fractal effect in the constitutive model of concrete. The present model considering the fractal effect is used to analyze a notched plain concrete beam under 3-point bending. Mesh sensitivity is also concerned. The numerical results show the efficiency and validation of the present model for structural analysis.

7. Acknowledgements

The present research is partly supported by the National Natural Science Foundation of China (No. 90815012), which is gratefully acknowledged.

8. References

- [1] B. B. “Mandelbrot, Fractals: Form, Chance, and Dimension,” W. H. Freeman and Company, San Francisco, 1977.
- [2] B. B. Mandelbrot, “The Fractal Geometry of Nature,” W. H. Freeman and Company, San Francisco, 1983.
- [3] H. P. Xie and F. Gao, “The Mechanics of Cracks and a Statistical Strength Theory for Rocks,” *International Journal of Rock Mechanics and Mining Sciences*, Vol. 37, 2000, pp. 477-488.
- [4] A. Yavari, “Generalization of Barenblatt’s Cohesive Fracture Theory for Fractal Cracks,” *Fractals*, Vol. 10, No. 2, 2002, pp. 189-198.
- [5] A. Carpinteri, B. Chiaia and P. Cornetti, “On the Mechanics of Quasi-Brittle Materials with a Fractal Microstructure,” *Engineering Fracture Mechanics*, Vol. 70, 2003, pp. 2321-2349.
- [6] V. E. Saouma, J. J. Broz, E. Bruhwiler, *et al.*, “Effect of Aggregate and Specimen Size on Fracture Properties of Dam Concrete,” *Journal of Materials in Civil Engineering*, Vol. 3, No. 3, 1991, pp. 204-218.
- [7] M. A. Issa, A. M. Hammad and A. Chudnovsky, “Correlation between Crack Tortuosity and Fracture Toughness in Cementitious Materials,” *International Journal of Fracture*, Vol. 60, 1993, pp. 97-105.
- [8] Y. H. Zhao, “Crack Pattern Evolution and a Fractal Damage Constitutive Model for Rock,” *International Journal of Rock Mechanics and Mining Sciences*, Vol. 35, No. 3, 1996, pp. 349-366.
- [9] L. Guarracino, “A Fractal Constitutive Model for Unsaturated Flow in Fractured Hard Rocks,” *Journal of Hydrology*, Vol. 324, 2006, pp. 154-162.
- [10] T. Yokobori, “Scientific Foundations for Strength and Fracture of Materials,” *Naukova Dumka*, Kiev, 1978.
- [11] M. P. Wnuk and A. Yavari, “On Estimating Stress Intensity Factors and Modulus of Cohesion for Fractal Cracks,” *Engineering Fracture Mechanics*, Vol. 70, 2003, pp. 1659-1674.
- [12] A. Carpinteri, B. Chiaia and K. M. Nematì, “Complex Fracture Energy Dissipation in Concrete under Different Loading Conditions,” *Mechanics of Materials*, Vol. 26, 1997, pp. 93-108.
- [13] Z. M. Ye, *Fracture Mechanics of Anisotropic and Concrete Materials*, China Railway Publishing House, Beijing, in Chinese, 2000.
- [14] T. H. Antoun, *Constitutive/Failure Model for the Static and Dynamic Behaviors of Concrete Incorporating Effects of Damage and Anisotropy*, Ph. D. Thesis, The University of Dayton, Dayton, Ohio, 1991.
- [15] H. P. Xie and Y. Ju, “A Study of Damage Mechanics Theory in Fractional Dimensional Space,” *Acta Mechanica Sinica*, in Chinese, Vol. 31, No. 3, 1999, pp. 300-310.
- [16] U. Cicekli, G. Z. Voyiadjis and R. K. A. Al-Rub, “A Plasticity and Anisotropic Damage Model for Plain Concrete,” *International Journal of Plasticity*, Vol. 23, 2007, pp. 1874-1900.
- [17] M. Ortiz, “A Constitutive Theory for Inelastic Behavior of Concrete,” *Mechanics of Materials*, Vol. 4, 1985, pp. 67-93.
- [18] R. Faria, J. Oliver and M. Cervera, “A Strain-Based Plastic Viscous-Damage Model for Massive Concrete Structures,” *International Journal of Solids and Structures*, Vol. 35, No. 14, 1998, pp. 1533-1558.
- [19] R. Faria, J. Oliver and M. Cervera, “On Isotropic Scalar Damage Models for the Numerical Analysis of Concrete Structures,” *In: CIMNE Monograph*, Barcelona, Spain, Vol. 198, 2000, pp. 422-426.
- [20] J. Lee and G. L. Fenves, “Plastic-Damage Model for Cyclic Loading of Concrete Structures,” *Journal of Engineering Mechanics, ASCE*, Vol. 124, 1998, pp. 892-900.
- [21] J. Y. Wu, J. Li and R. Faria, “An Energy Release Rate-Based Plastic-Damage Model for Concrete,” *International Journal of Solids and Structures*, Vol. 43, 2006, pp. 583-612.
- [22] J. C. Simo and T. J. R. Hughes, “Computational Inelasticity,” *Springer-Verlag*, New York, 1998.
- [23] A. I. Karsan and J. O. Jirsa, “Behavior of Concrete under Compressive Loadings,” *Journal of the Structural Division, ASCE*, Vol. 95, 1969, pp. 2535-2563.
- [24] H. Kupfer, H. K. Hilsdorf and H. Rusch, “Behavior of Concrete under Biaxial Stresses,” *ACI Materials Journal*, Vol. 66, 1969, pp. 656-666.
- [25] P. E. Petersson, “Crack Growth and Development of Fracture Zones in Plain Concrete and Similar Materials,” *In: Report TVBM 1006, Division of Building Materials*, University of Lund, Sweden, 1981.
- [26] R. Meyer, H. Ahrens and H. Duddeck, “Material Model for Concrete in Cracked and Uncracked States,” *Journal of the Engineering Mechanics Division, ASCE*, Vol. 120, No. 9, 1994, pp. 1877-1895.

Supertranslations to All Orders*

Rainer Dick

Department of Physics and Engineering Physics, University of Saskatchewan, Saskatoon, Canada

E-mail: rainer.dick@usask.ca

Received December 29, 2009; revised February 25, 2010; accepted March 18, 2010

Abstract

We calculate the transformation laws of the general linear superfield $V(x, \theta, \bar{\theta})$ and chiral superfields under $N=1$ supertranslations $\exp[i(\zeta \cdot Q + \bar{Q} \cdot \bar{\zeta})]$ to all orders in the translation parameters $\zeta, \bar{\zeta}$. We use the superfield formalism with complete expansions of the component fields in the coordinate shifts $\Delta x^\mu = -i(\zeta \cdot \sigma^\mu \cdot \bar{\theta} - \theta \cdot \sigma^\mu \cdot \bar{\zeta})$. The results show in particular how a general supertranslation transforms each component field of a supermultiplet into a complete superfield. The results also provide complete parametrizations of orbits of component fields under supertranslations.

Keywords: Supersymmetry, Supermultiplets, Supertranslations

1. Introduction

Quantum field theories with exact correspondences between bosonic and fermionic helicity states are not only basic ingredients for superstring theories, but have dominated both theoretical investigations and experimental searches for particle physics beyond the current “Standard Model” of particle physics for over three decades now.

The minimal version of supersymmetric extensions of the Standard Model extends the generators $M_{\mu\nu}, p_\mu$ of the Poincaré group by a set of fermionic generators Q_α and $\bar{Q}_{\dot{\alpha}}$ in the (1/2,0) and (0,1/2) representations of the proper orthochronous Lorentz group in four dimensions. It has been recognized early on that this extension of the Poincaré algebra can be represented linearly (and in a reducible, but not fully reducible manner) on a set comprising 4 complex spin-0 fields, 4 Weyl spinors and one complex spin-1 field. This set constitutes the so called general linear multiplet or general linear superfield V and its irreducible subsets had also been identified.

It is sufficient to know the action of the supertranslation generators Q_α and $\bar{Q}_{\dot{\alpha}}$ on the components of V , or equivalently the action of the supertranslation $\exp[i(\zeta \cdot Q + \bar{Q} \cdot \bar{\zeta})]$ to first order in the parameters $\zeta_\alpha, \bar{\zeta}_{\dot{\alpha}}$, to construct supersymmetric action principles and

the related supercurrents. Therefore the first order transformation laws for the components of V have been calculated a long time ago and can be found in many books and review articles on supersymmetry and with our current understanding this is all that is needed to discuss the physical implications of supersymmetry. Recent research in supersymmetry focuses on important applications like new solutions and structure of supergravity theories [1-5], impact of supersymmetry on perturbative calculations [6], cosmological implications of supersymmetry [7-9] and in particular the interesting problem how supersymmetry can be reconciled with a cosmological constant and help to explain it [10,11]. The structure of superpotentials in theories with broken supersymmetry is also an active area of research with phenomenological relevance [12-14].

From a mathematical point of view it is clearly desirable to also have the full transformation properties of the general linear multiplet readily available for reference. To provide such a reference is the purpose of this paper. To make these results also easily accessible for beginners in supersymmetry, the super-Poincaré algebra and the basic techniques of superspace calculations are also reviewed. Therefore the outline of the paper is as follows.

Our conventions for spinor representations of the Lorentz group and the super-Poincaré algebra are introduced in Section 2. Superspace is reviewed in Section 3 and the full supertranslation properties of the component fields of the general linear multiplet are calculated in Section 4.

*This work was supported by NSERC Canada.

Chiral superfields provide a particular irreducible representation within the reducible linear multiplet. Due to their practical relevance for the supersymmetrization of matter fields, the resulting supertranslation properties of the components of chiral superfields are listed in Section 5.

Appendix 1 contains a translation of our results into the conventions of Wess and Bagger [15]. The relevant spinor indices are reviewed in Appendix 2.

Our conventions for spinor representations and super-space calculations differ from Wess and Bagger only with regard to the definition of superderivatives and the definition of the 2nd order epsilon spinors with lower indices. Sections 2 and 3 are included to make the paper self-contained and easily accessible and to clarify conventions. However, the new results in Sections 4 and 5 are not affected by the different definitions. The *cognoscenti* should therefore go straight to Section 4.

2. The Super-Poincaré Algebra

The basic methodology for calculations with linear super-multiplets in four dimensions was developed some 35 years ago by Wess, Zumino, Salam, Strathdee and Ferrara [16-19]. This section and the following section provide a brief but concise introduction to the calculational techniques of supersymmetry and its linear representations in four dimensions.

We use $\eta_{00} = -1$ for the Minkowski metric and standard notation $\sigma_{\mu\alpha\dot{\alpha}}$ with

$$\begin{aligned}\sigma_0 &= (\sigma_{0\alpha\dot{\alpha}}) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = (\sigma_{1\alpha\dot{\alpha}}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\ \sigma_2 &= (\sigma_{2\alpha\dot{\alpha}}) = \begin{pmatrix} 0 & -i \\ i & 1 \end{pmatrix}, \quad \sigma_3 = (\sigma_{3\alpha\dot{\alpha}}) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\end{aligned}$$

for the Pauli matrices.

Complex conjugation turns undotted indices into dotted indices and *vice versa*,

$$(\psi_a)^* = \bar{\psi}_{\dot{a}},$$

and hermiticity of the Pauli matrices implies for the complex conjugate matrices

$$\bar{\sigma}^{\mu}_{\dot{\alpha}\alpha} = \sigma^{\mu}_{\alpha\dot{\alpha}}. \quad (1)$$

We pull spinor indices with the two-dimensional epsilon spinors

$$\begin{aligned}\varepsilon^{12} &= \varepsilon_{12} = 1, \quad \varepsilon^{i2} = \varepsilon_{i2} = 1, \\ \psi^{\alpha} &= \varepsilon^{\alpha\beta} \psi_{\beta}, \quad \psi_{\beta} = \psi^{\alpha} \varepsilon_{\alpha\beta}, \\ \bar{\psi}^{\dot{\alpha}} &= \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\psi}_{\dot{\beta}}, \quad \bar{\psi}_{\dot{\beta}} = \bar{\psi}^{\dot{\alpha}} \varepsilon_{\dot{\alpha}\dot{\beta}}.\end{aligned} \quad (2)$$

The Equations (1) then imply that the conjugate Pauli matrices with upper spinor indices are

$$\bar{\sigma}^{\mu\dot{\alpha}\alpha} = \varepsilon^{\dot{\alpha}\dot{\beta}} \varepsilon^{\alpha\beta} \sigma^{\mu}_{\beta\dot{\beta}}. \quad (3)$$

Numerically, we have with the upper index positions for the barred matrices and lower index positions for the unbarred matrices

$$\bar{\sigma}_0 = \sigma_0, \quad \bar{\sigma}_i = -\sigma_i.$$

Although not formally required, use of upper indices for barred Pauli matrices and lower indices for unbarred Pauli matrices is a useful and very common convention.

Relations for Pauli matrices are meticulously compiled in [15]. For convenience, we recall those relations which are directly relevant for the derivation of supertranslations to all orders,

$$\bar{\sigma}_{\mu}^{\dot{\alpha}\alpha} \sigma^{\mu}_{\beta\dot{\beta}} = -2\delta^{\dot{\alpha}}_{\dot{\beta}} \delta^{\alpha}_{\beta}, \quad (4)$$

$$(\sigma_{\mu} \cdot \bar{\sigma}_{\nu} + \sigma_{\nu} \cdot \bar{\sigma}_{\mu})_{\alpha}^{\beta} = -2\eta_{\mu\nu} \delta_{\alpha}^{\beta}, \quad (5)$$

$$(\bar{\sigma}_{\mu} \cdot \sigma_{\nu} + \bar{\sigma}_{\nu} \cdot \sigma_{\mu})_{\dot{\beta}}^{\dot{\alpha}} = -2\eta_{\mu\nu} \delta_{\dot{\beta}}^{\dot{\alpha}}, \quad (6)$$

$$\text{Tr}(\sigma_{\mu} \cdot \bar{\sigma}_{\nu}) = \sigma_{\mu\alpha\dot{\alpha}} \bar{\sigma}_{\nu}^{\dot{\alpha}\alpha} = -2\eta_{\mu\nu}, \quad (7)$$

$$\text{and } \sigma^{\lambda} \cdot \bar{\sigma}^{\mu} \cdot \sigma^{\nu} = \eta^{\lambda\nu} \sigma^{\mu} - \eta^{\lambda\mu} \sigma^{\nu} - \eta^{\mu\nu} \sigma^{\lambda} - i\varepsilon_{0123} \varepsilon^{\lambda\mu\nu\rho} \sigma_{\rho}. \quad (8)$$

The factor $\varepsilon_{0123} = \pm 1$ was included to allow for ready use of both conventions for the four-dimensional epsilon tensor.

We will briefly recall below that pulling spinor indices with the 2nd order epsilon spinors is motivated by the fact that this yields Lorentz invariant spinor products

$$\psi \cdot \chi \equiv \psi^{\alpha} \chi_{\alpha} = \varepsilon^{\alpha\beta} \psi_{\beta} \chi_{\alpha} = -\psi_{\alpha} \chi^{\alpha} = \chi \cdot \psi, \quad (9)$$

where the anti-commutation property of spinors was used. Conjugation also implies re-ordering of spinor quantities, such that conjugation of (9) yields

$$\bar{\chi} \cdot \bar{\psi} \equiv \bar{\chi}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}} = \bar{\chi}_{\dot{\alpha}} \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\psi}_{\dot{\beta}} = \bar{\psi} \cdot \bar{\chi}. \quad (10)$$

The vector representation matrices of the Lorentz algebra,

$$(L_{\mu\nu})_{\rho}^{\sigma} = \eta_{\mu\rho} \eta_{\nu}^{\sigma} - \eta_{\nu\rho} \eta_{\mu}^{\sigma},$$

appear as structure constants in the Poincaré algebra. The spinor representations of a proper orthochronous Lorentz transformation

$$\Lambda = \exp\left(\frac{i}{2} \omega^{\mu\nu} M_{\mu\nu}\right) = \exp\left(\frac{1}{2} \omega^{\mu\nu} L_{\mu\nu}\right)$$

are given by

$$U(\Lambda) = \exp\left(\frac{i}{2} \omega^{\mu\nu} S_{\mu\nu}\right)$$

and

$$\tilde{U}(\Lambda) = \exp\left(\frac{i}{2} \omega^{\mu\nu} \bar{S}_{\mu\nu}\right),$$

with generators

$$(S_{\mu\nu})_{\alpha}^{\beta} = \frac{i}{4} (\sigma_{\mu} \cdot \bar{\sigma}_{\nu} - \sigma_{\nu} \cdot \bar{\sigma}_{\mu})_{\alpha}^{\beta},$$

$$(\bar{S}_{\mu\nu})_{\dot{\beta}}^{\dot{\alpha}} = \frac{i}{4} (\bar{\sigma}_{\mu} \cdot \sigma_{\nu} - \bar{\sigma}_{\nu} \cdot \sigma_{\mu})_{\dot{\beta}}^{\dot{\alpha}}.$$

The relations

$$\text{Tr}(S_{\mu\nu} \cdot S_{\rho\sigma}) = \frac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho} - i\varepsilon_{0123}\varepsilon_{\mu\nu\rho\sigma}), \quad (11)$$

$$\text{Tr}(\bar{S}_{\mu\nu} \cdot \bar{S}_{\rho\sigma}) = \frac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho} + i\varepsilon_{0123}\varepsilon_{\mu\nu\rho\sigma}). \quad (12)$$

are used in the derivation of supersymmetric Maxwell or Yang-Mills actions.

The spinor products (9,10) are invariant because the matrices $U(\Lambda)$ and $\tilde{U}(\Lambda)$ are $SL(2, \mathbb{C})$ matrices,

$$\begin{aligned} \varepsilon^{\alpha\beta} U(\Lambda)_\alpha^\gamma U(\Lambda)_\beta^\delta &= \varepsilon^{\gamma\delta}, \\ \varepsilon_{\dot{\alpha}\dot{\beta}} \tilde{U}(\Lambda)_{\dot{\gamma}}^{\dot{\alpha}} \tilde{U}(\Lambda)_{\dot{\delta}}^{\dot{\beta}} &= \varepsilon_{\dot{\gamma}\dot{\delta}}. \end{aligned} \quad (13)$$

Stated differently, the epsilon spinors are Lorentz invariant.

We can now write down the super-Poincaré algebra in the form

$$\begin{aligned} [M_{\mu\nu}, M_{\rho\sigma}] &= i(L_{\mu\nu})_\rho^\lambda M_{\lambda\sigma} + i(L_{\mu\nu})_\sigma^\lambda M_{\rho\lambda}, \\ [M_{\mu\nu}, p_\rho] &= i(L_{\mu\nu})_\rho^\lambda p_\lambda, \\ [M_{\mu\nu}, Q_\alpha] &= -(S_{\mu\nu})_\alpha^\beta Q_\beta, \\ [M_{\mu\nu}, \bar{Q}_{\dot{\alpha}}] &= -(\bar{S}_{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}} \bar{Q}_{\dot{\beta}}, \\ \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= 2p_{\alpha\dot{\alpha}} = 2p_\mu \sigma^\mu_{\alpha\dot{\alpha}}, \\ \{Q_\alpha, Q_\beta\} &= 0, \quad \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0, \\ [p_\mu, Q_\alpha] &= 0, \quad [p_\mu, \bar{Q}_{\dot{\alpha}}] = 0. \end{aligned} \quad (14)$$

The $SL(2, \mathbb{C})$ property (13) reads in first order

$$(\bar{S}_{\mu\nu})_{\dot{\gamma}\dot{\delta}} = (\bar{S}_{\mu\nu})_{\dot{\delta}\dot{\gamma}}$$

and implies that Equation (14) can also be written as

$$[M_{\mu\nu}, \bar{Q}_{\dot{\alpha}}] = \bar{Q}_{\dot{\beta}} (\bar{S}_{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}}.$$

The super-Poincaré algebra satisfies all the pertinent super-Jacobi identities as a consequence of the representation properties of the vector and spinor representations of the Lorentz algebra. The particular super-Jacobi identity

$$[M_{\mu\nu}, \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\}] = \{[M_{\mu\nu}, Q_\alpha], \bar{Q}_{\dot{\alpha}}\} + \{[M_{\mu\nu}, \bar{Q}_{\dot{\alpha}}], Q_\alpha\}$$

holds as a consequence of the fact that the Pauli matrices have the same form in every inertial frame,

$$\begin{aligned} (L_{\mu\nu})_\kappa^\lambda \sigma^\kappa_{\alpha\dot{\alpha}} &= \eta_\mu^\lambda \sigma_{\nu\alpha\dot{\alpha}} - \eta_\nu^\lambda \sigma_{\mu\alpha\dot{\alpha}} \\ &= \frac{1}{4}(\sigma_\mu \cdot \bar{\sigma}_\nu - \sigma_\nu \cdot \bar{\sigma}_\mu)_\alpha^\beta \sigma^\lambda_{\beta\dot{\alpha}} \\ &\quad - \sigma^\lambda_{\alpha\dot{\beta}} \frac{1}{4}(\bar{\sigma}_\mu \cdot \sigma_\nu - \bar{\sigma}_\nu \cdot \sigma_\mu)_{\dot{\alpha}}^{\dot{\beta}}. \end{aligned} \quad (15)$$

This can be verified from Equations (5,6) by commuting the σ^λ matrices into the middle positions in the products on the right hand side. It can also be verified as a direct consequence of Equation (8).

