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Simulation of Gas-Liquid Mixture Movement through a Pipeline on the Seabed, Taking into Account the Heat Exchange Process

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Abstract

In the paper, a dynamical model of gas-liquid mixture motion through a pipeline with regard to change of rheological properties that occur as a result of heat-exchange process, is constructed, and the solutions of the obtained connected differential equations are given. Analytic expression allowing to determine pressure change along the length of a pipeline, is obtained.

Keywords

Equations, Mathematical Model, Gas-Liquid System, Heat-Exchange Process, Rheological Properties

1. Introduction

In most cases, in offshore fields, transportation of produced hydrocarbons is carried out along the seabed. With such transportation, an intensive heat exchange process takes place between the environment and gas-liquid mixture.

Currently, the issues related to non-stationary head exchange and the movement of oil and gases in underground pipelines, remain insufficiently studied, and this is associated with the complexity of the thermal and hydrodynamic processes occurring in the system oil pipeline-environment. In a theoretical study of the problem, the greatest difficulties arise due to the need of simultaneous accounting of such factors as complexity of the considered rheological parameters, strong temperature dependence of rheological parameters, need of accounting mutual thermal influence of oil pipeline and external environment, *i.e.* to consider the thermal process as a conjugate one [1].

A great majority of papers on the laminar motion of liquid with variable rheological properties are connected with the study of heat exchange and motion of non-Newtonian liquid in the stationary mode (Bussel-Gretz problem and some of its generalizations). In the classical assumptions of the latter, among which there is an assumption on constancy of physico-mechanical properties of liquid, the problem for the case of pumped flow of viscoplastic and many other non-Newtonian media, received its completed solution in the papers of Sellars, Traibus, Klein, A.K. Mirzajanzade, Z.P. Shulman, E.L. Smorodinskii, G.B. Freustater and other researchers. In view of relative complexity of the obtained solutions, that in the given case [2]-[13] is associated first of all with "anomaly" of the rheological properties, in future searches were made for finding simplifying methods for approximate solution of the problem and limits for their applicability was established. For the first time, the problem of steady heat exchange during the motion of viscous liquid with regard to temperature dependence of the viscosity factor under first kind boundary conditions on the pipe wall, was considered by L.S. Leibenzon [14]. The first approximate analytic solution of this problem obtained under the assumption that temperature and viscosity of liquid are constant over the cross section and change only along the length of the pipe, belongs to him. A further study of non-Newtonian liquids whose rheological parameters change due to temperature change, transportation of these fluids on very large scales contributed to the emergence of a number of studies of heat exchange problems under laminary motion in pipes. The solution of the problem with regard to temperature dependence of rheological characteristics meets great mathematical difficulties due to nonlinearity of the initial system of energy and motion equations and necessity of joint solution of these equations. In this connection, the known analytic solutions were obtained as a rule by approximate methods and under serious assumptions. R.M. Sattarov and R.M. Mamedov offered approximate solution of the Nusselt-Gretz problem in a plane channel for a nonlinear-viscoplastic medium whose behavior was described by Kesson's generalized equation with regard to dependence of plastic viscosity and ultimate shear stress on temperature [5].

Significant successes in the study of the problem of steady heat exchange when non-Newtonian media move in channels with regard to heat dependence of rheological parameters were achieved owing to the use of numerical methods. The latter ones have great advantage before analytical methods, they allow to avoid many simplifications and achieve significant commonality of results. Some quality and quantity estimations of the influence of variability of rheological properties on heat exchange and motion are given based on analysis of numerical solutions. The essential influence on specific form of temperature dependence of rheological parameters on termohydrodynamical process, is shown.

In spite of numerous investigations, the study of gas-liquid mixture motion through a pipeline on the seabed with regard to profile and thermal dependence of rheological properties, remains poorly studied and urgent. Therefore, simulation of gas-liquid mixture motion through a pipeline on the seabed with regard to heat exchange and pipeline profile has both important practical and scientific value and this problem is devoted to this issue.