3. $N=1$ Superspace

The Poincaré algebra is realized on spacetime coordinates x^μ through derivative operators

$$M_{\mu\nu} = -i(x_\mu \partial_\nu - x_\nu \partial_\mu), \quad p_\mu = -i\partial_\mu. \quad (16)$$

In a nutshell, superspace is based on the observation that this construction can be extended to the super-Poincaré algebra by supplementing Minkowski spacetime with fermionic coordinates θ^α and $\bar{\theta}^{\dot{\alpha}}$ and corresponding fermionic derivatives

$$\partial_\alpha \theta^\beta = \delta_\alpha^\beta, \quad \bar{\partial}_{\dot{\alpha}} \bar{\theta}^{\dot{\beta}} = \delta_{\dot{\alpha}}^{\dot{\beta}}. \quad (17)$$

The super-Poincaré algebra is then realized on the superspace coordinates $(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$ by amending the representations (16) of the bosonic operators with the realizations

$$Q_\alpha = -i\partial_\alpha - \sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_\mu, \quad \bar{Q}_{\dot{\alpha}} = i\bar{\partial}_{\dot{\alpha}} + \theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu, \quad (18)$$

for the fermionic operators and complementing the Lorentz generators to include the action on Q_α and $\bar{Q}_{\dot{\alpha}}$,

$$M_{\mu\nu} = -i(x_\mu \partial_\nu - x_\nu \partial_\mu) + \theta^\alpha (S_{\mu\nu})_\alpha^\beta \partial_\beta - (\bar{S}_{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}} \bar{\theta}^{\dot{\alpha}} \bar{\partial}_{\dot{\beta}}.$$

A superfield $V(x, \theta, \bar{\theta})$ maps a spacetime point x into the algebra over $\mathbb{C}$ which is generated by the five elements $(1, \theta, \bar{\theta})$, subject to the relations (35,36) in the Appendix A.2. Note that this definition explicitly refers to the fermionic arguments of V . Supertranslations are based on the concept that there are infinitely many incarnations of the four fermionic generators θ^α and $\bar{\theta}^{\dot{\alpha}}$ and that we can freely move between these infinitely many copies of the same algebra. In particular, if $(1, \theta, \bar{\theta})$ and $(1, \zeta, \bar{\zeta})$ generate the same algebra, we

require that $(1, \theta + \zeta, \bar{\theta} + \bar{\zeta})$ also generates the same algebra. This requirement is equivalent to anti-commutation properties

$$\theta_\alpha \zeta_\beta = -\zeta_\beta \theta_\alpha, \quad \bar{\theta}_{\dot{\alpha}} \bar{\zeta}_{\dot{\beta}} = -\bar{\zeta}_{\dot{\beta}} \bar{\theta}_{\dot{\alpha}},$$

and the corresponding conjugate equations, *i.e.* the algebra generated by $(1, \theta, \bar{\theta})$ is a subalgebra of a corresponding infinite-dimensional algebra.

The relations in Appendix A.2 imply that the expansion of every superfield with respect to the fermionic

elements θ^α and $\bar{\theta}^{\dot{\alpha}}$ can be written in terms of four scalars $\phi(x)$, $M(x)$, $N(x)$, $D(x)$, four Weyl fermions $\psi(x)$, $\chi(x)$, $\kappa(x)$, $\lambda(x)$, and a vector field $A_\mu(x)$,

$$V(x, \theta, \bar{\theta}) = \phi(x) + \theta \cdot \psi(x) + \bar{\chi}(x) \cdot \bar{\theta} + \theta \cdot \sigma^\mu \cdot \bar{\theta} A_\mu(x) + \theta^2 M(x) + \bar{\theta}^2 N(x) + \bar{\theta}^2 \theta \cdot \kappa(x) + \theta^2 \bar{\lambda}(x) \cdot \bar{\theta} + \theta^2 \bar{\theta}^2 D(x).$$

The commutation relations

$$i[\zeta \cdot Q, \theta^\alpha] = \zeta^\alpha, \quad i[\bar{Q} \cdot \bar{\zeta}, \bar{\theta}^{\dot{\alpha}}] = \bar{\zeta}^{\dot{\alpha}}, \\ i[\zeta \cdot Q + \bar{Q} \cdot \bar{\zeta}, x^\mu] = -i(\zeta \cdot \sigma^\mu \cdot \bar{\theta} - \theta \cdot \sigma^\mu \cdot \bar{\zeta})$$

imply for unitary supertranslations

$$|x', \theta', \bar{\theta}'\rangle = \exp[i(\zeta \cdot Q + \bar{Q} \cdot \bar{\zeta})] |x, \theta, \bar{\theta}\rangle \\ = |x + i(\zeta \cdot \sigma \cdot \bar{\theta} - \theta \cdot \sigma \cdot \bar{\zeta}), \theta + \zeta, \bar{\theta} + \bar{\zeta}\rangle,$$

and therefore

$$V'(x, \theta, \bar{\theta}) = \langle x, \theta, \bar{\theta} | \exp[i(\zeta \cdot Q + \bar{Q} \cdot \bar{\zeta})] | V \rangle \\ = V(x - i(\zeta \cdot \sigma \cdot \bar{\theta} - \theta \cdot \sigma \cdot \bar{\zeta}), \theta + \zeta, \bar{\theta} + \bar{\zeta}). \quad (19)$$

We can calculate the transformation properties of the component fields by comparing

$$V'(x, \theta, \bar{\theta}) = \phi'(x) + \theta \cdot \psi'(x) + \bar{\chi}'(x) \cdot \bar{\theta} + \theta \cdot \sigma^\mu \cdot \bar{\theta} A'_\mu(x) \\ + \theta^2 M'(x) + \bar{\theta}^2 N'(x) + \bar{\theta}^2 \theta \cdot \kappa'(x) + \theta^2 \bar{\lambda}'(x) \cdot \bar{\theta} \\ + \theta^2 \bar{\theta}^2 D'(x)$$

with the expansion of the right hand side of (19) with respect to the fermionic variables θ^α and $\bar{\theta}^{\dot{\alpha}}$.

4. Supertranslations of the General Linear Multiplet

Equation (19) implies in particular that supertranslations shift the argument x of component fields to

$$X^\mu = x^\mu - i(\zeta \cdot \sigma^\mu \cdot \bar{\theta} - \theta \cdot \sigma^\mu \cdot \bar{\zeta}). \quad (20)$$

We can calculate the transformation properties of the components of the supermultiplet V to all orders in the translation parameters ζ , $\bar{\zeta}$, by expanding the right hand side

$$V(X, \theta + \zeta, \bar{\theta} + \bar{\zeta}) = \phi(x - i(\zeta \cdot \sigma \cdot \bar{\theta} - \theta \cdot \sigma \cdot \bar{\zeta})) \\ + (\theta + \zeta) \cdot \psi(x - i(\zeta \cdot \sigma \cdot \bar{\theta} - \theta \cdot \sigma \cdot \bar{\zeta})) + \dots$$

of Equation (19) to all orders in θ and $\bar{\theta}$.

The first step requires the expansion of the component

fields with respect to the coordinate shifts

$$\Delta x^\mu = -i(\zeta \cdot \sigma^\mu \cdot \bar{\theta} - \theta \cdot \sigma^\mu \cdot \bar{\zeta}),$$

e.g.

$$\phi(X) = \phi(x) - i(\zeta \cdot \sigma^\mu \cdot \bar{\theta} - \theta \cdot \sigma^\mu \cdot \bar{\zeta}) \partial_\mu \phi(x) \\ - \frac{1}{2}(\zeta \cdot \sigma^\mu \cdot \bar{\theta} - \theta \cdot \sigma^\mu \cdot \bar{\zeta})(\zeta \cdot \sigma^\nu \cdot \bar{\theta} - \theta \cdot \sigma^\nu \cdot \bar{\zeta}) \partial_\mu \partial_\nu \phi(x) \\ + \frac{i}{6}(\zeta \cdot \sigma^\lambda \cdot \bar{\theta} - \theta \cdot \sigma^\lambda \cdot \bar{\zeta})(\zeta \cdot \sigma^\mu \cdot \bar{\theta} - \theta \cdot \sigma^\mu \cdot \bar{\zeta}) \\ \times (\zeta \cdot \sigma^\nu \cdot \bar{\theta} - \theta \cdot \sigma^\nu \cdot \bar{\zeta}) \partial_\lambda \partial_\mu \partial_\nu \phi(x) \\ + \frac{1}{24}(\zeta \cdot \sigma^\kappa \cdot \bar{\theta} - \theta \cdot \sigma^\kappa \cdot \bar{\zeta})(\zeta \cdot \sigma^\lambda \cdot \bar{\theta} - \theta \cdot \sigma^\lambda \cdot \bar{\zeta}) \\ \times (\zeta \cdot \sigma^\mu \cdot \bar{\theta} - \theta \cdot \sigma^\mu \cdot \bar{\zeta})(\zeta \cdot \sigma^\nu \cdot \bar{\theta} - \theta \cdot \sigma^\nu \cdot \bar{\zeta}) \partial_\kappa \partial_\lambda \partial_\mu \partial_\nu \phi(x) \\ = \phi(x) - i(\zeta \cdot \sigma^\mu \cdot \bar{\theta} - \theta \cdot \sigma^\mu \cdot \bar{\zeta}) \partial_\mu \phi(x) \\ + \frac{1}{4}(\zeta^2 \bar{\theta}^2 + \theta^2 \bar{\zeta}^2 + 2\zeta \cdot \sigma_\mu \cdot \bar{\zeta} \theta \cdot \sigma^\mu \cdot \bar{\theta}) \partial^2 \phi(x) \\ - \frac{1}{2}(\zeta \cdot \sigma^\nu \cdot \bar{\zeta} \theta \cdot \sigma^\mu \cdot \bar{\theta} + \zeta \cdot \sigma^\mu \cdot \bar{\zeta} \theta \cdot \sigma^\nu \cdot \bar{\theta}) \partial_\mu \partial_\nu \phi(x) \\ + \frac{i}{4}(\zeta^2 \bar{\theta}^2 \theta \cdot \sigma^\mu \cdot \bar{\zeta} - \theta^2 \bar{\zeta}^2 \zeta \cdot \sigma^\mu \cdot \bar{\theta}) \partial_\mu \partial^2 \phi(x) \\ + \frac{1}{16} \theta^2 \bar{\theta}^2 \zeta^2 \bar{\zeta}^2 \partial^2 \partial^2 \phi(x),$$

and corresponding expansions for combinations of the other eight component fields with various factors, which are different in each case due to the presence of fermionic variables in the extra factors. Altogether, this includes 35 more relations, e.g.

$$\theta \cdot \psi(X) = \theta \cdot \psi(x) - \frac{i}{2} \theta \cdot \sigma^\mu \cdot \bar{\theta} \zeta \cdot \sigma^\nu \cdot \bar{\sigma}_\mu \cdot \partial_\nu \psi(x) \\ + \frac{i}{2} \theta^2 \bar{\zeta} \cdot \bar{\sigma}^\mu \cdot \partial_\mu \psi(x) + \frac{1}{4} \zeta^2 \bar{\theta}^2 \theta \cdot \partial^2 \psi(x) \\ + \frac{1}{4} \theta^2 \zeta \cdot \sigma_\mu \cdot \bar{\zeta} \bar{\theta} \cdot \bar{\sigma}^\mu \cdot \partial^2 \psi(x) \\ - \frac{1}{4} \theta^2 (\zeta \cdot \sigma^\mu \cdot \bar{\zeta} \bar{\theta} \cdot \bar{\sigma}^\nu + \zeta \cdot \sigma^\nu \cdot \bar{\zeta} \bar{\theta} \cdot \bar{\sigma}^\mu) \cdot \partial_\mu \partial_\nu \psi(x) \\ - \frac{i}{8} \theta^2 \bar{\theta}^2 \zeta^2 \partial_\mu \partial^2 \psi(x) \cdot \sigma^\mu \cdot \bar{\zeta}.$$

Substitution of all the expansions in terms of standard words in the Grassmann variables into Equation (19) yields the full supertranslation properties of the component fields, which are reported in Equations (21-29). The transformation equations of the component fields are organized by contributions from the nine component fields $\phi(x)$, $\psi(x)$, $\bar{\chi}(x)$, $A_\mu(x)$, $M(x)$, $N(x)$, $\kappa(x)$,

$\bar{\lambda}(x)$, and $D(x)$, instead of organization by expansion in the supertranslation parameters ζ and $\bar{\zeta}$. In this way, supertranslations act on the component fields like matrices which have Grassmann valued differential

operators as entries. The reader can easily re-organize the transformation equations in terms of supertranslation parameters. The supertranslation equations are

$$\begin{aligned}\phi'(x) &= \phi(x) + \zeta \cdot \psi(x) + \bar{\chi}(x) \cdot \bar{\zeta} \\ &+ \zeta \cdot \sigma^\mu \cdot \bar{\zeta} A_\mu(x) + \zeta^2 M(x) + \bar{\zeta}^2 N(x) \\ &+ \bar{\zeta}^2 \zeta \cdot \kappa(x) + \zeta^2 \bar{\lambda}(x) \cdot \bar{\zeta} + \zeta^2 \bar{\zeta}^2 D(x),\end{aligned}\quad (21)$$

$$\begin{aligned}\psi'(x) &= i\sigma^\mu \cdot \bar{\zeta} \partial_\mu \phi(x) + \psi(x) \\ &+ \frac{i}{2} \sigma^\mu \cdot \bar{\sigma}_\nu \cdot \partial_\mu \psi(x) \zeta \cdot \sigma^\nu \cdot \bar{\zeta} \\ &- \frac{i}{2} \bar{\zeta}^2 \sigma^\mu \cdot \partial_\mu \bar{\chi}(x) + \sigma^\mu \cdot \bar{\zeta} A_\mu(x) \\ &+ \frac{i}{2} \sigma^\nu \cdot \bar{\sigma}^\mu \cdot \zeta \bar{\zeta}^2 \partial_\nu A_\mu(x) + 2\zeta M(x) \\ &+ i\zeta^2 \sigma^\mu \cdot \bar{\zeta} \partial_\mu M(x) + \kappa(x) \bar{\zeta}^2 \\ &- \sigma_\mu \cdot \bar{\lambda}(x) \zeta \cdot \sigma^\mu \cdot \bar{\zeta} - \frac{i}{2} \zeta^2 \bar{\zeta}^2 \sigma^\mu \cdot \partial_\mu \bar{\lambda}(x) \\ &+ 2\zeta \bar{\zeta}^2 D(x),\end{aligned}\quad (22)$$

$$\begin{aligned}\bar{\chi}'(x) &= -i\partial_\mu \phi(x) \zeta \cdot \sigma^\mu + \frac{i}{2} \zeta^2 \partial_\mu \psi(x) \cdot \sigma^\mu \\ &+ \bar{\chi}(x) - \frac{i}{2} \zeta \cdot \sigma^\nu \cdot \bar{\zeta} \partial_\mu \bar{\chi}(x) \cdot \bar{\sigma}_\nu \cdot \sigma^\mu \\ &+ A_\mu(x) \zeta \cdot \sigma^\mu - \frac{i}{2} \zeta^2 \partial_\nu A_\mu(x) \bar{\zeta} \cdot \bar{\sigma}^\mu \cdot \sigma^\nu \\ &+ 2\bar{\zeta} N(x) - i\bar{\zeta}^2 \partial_\mu N(x) \zeta \cdot \sigma^\mu \\ &- \zeta \cdot \sigma^\mu \cdot \bar{\zeta} \kappa(x) \cdot \sigma_\mu + \frac{i}{2} \zeta^2 \bar{\zeta}^2 \partial_\mu \kappa(x) \cdot \sigma^\mu \\ &+ \zeta^2 \bar{\lambda}(x) + 2\zeta^2 \bar{\zeta} D(x),\end{aligned}\quad (23)$$

$$\begin{aligned}M'(x) &= \frac{1}{4} \bar{\zeta}^2 \partial^2 \phi(x) - \frac{i}{2} \partial_\mu \psi(x) \cdot \sigma^\mu \cdot \bar{\zeta} \\ &+ \frac{1}{4} \bar{\zeta}^2 \zeta \cdot \partial^2 \psi(x) - \frac{i}{2} \bar{\zeta}^2 \partial_\mu A^\mu(x) + M(x) \\ &- i\zeta \cdot \sigma^\mu \cdot \bar{\zeta} \partial_\mu M(x) + \frac{1}{4} \zeta^2 \bar{\zeta}^2 \partial^2 M(x) \\ &+ \bar{\zeta} \cdot \bar{\lambda}(x) + \frac{i}{2} \bar{\zeta}^2 \zeta \cdot \sigma^\mu \cdot \partial_\mu \bar{\lambda}(x) + \bar{\zeta}^2 D(x),\end{aligned}\quad (24)$$

$$\begin{aligned}N'(x) &= \frac{1}{4} \zeta^2 \partial^2 \phi(x) + \frac{i}{2} \zeta \cdot \sigma^\mu \cdot \partial_\mu \bar{\chi}(x) \\ &+ \frac{1}{4} \zeta^2 \bar{\zeta} \cdot \partial^2 \bar{\chi}(x) + \frac{i}{2} \zeta^2 \partial_\mu A^\mu(x) + N(x) \\ &+ i\zeta \cdot \sigma^\mu \cdot \bar{\zeta} \partial_\mu N(x) + \frac{1}{4} \zeta^2 \bar{\zeta}^2 \partial^2 N(x) \\ &+ \zeta \cdot \kappa(x) - \frac{i}{2} \zeta^2 \partial_\mu \kappa(x) \cdot \sigma^\mu \cdot \bar{\zeta} + \zeta^2 D(x),\end{aligned}\quad (25)$$

$$\begin{aligned}A'_\mu(x) &= \frac{1}{2} \zeta \cdot \sigma_\mu \cdot \bar{\zeta} \partial^2 \phi(x) \\ &- \zeta \cdot \sigma^\nu \cdot \bar{\zeta} \partial_\mu \partial_\nu \phi(x) - \frac{i}{2} \zeta \cdot \sigma^\nu \cdot \bar{\sigma}_\mu \cdot \partial_\nu \psi(x) \\ &- \frac{1}{4} \zeta^2 \partial^2 \psi(x) \cdot \sigma_\mu \cdot \bar{\zeta} + \frac{1}{2} \zeta^2 \partial_\mu \partial_\nu \psi(x) \cdot \sigma^\nu \cdot \bar{\zeta} \\ &+ \frac{i}{2} \partial_\nu \bar{\chi}(x) \cdot \bar{\sigma}_\mu \cdot \sigma^\nu \cdot \bar{\zeta} - \frac{1}{4} \bar{\zeta}^2 \zeta \cdot \sigma_\mu \cdot \partial^2 \bar{\chi}(x) \\ &+ \frac{1}{2} \bar{\zeta}^2 \zeta \cdot \sigma^\nu \cdot \partial_\mu \partial_\nu \bar{\chi}(x) + A_\mu(x) \\ &+ \frac{i}{2} \zeta \cdot \sigma^\lambda \cdot \bar{\sigma}_\mu \cdot \sigma^\nu \cdot \bar{\zeta} (\partial_\nu A_\lambda(x) - \partial_\lambda A_\nu(x)) \\ &- \frac{1}{4} \zeta^2 \bar{\zeta}^2 (\partial^2 A_\mu(x) - 2\partial_\mu \partial_\nu A^\nu(x)) \\ &+ i\zeta^2 \partial_\mu M(x) - i\bar{\zeta}^2 \partial_\mu N(x) - \kappa(x) \cdot \sigma_\mu \cdot \bar{\zeta} \\ &- i\bar{\zeta}^2 \zeta \cdot \partial_\mu \kappa(x) - \frac{i}{2} \bar{\zeta}^2 \zeta \cdot \sigma^\nu \cdot \bar{\sigma}_\mu \cdot \partial_\nu \kappa(x) \\ &- \zeta \cdot \sigma_\mu \cdot \bar{\lambda}(x) + i\zeta^2 \bar{\zeta} \cdot \partial_\mu \bar{\lambda}(x) \\ &+ \frac{i}{2} \zeta^2 \partial_\nu \bar{\lambda}(x) \cdot \bar{\sigma}_\mu \cdot \sigma^\nu \cdot \bar{\zeta} - 2\zeta \cdot \sigma_\mu \cdot \bar{\zeta} D(x).\end{aligned}\quad (26)$$

For conversion of the last equation into standard words in the Grassmann variables ζ and $\bar{\zeta}$, note that from Equation (8)

$$\begin{aligned}\frac{i}{2} \zeta \cdot \sigma^\lambda \cdot \bar{\sigma}_\mu \cdot \sigma^\nu \cdot \bar{\zeta} F_{\nu\lambda}(x) &= \frac{1}{2} \varepsilon_{0123} \varepsilon^\lambda{}_\mu{}^\nu{}_\rho \zeta \cdot \sigma^\rho \cdot \bar{\zeta} F_{\nu\lambda}(x) \\ &= \varepsilon_{0123} \tilde{F}_{\mu\rho}(x) \zeta \cdot \sigma^\rho \cdot \bar{\zeta}.\end{aligned}$$