In the paper, we construct a model of gas-liquid mixture motion through a pipeline on the seabed with regard to heat exchange process and develop a technique for solving the obtained related differential equations.

2. Problem Statement

Let us consider gas-liquid mixture motion in a pipeline. To this end we partition the pipeline into three parts (**Figure 1**):

- The first part is the segment from the surface to the bottom of the sea;
- The second part is bottom length;
- The third part is the segment from the bottom to the sea surface.

In the first approximation, we accept mixture as homogeneous and consider the motion of homogeneous gas (or almost homogeneous).

The equation of motion of such gas is described by the Charniy equation [15]:

$$-\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial t} + 2aQ - \rho g \tag{1}$$

$$-\frac{1}{c^2}\frac{\partial P}{\partial t} = \frac{\partial Q}{\partial x}.$$

In the case when the flow is stationary, we have

$$\frac{\partial Q}{\partial t} = 0, \frac{\partial P}{\partial t} = 0 \Rightarrow \frac{\partial Q}{\partial x} = 0, Q = const.$$

From the first equation of (1) we get

$$-\frac{\mathrm{d}P}{\mathrm{d}x} = 2aQ - \rho g. \tag{2}$$

The gas density can be determined from the formula

$$\rho = \frac{\rho_{\text{atm}}}{P_{\text{atm}}}\frac{T_0}{T}P. \tag{3}$$

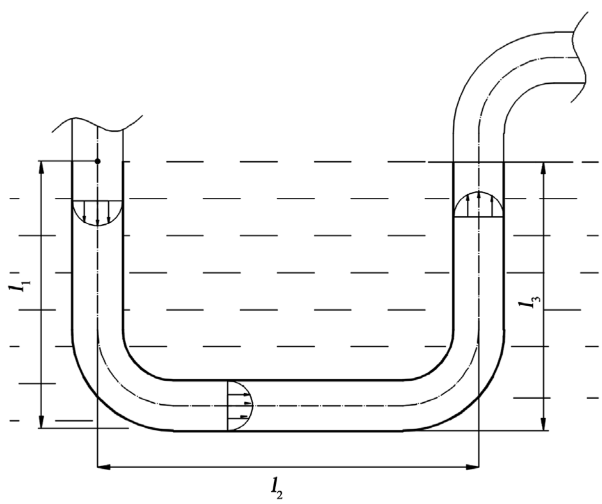

Figure 1. Pipeline profile.

Substituting expression (3) in Equation (2), we get

$$-\frac{\mathrm{d}P}{\mathrm{d}x}=2aQ-\frac{\rho_{\text{atm}}}{P_{\text{atm}}}\frac{T_0}{T}Pg\,, \tag{4}$$

or

$$\frac{\mathrm{d}P}{\mathrm{d}x}-\frac{\rho_{\text{atm}}}{P_{\text{atm}}}\frac{T_{\text{atm}}}{T}Pg=-2aQ,\ \ Q=const\,. \tag{5}$$

Here P is mixture pressure at any cross-section, P_{atm} is atmospheric pressure, ρ_{atm} is mixture's density under atmospheric pressure, g is the free fall acceleration, T is mixture's temperature in any cross-section, T_{atm} is mixture's temperature under atmospheric pressure, Q is consumption, a is resistance factor.

Accepting the external medium temperature change along the water thickness as linear, we get (**Figure 2**).

$$T_1=T_3-\frac{T_3-T_2}{l}x\,, \tag{6}$$

where l is the length of setting of the pipeline into water, T_2 is water temperature in the seabed, T_3 is water temperature in the surface.

Taking into account only convective part of heat emission for heat transfer equation we have [16] [17] [18]:

$$\frac{\mathrm{d}T}{\mathrm{d}x}=-\frac{\beta}{R}\left(T-T_1\right) \tag{7}$$

$$\beta=\frac{2k_0}{RV^*\left[c_g\rho_g\varphi+c_l\rho_l\left(1-\varphi\right)\right]}.$$

R is the inner radius of the pipeline, V^* is averaged velocity of gas flow through the piper's cross section.

ρ_g is gas density.