The remaining transformation equations are

$$\begin{aligned}\kappa'(x) &= \frac{i}{4} \zeta^2 \sigma^\mu \cdot \bar{\zeta} \partial_\mu \partial^2 \phi(x) + \frac{1}{4} \zeta^2 \partial^2 \psi(x) \\ &- \frac{1}{4} (\zeta \cdot \sigma_\mu \cdot \bar{\zeta}) \sigma^\mu \cdot \partial^2 \bar{\chi}(x) \\ &+ \frac{1}{2} (\zeta \cdot \sigma^\mu \cdot \bar{\zeta}) \sigma^\nu \cdot \partial_\mu \partial_\nu \bar{\chi}(x) \\ &- \frac{i}{8} \zeta^2 \bar{\zeta}^2 \sigma^\mu \cdot \partial_\mu \partial^2 \bar{\chi}(x) - \frac{i}{2} \sigma^\mu \cdot \bar{\sigma}^\nu \cdot \zeta \partial_\nu A_\mu(x) \\ &+ \frac{1}{4} \zeta^2 \sigma^\mu \cdot \bar{\zeta} (\partial^2 A_\mu(x) - 2\partial_\mu \partial_\nu A^\nu(x)) \\ &+ i\sigma^\mu \cdot \bar{\zeta} \partial_\mu N(x) + \frac{1}{2} \zeta \bar{\zeta}^2 \partial^2 N(x) + \kappa(x) \\ &+ \frac{i}{2} (\zeta \cdot \sigma^\nu \cdot \bar{\zeta}) \sigma^\mu \cdot \bar{\sigma}_\nu \cdot \partial_\mu \kappa(x) \\ &+ i(\zeta \cdot \sigma^\mu \cdot \bar{\zeta}) \partial_\mu \kappa(x) - \frac{i}{2} \zeta^2 \sigma^\mu \cdot \partial_\mu \bar{\lambda}(x) \\ &+ 2\zeta D(x),\end{aligned}\quad (27)$$

$$\begin{aligned}
\bar{\lambda}'(x) = & -\frac{i}{4}\bar{\zeta}^2\partial_\mu\partial^2\phi(x)\zeta\cdot\sigma^\mu \\
& -\frac{1}{4}(\zeta\cdot\sigma^\mu\cdot\bar{\zeta})(\partial^2\psi(x)\cdot\sigma_\mu-2\partial_\mu\partial_\nu\psi(x)\cdot\sigma^\nu) \\
& +\frac{i}{8}\zeta^2\bar{\zeta}^2\partial_\mu\partial^2\psi(x)\cdot\sigma^\mu+\frac{1}{4}\bar{\zeta}^2\partial^2\bar{\chi}(x) \\
& +\frac{i}{2}\partial_\nu A_\mu(x)\bar{\zeta}\cdot\bar{\sigma}^\nu\cdot\sigma^\mu+\frac{1}{4}\bar{\zeta}^2\partial^2 A_\mu(x)\zeta\cdot\sigma^\mu \\
& -\frac{1}{2}\bar{\zeta}^2\zeta\cdot\sigma^\mu\partial_\mu\partial_\nu A^\nu(x)-i\partial_\mu M(x)\zeta\cdot\sigma^\mu \\
& +\frac{1}{2}\zeta^2\bar{\zeta}^2\partial^2 M(x)+\frac{i}{2}\bar{\zeta}^2\partial_\mu\kappa(x)\cdot\sigma^\mu+\bar{\lambda}(x) \\
& -\frac{i}{2}(\zeta\cdot\sigma^\nu\cdot\bar{\zeta})\partial_\mu\bar{\lambda}(x)\cdot\bar{\sigma}_\nu\cdot\sigma^\mu \\
& -i(\zeta\cdot\sigma^\mu\cdot\bar{\zeta})\partial_\mu\bar{\lambda}(x)+2\bar{\zeta}D(x),
\end{aligned} \tag{28}$$

$$\begin{aligned}
D'(x) = & \frac{1}{16}\zeta^2\bar{\zeta}^2\partial^2\partial^2\phi(x) \\
& -\frac{i}{8}\zeta^2\partial_\mu\partial^2\psi(x)\cdot\sigma^\mu\cdot\bar{\zeta}+\frac{i}{8}\bar{\zeta}^2\zeta\cdot\sigma^\mu\cdot\partial_\mu\partial^2\bar{\chi}(x) \\
& -\frac{1}{4}\zeta\cdot\sigma^\mu\cdot\bar{\zeta}(\partial^2 A_\mu(x)-2\partial_\mu\partial_\nu A^\nu(x)) \\
& +\frac{1}{4}\zeta^2\partial^2 M(x)+\frac{1}{4}\bar{\zeta}^2\partial^2 N(x)-\frac{i}{2}\partial_\mu\kappa(x)\cdot\sigma^\mu\cdot\bar{\zeta} \\
& +\frac{i}{2}\zeta\cdot\sigma^\mu\cdot\partial_\mu\bar{\lambda}(x)+D(x).
\end{aligned} \tag{29}$$

These transformation laws are compatible with the reality constraints $V(x, \theta, \bar{\theta}) = V^\dagger(x, \theta, \bar{\theta})$ which define the vector multiplet,

$$\begin{aligned}
\phi(x) &= \phi^\dagger(x), \quad \psi(x) = \chi(x), \quad A_\mu(x) = A_\mu^\dagger(x), \\
M(x) &= N^\dagger(x), \quad \kappa(x) = \lambda(x), \quad D(x) = D^\dagger(x).
\end{aligned}$$

5. Supertranslations of the Chiral Multiplet

Besides the superderivatives (18) one can also define supercovariant derivatives [18,19]

$$D_\alpha = -i\partial_\alpha + \sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, \quad \bar{D}_{\dot{\alpha}} = i\bar{\partial}_{\dot{\alpha}} - \theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu, \tag{30}$$

such that

$$\{D_\alpha, D_\beta\} = 0, \quad \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0, \quad \{D_\alpha, \bar{D}_{\dot{\alpha}}\} = 2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu,$$

and

$$\{Q_\alpha, D_\beta\} = 0, \quad \{\bar{Q}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0, \quad \{Q_\alpha, \bar{D}_{\dot{\alpha}}\} = 0, \quad \{\bar{Q}_{\dot{\alpha}}, D_\alpha\} = 0.$$

The condition for chiral superfields

$$(i\bar{\partial}_{\dot{\alpha}} - \theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu)\Phi(x, \theta, \bar{\theta}) = 0$$

is therefore invariant under supertranslations.

The basic solutions

$$\bar{D}_{\dot{\alpha}}\theta^{\dot{\alpha}} = 0, \quad \bar{D}_{\dot{\alpha}}(x^\mu + i\theta\cdot\sigma^\mu\cdot\bar{\theta}) = 0,$$

imply [15]

$$\begin{aligned}
\Phi(x, \theta, \bar{\theta}) &= \Phi(x + i\theta\cdot\sigma\cdot\bar{\theta}, \bar{\theta}) = \phi(x + i\theta\cdot\sigma\cdot\bar{\theta}) \\
&+ \theta\cdot\psi(x + i\theta\cdot\sigma\cdot\bar{\theta}) + \theta^2 F(x + i\theta\cdot\sigma\cdot\bar{\theta}).
\end{aligned}$$

The relation

$$-(\theta\cdot\sigma^\mu\cdot\bar{\theta})(\theta\cdot\sigma^\mu\cdot\bar{\theta})\partial_\mu\partial_\nu = \frac{1}{2}\theta^2\bar{\theta}^2\partial^2$$

yields

$$\begin{aligned}
\Phi(x, \theta, \bar{\theta}) &= \phi(x) + i\theta\cdot\sigma^\mu\cdot\bar{\theta}\partial_\mu\phi(x) \\
&+ \frac{1}{4}\theta^2\bar{\theta}^2\partial^2\phi(x) + \theta\cdot\psi(x) \\
&- \frac{i}{2}\theta^2\partial_\mu\psi(x)\cdot\sigma^\mu\cdot\bar{\theta} + \theta^2 F(x).
\end{aligned} \tag{31}$$

The chiral superfield corresponds to the following substitutions in the general superfield V ,

$$\bar{\chi}(x) \rightarrow 0, \quad A_\mu(x) \rightarrow i\partial_\mu\phi(x), \quad M(x) \rightarrow F(x),$$

$$N(x) \rightarrow 0, \quad \kappa(x) \rightarrow 0, \quad \bar{\lambda}(x) \rightarrow -\frac{i}{2}\partial_\mu\psi(x)\cdot\sigma^\mu,$$

$$D(x) \rightarrow \frac{1}{4}\partial^2\phi(x).$$

It is clear from the construction, but can also be checked explicitly that these constraints are compatible with the transformation laws (21-29) of the full linear multiplet.

We find the following supertranslations of the components of the chiral multiplet,

$$\begin{aligned}
\phi'(x) &= \phi(x) + i\zeta\cdot\sigma^\mu\cdot\bar{\zeta}\partial_\mu\phi(x) + \frac{1}{4}\zeta^2\bar{\zeta}^2\partial^2\phi(x) \\
&+ \zeta\cdot\psi(x) - \frac{i}{2}\zeta^2\partial_\mu\psi(x)\cdot\sigma^\mu\cdot\bar{\zeta} + \zeta^2 F(x),
\end{aligned} \tag{32}$$

$$\begin{aligned}
\psi'(x) &= 2i\sigma^\mu\cdot\bar{\zeta}\partial_\mu\phi(x) + \zeta\bar{\zeta}^2\partial^2\phi(x) + \psi(x) \\
&+ 2(\zeta\cdot\sigma_\nu\cdot\bar{\zeta})S^{\mu\nu}\cdot\partial_\mu\psi(x) - \frac{1}{4}\zeta^2\bar{\zeta}^2\partial^2\psi(x) \\
&+ 2\zeta F(x) + i\zeta^2\sigma^\mu\cdot\bar{\zeta}\partial_\mu F(x),
\end{aligned} \tag{33}$$

$$\begin{aligned}
F'(x) &= \bar{\zeta}^2\partial^2\phi(x) - i\partial_\mu\psi(x)\cdot\sigma^\mu\cdot\bar{\zeta} \\
&+ \frac{1}{2}\bar{\zeta}^2\zeta\cdot\partial^2\psi(x) + F(x) \\
&- i\zeta\cdot\sigma^\mu\cdot\bar{\zeta}\partial_\mu F(x) + \frac{1}{4}\zeta^2\bar{\zeta}^2\partial^2 F(x).
\end{aligned} \tag{34}$$

Please note that this presentation does not involve the usual rescaling $\psi(x) \rightarrow \sqrt{2}\psi(x)$ of the spinor component of the chiral superfield, which is required for canonically normalized kinetic terms in supersymmetric Lagrangians.

6. Conclusions

The supertranslation properties of the component fields of a general linear supermultiplet and of a chiral multiplet were reported to all orders in the translation parameters ζ and $\bar{\zeta}$ in (21-29) and (32-34), respectively. On the one hand, one can think of these results as explicit parametrizations of orbits of supertranslations in the space of component fields of a supersymmetric theory. On the other hand, one can consider the transformed fields as superfields in the variables $(x, \zeta, \bar{\zeta})$, because *e.g.*

$$\begin{aligned}\Psi'_\alpha(x) &= \Psi'_\alpha(x, \zeta, \bar{\zeta}) \\ &= \left[\frac{\partial}{\partial \theta^\alpha} V(x - i(\zeta \cdot \sigma \cdot \bar{\theta} - \theta \cdot \sigma \cdot \bar{\zeta}), \theta + \zeta, \bar{\theta} + \bar{\zeta}) \right]_{\substack{\theta=0, \\ \bar{\theta}=0}} \\ &= \left(i\sigma^\mu_{\alpha\dot{\alpha}} \bar{\zeta}^{\dot{\alpha}} + \frac{\partial}{\partial \zeta^\alpha} \right) V(x, \zeta, \bar{\zeta}) = iD^{(\zeta)}_\alpha V(x, \zeta, \bar{\zeta}),\end{aligned}$$

and higher order derivatives with respect to the θ and $\bar{\theta}$ variables at $\theta=0$, $\bar{\theta}=0$ can also be expressed as supercovariant derivatives with respect to ζ and $\bar{\zeta}$. For example, the transformed vector field is

$$\begin{aligned}A'_{\alpha\dot{\alpha}}(x, \zeta, \bar{\zeta}) &= A'_\mu(x, \zeta, \bar{\zeta}) \sigma^\mu_{\alpha\dot{\alpha}} \\ &= \left[\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} \frac{\partial}{\partial \theta^\alpha} V(x - i(\zeta \cdot \sigma \cdot \bar{\theta} - \theta \cdot \sigma \cdot \bar{\zeta}), \theta + \zeta, \bar{\theta} + \bar{\zeta}) \right]_{\substack{\theta=0, \\ \bar{\theta}=0}}.\end{aligned}$$

We have

$$\begin{aligned}&\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} \frac{\partial}{\partial \theta^\alpha} V(x - i(\zeta \cdot \sigma \cdot \bar{\theta} - \theta \cdot \sigma \cdot \bar{\zeta}), \theta + \zeta, \bar{\theta} + \bar{\zeta}) \\ &= \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} \left[\left(\frac{\partial}{\partial \zeta^\alpha} + i\sigma^\nu_{\alpha\dot{\beta}} \bar{\zeta}^{\dot{\beta}} \frac{\partial}{\partial X^\nu} \right) V(X, \xi, \bar{\xi}) \right]_{\substack{X=x-i(\zeta \cdot \sigma \cdot \bar{\theta} - \theta \cdot \sigma \cdot \bar{\zeta}), \\ \xi=\theta+\zeta, \bar{\xi}=\bar{\theta}+\bar{\zeta}}} \\ &= \left[\left(\frac{\partial}{\partial \bar{\zeta}^{\dot{\alpha}}} - i\zeta^\beta \sigma^\mu_{\beta\dot{\alpha}} \frac{\partial}{\partial X^\mu} \right) \right. \\ &\quad \left. \times \left(\frac{\partial}{\partial \zeta^\alpha} + i\sigma^\nu_{\alpha\dot{\beta}} \bar{\zeta}^{\dot{\beta}} \frac{\partial}{\partial X^\nu} \right) V(X, \xi, \bar{\xi}) \right]_{\substack{X=x-i(\zeta \cdot \sigma \cdot \bar{\theta} - \theta \cdot \sigma \cdot \bar{\zeta}), \\ \xi=\theta+\zeta, \bar{\xi}=\bar{\theta}+\bar{\zeta}}},\end{aligned}$$

and therefore

$$A'_\mu(x, \zeta, \bar{\zeta}) = -\frac{1}{2} \bar{\sigma}^{\dot{\alpha}\alpha}_\mu \bar{D}^{(\zeta)}_{\dot{\alpha}} D^{(\zeta)}_\alpha V(x, \zeta, \bar{\zeta}).$$

From this point of view, (21-29) and (32-34) tell us explicitly how supertranslation of the components of V induces corresponding superfields.

7. References

- [1] R. Clarkson, A. M. Ghezelbash and R. B. Mann, "New Reducible Membrane Solutions in D=11 Supergravity," *Journal of High Energy Physics*, 2004.
- [2] A. M. Ghezelbash, "Supergravity Solutions without Tri-Holomorphic U (1) Isometries," *Physical Review D*, Vol. 78, 2008, p. 126002.
- [3] de B. Wit, H. Nicolai and H. Samtleben "Gauged Supergravities, Tensor Hierarchies and M-Theory," *Journal of High Energy Physics*, Vol. 2, 2008, p. 044.
- [4] G. Compère, K. Copsey, de S. Buyl and R. B. Mann, "Solitons in Five-Dimensional Minimal Supergravity: Local Charge, Exotic Ergoregions and Violations of the BPS Bound," *Journal of High Energy Physics*, Vol. 12, No. 47, 2009, p. 46.
- [5] U. Gran, J. Gutowski and G. Papadopoulos, "M-Theory Backgrounds with 30 Killing Spinors are Maximally Supersymmetric," ARXIV, 2010.
- [6] R. Britto, E. Buchbinder, F. Cachazo and B. Feng, "One-Loop Amplitudes of Gluons in SQCD," *Physical Review D*, Vol. 72, 2005, p. 27.
- [7] R. Dick, "Implications of a Dilaton in Gauge Theory and Cosmology," *Fortschr der Physik*, Vol. 45, 1997, pp. 537-587.
- [8] R. Dick, "Heterotic String Dark Matter from the Graviton Multiplet," *Physics Letters B*, Vol. 510, 2001, pp. 10-16.
- [9] J. McDonald and N. Sahu, "Kev Warm Dark Matter Via the Supersymmetric Higgs Portal," *Physical Review D*, Vol. 79, 2009, p. 6.
- [10] A. A. Bytsenko, S. D. Odintsov and S. Zerbini, "The Effective Action in Gauged Supergravity on Hyperbolic Background and Induced Cosmological Constant," *Physics Letters B*, Vol. 336, 1994, pp. 355-361.
- [11] E. Elizalde, S. Jhingan, S. Nojiri, S. D. Odintsov, M. Sami and I. Thongkool, "Dark Energy Generated From a (Super) String Effective Action with Higher Order Curvature Corrections and a Dynamical Dilaton," *European Physical Journal C*, Vol. 53, 2008, pp. 447-457.
- [12] A. E. Nelson and N. Seiberg, "R-symmetry Breaking Versus Supersymmetry Breaking," *Nuclear Physics B*, Vol. 416, 1994, 46-62.
- [13] K. A. Intriligator, N. Seiberg and D. Shih, Supersymmetry Breaking, R-symmetry Breaking and Metastable Vacua," *Journal of High Energy Physics*, Vol. 7, 2007, p. 19.
- [14] M. Dine, G. Festuccia and Z. Komargodski, "A Bound on the Superpotential," ARXIV, 2009, p. 2527.
- [15] J. Wess and J. Bagger, "Supersymmetry and Supergravity, 2nd Edition," Princeton University Press, 1992.
- [16] J. Wess and B. Zumino, "Supergauge Transformations in four Dimensions," *Nuclear Physics B*, Vol. 70, pp. 39-50; "A Lagrangian Model Invariant under Supergauge Transformations," *Physics Letters B*, Vol. 49, 1974, pp. 52-54.
- [17] A. Salam and J. Strathdee, "Supergauge Transformations," *Nuclear Physics B*, Vol. 76, 1974, pp. 477-482.
- [18] S. Ferrara, J. Wess and B. Zumino, "Supergauge Multiplets and Superfields," *Physics Letters B*, Vol. 51, 1974, pp. 239-241.
- [19] A. Salam and J. Strathdee, "On Superfields and Fermi-Bose Symmetry," *Physical Review D*, Vol. 11, 1975, pp. 1521-1535.

Appendix

A.1. Translation of Our Results Into the Conventions of Wess and Bagger

Our superspace realizations (18) and (30) are related to the realizations in Wess and Bagger [15] according to

$$Q_\alpha = -iQ_\alpha^{(WB)}, \quad \bar{Q}_\alpha = -i\bar{Q}_\alpha^{(WB)}, \\ D_\alpha = -iD_\alpha^{(WB)}, \quad \bar{D}_\alpha = -i\bar{D}_\alpha^{(WB)}.$$

With these conventions the component field expansions for chiral superfields agree and the generators for supertranslations are also the same (cf. (4.11,4.12) in [15]),

$$i(\zeta \cdot Q + \bar{Q} \cdot \bar{\zeta}) = \zeta \cdot Q^{(WB)} + \bar{Q}^{(WB)} \cdot \bar{\zeta},$$

i.e. our results for supertranslations to all orders also directly apply as generalizations of the first order transformation laws reported in [15].

Note that Wess and Bagger use an operator representation of the super-Poincaré algebra with the same signature of the Minkowski metric but $p_\mu^{(WB)} = i\partial_\mu = -p_\mu$. This comes from the familiar sign difference between field theoretic and quantum mechanical operator realizations of Noether charges. If $\phi(x) = \langle x | \phi \rangle$ is a field operator, the momentum operators

$$P_\mu = \int d^3x \left(\eta_\mu^0 \mathcal{L} - \partial_\mu \phi \cdot \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)} \right)$$

satisfy

$$[P_\mu, \phi(x)] = i\partial_\mu \phi(x)$$

and generate translations according to

$$\begin{aligned} \phi'(x) &= \exp(-i\varepsilon \cdot P) \phi(x) \exp(i\varepsilon \cdot P) = \phi(x + \varepsilon) \\ &= \langle x + \varepsilon | \phi \rangle = \langle x | \exp(i\varepsilon \cdot p) | \phi \rangle = \langle x | \exp(-i\varepsilon \cdot p^{(WB)}) | \phi \rangle. \end{aligned}$$

Similar relations hold for Lorentz and gauge charges. Another way to look at the sign difference is through Jacobi identities. If the generators X_a satisfy the Lie

algebra,

$$[X_a, X_b] = iC_{ab}{}^c X_c,$$

then the adjoint matrix representation is given by

$$(X_a)_b{}^c = -iC_{ab}{}^c.$$

A.2. Spinor Identities

There are several useful identities for products of spinors which are used in the determination of the general linear multiplet and its transformation laws.

The following identities are a direct consequence of the anti-commutation properties of spinors and the definitions (9) and (10) of spinor products,

$$\begin{aligned} \theta^\alpha \theta^\beta &= \varepsilon^{\alpha\beta} \theta^1 \theta^2 = \frac{1}{2} \varepsilon^{\alpha\beta} (-\theta^1 \theta_1 + \theta_2 \theta^2) \\ &= -\frac{1}{2} \varepsilon^{\alpha\beta} \theta^\gamma \theta_\gamma = -\frac{1}{2} \varepsilon^{\alpha\beta} \theta^2, \end{aligned} \quad (35)$$

$$\bar{\theta}^{\dot{\alpha}} \bar{\theta}^{\dot{\beta}} = \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\theta}^{\dot{1}} \bar{\theta}^{\dot{2}} = \frac{1}{2} \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\theta}^2, \quad (36)$$

$$\theta_\alpha \theta^\beta = -\theta^\beta \theta_\alpha = -\frac{1}{2} \delta_\alpha{}^\beta \theta^2, \quad \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\beta}} = -\bar{\theta}^{\dot{\beta}} \bar{\theta}_{\dot{\alpha}} = \frac{1}{2} \delta_{\dot{\alpha}}{}^{\dot{\beta}} \bar{\theta}^2.$$

Please note that θ^2 in the first line in Equation (35) denotes the $\theta^{\alpha=2}$ component of the spinor θ , but in the second line it is $\theta^2 = \theta \cdot \theta$. In every equation in supersymmetry it is clear from the context what θ^2 means. In all equations in previous sections of this paper θ^2 always refers to $\theta^2 = \theta \cdot \theta$.

The following relations also use the properties (3-7) of the Pauli matrices,

$$\psi \cdot \sigma^\mu \cdot \bar{\chi} = -\bar{\chi} \cdot \bar{\sigma}^\mu \cdot \psi, \quad (37)$$

$$\psi \cdot \sigma^\mu \cdot \bar{\sigma}^\nu \cdot \chi = \chi \cdot \sigma^\nu \cdot \bar{\sigma}^\mu \cdot \psi, \quad (38)$$

$$\bar{\theta}^{\dot{\alpha}} \psi^\alpha = \frac{1}{2} \bar{\sigma}_\mu^{\dot{\alpha}\alpha} \psi \cdot \sigma^\mu \cdot \bar{\theta}, \quad (39)$$

$$\zeta \cdot \sigma^\mu \cdot \bar{\theta} \zeta \cdot \sigma^\nu \cdot \bar{\theta} = -\frac{1}{2} \eta^{\mu\nu} \zeta^2 \bar{\theta}^2. \quad (40)$$

Tilted Bianchi Type VI₀ Cosmological Model in Saez and Ballester Scalar Tensor Theory of Gravitation

Subrata Kumar Sahu

Post Graduate Department of Mathematics, Lingaya's University, Faridabad, India

E-mail: subrat_sahoo2002@rediffmail.com

Received February 17, 2010; revised March 17, 2010; accepted March 20, 2010

Abstract

Tilted Bianchi type VI₀ cosmological model is investigated in a new scalar tensor theory of gravitation proposed by Saez and Ballester (Physics Letters A 113:467, 1986). Exact solutions to the field equations are derived when the metric potentials are functions of cosmic time only. Some physical and geometrical properties of the solutions are also discussed.