ρ_l is liquid's density, c_g is specific heat-capacity of gas under the given pressure, c_l is specific heat capacity of liquid, φ is volume fraction of gas in mixture.

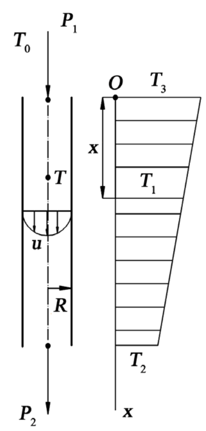

Figure 2. Design scheme.

k_0 is heat emission between fluid and pipeline's wall.

Substituting the expression (6) in (7), we get

$$\frac{\mathrm{d}T}{\mathrm{d}x}+\frac{\beta}{R}T=\frac{\beta}{R}\left(T_3-\frac{T_3-T_2}{l}x\right). \tag{8}$$

For the boundary condition $T|_{x=0}=T_0$ we get the solution of the equation in the form:

$$T=\left(T_0-T_3\right)\exp\left(-\frac{\beta}{R}x\right)+T_3+\frac{R}{\beta}\frac{T_3-T_2}{l}\left(1-\exp\left(-\frac{\beta}{R}x\right)\right)-\frac{T_3-T_2}{l}x. \tag{9}$$

As $\frac{\beta}{R}x<1$ even for $x=l$ (limit value *x*), then

$$\exp\left(-\frac{\beta}{R}x\right)\cong 1-\frac{\beta}{R}x.$$

Then

$$T=T_0\left(1-\frac{T_0-T_3}{T_0}\frac{\beta}{R}x\right), \tag{10}$$

where T_0 is temperature of gas-liquid mixture in the entry.

Substituting expression (10) in formula (5), we get

$$\frac{\mathrm{d}P}{\mathrm{d}x}-\frac{\rho_{\mathrm{atm}}}{P_{\mathrm{atm}}}\frac{T_{\mathrm{atm}}}{T_0\left(1-\frac{T_0-T_3}{T_0}\frac{\beta}{R}x\right)}Pg=-2aQ. \tag{11}$$

In the first approximation, we accept that dependence of kinematic viscosity of mixture on temperature is linear:

$$\nu=\nu_0-\frac{T-T_0}{T_0-T_4}\left(\nu_T-\nu_0\right), \tag{12}$$

where T_4 is the temperature of gas-liquid mixture at the end of the first area, ν_0 is kinematic viscosity of mixture at temperature T_0, ν_T is kinematic viscosity of mixture at temperature T_4.

Substituting expression (10) in (12), we get

$$\nu=\nu_0+\frac{T_0-T_3}{T_0-T_4}\left(\nu_T-\nu_0\right)\frac{\beta}{R}x. \tag{13}$$

The resistance factor is determined by the formula [15]:

$$a=\frac{16\nu}{d^2}, \tag{14}$$

where d is the inner diameter of the pipe.

As $\frac{T_0-T_3}{T_0}\frac{\beta}{R}x\ll 1$ even for l (limit value of *x*), then

$$\frac{1}{T_0\left(1-\frac{T_0-T_3}{T_0}\frac{\beta}{R}x\right)}\cong\frac{1}{T_0}\left(1+\frac{T_0-T_3}{T_0}\frac{\beta}{R}x\right). \tag{15}$$

Then allowing for expressions (13), (14) and (15), from Equation (11) we get

$$\frac{\mathrm{d}P}{\mathrm{d}x} - M(x)P = Q(x), \tag{16}$$

where

$$Q(x) = a_1 + a_2 x,\ \ M(x) = a_0 + b_0 x \tag{17}$$

$$a_1 = -32\frac{Q}{d^2}\nu_0,\ \ a_2 = -32\frac{Q}{d^2}\frac{T_0 - T_3}{T_0 - T_4}(\nu_T - \nu_0)\frac{\beta}{R}$$

$$a_0 = g\frac{\rho_{\text{atm}}}{P_{\text{atm}}}\frac{T_{\text{atm}}}{T_0},\ \ g\frac{\rho_{\text{atm}}}{P_{\text{atm}}}\frac{T_{\text{atm}}}{T_0}\frac{T_0 - T_3}{T_0}\frac{\beta}{R} = b_0.$$

The solution of Equation (16) will be of the form:

$$P = \exp\left(\int M(x)\mathrm{d}x\right)\int Q(x)\exp\left(-\int M(x)\mathrm{d}x\right)\mathrm{d}x + c_3\exp\left(\int M(x)\mathrm{d}x\right), \tag{18}$$

where c_3 is an integration constant.

For the values of parameters of the system entering the parameter b_0, $\frac{b_0 x^2}{2} < 1$.

Then in the first approximation

$$\exp\left(\frac{b_0 x^2}{2}\right) \approx 1 + \frac{b_0 x^2}{2}. \tag{19}$$

Allowing for expressions (17) and (19), from expression (18) we get

$$\begin{aligned} P = \left(1 + \frac{b_0 x^2}{2}\right)\exp(a_0 x)\left(a_1\int\exp(-a_0 x)\left(1 - \frac{b_0 x^2}{2}\right)\mathrm{d}x\right. \\ \left. + a_2\int x\exp(-a_0 x)\left(1 - \frac{b_0 x^2}{2}\right)\mathrm{d}x\right) + c_3\left(1 + \frac{b_0 x^2}{2}\right)\exp(a_0 x) \end{aligned}. \tag{20}$$

Having differentiated expression (20), we get

$$\begin{aligned} P_2 = \left(1 + \frac{b_0 x^2}{2}\right)&\left(\frac{a_1}{a_0} - \frac{b_0 a_1}{2a_0^3}\left(a_0^2 x^2 + 2a_0 x + 2\right)\right. \\ &\left. - \frac{a_2}{2a_0^4}\left(a_0^3 b_0 x^3 - 3a_0^2 b_0 x^2 - 2a_0^3 x + 6a_0 b_0 x + 2a_0^2 - 6b_0\right)\right). \\ &+ c_3\left(1 + \frac{b_0 x^2}{2}\right)\exp(a_0 x) \end{aligned} \tag{21}$$

We find c_3 from the boundary condition $P|_{x=0} = P_1$.

$$c_3 = P_1 - \frac{a_1}{a_0} + \frac{b_0 a_1}{a_0^3} + \frac{a_2}{a_0^2} - \frac{3a_2 b_0}{a_0^4}.$$

Now we consider the horizontal part of the pipeline (**Figure 3**).

As in the seabed the temperature of gas-liquid mixture and environment is balanced, the heat exchange between them comes to an end. Then the equation of gas motion will be of the form:

$$\frac{\mathrm{d}P}{\mathrm{d}x} = -2aQ \tag{22}$$

$$P = P_2 - 2aQx \tag{23}$$

$$P|_{x=l_2} = P_3 \tag{24}$$

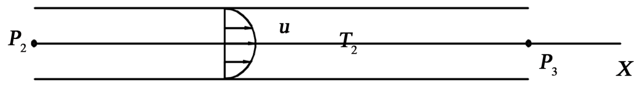

Figure 3. Design scheme.

$$P_3 = P_2 - 2aQl_2, \tag{25}$$

where P_2 is pressure at the end of the first area, l_2 is the length of the second area of the pipeline, P_3 is pressure at the end of the second area.

Similar (6) for the third area the temperature change will occur by the law (we will locate the origin of the coordinate axis x in the lower section of the pipeline)

$$T_1 = T_2 + \frac{T_3 - T_2}{l} x. \tag{26}$$

The heat transfer equation similar to (7) will be in the form:

$$\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{\beta}{R}(T - T_1). \tag{27}$$

Allowing for formula (26), from expression (27) we have:

$$T = T_2\left(1 + 2\frac{\beta}{R}x\right). \tag{28}$$

Substituting expression (28) in formula (5), we get

$$\frac{\mathrm{d}P}{\mathrm{d}x} + \frac{\rho_{\text{atm}}}{P_{\text{atm}}}\frac{T_{\text{atm}}}{T_2}\left(1 - 2\frac{\beta}{R}x\right)Pg = -2aQ. \tag{29}$$