Keywords: Saez and Ballester Theory, Tilted Cosmological Model, Scalar Field

1. Introduction

In recent years, there has been a considerable interest in the investigation of cosmological models in which the matter does not move orthogonally to the hyper surface of homogeneity. These are called tilted cosmological models. The general behaviors of tilted cosmological models have been studied by King and Ellis [1], Ellis and King [2], Collins and Ellis [3], Bali and Sharma [4,5], Bali and Meena [6].

Bradely and Sviestins [7] investigated that heat flow is expected for cosmological models. Following the development of inflationary models, the importance of scalar fields (mesons) has become well known. Saez and Ballester [8] have developed a new scalar tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field, an anti-gravity regime appears. This theory suggests a possible way to solve the missing matter problem in non-flat FRW cosmologies. Sing and Agrawal [9], Reddy and Rao [10], Reddy *et al.* [11], Mohanty and Sahu [12,13], Adhav *et al.* [14], Tripathy *et al.* [15] are some of the authors who have studied the various aspects of Saez and Ballester [8] scalar tensor theory.

We derived the field equations for Bianchi type VI₀ metric in Section 2. We solved the field equations in Section 3. We mentioned some physical and geometrical properties of the solutions in Section 4 and also mentioned the concluding remark in Section 5.

2. Field Equations

Here we consider the Bianchi type VI₀ metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2qx} dy^2 + C^2 e^{2qx} dz^2 \quad (1)$$

where A, B and C are functions of cosmic time t only and q is a non-zero constant.

The field equations given by Saez and Ballester [8] for the combined scalar and tensor fields are

$$G_i^j - \omega V^n \left(V_{,i} V^{,j} - \frac{1}{2} g_i^j V_{,a} V^{,a} \right) = -T_i^j \quad (2)$$

and the scalar field satisfies the equation

$$2V^n V_{,i} + n V^{n-1} V_{,a} V^{,a} = 0 \quad (3)$$

where $G_i^j \equiv R_i^j - \frac{1}{2} g_i^j R$ is the Einstein tensor; n, an arbitrary exponent; and ω , a dimensionless coupling constant; T_i^j is the stress tensor of the matter. The energy momentum tensor for a perfect fluid distribution with heat conduction given by Ellis [2] as

$$T_i^j = (\rho + p) u_i u^j + p g_i^j + q_i u^j + u_i q^j \quad (4)$$

together with

$$g_{ij} u^i u^j = -1 \quad (5)$$

$$q_i q^i > 0 \quad (6)$$

and

$$q_i u^i = 0, \quad (7)$$

where p is the pressure, ρ is the energy density, q_i is the heat conduction vector orthogonal to u^i . The fluid vector u^i has the components

$\left(\frac{\sinh \lambda}{A}, 0, 0, \cosh \lambda\right)$ satisfying Equation (5) and λ is the tilt angle. Here comma and semicolon denote ordinary and co-variant differentiation respectively.

With the help of Equations (3-7), the field Equation (2) for the metric (1) in the commoving co-ordinate system take the following explicit forms:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{q^2}{A^2} - \frac{\omega V^n V_4^2}{2} = - \left[(\rho + p) \sinh^2 \lambda + p + 2q_1 \frac{\sinh \lambda}{A} \right] \quad (8)$$

$$\frac{C_{44}}{C} + \frac{A_{44}}{A} + \frac{C_4 A_4}{CA} - \frac{q^2}{A^2} - \frac{\omega V^n V_4^2}{2} = -p \quad (9)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{q^2}{A^2} - \frac{\omega V^n V_4^2}{2} = -p \quad (10)$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA} - \frac{q^2}{A^2} + \frac{\omega V^n V_4^2}{2} = (\rho + p) \cosh^2 \lambda - p + 2q_1 \frac{\sinh \lambda}{A} \quad (11)$$

$$\begin{aligned} & (\rho + p) \cosh^2 \lambda - p + 2q_1 \frac{\sinh \lambda}{A} \\ & (\rho + p) A \sinh \lambda \cosh \lambda + q_1 \cosh \lambda \\ & + q_1 \frac{\sinh^2 \lambda}{\cosh \lambda} = \frac{B_4}{B} - \frac{C_4}{C} \end{aligned} \quad (12)$$

$$V_{44} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) V_4 + \frac{n V_4^2}{2 V} = 0 \quad (13)$$

Hereafterwards the suffix 4 after a field variable represents ordinary differentiation with respect to time.

3. Solutions

Equations (8-13) are six equations with eight unknowns $A, B, C, p, \rho, V, \lambda$ and q_1 , therefore, we require two more conditions.

First we assume that the model is filled with stiff fluid which leads to

$$p = \rho \quad (14)$$

We also assume that

$$A = B^n \quad (15)$$

In order to derive exact solutions of the field Equations (8-13) easily, we use the following scale transformations:

$$A = e^{n\beta}, \quad B = e^\beta, \quad C = e^\gamma, \quad dt = ABC dT \quad (16)$$

The field Equations (8-13) reduce to

$$\begin{aligned} & \beta'' - n\beta'^2 - (n+1)\beta'\gamma' + \gamma'' + q^2 e^{2(\beta+\gamma)} - \frac{\omega V^n V'^2}{2} = \\ & - \left[(\rho + p) \sinh^2 \lambda + p + \frac{2q_1 \sinh \lambda}{e^{n\beta}} \right] e^{2[(n+1)\beta+\gamma]} \end{aligned} \quad (17)$$

$$\begin{aligned} & \gamma'' - (n+1)\beta'\gamma' + n\beta'' - n\beta'^2 - \\ & q^2 e^{2(\beta+\gamma)} - \frac{\omega V^n V'^2}{2} = -p e^{2[(n+1)\beta+\gamma]} \end{aligned} \quad (18)$$

$$\begin{aligned} & n\beta'' - n\beta'^2 - (n+1)\beta'\gamma' + \\ & \beta'' - q^2 e^{2(\beta+\gamma)} - \frac{\omega V^n V'^2}{2} = -p e^{2[(n+1)\beta+\gamma]} \end{aligned} \quad (19)$$

$$\begin{aligned} & n\beta'^2 + (n+1)\beta'\gamma' - q^2 e^{2(\beta+\gamma)} + \frac{\omega V^n V'^2}{2} = \\ & \left[(\rho + p) \cosh^2 \lambda - p + \frac{2q_1 \sinh \lambda}{e^{n\beta}} \right] e^{2[(n+1)\beta+\gamma]} \end{aligned} \quad (20)$$

$$\begin{aligned} & (\rho + p) e^{n\beta} \sinh \lambda \cosh \lambda + q_1 \cosh \lambda + \\ & q_1 \frac{\sinh^2 \lambda}{\cosh \lambda} = (\beta' - \gamma') e^{-2[(n+1)\beta+\gamma]} \end{aligned} \quad (21)$$

$$V'' + \frac{n V'}{2 V} = 0 \quad (22)$$

In view of Equation (14), Equation (17) and Equation (21), Equations (18,19), yield

$$\beta(=\gamma) = K_1 T + K_2 \quad (23)$$

and

$$\alpha = (K_1 T + K_2)^n \quad (24)$$

where $K_1 (\neq 0)$, K_2 are arbitrary constants.

Thus the corresponding metric of our solution can be written as

$$ds^2 = -T^{2n+2} dT^2 + T^{2n} dX^2 + T(e^{-2qx} dY^2 + e^{2qx} dZ^2) \quad (25)$$

4. Some Physical and Geometrical Properties of the Solutions

On integration Equation (22) yields

$$V = \left(\frac{n+2}{2} K_3 T + K_4 \right)^{\frac{2}{n+2}} \quad (26)$$

where $K_3 (\neq 0)$, K_4 are arbitrary constants.

Using Equations (23,24) and Equation (26) in Equations (19, 20), we get

$$p(=\rho) = \left[K_5^2 + q^2 e^{4(K_1 T + K_2)} \right] e^{-(2n+4)(K_1 T + K_2)} \quad (27)$$

where $K_5 = \left((2n+1)K_1^2 + \frac{\omega K_3^2}{2} \right)$ is a constant.

Substituting Equations (23),(26) and (27) in Equation (17) we get

$$\sinh^2 \lambda = \frac{1}{2} \left[\left\{ K_5^2 - q^4 e^{16(K_1 T + K_2)} \right\}^{-1} e^{-(2n+4)(K_1 T + K_2)} - 1 \right] \quad (28)$$

Further substituting Equations (23), (27) and (28) in Equation (21), we get

$$q_1 = -e^{n(K_1 T + K_2)} \left[K_5^2 + q^2 e^{4(K_1 T + K_2)} \right] \times \left[\frac{1}{2} \left[\left\{ K_5^2 - q^4 e^{16(K_1 T + K_2)} \right\}^{-1} e^{-(2n+4)(K_1 T + K_2)} - 1 \right]^{\frac{1}{2}} \right] \times \left[e^{-(2n+4)(K_1 T + K_2)} + \left\{ K_5^2 - q^4 e^{16(K_1 T + K_2)} \right\} \right] \quad (29)$$

The spatial volume for the model (25) is given by

$$\text{Vol.} = (K_1 T + K_2)^{n+1} \quad (30)$$

From Equations (27-29) we find that the pressure, energy density, tilt angle, heat conduction vector of the fluid distribution are constants at time $T=0$ and gradually decreases in the course of evolution. Equation (26) shows that the scalar field V changes with time and at time $T=0$, the scalar field is found to be a constant. Equation (30) implies the anisotropic expansion of the universe with time. It is interesting to note that the model does not admit singularity throughout evolution.

5. Conclusions

In this paper we have solved Saez and Ballester field equations for the tilted Bianchi VI₀ cosmological model. It is observed that the pressure, energy density, tilt angle, heat conduction vector of the fluid distribution are constants at time $T=0$ and gradually decrease with the increase of the age of the universe. It is interesting to note that the models we have constructed here is free from singularity at time $T=0$ and for $\omega = 0$ the Saez and Ballester [8] theory approaches general relativity. This supports the analysis that the introduction of scalar field avoids initial singularity.

6. Acknowledgements

The author would like to convey his sincere thanks and gratitude to the anonymous referee for his useful and kind suggestions for the improvement of this paper.

7. References

- [1] A. R. King and G. F. R. Ellis, "Tilted Homogeneous Cosmological Models," *Communications in Mathematical Physics*, Vol. 31, 1973, p. 209.
- [2] G. F. R. Ellis and A. R. King, "Was a Big Bang a Whimper," *Communications in Mathematical Physics*, Vol. 38, 1974, p. 119.
- [3] C. B. Collins and G. F. R. Ellis, "Alternatives to the Big Bang," *Physics Reports*, Vol. 56, 1979, p. 65.
- [4] R. Bali and K. Sharma, "Tilted Bianchi Type-I Models with Heat Conduction Filled with Disordered Radiations of Perfect Fluid in General Relativity," *Astrophysics and Space Science*, Vol. 271, 2000, p. 227.
- [5] R. Bali and K. Sharma, "Tilted Bianchi Type-I Dust Fluid Cosmological Model in General Relativity," *Pramana*, Vol. 58, No. 3, 2002, p. 457.
- [6] R. Bali and B. L. Meena, "Tilted Cosmological Models Filled with Disordered Radiations in General Relativity," *Astrophysics and Space Science*, Vol. 281, 2002, p. 565.
- [7] J. M. Bradely and E. Sviestins, "Some Rotating Time-Dependent Bianchi Type VIII Cosmologies with Heat Flow," *General Relativity Gravitation*, Vol. 16, 1984, p. 1119.
- [8] D. Saez and V. J. Ballester, "A Simple Coupling with Cosmological Implications," *Physics Letters A*, Vol. 133, 1986, p. 477.
- [9] T. Sing and A. K. Agrawal, "Some Bianchi Type Cosmological Models in a New Scalar-Tensor Theory," *Astrophysics and Space Science*, Vol. 182, 1991, p. 289.
- [10] D. R. K. Reddy and N. V. Rao, "Some Cosmological Models in Scalar-Tensor Theory of Gravitation," *Astrophysics and Space Science*, Vol. 277, 2001, p. 461.
- [11] D. R. K. Reddy, R. L. Naidu and V. U. M. Rao, "Axially Symmetric Cosmic Strings in a Scalar Tensor Theory," *Astrophysics and Space Science*, Vol. 306, 2006, p. 185.
- [12] G. Mohanty and S. K. Sahu, "Bianchi Type-I Cosmological Effective Stiff Fluid Model in Saez and Ballester Theory," *Astrophysics and Space Science*, Vol. 291, 2004, p. 75.
- [13] G. Mohanty and S. K. Sahu, "Effective Stiff Fluid Model in Scalar-Tensor Theory Proposed by Saez and Ballester," *Communications in Physics*, Vol. 14, No. 3, 2004, p. 178.
- [14] K. S. Adhav, M. R. Ugale, C. B. Kale and M. P. Bhende, "Bianchi Type VI Sring Cosmological Model in Saez and Ballester's Scalar Tensor Gravitation," *International Journal of Theoretical Physics*, Vol. 46, 2007, p. 3122.
- [15] S. K. Tripathy, S. K. Nayak, S. K. Sahu and T. R. Routray, "String Cloud Cosmologies for Bianchi Type-III Models with Electromagnetic Field," *Astrophysics and Space Science*, Vol. 315, 2008, p. 105.

The Possible Dimension, Additional to Space-Time, which Physicists Ignore

Arie S. Issar, Shaul Sorek

Ben-Gurion University of the Negev Israel, Blaustein Institutes for Desert Research, Israel

Zuckerberg Institute for Water Research, Environmental Hydrology & Microbiology

E-mail: {issar, sorek}@bgu.ac.il

Received February 23, 2010; revised March 25, 2010; accepted March 25, 2010

Abstract

In response to Wheeler's challenge to find an element that is: "*something that itself has no localization in space and time...pure knowledge ... an atom of information*" we suggest to account for Information as a dimension. Its degrees of freedom are arithmetical (+-) and logical (if-then) forward and backward steps. While Space refers to gaps in distance, Time refers to change in instances, Information refers to a sequence of notions measured by the number of steps made (or bytes used) by a computer in order to perform (describe or solve) a certain logical sentence or a sequence of logical sentences. In the attempt to quantifiably formulate the incorporation of Information into physical laws, we refer to Hamiltonian extended stationary principle in terms of Space, Time and an additional degree of freedom, suggested as an information state. The obtained Euler equation is demonstrated for the case of a thin rod under longitudinal vibrations, investigated by dimensionless analysis. It is shown that depending on the value of information and its rate, one may obtain dominant forms conforming to Poisson's equation in Space vs Information, wave equation in Time vs Information and the expected wave equation in Time vs Space.

Keywords: Information, Hamiltonian, Dimensionless-Analysis

1. The New Dimension Wheeler Foresaw: Information

"If we're ever going to find an element of nature that explains space and time, we surely have to find something that is deeper than space and time—something that itself has no localization in space and time. The amazing feature of the elementary quantum phenomenon—the Great Smoky Dragon—is exactly this. It is indeed something of a pure knowledge-theoretical character, an atom of information, which has no localization in between the point of entry and the point of registration. This is the significance of the delayed-choice experiment." [1].

Following Wheeler's call [1] we maintain that Information should be (along which intelligence is measured as well as instinctive knowledge) added, independently, to space and time. At first it was thought by Issar to call this dimension¹ either thinking or 'intelligence', but after investigating these options it was found that thinking is

the description of moving and 'intelligence' are structures along degrees of freedom along a more substantial dimension. The introduction of a new basic dimension will enable to describe intelligence as a feature or even a structure constructed from the addition of fundamental steps of observation and logical conclusion. These, are in fact the basic steps of arithmetic addition (and subtraction) and the basic logical conclusion of 'if-then', forward and backwards. Generalizing this suggestion will say that the adding one to another of a few basic conclusions, which brings to the arriving of a more general structure, is actually a construction along a dimension, which constitutes 4 degrees of freedom. The more intelligent is the living being, the more will be its ability to increase the building composed of elaborate structures of knowledge added one to another and put together by logical steps, to form wider and higher structures of intelligent thinking and behavior.

This conclusion brought to search for an altogether new dimension along which the movement while taking these fundamental steps can be described and quantified. After investigating various options the conclusion was that the most appropriate term will be: 'Information'. Thus, change in location on space is quantified by steps

¹An intrinsic independent property, representing the minimum number of directions needed to specify either a point on space, an instant on time and a notion when considering Information.

of the foot or by a meter, duration of time is quantified by the movement of shade on the sun-dial or by a clock, while progress on the Dimension of Information is quantified by the length of mathematical or logical sentences needed to describe a certain idea or number of bytes used by a computer to perform a certain logical sentence.

Space has six degrees of freedom namely along or spins around three coordinates, in two directions *i.e.* forward and backwards, Time one degree of freedom from present to future (the perspective from present to past actually refers to knowledge and thus is regarded as the Dimension of Information), while the additional dimension of Information has four degrees of freedom [2], which are: addition and subtraction and induction and deduction (*i.e.* 'if-then' and 'when-then'). Altogether the evolution of the universe is along a three dimensional continuum of Space-Time-Information, having 11 degrees of freedom.

The introduction of this new dimension enables to answer a question that many outstanding physicists came up with and which Albert Einstein [3] brought up in his lecture before the Prussian Academy of Sciences, namely: "How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality? His answer to this enigma was by accepting as a fact that Mathematics is intrinsic both in Nature and in the human mind. Yet while the human mind can build wonderful logical structures with the aid of this mysterious tool, these structures are not factual if not cross-checked by empirical observations. In Einstein's words: "In my opinion the answer to this question is, briefly, this: as far as the propositions of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality [3].

In his Herbert Spencer lecture at Oxford, Einstein [4] manifested his faith in mathematics as the skeleton of the edifice of nature, and thus the power of abstract mathematical thought to reveal the secrets of the laws interconnecting our observation of natural phenomena. He stressed the role of mathematics as a bridge between mind and nature. In his words: "Our experience hitherto justifies us in believing that nature is the realization of the simplest conceivable mathematical ideas. I am convinced that we can discover by means of purely mathematical constructions the concepts and the laws connecting them with each other, which furnish the key to the understanding of natural phenomena. Useful mathematical concepts may well be suggested by experience, but in no way can they be derived from it. Experience naturally remains the sole criterion of the usefulness of a mathematical construction for physics. But the actual creative principle lies in mathematics. Thus, in a certain sense, I take it to be true that pure thought can grasp the real, as the ancients had dreamed

[4].

He did not touch, however, upon the basic question: How comes that mathematics is on one hand the brain-child of the human being and on the other hand is intrinsic in the framework of the universe? When he did refer to this question he admitted failure from the start "One may say "the eternal mystery of the world is its comprehensibility" [5].

It can be concluded, thus, that for Einstein, this question was solved once he accepted the philosophical world view and thus the "God" of Baruch-Benedictus Espinoza, who argued for the uniformity of the terms of "God" and "Nature"². Espinoza and thus Einstein, took it as granted that being a Supreme Mathematician is one of the infinite attributes of "God" *i.e.* Nature.

Eugene Wigner [6], not being a Spinozist, brought this enigma to the level of an absurd in the title in his paper "The Unreasonable Effectiveness of Mathematics in the Physical Science", in which he restated the problem by touching on the super-natural. His claim was that "The miracle of the appropriateness of the language of mathematics to the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve." [6].

Indeed the achievements of the theories of Einstein, which were applauded by the world of science on the occasion of the hundred anniversary of Annum Mirabelis, had confirmed his trust in the power of mathematical thinking to unveil the many faces of nature.

One of the famous cases of forecasting, which was confirmed by many observations, is Einstein's General Theory of Relativity. In 1922 by developing the equations of this theory the Russian physicists Alexander Friedman [7] showed that these equations demand either a contracting or an expanding universe along space-time dimensions, which Einstein tried to stabilize by introducing his 'cosmological constant' [7]. Hubble's observations showed that this constant is redundant. Running back the "motion picture", namely turning expansion to contraction on space-time dimensions brought cosmologists to conclude that all started with a singular event, namely the Big Bang, after which the cosmos expands continuously to this day and into the future.

2. The 5th Dimension along Which Our Universe Expands

All the computerized models used by the physicists to contract the universe to reach the pre-Big Bang singular point and expand it to its present dimensions and beyond were dictated by mathematical procedures, demanding

²Einstein's response the telegraphed question of New York's Rabbi Herbert S. Goldstein in (24 April 1929): "Do you believe in God? I believe in Spinoza's God, Who reveals Himself in the lawful harmony of the world, not in a God Who concerns Himself with the fate and the doings of mankind.

various assumptions at various stages. The basic assumption, following Einstein's basic conceptual model is that our universe is four-dimensional, (three orthogonal spatial coordinate system and time which has only one degree of freedom from past to future). This continuum is measurable by an observer using a meter and a clock.

Yet, Einstein's, Friedman's and their colleagues, investigating the implication of the General Theory of Relativity, like the Dutch Willem de Sitter [8], and the French Georges Lemaitre [9], were all following the same laws of mathematics, which they assumed that the universe is following. However, it never occurred to these physicists to ask the following questions:

1) Along which dimension evolve the mathematical-logical structures they have constructed and were running in their brains and later on their computers.

2) Is it not possible that this sequence of ideas in their brain while thinking and trying to understand these theories is running along a dimension addition to space-time?

One will not be surprised, however, that any physicist adherent to the mandate of his profession *i.e.* investigating the physical world along the physical spatial-temporal dimensions, using a meter and a clock will revoke these questions, claiming that they belong to the field of 'meta-physics' and endangers the objective approach to science. In other words, once the subjective individual sitting in front of his computer becomes involved in the program being run on this objective machine, the results may be biased. This approach makes further questions redundant, like: The knowledge gained allowing the execution of a program by a computer every few years becoming faster than its previous ancestor, is it not due to progress in information (hardware and software)?

It goes without saying that we suggest that once such advancement is noticeable, it should be measured along a dimension of information.

Speaking about this additional dimension one comes to the 'fifth dimension' (*i.e.* the three spatial directions and the direction of time) introduced in by Kaluza in 1919 [10]. Although Kaluza was able to show that by introducing a fifth dimension then both gravity and electromagnetism can be described from the same underlying framework, and albeit Einstein's interest and preliminary acceptance [10], this 5th dimension was not acceptable among physicists. The main reason, to the present authors' opinion, being that the 5th dimension was a mathematical innovation and the physicists could not 'see it' 'measure it with their rulers' and clocks. No physicist, including Einstein, who pondered about the mystery of mathematics, dared to ask, how is it that a mathematical 5th dimension is capable of unifying the electro-magnetic and the gravity fields. Is it not possible that the 5th dimension is along the dimension of mathematics? Klein [10], a theoretical-physicist, proposed a solution to the physical deficiency in Kaluza's (1919) suggestion by attributing to the mathematical 5th dimension

a spatial character of curling it up into a small enough space to escape ordinary detection.

Curling up dimensions and thus making them "physical", is a plausible solution from the point of view of the physicist, who configures with "four dimensional" universe. Yet his brain is free to roam multi-dimensional universes. In other words what Kaluza showed the physicists is that mathematics is a vehicle enabling to add dimensions and thus expanding beyond Space-Time.

The curling up of space, was also criticized by Hawking [11], in the case of discussing the multi dimensional (ten or twenty-six) space-time universe, suggested by the string theory [12,13]. His question is: Why should some, but not all, of the dimensions be curled up into a small ball? It is beyond the scope of the present article to discuss the answers that Hawking [11] suggests, the common factor of which is that these are all physical space-time dimensions. On the other hand the need to invent the conceptual model of the 'string theory' [12,13] and add a non-observable dimension to the space-time continuum, may be regarded as a hint, what sort of a dimension it should be. More-overformulations accounting for Information as an additional dimension will also address the 11-dimensional M-theory [13] that requires space-time to have eleven dimensions.

3. The Dimension not Accounted by Darwin

While investigating the geology of the Quaternary, *i.e.* the layers deposited during the last two million years, of the Coastal Plain of Israel, the first author of the present article got acquainted with the evolution of the stone tools. These tools show evolution from primitive pebble tools, which were just pebbles etched at one end to become sharp and pointed, to the evolved flint arrow heads. In other words: a body showing a rise of spatial-complexity along the dimension of time. The evolution of this complexity expressed, as we know, also progress in the intelligence of its manufacturers. The question then arises whether this trend of evolution could be explained in the framework of the conventional Darwinian to Neo-Darwinian paradigms, *i.e.* evolution through the process of random mutations filtered by the constraints of the hostile environment, which condemned to disappearance form the stage of existence the less fitted.

Thus, while success in the process of Darwinian selection, *i.e.* survival of the fittest, can be measured according to the number of similar forms of life and their distribution in space-time. The question which bothered the first author was: What about the ability of these forms of life who were able to change the hostile environment. As the ability to change the environment, in the case of the hominids, was a function of the evolution of their intelligence, the following question was: How should the survival of the more intelligent forms be measured? In other

words, who are more successful from the point of view of the Darwinian paradigm, the hominids or the beetles?

The general question which follows is: By which units of measurements the evolution of intelligence can be quantified (when IQ tests are not feasible) and on what dimension can evolution of intelligence be measured?

Generalizing these questions, it can be said that the Darwinian paradigm explained beautifully the evolution of forms, *i.e.* spatial changes and the spread out of the forms along the space dimension as time passed, *i.e.* the time dimension. Now, at a certain point on the dimension of time the hominids branched off from the primates and started to produce tools. These tools became more and more sophisticated as time progressed and as the hominids multiplied and spread over the globe. The dimensions on which this progress on the tree of evolution is described, by conventional measures, are either temporal, *i.e.* the time passed since the first pebble-tools were produced, or spatial, *i.e.* the spatial features of the hominids and their tools, as well as their geographical range. All these data are along space-time dimensions and the question is: Once progress of intelligence became dominant in the evolutionary process should not an additional dimension be added to properly address this progress and evaluate it?

While the questions started with relation to hominids, in due time this question was generalized for the entire bio-world. This happened after coming upon the results of the research carried out by the psychologist Morton E. Bitterman [14], who found that the evolution of intelligent behavior in the bio-world correlates with the place of the species on the evolutionary tree. This meant that the increase of intelligence is parallel with the appearance of new forms of life on the geological timetable. Thus vertebrates are more intelligent than invertebrates, saurians than fishes, mammals than saurians, *etc.* Bitterman [14] investigated the level of intelligence by “the ability to develop a new way of reaction when an entirely new situation comes up.” The question, which came up after reading this conclusion was whether there exists a dimension on which intelligence can be measured, except by the time needed to learn to push a button or find food in a maze? Moreover, once experience is gained and turned into instinctive behavior or abstract knowledge on what dimension can this be presented and evaluated, in addition to the spatial-temporal scales?

The answer to these questions is: The Dimension of Information”. Yet this dimension is not necessary just to describe the evolution of intelligence in the bio-world in general and that of the *Homo sapiens-sapientia* in particular, but is also essential to describe the rise in the complexity of the physical world [15], namely the growth of the complexity of the structures composed of information bits, which means on one hand more information bits, as well as higher levels of organization of algebraic-logical sentences.

The above definition of Information actually describes the coordinates along which the intelligibility of a messages sent through any system of tele-communication can be measured either by telephone, telegraph, e-mail or internet. In this context it is generalizing the term of ‘information’ as defined in the theory developed by Shannon and Weaver [16]. In their theory they suggested that the loss in the intelligibility of a messages sent through any system of communication, namely its increasing distortion by ‘noise’, can be described in a similar formulation to Boltzman-Maxwell’s formulation of the physical Second Law of Thermodynamics. Thus, one cannot avoid the general conclusion that the loss of meaning (decrease in the number of ordered sets of bits of information) of a certain message is through a process similar to that which determines the increase of entropy in a thermodynamic system. The addition of the Dimension of Information measured along the - information/time/space - dimensions, enables this loss of meaning to be expressed in physical-mathematical terms.

Physically-mathematically speaking, an increase in order is equated with increase in complexity and with organization, and is defined as negative entropy. Thus the Boltzman and Planck expression for the entropy of a system $S_{BP} = K \ln W$ (where S_{BP} denotes the statistical entropy of a closed system, K denotes Boltzman’s constant and W denotes the number of independent quantum states) can be regarded as strictly a thermodynamic statement.

On the other hand when the same expression is presented as $S_{BP} = -K \sum_{i=1}^{i=n} P_i \ln P_i$ (where P denotes the probability that the system exists in the microstate i) it becomes a measure of the probability of the system, *i.e.* the measure of our ignorance of the actual quantum state of the system. Such a measurement is also along the dimension of information, or more correctly, the exponential value on the dimension of information. This is proportional to the level of organization of the system. In other words, the more information and the higher is its exponential value so is the system of lower probability, and thus lowers entropy.

Instead of the expression for S_{BP} , a more general form [16] can be $I_n = -K \sum_{i=1}^{i=n} P_i \ln P_i$ where I_n denotes the total information derived from a system where $\ln P_i$ expresses the total contributions of each subset of which the system is composed of, weighted by its probability.

In conclusion, the adding of the Dimension of Information enables to understand better the physical observation made by Shannon that the noise (reciprocal to the quantity of Information) in a communication system becomes greater the longer are the dimensions of space-time.

4. A Few Words of Encouragement from Physicists

Although we foresee the difficulties facing the undertaking of building a bridge between the space-time measurable world of the physicists and the one containing also the dimension of Information, still a blink of hope exists. This emerges from the fact that quantum physicists are becoming aware of the need to introduce the observer and an additional dimension, similar to that of Information, to their conceptual model in order to explain “bizarre” phenomena, in the micro-world. This can be seen in the following citations, which we believe can also be regarded as a support to the space-time-Information conceptual model:

1) “We have already considered with disfavour the possibility of the universe having been planned by a biologist or an engineer; from the intrinsic evidence of his creation, the Great Architect of the Universe now begins to appear as a pure mathematician.” [17].

2) “No, it’s a new kind of wave which I call ‘active information’. The notion of active information is already familiar to us from computers. Also, if I tell you something and you do something, that’s obviously active information. If I shouted ‘fire’, everybody would move, so we know that in living intelligent systems, and in computers, active information is a useful concept. Now what I am proposing is that matter in general is not so different.” [1].

5. Formulation

As already mentioned physicists and mathematicians strive to formulate the multi-dimensional continuum in the framework of the conceptual model of the multi dimensional string theory. The following formulation is more modest and is exercised in the framework of the Hamiltonian extended stationary principle in terms of Space, Time and an additional degree of freedom, all as independent stationary variables. It is suggested that this degree of freedom may be regarded as an information state.

Thus, let us denote the information state by I , we postulate that similar to Hamilton’s extended stationary principle, there exists a functional $\chi(f)$ being a function of its integrant f between an initial information level I_0 and that of a final one I_f , in the form

$$\chi = \int_{I_0}^{I_f} \int_{\Omega} \int_{\phi} f(I, t, \Omega, \phi, \phi_t, \phi_x, \phi_{\Omega}) dI dt d\Omega \quad (1)$$

Note that in (1) we consider time $t \in [t_0, t_f]$ and space Ω , to be independent of information. Accordingly, $\phi = \phi(I, t, \Omega)$ denotes the dependent function with

its: information derivative $\phi_I (\equiv \partial\phi/\partial I)$, time derivative $\phi_t (\equiv \partial\phi/\partial t)$ and spatial derivative $\phi_{\Omega} (\equiv \partial\phi/\partial\Omega)$.

The spatial domain is assumed to be fixed. The necessary and sufficient condition to obtain minimum for χ , is that the dependent function $\phi(I, t, \Omega)$ satisfy Euler’s equation, namely

$$\frac{\partial}{\partial I} \frac{\partial f}{\partial \phi_I} + \frac{\partial}{\partial t} \frac{\partial f}{\partial \phi_t} + \frac{\partial}{\partial \Omega} \frac{\partial f}{\partial \phi_{\Omega}} - \frac{\partial f}{\partial \phi} = 0 \quad (2)$$

In what follows, we will investigate the implementation of the theory to a 1D problem.

6. Example

Let us consider a thin rod under vibrations along $x \in [0, \ell]$ the longitudinal direction with $U = U(I, t, x)$ as its wave amplitude.

For this proposed example, we relate U with ϕ , and choose f in the form

$$f = \frac{\mathcal{K}_I}{2} \left(\frac{\partial U}{\partial I} \right)^2 + \frac{\mathcal{K}_t}{2} \left(\frac{\partial U}{\partial t} \right)^2 + \frac{\mathcal{K}_x}{2} \left(\frac{\partial U}{\partial x} \right)^2 + FU \quad (3)$$

where $F = F(I, t, x)$ denotes the specific external driving force over a unit area, $\mathcal{K}_I, \mathcal{K}_t$ and $\mathcal{K}_x$ denote coefficients associated with the second partial derivative of U .

For example, let

$$\left. \begin{aligned} \mathcal{K}_I &= 1 \\ \mathcal{K}_t &= -1/\mathcal{G}^2 \\ \mathcal{K}_x &= 1/V^2 \end{aligned} \right\} \quad (4)$$

where $\mathcal{G}$ denotes the travelling speed of the information wave in the I -vs- t plan and $V \left(= \frac{\mathcal{G}}{C} \right)$ accounts for the ratio $\mathcal{G}$ and C the speed of a traveling wave in the x -vs- t plan. In view of (1), (3) and (4) the functional χ to be varied, will be in the form

$$\begin{aligned} \chi = \int_{I_0}^{I_f} \int_{t_0}^{t_f} \int_0^{\ell} & \left[\frac{(\partial U / \partial I)^2}{2} + \frac{(\partial U / \partial x)^2}{2V^2} \right. \\ & \left. - \frac{(\partial U / \partial t)^2}{2\mathcal{G}^2} - FU dI dt dx \right] \\ & + \int_{t_0}^{t_f} \int_0^{\ell} (QU + ZU^2/2) dt dx \end{aligned} \quad (5)$$

where Q and Z denote, respectively, generalized source and conductance terms on the boundary envelope in space and time.

Upon varying χ , we describe an extremum process to define the U function in I, t, x terms that will minimize the χ functional. Assuming that the bound-

ary conditions in (5) are fulfilled, we follow (2), Euler's equation, and obtain a modified wave equation in the form,

$$\frac{\partial^2 U}{\partial I^2} + \frac{1}{V^2} \frac{\partial^2 U}{\partial x^2} - \frac{1}{g^2} \frac{\partial^2 U}{\partial t^2} + F = 0 \quad (6)$$

In what follows, we will consider different dominant forms that may be obtained from (6). To do this we will refer to nondimensional analysis.

6.1. Dimensionless Analysis

Let us denote the characteristic value of a property by $(\cdot)_C$. We choose the characteristic value so as to allow the dimensionless terms, $[\cdot]^*$, be of a unit order. This will allow the comparison between the scalar factors multiplying the dimensionless terms.

The dimensionless form of (6) reads

$$\left[\frac{\partial^2 U}{\partial I^2} \right]^* + \frac{I_C C}{g L_C} \left[\frac{\partial^2 U}{\partial x^2} \right]^* - \frac{I_C}{g \Delta t_C} \left[\frac{\partial^2 U}{\partial t^2} \right]^* + \frac{I_C^2 F_C}{U_C} [F]^* = 0 \quad (7)$$

where L_C denotes a characteristic spatial increment and Δt_C denotes a characteristic time step. In view of (7) and the relative order of magnitude of its scalar numbers, we may obtain different dominant forms. To investigate these, let us also define:

$$q \equiv \frac{L_C}{\Delta t_C} \quad (8)$$

as a characteristic velocity,

$$\alpha \equiv \frac{U_C}{\Delta t_C} \quad (9)$$

as the characteristic amplitude rate,

$$\rho_I \equiv \frac{I_C}{L_C} \quad (10)$$

as the information density,

$$g_C \equiv \frac{I_C}{\Delta t_C} \quad (11)$$

as the characteristic traveling velocity of information, and

$$\epsilon_C \equiv \frac{U_C}{L_C} \quad (12)$$

as the characteristic aspect ratio.

6.2. Traveling Wave in Space and Time

Consider the possibility that,

$$\left. \begin{aligned} \frac{\rho_I}{V} &\gg 1 \\ \frac{g_C}{g} &\gg 1 \\ \frac{\rho_I}{\epsilon_C} I_C F_C &\gg 1 \end{aligned} \right\} \quad (13)$$

In view of (13), the approximate form of (6) conforms to a traveling wave in the form,

$$\frac{\partial^2 U}{\partial x^2} - \frac{1}{C^2} \frac{\partial^2 U}{\partial t^2} + V^2 F = 0 \quad (14)$$

Note that the driving force is amplified by the square of the ratio between traveling velocity of information and the velocity of the aforementioned traveling wave.

6.2.1. Traveling wave in information and time

Consider the possibility that,

$$\left. \begin{aligned} \frac{C}{q} &\ll 1 \\ \frac{g F_C}{\alpha} &\ll 1 \end{aligned} \right\} \quad (15)$$

In view of (13), the approximate form of (6) conforms to a traveling wave equation in the I -vs- t plane

$$\frac{\partial^2 U}{\partial I^2} - \frac{1}{g^2} \frac{\partial^2 U}{\partial t^2} + F = 0 \quad (16)$$

It is worthy to stress that (16) occurs when the characteristic velocity is of greater magnitude than that of the traveling wave velocity and the ratio between the amplitude rate and the driving force be also of greater magnitude than the traveling velocity of the considered wave.

6.2.2. Stagnation in Space and Information

Consider the possibility that,

$$\left. \begin{aligned} \frac{C}{q} &\gg 1 \\ \frac{g F_C}{\alpha} &\gg 1 \end{aligned} \right\} \quad (17)$$

By virtue of (17) the dominant form of (6) will become,

$$\frac{\partial^2 U}{\partial I^2} + \frac{1}{V^2} \frac{\partial^2 U}{\partial x^2} + F = 0 \quad (18)$$

The amplitude surface which is obtained by the solution of (18), may exhibit craters and/or peaks, depending on the driving force which, as a function also of information, acts as a sink/source term.

7. Questions and Conclusions

The mathematician Kaluza in 1919 suggested that if the world was five dimensional (3 spatial + time + 5th) then electromagnetism and gravitation can be described by a 5 dimensional geometry. The physicist Klein [10] ex-

plained the invisibility of this extra dimension, by adding to this geometric description, the principle of “perspective” namely that this dimension which we do not observe is because it is rolled up to a tiny size. Klein [10] computed the circumference of this tiny corpuscle to be about twenty powers of ten smaller than an atomic nucleus. The revolution of adding dimensions continued when the nuclear forces were discovered and the question rose whether to achieve a general theory these forces should not be incorporated into a Kaluza-Klein’s [10] theory by this reducing all the forces of nature to pure geometry? This brought to the multi-dimensional string theory [12,13] which brought to the formulation of the 11-dimensional M-theory [13]. Yet in the various articles discussing the physics of a multi-dimensional continuum, one can not find any suggestion of a dimension which is not spatial-temporal.

Focusing just on the period from Newton to Einstein to Kaluza-Klein [10] theories, every physicist will probably agree that there was evolution in the complexity of the physical conceptual model. Yet, the question, which should be asked, is: What about the evolution of human thinking, which became more complex since it had to address additional concepts. On what dimension did human thought evolve? Was it just on space-time?

The evolution of the biological sciences and especially that of the processes of heredity and genetics has even made these questions more relevant. Does the information contained in the DNA molecule described just by its space-time structure? Or just by its chemical configuration? Moreover, even if these descriptions are sufficient to pin-point a certain congenital trait, do they describe the past history of the evolution of these traits? These questions are now hotly debated, between the proponents of intelligent design, creationism, and Darwinism [18].

Generalizing these questions will be: Isn’t it possible that there exists a non spatial-temporal dimension, which the physicists and biologists ignore because it is not observed, yet science on the whole is built and is continuously progressing along it? Our suggestion is: Indeed this is the Dimension of Information.

8. References

- [1] P. C. W. Davies and J. R. Brown “The Ghost in the Atom Interviews with John Bell, David Bohm and Basil Hile,” Cambridge University Press, Cambridge, 1986, p. 128
- [2] A. S. Issar, “Evolution of Intelligence in the Bio-World. Divine Action and Natural Selection: Questions of Science and Faith in Biological Evolution,” In: R. Gordon and J. Seckbach, Eds., World Scientific, Singapore, 2009.
- [3] A. Einstein, “Geometry and Experience: Lecture before the Prussian Academy of Sciences on January 27, 1921,” In: A. Einstein, Ed., *Ideas and Opinions*, Crown Publishers, New York, 1954.
- [4] A. Einstein, “On the Method of Theoretical Physics,” In: A. Einstein, Ed., *Ideas and Opinions*, Crown Publishers, New York, 1954, pp. 270-276.
- [5] A. Einstein, “Physics and Reality,” *Franklin Institute Journal* Ditto, 1936, pp. 290-322.
- [6] E. Wigner, “The Unreasonable Effectiveness of Mathematics in the Natural Sciences,” *Communications on Pure and Applied Mathematics*, Vol. 13, No. 1, 1960, pp. 1-14.
- [7] A. Friedman, “English Translation in: On the Curvature of Space,” *General Relativity and Gravitation*, Springer Netherlands, Vol. 31, 1999. Original article in: T. Kaluza, “Über die Krümmung des Raumes,” *Zeitschrift für Physik*, Vol. 10, No. 1, 1999, pp. 377-386.
- [8] P. Kerszberg, “The Invented Universe: The Einstein-De Sitter Controversy 1916 and the Rise of Relativistic Cosmology,” Oxford University Press, 1989.
- [9] J. Farrell, “The Day without Yesterday: Lemaitre, Einstein and the Birth of New Cosmology,” Thunder’s Mouth Press, New York, 2005.
- [10] P. S. Wesson, “Five-Dimensional Physics: Classical and Quantum Consequences of Kaluza-Klein Cosmology,” World Scientific Publishing, Singapore, 2006.
- [11] S. W. Hawking, “A Brief History of Time—From the Big Bang to Black Holes,” A Bantam Book, 1988.
- [12] J. Polchinski, “String Theory,” Cambridge University Press, Cambridge, 1998.
- [13] M. Kaku, “Strings, Conformal Fields, and M-Theory,” Springer-Verlag, New York, 1999.
- [14] M. E. Bitterman, “Phyletic Differences in Learning,” *American Psychologist*, Vol. 20, 1965, pp. 396-410.
- [15] S. A. Kauffman, “The Origins of Order,” *Self Organization and Selection in Evolution*, Oxford University Press, New York, Oxford, 1993.
- [16] W. Weaver and C. E. Shannon, “The Mathematical Theory of Communication,” University of Illinois Press, Urbana, 1963.
- [17] H. J. Jeans, “The Mysterious Universe,” Cambridge University Press, Cambridge, 1930, p. 134.
- [18] A. S. Issar, “Evolution of Intelligence in the Bio-World Divine Action and Natural Selection: Questions of Science and Faith in Biological Evolution,” In: R. Gordon and J. Seckbach, Eds., World Scientific, Singapore, 2009.

Instability of Thermally Conducting Self-Gravitating Systems

Shaista Shaikh, Aiyub Khan

Department of Mathematics and Statistics, Jai Narain Vyas University, Jodhpur, India

E-mail: shaikh_shaista@yahoo.com

Received February 23, 2010; revised March 25, 2010; accepted March 25, 2010

Abstract

The gravitational instability of a thermally conducting self-gravitating system permeated by a uniform and oblique magnetic field has been analyzed in the framework of Tsallis' nonextensive theory for possible modifications in the Jeans' instability criterion. It is concluded that the instability criterion is indeed modified into one that depends explicitly on the nonextensive parameter. The influence of thermal conductivity on the system stability is also examined.

Keywords: Jeans' Criterion, Nonextensivity, Thermally Conducting, Self-Gravitating

1. Introduction

In any subject of astrophysics and cosmology, many-body gravitating systems play an essential role. Globular clusters and elliptical galaxies, which are recognized as self-gravitating stellar systems, are typical examples. Hence, the study of stability of self-gravitating systems becomes very essential.

The condition of gravitational instability of self-gravitating systems is determined by the Jeans' criterion put forward by James Jeans [1] in 1902. In terms of wavenumber, the criterion reads: "An infinite homogenous self-gravitating atmosphere is unstable for all wavenumbers k less than the Jeans' wavenumber $k_J = \sqrt{4\pi G \rho_0} / v_s$, where ρ_0 is the density, G is the gravitational constant, $v_s = \sqrt{k_B T / m}$ is the speed of sound, k_B is the Boltzmann's constant, T is the physical temperature and m is the mass of the particle."

The Jeans' problem has been extensively studied under varying assumptions. A comprehensive account of these studies has been given by Chandrasekhar [2] in his monograph on hydrodynamic and hydromagnetic stability. The Hall Effect on plasma stability has been analyzed by several researchers (Ariel [3], Bhowmik [4] and Ali & Bhatia [5]) leading to the conclusion that Hall Currents are destabilizing in nature. Vyas & Chhajlani [6], Sharma & Chand [7], Khan & Bhatia [8] have investigated the influence of permeability of porous medium on plasma instability due of the impor-

tance of such studies in geology and heavy oil recovery. In view of the role played by thermally conducting fluids in various astrophysical and geophysical phenomena as well as industrial and engineering processes, the stability of such fluids has been the center of numerous analyses (Kumar [9], Chhajlani and Vaghela [10], Mehta and Bhatia [11]).

In all these investigations, the Boltzmann-Gibbs statistical mechanics have been employed to study the thermodynamics of the system. However, the physical restrictions of this formalism have been recently pointed out in different literatures based on various studies involving long-range interacting systems (Padmanabhan [12], Taruya & Sakagami [13]). As an alternative, the nonextensive theory proposed by Tsallis [14] is gaining considerable attention.

The new framework for thermodynamics based on Tsallis' nonextensive theory has been applied extensively to deal with a variety of interesting problems to which the standard B-G statistical mechanics cannot be applied. Examples include the study of waves and instability phenomena, such as the plasma oscillations [15,16], the relativistic Langmuir waves [17], the linear or nonlinear Landau damping in plasmas [18] and dark matter and gas density profiles observed in galaxies and clusters [19]. The study of self-gravitating stellar systems has been one of the most interesting applications of Tsallis' nonextensive thermodynamics [20-24].

In this paper, we analyze the stability of a thermally conducting self-gravitating system embedded by a uniform and oblique magnetic field for possible modification

in the Jeans' instability criterion due to the presence of nonextensive effects. The influence of thermal conductivity on the growth rate of the system is also examined.

2. Nonextensive Theory

The physical restrictions of the Boltzmann-Gibbs statistical mechanics have stressed the need for a possible generalization of this formalism. Such a generalization was proposed by Tsallis in 1988 (known as "Tsallis' Statistics") by constructing a new form of entropy written as

$$S_q = k_B \left(\sum_i p_i^q - 1 \right) / (1 - q) \quad (1)$$

where p_i is the probability of the i th microstate and q is a parameter quantifying the degree of nonextensivity of the system. In the limit $q \rightarrow 1$, the celebrated B-G extensive formula, namely

$$S = -k_B \sum_i p_i \ln p_i \quad (2)$$

is recovered.

Various literatures involving the thermo-statistical analysis of many astrophysical systems and processes (Plastino & Plastino [25], Abe [26]) make it clear that Tsallis' statistics may be the appropriate theory for description of astrophysical systems with long-range interaction of gravitation.

The nonextensivity in the Jeans' problem is introduced through the equation of state of an ideal gas. In the framework of nonextensive theory, the q -nonextensive velocity distribution function for free particles is given by

$$f(v) = B_q \left[1 - (1 - q) \frac{mv^2}{2k_B T} \right]^{\frac{1}{1-q}} \quad (3)$$

where B_q is a normalization constant and the other variables have their usual meanings.

If N denotes the particle number density, pressure is defined by $P = \frac{1}{3} Nm \langle v^2 \rangle$ with $\langle v^2 \rangle$ the mean square velocity of the particle defined in Tsallis' statistics by

$$\langle v^2 \rangle = \frac{\int v^2 [f(v)]^q d^3v}{\int [f(v)]^q d^3v} \quad (4)$$

In 2003, Silva & Alcaniz [27] calculated the q expectation value for the square velocity of the particle as

$$\langle v^2 \rangle_q = \frac{6}{5-3q} \frac{k_B T}{m}, (0 < q < 5/3) \quad (5)$$

Clearly, the standard mean square velocity $\langle v^2 \rangle = 3k_B T/m$ is perfectly recovered when $q \rightarrow 1$. Thus, the equation of state of an ideal gas in the nonextensive kinetic theory is obtained as

$$P_q = \frac{1}{3} Nm \langle v^2 \rangle_q = \frac{2}{5-3q} \frac{k_B \rho T}{m} \quad (6)$$

where we have written $N = \rho/m$. Note that the standard equation of state is correctly recovered in the limit $q \rightarrow 1$. The above equation can also be written in the form $P_q = Nk_B T_q$, with the physical temperature T_q , a variable that depends on the nonextensive parameter q as $T_q = \frac{2T}{5-3q}$. Consequently, the speed of sound can be written as

$$S_q = \sqrt{\frac{k_B T_q}{m}} = S \sqrt{\frac{2}{5-3q}} \quad (7)$$

significantly different from the one in B-G statistics ($q = 1$, $T_q = T$). We shall use this modified form while writing the perturbation equations of the self-gravitating system considered in this paper.

3. Perturbation Equations

Following standard lines, we write the linearized perturbation equations characterizing the flow of a thermally conducting self-gravitating fluid embedded by a uniform and oblique magnetic field denoted by $\vec{H} = (H_x, 0, H_z)$.

$$\frac{\partial \rho_1}{\partial t} + \rho \nabla \cdot \vec{u} = 0 \quad (8)$$

$$\frac{\partial \vec{u}}{\partial t} = -\frac{1}{\rho} \nabla p_1 - \nabla \phi_1 + (\nabla \times \vec{h}_1) \times \vec{H} \quad (9)$$

$$\frac{\partial \vec{h}_1}{\partial t} = \nabla \times (\vec{u} \times \vec{H}) \quad (10)$$

$$\nabla^2 \phi_1 = -G \rho_1 \quad (11)$$

$$\frac{\partial}{\partial t} (p_1 - \gamma S_q^2 \rho_1) = \chi \nabla^2 (p_1 - S_q^2 \rho_1) \quad (12)$$

where $\vec{u}(u, v, w)$, $\vec{h}_1(h_x, h_y, h_z)$, ρ_1 , p_1 and ϕ_1 are respectively the perturbations in velocity $\vec{u}$, magnetic field $\vec{H}$, density ρ , pressure p and gravitational potential ϕ , G is the gravitational constant, γ denotes an adiabatic exponent and χ is the coefficient of thermal conductivity.

We seek the solutions of the Equations (8)-(12) whose dependence on the space coordinates (x, y, z) and time t is of the form

$$\exp(ik \sin \theta x + ik \cos \theta z + i\omega t) \quad (13)$$

where $\vec{k} = (k \sin \theta, 0, k \cos \theta)$ is the wavenumber of perturbation making angle θ with the x -axis and ω is the frequency of perturbation. Eliminating ρ_1 , ϕ_1 & p_1 from the above equations, we get six equations govern-

ing the perturbation of velocity and magnetic field which can be written in the matrix form

$$[A][B] = 0 \quad (14)$$

where $[A]$ is a sixth order square matrix and $[B]$ is a single column matrix in which the elements are $(u, v, w, h_x, h_y, h_z)^T$. The elements of $[A]$ are

$$\begin{aligned} A_{11} &= i\omega - iD \sin^2 \theta, \\ A_{12} &= 0, A_{13} = (-iD) \sin \theta \cos \theta, \\ A_{14} &= -\frac{H}{\rho} ik \cos \theta, A_{15} = 0, A_{16} = \frac{H}{\rho} ik \sin \theta, \\ A_{21} &= 0, A_{22} = i\omega, A_{23} = 0, \\ A_{24} &= 0, A_{25} = -\frac{ik}{\rho} (H_x \sin \theta + H_z \cos \theta), A_{26} = 0, \\ A_{31} &= (-iD) \sin \theta \cos \theta, A_{32} = 0, A_{33} = i\omega - iD \cos^2 \theta, \\ A_{34} &= \frac{H_x}{\rho} ik \cos \theta, A_{35} = 0, A_{36} = -\frac{H_x}{\rho} ik \sin \theta, \\ A_{41} &= -H_z ik \cos \theta, A_{42} = 0, A_{43} = H_x ik \cos \theta, \\ A_{44} &= i\omega, A_{45} = 0, A_{46} = 0, \\ A_{51} &= 0, A_{52} = -ik(H_x \sin \theta + H_z \cos \theta), A_{53} = 0, \\ A_{54} &= 0, A_{55} = i\omega, A_{56} = 0, \\ A_{61} &= H_z ik \sin \theta, A_{62} = 0, A_{63} = -H_x ik \sin \theta, \\ A_{64} &= 0, A_{65} = 0, A_{66} = i\omega \end{aligned} \quad (15)$$

where we have written

$$D = \frac{S_q^2 k^2 \gamma (i\omega + \chi k^2) - G\rho (i\omega + \gamma \chi k^2)}{\omega (i\omega + \gamma \chi k^2)} \quad (16)$$

4. Dispersion Relation

The vanishing of $|A|$ gives the dispersion relation as the product of three factors:

$$(i\omega) \cdot \left\{ (i\omega)^2 + k^2 V^2 \right\} \cdot \left\{ (i\omega)^2 (i\omega - iD) + (i\omega) \frac{(H_x^2 + H_z^2)}{\rho} k^2 - iD k^2 V^2 \right\} \quad (17)$$

By writing $\omega = in$ and using the value of D in the third factor of Equation (17), we obtain the resulting dispersion relation, which is an equation of degree five in n of the form

$$n^5 - c_4 n^4 + c_3 n^3 - c_2 n^2 + c_1 n - c_0 = 0 \quad (18)$$

with the coefficients c_4 to c_0 given by

$$\begin{aligned} c_4 &= \gamma \chi k^2 \\ c_3 &= S_q^2 k^2 \gamma - G\rho - \frac{(H_x^2 + H_z^2)}{\rho} k^2 \\ c_2 &= \gamma \chi k^2 \left(S_q^2 k^2 - G\rho - \frac{(H_x^2 + H_z^2)}{\rho} k^2 \right) \\ c_1 &= k^2 V^2 (S_q^2 k^2 \gamma - G\rho) \\ c_0 &= \gamma \chi k^4 V^2 (S_q^2 k^2 - G\rho) \end{aligned} \quad (19)$$

where we have taken

$$V^2 = \frac{(H_x \sin \theta + H_z \cos \theta)^2}{\rho}$$

5. Analysis of Dispersion Relation

In the study of Jeans' instability, the boundary between stable and unstable solutions is achieved by setting $n=0$ in the dispersion relation (Equation (17)). The result is a family of q -parameterized critical wave-numbers k_q given by

$$k_q = \frac{\sqrt{G\rho}}{S_q} = k_J \sqrt{\frac{5-3q}{2}} \quad (20)$$

Note that the standard values as obtained from fluid theory are recovered only if $q=1$. We have, thus, obtained a modified form of Jeans' Criterion which shall now be analyzed for different values of q . As we know, the value of nonextensive parameter q lies between 0 and 5/3. Hence, we will analyze the Jeans' criterion for different values of q in this range. Let us calculate the critical wave numbers for $q=1$, $q=0.3$, i.e. $0 < q < 1$ and $q=1.6$, i.e. $1 < q < 5/3$. For these calculations, we take numerical values for conditions prevailing in magnetic collapsing clouds:

$$\rho = 1.7 \times 10^{-21} \text{ kg m}^{-3},$$

$$G = 6.658 \times 10^{-11} (\text{kg})^{-1} \text{ m}^3 \text{ s}^{-2},$$

$$S^2 = 2.5 \times 10^8 \text{ m}^2 \text{ s}^{-2},$$

$$V^2 = 5 \times 10^8 \text{ m}^2 \text{ s}^{-2}. \quad (21)$$

The following critical wave numbers are obtained through numerical calculations

$$k_{q=1.0} = 2.12 \times 10^{-20} \text{ m}^{-1} \quad (22)$$

$$k_{q=0.3} = 3.04 \times 10^{-20} \text{ m}^{-1} \quad (23)$$

$$k_{q=1.6} = 0.67 \times 10^{-20} \text{ m}^{-1} \quad (24)$$

Let us discuss the Jeans' Criterion in light of the above.

a) When $q=1$, the Jeans' Criterion as obtained through fluid theory is recovered perfectly. The system is unstable for wavenumbers $k < k_{q=1.0}$ and stable for wavenumbers $k > k_{q=1.0}$.

b) When $q=0.3$ i.e. $0 < q < 1$, the system is unstable for $k < k_{q=0.3}$ and stable for wave numbers $k > k_{q=0.3}$. Hence, the Jeans' Criterion is modified as $k < k_{q=0.3}$ and $k > k_{q=1.0}$ i.e. the system may now be unstable even for the wave numbers greater than $k_{q=1.0}$ provided that they are less than $k_{q=0.3}$.

c) When $q=1.6$ i.e. $1 < q < 5/3$, the system is unstable for $k < k_{q=1.6}$ and stable for wave numbers $k > k_{q=1.6}$. Hence, the Jeans' Criterion is modified as $k > k_{q=1.6} < k_{q=1.0}$ i.e. the system which was believed to be unstable for wave numbers less than $k_{q=1.0}$ according to fluid theory, may now be stable for wave numbers less than $k_{q=1.0}$ but greater than $k_{q=1.6}$.

We have demonstrated the effect of nonextensive parameter q on the system stability by plotting wavenumber against growth rate for the values of q mentioned above. The result is as shown in **Figure 1**. The same conclusions, as outlined in a)-c), are drawn by studying the plot.

In order to gauge the influence of thermal conductivity on the growth rate of the system, we have plotted wavenumber against growth rate for varying values of thermal conductivity in **Figure 2** for a fixed value of nonextensive parameter $q = 1$. We notice that as the value

of χ (taken as X in the figure) increases, the value of growth rate initially increases in the unstable region. However, as the system moves from unstable to stable region, the growth rate decreases with increase in thermal conductivity for a fixed wave number. Hence, we conclude that thermal conductivity has a mixed, but predominantly stabilizing, influence on the system stability.

6. Results

The Jeans' gravitational instability of a thermally conducting self-gravitating fluid permeated by a uniform and oblique magnetic field has been analyzed in the framework of nonextensive theory. It is concluded that thermal conductivity has a predominantly stabilizing influence on the growth rate of the system. The presence of nonextensive effects modifies the standard Jeans' Criterion into one that depends explicitly on the nonextensive parameter q . However, in spite of this modification, the basic instability criterion is maintained: perturbations with $k > k_q$ do not grow while instability takes place for $k < k_q$.

7. Concluding Remarks

We have studied the stability of a large-scale self-gravitating system in the framework of Tsallis' Nonextensive Statistical Mechanics (NSM). Our approach differs from the kinetic theoretical approach based on the Vlasov equation, where the evolution of the system is described by perturbing the equilibrium Max-wellian velocity distribution function. We have, instead, considered the non-Maxwellian (power-law) equilibrium distribution function (Equation (3)) which is a nonextensive generalization of the standard distribution function. Considerable

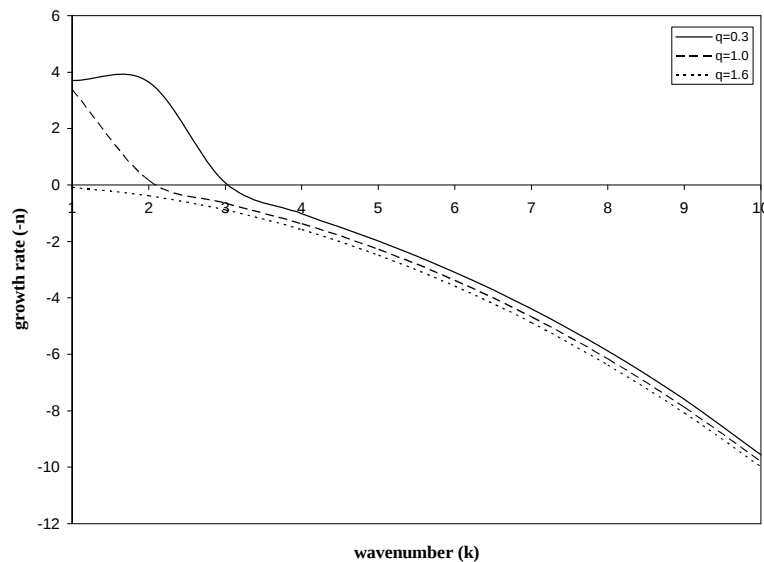


Figure 1. Graph of wavenumber vs. growth rate of a thermally conducting fluid for varying values of nonextensive parameter.

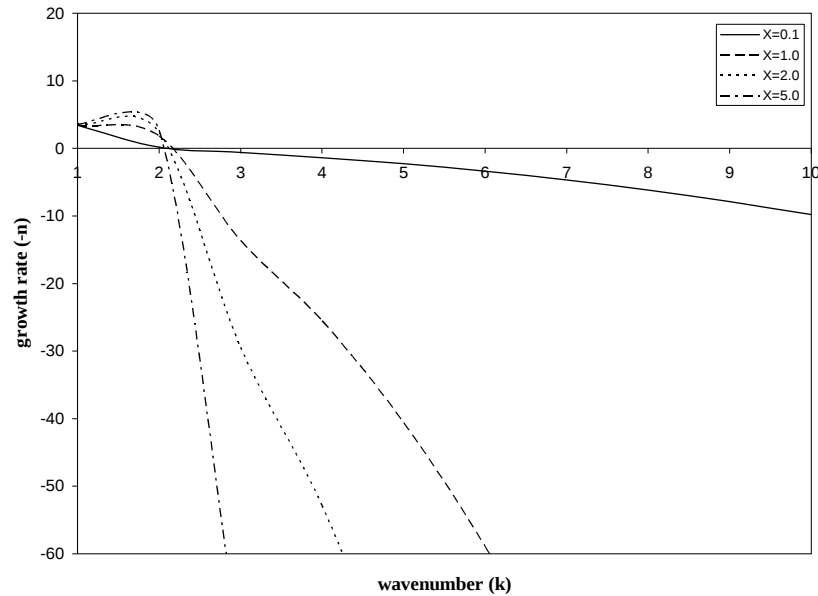


Figure 2. Graph of wavenumber vs. growth rate of a thermally conducting fluid for varying values of coefficient of thermal conductivity.

amount of experimental evidence supports the employment of such a distribution (e.g. Liu *et al.* [28]), clearly indicating that the standard Maxwellian velocity distribution might provide only a very crude description of the velocity distribution for a self-gravitating gas, or generally for any system endowed with long range interactions. Infact, a well determined criterion for gravitational instability is not a privilege of the exponential velocity distribution function, but is shared by an entire family of power-law functions (named q -exponentials) which includes the standard Jeans' result for the Maxwellian distribution as a limiting case ($q = 1$). This being said, it must also be stressed that all nonextensive systems need not require the Tsallis' statistics to understand their behaviors (Cohen [29]). In the light of present understanding, it is still unclear which class of nonextensive systems requires Tsallis' statistics for its statistical description, mainly due to the fact that the physical meaning of the nonextensive parameter q is yet to be settled. Although some progress in this regard is being made (Du [22,23]), the Nonextensive Statistical Mechanics remains open to further verification and deeper understanding.

Further Reading: Interested readers may refer to similar works by the authors [30,31,32] on the instability of thermally conducting self-gravitating systems in the framework of nonextensive statistics.

Acknowledgements: This work was carried out as part of a major research project (F. No. 36-103/2008 (SR)) awarded by University Grants Commission (UGC), India to the one of the authors, Aiyub Khan. The financial assistance from UGC is gratefully acknowledged. The authors are also grateful to the reviewer for useful suggestions, which have helped in improving the overall presentation and quality of the paper.

8. References

- [1] J. H. Jeans, "The Stability of Spherical Nebulae," *Philosophical Transactions of the Royal Society of London*, A 199, 1902, pp. 1-53.
- [2] S. Chandrasekhar, "Hydrodynamics & Hydromagnetic Stability," Clarendon Press, Oxford, 1961.
- [3] P. D. Ariel, "The Character of Equilibrium of an Inviscid Infinitely Conducting Fluid of Variable Density in the Presence of a Horizontal Magnetic Field with Hall-Current," *Journal of Plasma Physics*, Vol. 4, 1970, pp. 523-530.
- [4] G. Bhowmik, "Rayleigh Taylor Instability of a Viscous Hall Plasma with Magnetic Field," *Journal of Plasma Physics*, Vol. 7, 1972, pp. 117-132.
- [5] A. Ali and P. K. Bhatia, "Magnetic Resistivity and Hall Currents Effects on the Stability of a Self-Gravitating Plasma of Varying Density in Variable Magnetic Field," *Astrophysics and Space Science*, Vol. 201, 1993, pp. 15-27.
- [6] M. K. Vyas and R. K. Chhajlani, "Gravitational Instability of a thermally-conducting Plasma Flowing through a Porous Medium in the Presence of Suspended Particles," *Astrophysics and Space Science*, Vol. 149, 1988, pp. 323-342.
- [7] R. C. Sharma and T. Chand, "Gravitational Instability for Some Astrophysical Systems," *Astrophysics and Space Science*, Vol. 183, 1991, pp. 215-224.
- [8] A. Khan and P. K. Bhatia, "Stability of Two Superposed Viscoelastic Fluid in a Horizontal Magnetic Field," *Indian Journal of Pure and Applied Mathematics*, Vol. 32, 2001, pp. 98-108.
- [9] S. S. Kumar, "On Gravitational Instability, III," *Publications of the Astronomical Society of Japan*, Vol. 13, 1961,

- pp. 121-124.
- [10] R. H. Chhajlani and D. S. Vaghela, "Magnetogravitational Stability of Self-Gravitating Plasma with Thermal Conduction and Finite Larmor Radius through Porous Medium," *Astrophysics and Space Science*, Vol. 134, 1987, pp. 301-315.
 - [11] V. Mehta and P. K. Bhatia, "Gravitational Instability of a Rotating Viscous Thermally Conducting plasma," *Contributions to Plasma Physics*, Vol. 29, 1989, pp. 617-626.
 - [12] T. Padmanabhan, "Statistical Mechanics of Gravitating Systems," *Physics Reports*, Vol. 188, 1990, pp. 285-362.
 - [13] A. Taruya and M. Sakagami, "Thermodynamic Properties of Stellar Polytrope," *Physica A*, Vol. 318, 2003, pp. 387-413.
 - [14] C. Tsallis, "Possible Generalization of Boltzmann-Gibbs Statistics," *Journal of Statistical Physics*, Vol. 52, 1988, pp. 479-487.
 - [15] R. Silva, J. A. Alcaniz and J. A. S. Lima, "Constraining Nonextensive Statistics with Plasma Oscillation Data," *Physica A*, Vol. 356, 2005, pp. 509-516.
 - [16] S. Shaikh, A. Khan and P. K. Bhatia, "Jeans' Gravitational Instability of a Thermally Conducting, Unbounded, Partially Ionized Plasma," *Zeitschrift für Naturforschung*, Vol. 61, 2006, pp. 275-280.
 - [17] V. Munoz, "A Nonextensive Statistics Approach for Langmuir Waves in Relativistic Plasmas," *Nonlinear Processes in Geophysics*, Vol. 13, 2006, pp. 237-241.
 - [18] F. Valentini, "Nonlinear Landau Damping in Nonextensive Statistics," *Physics of Plasmas*, Vol. 12, 2005, pp. 1-7.
 - [19] M. P. Leubner, "Nonextensive Theory of Dark Matter and Gas Density Profiles," *The Astrophysical Journal*, Vol. 632, 2005, pp. L1-L4.
 - [20] J. A. S. Lima, R. Silva and J. Santos, "Jeans' Gravitational Instability and Nonextensive Kinetic Theory," *Astronomy and Astrophysics*, Vol. 396, 2002, pp. 309-313.
 - [21] J. L. Du, "Jeans' Criterion and Nonextensive Velocity Distribution Function in Kinetic Theory," *Physics Letters A*, Vol. 320, 2004, pp. 347-351.
 - [22] J. L. Du, "Nonextensivity in Nonequilibrium Plasma Systems with Coulombian Long-Range Interactions," *Physics Letters A*, Vol. 329, 2004, pp. 262-267.
 - [23] J. L. Du, "What Does the Nonextensive Parameter Stand for in Self-Gravitating Systems?" *Astrophysics and Space Science*, Vol. 305, 2006, pp. 247-251.
 - [24] J. L. Du, "Nonextensivity and the Power-Law Distributions for the Systems with Self-Gravitating Long-Range Interactions," *Astrophysics and Space Science*, Vol. 312, 2007, pp. 47-55.
 - [25] A. Plastino and A. R. Plastino, "Stellar Polytropes and Tsallis' Entropy," *Physics Letters A*, Vol. 174, 1993, pp. 384-386.
 - [26] S. Abe, "Thermodynamic Limit of a Classical gas in Nonextensive Statistical Mechanics: Negative Specific Heat and Polytropism," *Physics Letters A*, Vol. 263, 1999, pp. 424-429.
 - [27] R. Silva and J. S. Alcaniz, "Negative Heat Capacity and Non-Extensive Kinetic Theory," *Physics Letters A*, Vol. 313, 2003, pp. 393-396.
 - [28] J. M. Liu, J. S. D. Groot, J. P. Matte, T. W. Johnston and R. P. Drake, "Measurements of Inverse Bremsstrahlung Absorption and Non-Maxwellian Electron Velocity Distributions," *Physical Review Letters*, Vol. 72, 1994, pp. 2717-2720.
 - [29] E. G. D. Cohen, "Statistics and Dynamics," *Physica A*, Vol. 305, 2002, pp. 19-26.
 - [30] S. Shaikh, A. Khan and P. K. Bhatia, "Stability of Thermally Conducting Plasma in a Variable Magnetic Field," *Astrophysics and Space Science*, Vol. 312, 2007, pp. 35-40.
 - [31] S. Shaikh, A. Khan and P. K. Bhatia, "Thermally Conducting Partially Ionized Plasma in a Variable Magnetic Field," *Contributions to Plasma Physics*, Vol. 47, No. 3, 2007, pp. 147-156.
 - [32] S. Shaikh, A. Khan and P. K. Bhatia, "Jeans' Gravitational Instability of a Thermally Conducting Plasma," *Physics Letters A*, Vol. 372, 2008, pp. 1451-1457.

Measurement of Relative Metastable Level Population of Gd Atoms in Hollow Cathode Lamp with LIF Method

Seyed Hassan Nabavi, Ata Koohian

Faculty of Physics, Tehran University, Tehran, Iran

E-mail: koohian@ut.ac.ir

Received February 5, 2010; revised March 29, 2010; accepted March 31, 2010

Abstract

Relative metastable level population of metal plasma having low-lying metastable states departs from equilibrium value. It needs to be experimentally investigated. This paper reports the use of hollow cathode lamp based Laser Induced Fluorescence (LIF) spectroscopy technique to measure Relative metastable level population of metal in a plasma produced by a hollow cathode lamp. The relative population of ground state and 533 cm^{-1} levels of Gd atoms in hollow cathode lamp is measured with LIF method.

Keywords: Gadolinium, Fluorescence, Population

1. Introduction

Gd metal has widespread applications in medical, astronomy and nuclear industries. Work on various parameters of this metal has drawn the attention of many researchers [1,2]. Hollow cathode lamps are mainly used for investigating various parameters of Gd Metals [3]. The scattered atoms of the Gd metal in the lamp have metastable levels, with low energy and high life time. These levels will be populated due to number of collisions such as the collisions of atoms with each other, collision of atoms with electrons, and collision of atoms with lamp's wall. The population's measurements at these levels are of a significant importance, especially in those experiments related to laser and material interactions where atoms from metastable levels are excited to other levels which have higher energies. Therefore the knowledge of knowing when the states are fully populated is essential in analyzing these types of activities. There are different methods for the measurement of level's populations, like absorption spectrum measurement which is the most current method. However one of the main problems with this method is its high optical noise [4]. In this paper, the use of inductive fluorescence method is proposed for the calculation of the relative population of the Gd metastable levels.

2. Laser Induced fluorescence (LIF) in lamp

The Gd metal in a hollow cathode lamp is scattered from the cathode by buffer gas atoms and is vaporized in the

lamp. The energetic electrons, ions, excited neutral atoms of the buffer gas collide with Gd vapor and cause the population and depopulation Gd levels. These processes create a new distribution of levels population in Gd atoms [5]. If there were no electrons in the lamp, Boltzmann distribution could be used for the estimation of the population distribution. However the existence of electrons in the lamp makes the Boltzmann distribution to be a void distribution for the population. This suggests that a different method should be used for obtaining the distribution.

The LIF method is one of the best methods for obtaining the population distribution in such cases with low atomic density and high disorderly. LIF is a process where atoms are excited to higher electronic energy states via laser absorption and induces fluorescence radiation.

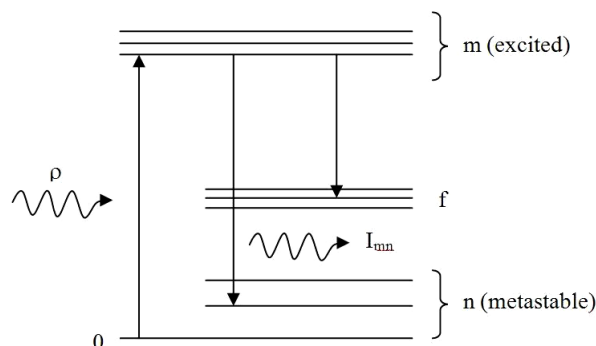


Figure 1. Some possible transitions in LIF.

The intensity of this fluorescence is dependent on the absorption density. Typically fluorescence occurs at wavelengths greater than or equal to laser wavelength. Metastable levels (**Figure 1**) can be excited to higher levels by a visible laser lines and then the fluorescence (**Figure 1**) can be detected by a monochromator. Fluorescence intensity is [6]:

$$I_{mn} = A_{mn} N_m h \nu_{mn} \eta \quad (1)$$

where N_m is the Upper level population, A_{mn} is transition probability from m to n, $h \nu_{mn}$ is the energy of this transition and η is the correction coefficient related to detection systems like photon multiplier (PMT) and the grating in monochromator.

$$\eta = \eta_{PMT} \times \eta_{grating} \quad (2)$$

where η_{PMT} and $\eta_{grating}$ are the correction coefficients. N_n can be obtained by rate equations,

$$\begin{aligned} \frac{dN_m}{dt} &= -N_m R_m - N_m B_{nm} \rho \\ \frac{dN_n}{dt} &= N_m B_{nm} \rho - N_m B_{mn} \rho + N_m A_{mn} - \frac{N_n}{\tau_n} \end{aligned} \quad (3)$$

where, B is the Einstein coefficient of the transitions, ρ is the density of the photons that react with atoms, τ_n is the life time of the low level, and R_m is the rate of radiation and non radiation fall downs from upper level m (**Figure 1**).

$$R_m = A_{mn} + A_m \rightarrow f \neq n + R_{collision} \quad (4)$$

By considering the fact that the lower level n, is metastable level, so $1/\tau_n$ goes to zero and the relative population of levels will be calculated from the numerical solutions of (3).

3. Experimental Method

Figure 2 shows the experimental arrangement for the measuring the relative population. a Ring dye laser beam which is capable of scanning 30 GHz around a wavelength is focused into hollow cathode lamp with Gd metals as its cathode. The buffer gas inside this lamp is Ne with the pressure 3torr and the maximum current that can pass through this lamp is 15 mA. With the gas pressure of 3torr and 10mA electrical current the $R_{collision}^{-1}$ in (4) is about 50 ns [6].

By sweeping a range of wavelengths and setting the system wavelength to a desirable value, the Gd atoms are excited to upper levels and then transit to lower levels (**Figure 1**). Fluorescence induced by these transitions are focused into entrance window of monochromator by a short focal length lens. The focused beam will exit the monochromator after hitting the holographic grating

(1200 1/mil). The angel of grating relative to the entrance light must be arranged in a way that the desired Florescence line is selected. This beam will enter PMT and will be amplified electrically. This will allow the observation of the beam on oscilloscope. The measurements of the relative population of the ground state and the 533 cm^{-1} level are required. These levels have wavelengths of $5618 \text{ \AA}$ and $5791 \text{ \AA}$ respectively and will be excited to 17795 cm^{-1} level and then the radiation from the atomic transition to level 215 cm^{-1} can be observed on the PMT (**Figure 3**).

By observing the number of fluorescence photon at the excited wavelengths of levels 0 and 533 cm^{-1} and by putting these numbers into (3) and solving these equations simultaneously in the steady state conditions, the relative population of the two levels is obtained.

4. Conclusions

HCL based LIF has been implemented to measure relative level population of Gadolinium. the ground and 533 cm^{-1} metastable states. in a HCL. Utilizing the emission of HCL provides for relative atom density measurement. This becomes particularly useful when there is a low fluorescence signal in the atomic data. It is a good technique using the LIF emission from a hollow cathode that can be utilized to measure the relative atom density. By using the experimental results and standard tables [7] a value around 2, i.e. $N_{533}/N_0 = 2 \pm 15\%$ for relative population of the levels 533 cm^{-1} and 0 cm^{-1} , was obtained, which means due to existence of electrons and their collisions with Gd atoms the population distribution pattern of level's does not follow the Boltzmann law any more.

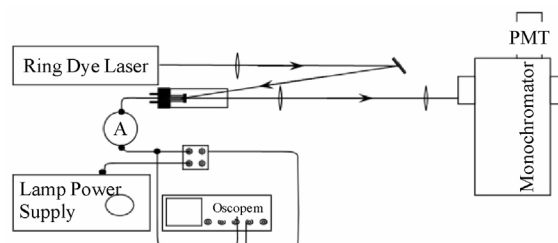


Figure 2. Experimental arrangement for measuring LIF signal.

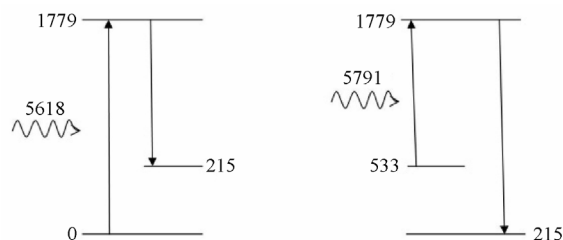


Figure 3. Engaged levels in experiment.

The LIF method with its simple experimental arrangement has a significantly high signal to noise ratio compared to similar methods like optogalvanic or absorption methods. In this experiment the measurements have been conducted using Gd hollow cathode lamp for the first time and it is suggested that this measurements can be obtained for other states by using lasers with different wavelength.

5. References

- [1] E. Bi'émont, G. Kohnen and P. Quinet, "Transition Probabilities in Gd III," *Astronomy and Astrophysics*, Vol. 393, No. 2, 2002, pp. 717-720.
- [2] S. A. Enger, A. Rezaei and P. Munck, "Gdolinium Neutron Capture Brachytherapy (GDNCB), A New Treatment Method for Intravascular Brachytherapy," *Medical Physics*, Vol. 33, No. 1, 2006, pp. 46-51.
- [3] A. Majumder, B. Dikshit, M. S. Bhatia and V. K. Mago, "Use of Multiwavelength Emission from Hollow Cathode Lamp for Measurement of State Resolved Atom Density of Metal Vapor Produced by Electron Beam Evaporation," *Review of Science Instrument*, Vol. 79, No. 9, 2008, pp. 1-7.
- [4] A. Okamoto, T. Kobuchi, S. Kitajima and M. Sasao, "Study of Metastable Population Density in a Hollow Cathode Helium Discharge," *Plasma and Fusion Research*, Vol. 2, 2007, p. 29.
- [5] R. Payling, D. G. Gones and A. Bengtson, "Glow Discharge Optical Emission Spectroscopy," John Wiley, 1997, pp. 343-347.
- [6] W. Demtroder, "Laser Spectroscopy," Springer, 1996.
- [7] C. H. Corliss and W. A. Bozman, "Experimental Transition Probabilities for Spectral Line of Seventy Elements," 1962

Bohr Correspondence Principle and Multiphoton Nature Raleigh Light Scattering

Valeriy E. Ogluzdin

Prokhorovs General Physics Institute, Russian Academy of Sciencesul, Moscow, Russia

E-mail: ogluzdin@kapella.gpi.ru

Received January 20, 2010; revised April 6, 2010; accepted April 10, 2010

Abstract

The correspondence principle and the condition of supplementation were introduced by N. Bohr for the submission of light phenomena, taking into account the wave nature of electromagnetic radiation on one hand, and its quantum structures on the other. In this paper, correspondence principle combines two models of matter, namely, the classical point of view of environment can be considered as an ensemble of equally-frequencies oscillators, *i.e.* electrons in the surrounding various atoms (molecules) of the matter and characterized by its own set of frequencies (but not hesitant in the absence of an energy source) and the quantum - environment could be presented as a set (ensemble) two-level systems, a wide range of Bohr frequencies. According to the correspondence principle Bohr jump-frequencies of atoms (molecules or nanoparticles) and natural frequencies oscillations of electrons of the same environment - oscillators are equal to each other. The dispersion characteristics of the environment in the every study range of optical frequencies correspond to the model of the classical harmonic oscillator of Lorenz, capable oscillates with Bohr frequency. Using the laws of classical mechanics to describe the environment and its dispersion properties, and the simultaneous presentation of light radiation in the form of a beam interacting with the environment of photons (quanta, corpuscles) helps explain peculiarities of the spectral composition Raleigh light scattered.

Keywords: Correspondence Principle, Complementarily Condition, Raleigh Light Scattering, Classical Harmonic Oscillator of Lorenz, Bohr Jump-Frequencies

1. Experimental Research RLS in Organic Liquids by Fabelinskiy I. L. (1957)

The purpose of this message—the description of multiphoton model of Raleigh light scattering (RLS), using the correspondence principle of N. Bohr.

Before turning to interpret RLS through multiphoton model, we should remember the results of early experiments, performed by I. L. Fabelinskiy (1957) [1], in which it was found, that the fine structure of the spectrum of monochromatic light, scattered in pure organic liquids, as a rule, consists of three spectral components. Recall that to observe RLS in these experiments as a source of radiation is used commonly available for such studies mercury low pressure spectral lamp (line $\lambda = 4358 \text{ \AA}$). Through prolonged exposure photo plates could benefit from economies of accumulation, which even in the case of a weak signal was guaranteed for black photo recording. In the article was presented an overview of theories of molecular scattering of light in

pure liquids as well as a list of experimental works on this subject (before 1957).

Typically, the structure of the spectrum RLS of pure organic liquid contains three spectral components. The spectral shifts Stocks and anti-Stocks components concerning of the central, unbiased, the most glaring components usually are not equal to one another. In toluene, benzene and CS_2 shifts Stocks and anti-Stocks components of the triplet in different series of experiments for each substance do not coincide with each other. This asymmetry was noted by I. L. Fabelinskiy, and also attention was drawn to the fact that anti-Stocks line less intense than Stocks. According to I. L. Fabelinskiy, is the real reason for asymmetry shifts Stokes and anti-Stocks components remained unknown while writing the work? Disagreement Stocks and anti-Stocks components on the frequency of the incident radiation may indicate that the emergence of the spread of radiation components are not connected to each other and are independent of each other.

This difference is due to excitation of independent os-

cillators in the environment, some oscillator's environment is responsible for Stocks frequency components, while others—for anti-Stocks; detection of natural frequencies of these oscillators and their search—is our goal.

The only simultaneous recognition dispersion characteristics of refractive indexes in this spectral field of different numerous (equal and unequal frequencies) oscillators [2,3], uniformly filling the entire volume of the environment can provide an answer to the question of different intensity Stocks, anti-Stocks components of the fine structure of RLS.

2. Correspondence Principle of N. Bohr [4]; Bohr Jump-Frequencies [5,6].

In the article, in the future statement considerable attention will be given to the simple oscillators of the medium. It is their presence in the environment can be attributed to our attention to the correspondence principle and to the complementarily condition N. Bohr. The correspondence principle and the complementarily condition has been proposed by N. Bohr (1932) in order to remain as long as possible in terms of the concepts of classical physics, and as long as possible to explain physical regularities with simple graphic models. Recall that the correspondence principle (principle of supplementary, complementarily, subsidiary) has been used Bohr to explain the relationship between the electromagnetic field and light quanta [4].

In such system we have in accordance with the principle of correspondence (supplementary, complementarily, subsidiary) different oscillators of environment associated with the electrons among the various two-level formations (with atomic skeletons, fragments of molecules, nanoparticles), the distance between the levels which can be uniquely represented through the Bohr jump-frequency ν_{0i} (see Fermi E. 1965) [5]:

$$\nu_{0i} = (E_i - E_0)/h \quad (1).$$

here E_i , E_0 —energy of excited and ground levels of medium; h —Planck constant.

Important is the fact that the electrons in atomic (molecular) environment can be represented in the form of classic oscillators, each depending on the position in the atom (molecule, nanoparticle) situation characterized by some intrinsic of frequency vibrations. The frequency ν_{0i} can be determined from the relationship between mass of the electron m and the coefficient of elasticity G_j , describing stiffness connection of electrons with the skeleton atom or molecule (Garbuny M. 1965) [7].

These frequencies (indices i and j denotes multiple frequencies and communications within the molecule or nanoparticle: $i = 1, 2, 3 \dots \infty$; $j = 1, 2, 3 \dots \infty$) easy to detect and identify if we have the absorption or radiation spectra of the investigated substance [6]. The degree of absorption (no transparency) of weak radiation by envi-

ronment in different parts of the spectrum and there are ultimately a set of characteristic frequencies ν_{0i} for the given medium.

Narrow-lines nature of atomic spectra and stripe-lines of molecular evidence that in the case of atoms electron interaction with atomic skeleton (for the hydrogen - kernel) is more simple than in the case of molecules or different-sized nanoparticles consisting of the ensemble of molecules or atoms connected to one another.

At the same time, note that in the absence of a source of excitation (that is, spectral lamps, arc, solar radiation or laser) of investigated environment, its own characteristics (Bohr jump-frequencies) do not appear explicitly, and their detection is difficult.

As our research on scattering almost resonance radiation in atomic vapor metals [8,9] or photoluminescence phenomenon of *Si*-powder in the ethanol [10,11] in these cases, direct observation characteristic Bohr jump-frequencies ν_{0i} environment with help of one-frequency ν laser is made difficult and it is necessary additional measurements spectrum. Their location can be calculated only on the basis of processing of spectrograms, using the ratio, follows from the conservation energy law, that takes into account the spectral characteristics of environment and has a kind of:

$$\nu_{0i} = 2\nu - \nu_m \quad (2).$$

here ν is the frequency of the radiation, that affects the medium (environment); ν_{0i} -frequencies oscillators environment, they are Bohr jump-frequencies; h - Planck constant is omitted;

index m is replaced by s (stoks), if $\nu < \nu_{0i}$,
index m is replaced by as (anti-stokes), if $\nu > \nu_{0i}$.

Indexes s and as suited to the observed in the experiment Stocks and anti-Stocks components radiation, scattered by atomic medium [8,9], and can be applied to photoluminescence [10,11].

If environment has continuous spectrum of absorption, then using harmonics Fourier decomposition, we can give all set of lines, each being determined her proper Bohr jump-frequency.

A few words about the relationship (2):

$$\nu_m = 2\nu - \nu_{0i} \quad (3).$$

According to the theory [12] the probability of such processes is low. However, all may be significantly simplified, if one recalls the principle of correspondence and classic dispersion theory, which describes the behavior of the refractive index medium $n(\nu)$ near natural frequencies ν_{0i} classic Lorenz harmonic oscillators. Since in this case the refractive index medium or less than unit, if $\nu > \nu_{0i}$, or many more units, if $\nu < \nu_{0i}$, the processes, described by relationship (3), are playing a decisive role in these areas spectra, and, consequently; in this time the the lower-order processes are ineffective. It is from this perspective, we try to understand the nature RLS and its complex spectral correct structure [1].

3. Dispersion of the Refractive Index $N(\nu)$ of Environment, Consisting of a Classical Harmonic Oscillators Lorenz [2,3]; Multiphoton Nature RLS.

The purpose of this work is to explain the nature RLS, based on attracting mechanisms suited to the role of characteristic frequency oscillators ν_{0i} environment, as well as contribute to the scattering processes multiphoton interaction. Note that these characteristic frequencies oscillators ν_{0i} we can compare (or equate them) Bohr frequencies and use the condition of complementarity of Bohr. The correspondence principle and condition of complementarity of Bohr in this case connect the conclusions follow from the model of classical harmonic oscillator of Lorenz, on the one hand, and, on the other hand, allow considering a two-level environments model and the consequences arising from this model. We are in this regard, in particular, would be interested in the correspondence between the states of the **classical** harmonic oscillator Lorenz and the electronic levels of atoms (molecules) of a two-level environment (in **quantum** model).

It should be noted, that in the **classical** model oscillator can be hesitate - after receiving a portion of energy, or be able to stop; and one quarter period vibration oscillators corresponds to a single act of converting kinetic energy into potential, or vice versa. Such a portion of energy in the quantum model of a two-level atomic system corresponds to the transition of electron from the level to level. This portion energy can either be absorbed or be emitted. Once again, we remind you that if power to the system is not introduced, the identification of frequencies ν_{0i} , characterizing environment, difficult.

But in this case, we must remember that all frequencies ν_{0i} environment, their full "virtual" spectrum is the calling card of this media. And all frequencies ν_{0i} —unique and independent performances of the environment.

Due to the introduction of classical oscillators naturally becomes our approach to the classical theory of dispersion of the refractive index $n(\nu)$ of environment, consisting of an ensemble of classical harmonic oscillators Lorenz. According to the theory of dispersion Lorenz [2,3] for the frequency of the incident radiation is less than the natural frequency oscillators ν_{0i} , the refractive index of environment more unit and with the reduction of the difference between frequency of the incident radiation ν and frequency oscillators ν_{0i} the refractive index $n(\nu)$ can grow indefinitely: $n(\nu) \gg 1$ [2].

Experiments on slow light, performed most recently perfectly illustrate this, and the effect of slowing light can be used to explain the long persistence of photoluminescence radiation [10,11]. Let us mark, that in these same areas of the spectrum it is possible to create the conditions for the spread of photons with "above light" speed and they form a cone of Vavilov-Tcherenkov radiation [8,9].

If the frequency of the radiation ν higher than the natural frequency oscillators ν_{0i} , we face a situation, where the refractive index of the environment becomes less than unit: $n(\nu) < 1$. The difference in the refractive index of the unit serves as a natural barrier to the spread of photons of light radiation, affects on the environment.

That obstacle can be overcome, assuming, that only part of photons incident on environment almost resonance radiation ($\nu \neq \nu_{0i}$) will be utilize on dynamic compensation dispersion of the refractive index depending on the frequency for protect: $n(\nu) \rightarrow 1$. Populations lower and upper levels in the atoms of a two-level environment through the process (3) will be equalized and then we will be $n(\nu) \rightarrow 1$, which corresponds to enlightenment environment, while another portion of the photons of the same beam will be distributed through the medium without signs of slowing down, which tends to be happening in physical experiments.

Multiphoton mechanism of dynamic compensation of dispersion is well exemplified by V. E. Ogluzdin [9] for the near-resonant propagation of light through atomic vapours potassium, as well as in the case application by V. E. Ogluzdin [10,11] of this model to explain the phenomenon of photoluminescence.

Before turning directly to the interpretation RLS based on multiphoton model, we once again remind, that the spectral structure RLS radiation is usually three spectral component, and shifts Stokes and anti-Stokes components relative to the central, unbiased frequency component ν_{0i} is not equal to each other. Typically, the intensity Stokes components RLS, according I. L. Fabelinskiy, exceeds intensity anti-Stokes components and in the spectrum of scattered radiation can abundant a continuum that extends to 100-150 cm^{-1} in both sides of the frequency of line of exciting radiation.

If the proposed model (see Equations (2) and (3)) is true, then this scenario suggests, that the emergence of Stokes and anti-Stokes components of RLS is associated with the excitement of independent sets of oscillator environment and its opening on the frequency $\nu = \nu_{01}$.

According to (3) Stokes and anti-Stokes components ν_s , ν_{as} of RLS and their corresponding Bohr jump-frequencies ν_{01} , $\nu_{02} \dots \nu_{0i}$ are on different sides on the frequency of the exciting radiation ν . Naturally, the emergence of Stokes and anti-Stokes components can only appear in an area occupied by light beam. Where excited radiation is lacking, properties of environment do not change.

Under the experiment, this will not preclude the distribution of new frequency components ν_s , ν_{as} across imperturbable environment and registering them at photoplate. Generally speaking, this radiation is dispersed into a 4π steradian. Recall, that in the paper (I. L. Fabelinskiy 1957), the registration spectrum RLS realize in the transverse direction concerning direction of the incident radiation.

The frequency ν of incident radiation may by accident

coincide with the frequency of the oscillator of environment ($\nu = \nu_{0i}$). Since the refractive index [2] in this case $n(\nu) = 1$, the portion of the incident radiation at a frequency ν can easily pass through medium. But we must remember about of oscillators environment, Bohr jump-frequencies of which are shifted to the frequency of the incident radiation ν in the blue and red regions of the spectrum. Ultimately, their presence, the dispersion characteristics of the refraction indexes therefore: $n(\nu) > 1$ or $n(\nu) < 1$ determine process RLS, according to the ratio of (3).

It is commonly supposed that RLS process is due microfluctuations density of the environment and the orientation of its species. But the self influence of the radiation to alter the optical properties and, in particular, the emergence of these fluctuations is not considered.

However, microfluctuations of the refractive index of the environment may arise from the fact, that the same radiation is acted on by the environment, could be a source of such microfluctuations and thus cause deviations (or diffraction), as the radiation and its of Stokes and anti-Stokes components, which is generally observed in such experiments.

Incidentally, it is understandable situation, which occurs in the case of the Stimulated (Mandelstam-) Brillouin scattering (SBS), when the Stokes components of radiation can be traced in the opposite direction; they are reflected [13]. Really, in the moment of the generation of the inversion of oscillators of medium for frequencies $\nu < \nu_{0i}$ at the front pulse of light can create conditions, protecting to a change in the refractive index of the environment: $n(\nu) < 1$.

The inversion of a two-level environment produce in the region Stocks of frequencies ($\nu_s < \nu_{0i}$) reduction in the refractive index $n(\nu)$ of medium [14]. If will be realized condition $n(\nu_s) < 1$, then for SBS component take place high reflectivity and this condition determine the opposite direction of its spread.

4. Conclusions

A agreement between own frequencies different oscillators of environment and Bohr jump-frequencies of this environment has brought to the interpretation of the fine structure RLS mechanism, on the one hand, based on the classical theory of dispersion, and on the other, based on the concept of quantum, the corpuscular nature of light.

The possibility of observing RLS a wide body angle (4π steradian) testifies to the absence of significant barriers to its spread.

The intensity of anti-Stokes components observed in the discussed experiment is smaller, than Stokes items and depend from the refractive index of oscillators of environment, whose frequencies correspond to a frequency of the incident radiation. Reverse direction of the reflected Stokes SBS due

to a modified dispersion dependence of oscillators of environment brought pumping radiation.

5. References

- [1] I. L. Fabelinski, "Molecular scattering of light," *Uspekhi Fizicheskikh Nauk*, Russian Academy of Sciences, Moscow, in Russian, Vol. 63, 1957, p. 355.
- [2] M. Born and E. Wolf, "Principles of Optics," Pergamon Press, Oxford, London, Edinburgh, New York, Paris, Frankfurt, 1970.
- [3] C. F. Bohren and D. R. Huffman, "Absorption and Scattering of Light by Small Particles," Wiley, New York, 1983.
- [4] N. Bohr, "Atomic Theory and the Description of Nature," Cambridge University Press, Cambridge, 1934.
- [5] E. Fermi, "Notes on Quantum Mechanics: A Course Given by Enrico Fermi at the University of Chicago," University of Chicago Press, Chicago, 1965
- [6] V. E. Ogluzdin, "The Role of Bohr Frequencies in the scattering, Luminescence, and Generation of Radiation in Different Media," *Physics-Uspekhi*, in English, Vol. 49, No. 4, 2006, p. 401; *Uspekhi Fizicheskikh Nauk*, Russian Academy of Sciences, Moscow, in Russian, Vol. 176, 2006, p. 415.
- [7] M. Garbuny, "Optical Physics," Academic Press, New York and London, 1965.
- [8] V. E. Ogluzdin, "Photons Traveling at the Speed of Light in Two-Level Atomic Medium as a Source of Cherenkov Radiation Cones," *Physics-Uspekhi*, in English, Vol. 47, 2004, p. 829; *Uspekhi Fizicheskikh Nauk*, Russian Academy of Sciences, Moscow, in Russian, Vol. 174, 2004, p. 895.
- [9] V. E. Ogluzdin, "Vavilov-Cherenkov Effect under Conditions of Almost Resonant Interaction between Intense Light Beams and Atomic Potassium Vapor," *Soviet Physics-Journal of Experimental and Theoretical Physics*, in English, Vol. 52, 1980, p. 181; *Zhurnal Eksperimentalnoi i Teoreticheskoi Fiziki*, Moscow, in Russian, Vol. 79, 1980, p. 361.
- [10] V. E. Ogluzdin, "Interpretation of the Visible Photoluminescence of Inequisized Silicon Nanoparticles Suspended in Ethanol," *Semiconductors*, in English, Vol. 39, 2005, p. 884; *Fizika i Tekhnika Poluprovodnikov*, S. Petersburg, in Russian, Vol. 70, 2005, p. 920.
- [11] V. E. Ogluzdin, "Bulletin of Russian Academy of Science," *Physics*, Vol. 3, 2006, p. 475.
- [12] R. H. Pantell and H. E. Puthoff, "Fundamentals of Quantum Electronics," John Wiley & Sons, New York, London, Sydney, Toronto, 1969.
- [13] V. I. Bespalov, A. A. Kubarev and G. A. Pasmanik, "Stimulated Brillouin Scattering," *Izv. Vysch. Uchebn. Zaved. Radiofiz.*, Vol. 13, 1970, p. 1433; *Soviet Radiophysics*, Gorkiy, in Russian.
- [14] F. A. Korolev, "Theoretical Optics: An Introduction," in Russian, High School, Moscow, 1966, p. 298.

The Double-Slit Experiment and Particle-Wave Duality: Toward a Novel Quantum Interpretation

Itzhak Orion¹, Michael Laitman²

¹Ben-Gurion University of the Negev, Beer-Sheva, Israel

²Ashlag Research Institute, Ramat-Gan, Israel

E-mail: iorion@bgu.ac.il

Received March 7, 2010; revised March 29, 2010; accepted April 12, 2010

Abstract

The double-slit experiment demonstrates the quantum physics particle-wave duality problem. Over the last decades many interpretations were introduced to the quantum theory perception problem. In most cases there was use of unclear terms, or obscure processes in these interpretations, such as particle splitting. In this paper we propose a novel concept to explain the experiment based on two postulates: The Equivalence of Form (EoF), and the particles connection to other particles, effectively functioning as a group. These two conditions are necessary to maintain wave qualities in the collective relations, and therefore cannot exist in a single particle. De Broglie introduced the mathematical relation of particle to wave; however, he did not specify the conditions for that. The proposed interpretation is a new way of looking at particles as a united group, the *Kevutsa*, which has a higher order level of matter. A series of identical particles maintain additional qualities to show a large united, correlated motion that we observe as waves transport through systems.

Keywords: Particle-Wave Duality; Interference; Quantum Theory; Electrons Diffraction

1. Introduction

1.1. The Huygens-Fresnel principle

Wave propagation can be well explained by the Huygens-Fresnel principle, as a whole space filled with a plane wave (at distance for example) advancing into a certain slit. The space is governed by the wave in a tight causality since a wave phenomenon can be described only if it is occupying a group of points or a geometrical place. According to this principle, each point along the wave-front can act as a source of secondary waves.

The Huygens-Fresnel principle is the very method to explain the formation of wave diffraction or interference behind single or double slits.

1.2. Quantum Interpretations

The problematic view of the quantum reality compared to the observed macroscopic reality lays on three principles:

- 1) The quantization of electron energy or levels in the atomic structure.
- 2) The particle-wave duality.

3) The Heisenberg uncertainty principle.

Although each of these principles is observed, quantum reality is still considered strange, and the interpretations usually define unique terminologies, not in use in other fields of physics. In this paper, we introduce a new interpretation for the main experiment that impresses the particle-wave duality; the double slit electron interference experiment.

The particle-wave duality was first introduced by L. de Broglie: A particle has a wavelength or a frequency due to its momentum (p) or its energy (E), respectively.

$$\lambda = \frac{h}{p}; \quad f = \frac{E}{h} \quad (1)$$

where, h is the Planck's constant.

De Broglie wave's expression was proved in observations of a wave's formation, such as diffraction of particles.

Since the late 1920s, many interpretations of the quantum reality have been suggested, the most famous among them is *The Copenhagen interpretation* [1] introduced by Niels Bohr and Werner Heisenberg. The Copenhagen interpretation is relying on the Max Born explanation of the wave functions having an abstract mathematical meaning, enabling us to reveal some statistical quantities

regarding the particles and their states. In the Copenhagen interpretation, as the “standard” quantum methodology, the particle has various probabilities to be measured in different states, and eventually the measurement is randomly obtained. This point of view opened the possibility of freedom since it holds the meaning of ‘unknown intermediate states’ which are all real until a measurement “chooses” one of these states to stay real.

Several quantum interpretations proposed that the observer is involved in the experimental results in case of quantum systems: *Consciousness causes collapse* [2], *Participatory Anthropic Principle* (PAP) [3], and the *Many minds interpretation* [4]. In this reality there is no way to separate between the experiment and the observer, which gives an air of “You see what you decide to see” to the whole physical reality. Therefore, there is no physical reality but a psychological reality (or we are asked to stop ‘doing physics’ from this point forth).

This paper aims to propose a new quantum interpretation by means of a new approach to look at electrons. In the following explanations, time is to be interpreted in terms of a series of changes in a given system, similar to a stationary state where there is no need to assume time-dependent variables.

2. Electron Double Slits Experiments

The first electron diffraction phenomenon was observed in an experiment performed by Davisson and Germer on nickel crystals in 1927 [5]. Slits system for the electron double slits experiment was developed later, and Jönsson published his experimental results on 1961 in *Zeitschrift für Physik* [161, 454 (1961)] (translation was published in 1974 in AJP [6]). R. P. Crease published this experiment under the title “The most beautiful experiment” in *Physics World* 2002 (Sep. 1) [7]. These physical experiments, which were repeated after the technology improvements show how interesting phenomenon is the double slits experiment.

2.1. “Gedanken Experiment”

In quantum physics textbooks, electron beams are described as though they are running in a “Gedanken Experiment”. The best description of that appears in Richard P. Feynman’s, *The Feynman Lectures on Physics – Volume 3 Quantum Mechanics*, (Chapter 1) [8]. The description starts with a double-slit wall and an electron gun shooting electrons toward the wall. Behind that wall is a screen that shows the intensity at each point. Instead of a typical distribution of balls on the screen behind, an interference pattern appeared on the screen, similar to waves interference. Analyzing the electrons making this pattern in a “Gedanken Experiment” is done by checking through which slit each electron traveled, assuming that

an electron can only travel through one slit at a time. However, if one keeps track by a flashlight where through each electron goes, the interference pattern is not observed anymore. Instead, a simple distribution similar to the case of the balls appears. Shooting electrons one at a time or as a whole fluency does not have any impact on the interference pattern.

The apparent problem of destroying the interference pattern by viewing the electrons through slit that they pass is explained by either of the followings:

- 1) The Heisenberg uncertainty principle.
- 2) ‘Hidden’ variables – inner properties of electron.

Einstein suspected that quantum theory is incomplete, which means that there had to be ‘hidden’ variables in quantum theory. The EPR (Einstein–Podolsky–Rosen) experiment was carried out in 1937 in order to inspect if quantum theory is a complete physical theory or not [9]. In contrast, John Bell, in his 1964 paper [10], very clearly showed that quantum mechanics and Einstein’s assumptions lead to different results; hence the ‘hidden’ variables assumption was neglected.

The special assumptions of the Copenhagen interpretation contained the following expressions:

- Single electron interferes with itself;
- Single electron goes through both slits;
- Superposition of possibilities with each other.
- The observer made the wave function collapse.

Physics, therefore, limited itself to computing probabilities, and omitted the ability to give common-sense explanations and a natural understanding of the nature of electrons.

3. The Group (Kevutsa) Interpretation

Group of electrons and their act as a whole group is the interpretation proposed in this paper. As in the case of a stationary system, a series of actions of electrons of the same group are not separate by a time interval, due to the system keeping its unity connection. The electrons system can assumed to be united, due to the Equivalence of Form (EoF) of elementary particles in the group. It is known that the electrons are completely identical in the double slits experiments, even their energy must be equal in order to produce a fair interference pattern on the screen. The identity of electrons is so high that it was noted by R. Feynman’s Nobel lecture in 1965 as “there is a single electron needed in order to describe the whole universe, since it can propagate through space and time in such a way as to appear in many places simultaneously”.

In macroscopic subjects, such as marbles or bullets, the Equivalence of Form (EoF) condition is not satisfied, and therefore they cannot act as a group. These bullets are always different from one another. The bullets are not having EoF among their bundle, and therefore will

not possess the **Kevutsa** (group) properties.

The **Kevutsa** interpretation introduces here a new property or properties that must be held by a complete group of particles, connected by EoF, to perform wave actions. The group of particles must be considered a united system that has all the variables at the same time to make a connected motion like a wave-form of light. The wave quality of the connected group leads it to obey the Huygens-Fresnel principle, which belongs only to waves. In the **Kevutsa** interpretation there are no 'hidden' variables in each electron of the experiment. Instead, the connection between electrons gives more degrees of freedom by which additional variables can be related.

The mathematical meaning of the connection between electrons can be illustrated if one defines three dimensional coordinates as the state values of a separated electron defined in a 3 electrons system:

$$a_1(x_1, y_1, z_1); a_2(x_2, y_2, z_2); a_3(x_3, y_3, z_3) \quad (2)$$

However adjoined 3 electrons will be defined by:

$$\begin{aligned} &a_1((x_3-x_1), (y_3-y_1), (z_3-z_1), (x_2-x_1), (y_2-y_1), (z_2-z_1)); \\ &a_2((x_1-x_2), (y_1-y_2), (z_1-z_2), (x_3-x_2), (y_3-y_2), (z_3-z_2)); \\ &a_3((x_1-x_3), (y_1-y_3), (z_1-z_3), (x_2-x_3), (y_2-y_3), (z_2-z_3)). \end{aligned} \quad (3)$$

Therefore, the grouping of electrons highly increases the numbers of variables, and each state can hold enough information needed to satisfy the 'hidden' variables problem. The 'hidden variables' in Einstein's assumption were located in each single particle, while here these variables can be occupied in the connection of group of particles.

Measuring each separate electron track using a flashlight is actually an act of separation of electrons out of their united group. A single electron aims to absorb a photon for itself alone, differing it from the group, leads to a lack of the united group description.

The **Kevutsa** group interpretation point-of-view is different from the *Copenhagen interpretation* that always strives to predict a single electron destiny, instead of looking at the whole group of non-separated particle system and their results.

De Broglie introduced the mathematical relation between particle properties and its wave properties. How-

ever, he did not introduce the conditions by which particles act as a wave. We recognize the EoF principle as a condition for wave formation of particles.

4. Conclusions

The Kevutsa interpretation is a new way of looking at particles as a whole, united group, which has a higher level of order of matter. A series of identical particles (with EoF) have more qualities to show a large united (even infinite) correlated motion that we observe as waves transport through systems.

5. References

- [1] W. Heisenberg, "Physics and Philosophy: The Revolution in Modern Science," George Allen & Unwin, London, 1958.
- [2] D. Albert, "Quantum Mechanics and Experience," Harvard University Press, Cambridge, 1994, pp. 80-111.
- [3] J. D. Barrow and F. J. Tipler, "The Anthropic Cosmological Principle," Oxford University Press, Oxford, 1986.
- [4] H. Everett, "Relative state formulation of quantum mechanics," *Reviews of Modern Physics*, Vol. 29, 1957, pp. 454-462.
- [5] C. Davisson and L. H. Germer, "Reflection of Electrons by a Crystal of Nickel," *Nature*, Vol. 119, 1927, pp. 558-560.
- [6] C. Jönsson, "Electron Diffraction at Multiple Slits," *American Journal of Physics*, Vol. 4, 1974, pp. 4-11.
- [7] R. P. Crease, "The Most Beautiful Experiment," *Physics World*, September 1 2002.
- [8] R. P. Feynman, "The Feynman Lectures on Physics," Addison-Wesley, Vol. 3, 1965, pp. 1-8.
- [9] A. Einstein, B. Podolsky and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?" *Physical Review*, Vol. 47, No. 10, 1935, pp. 777-780.
- [10] J. S. Bell, "On the Einstein-Podolsky-Rosen Paradox," *Physics*, Vol. 1, 1964, p. 195.

TABLE OF CONTENTS

Volume 1 Number 1

April 2010

Plastic Flow Macrolocalization: Autowave and Quasi-Particle

L. B. Zuev, S. A. Barannikova..... 1

Translational Motion of a Free Large Polaron and Broadening of Absorption Spectra

V. Mukhomorov..... 9

On the Nature of Dark Matter and Dark Energy

B. Y. Alexeevich, M. I. Fedorovich..... 17

**Studying Magnetization Distribution in Magnetic Thin Films under Transversal
Application of Magnetic Fields**

A. A. Zakharenko..... 33

**Computation of the Schrödinger Equation via the Discrete Derivatives Representation
Method: Improvement of Solutions Using Particle Swarm Optimization**

A. Zerarka, H. Saidi, A. Attaf, N. Khelil..... 44

Fracture and Damage Behaviors of Concrete in the Fractal Space

R. Dick..... 59

**Tilted Bianchi Type VI₀ Cosmological Model in Saez and Ballester Scalar Tensor
Theory of Gravitation**

S. K. Sahu..... 67

The Possible Dimension, Additional to Space-Time, which Physicists Ignore

A. S. Issar, S. Sorek..... 70

Instability of Thermally Conducting Self-Gravitating Systems

S. Shaikh, A. Khan..... 77

**Measurement of Relative Metastable Level Population of Gd Atoms in Hollow
Cathode Lamp with LIF Method**

S. H. Nabavi, A. Koohian..... 83

Bohr Correspondence Principle and Multiphoton Nature Raleigh Light Scattering

V. E. Ogluzdin..... 86

**The Double-Slit Experiment and Particle-Wave Duality: Toward a Novel Quantum
Interpretation**

I. Orion, M. Laitman..... 90