Allowing for expression (29) and linear dependence of kinematic dependence of mixture on temperature:

$$\nu = \nu_0 + \frac{T_0 - T_3}{T_0 - T_2}(\nu_T - \nu_0). \tag{30}$$

we get an equation similar to expression (16):

$$\frac{\mathrm{d}P}{\mathrm{d}x} + M_1(x)P = Q_1(x). \tag{31}$$

where

$$Q_1(x) = a_3 + a_4 x,\ M_1(x) = c_0 + d_0 x \tag{32}$$

$$a_3 = -32\frac{Q}{d^2}\nu_0,\ a_4 = -64\frac{Q}{d^2}\frac{T_2}{T_3 - T_2}(\nu_T - \nu_0)\frac{\beta}{R}$$

$$c_0 = g\frac{\rho_{\text{atm}}}{P_{\text{atm}}}\frac{T_{\text{atm}}}{T_2},\ d_0 = -2g\frac{\rho_{\text{atm}}}{P_{\text{atm}}}\frac{T_{\text{atm}}}{T_2}\frac{\beta}{R}.$$

The solution of Equation (31) has the form:

$$P = \exp\left(\int M_1(x)\mathrm{d}x\right)\int Q_1(x)\exp\left(-\int M_1(x)\mathrm{d}x\right) + c_3\exp\left(\int M_1(x)\mathrm{d}x\right) \tag{33}$$

$$\begin{aligned} P_4 = {} & \left(1 - \frac{b_0 x^2}{2}\right)\left(\frac{a_1}{a_0} + \frac{b_0 a_1}{2a_0^3}\left(a_0^2 x^2 - 2a_0 x + 2\right)\right. \\ & \left. + \frac{a_2}{2a_0^4}\left(a_0^3 b_0 x^3 - 3a_0^2 b_0 x^2 + 2a_0^3 x + 6a_0 b_0 x - 2a_0^2 - 6b_0\right)\right. \\ & \left. + c_4\left(1 - \frac{b_0 x^2}{2}\right)\exp(-a_0 x)\right). \end{aligned} \tag{34}$$

We find c_4 from the boundary condition $P\big|_{x=0} = P_3$

$$c_4 = P_3 - \frac{a_3}{c_0} + \frac{d_0 a_3}{c_0^3} + \frac{a_4}{c_0^2} + \frac{3a_4 d_0}{c_0^4}.$$

Using (34), we can find pressure at the end of the pipeline but already on the sea surface:

$$\begin{aligned} P_4 = &\left(1 - \frac{b_0 l_3^2}{2}\right)\left(\frac{a_1}{a_0} + \frac{b_0 a_1}{2a_0^3}\left(a_0^2 l_3^2 - 2a_0 l_3 + 2\right)\right. \\ &+ \frac{a_2}{2a_0^4}\left(a_0^3 b_0 l_3^3 - 3a_0^2 b_0 l_3^2 + 2a_0^3 l_3 + 6a_0 b_0 l_3 - 2a_0^2 - 6b_0\right) \\ &\left. + c_4\left(1 - \frac{b_0 l_3^2}{2}\right)\exp\left(-a_0 l_3\right)\right). \end{aligned} \tag{35}$$

3. Discussion of Results

Taking into account the values of practical parameters (**Table 1**) from the expression (22), (26), (34) we calculate P_2, P_3, P_4.

Table 1. The values of parameters.

Variable	Value
Atmospheric pressure P_{atm} (Pa)	10^5
Density under atmospheric pressure ρ_{atm} (kg/m^3)	0.668
ν_0 is kinematic viscosity of mixture at temperature T_0 (m^2/s)	3.4×10^{-5}
ν_T is kinematic viscosity of mixture at temperature T_4 (m^2/s)	1.14×10^{-4}
T_0 is temperature of gas-liquid mixture in the entry, (˚C)	20
T_2 is water temperature in the seabed, (˚C)	5
T_3 is water temperature in the surface (˚C)	30
T_4 is the temperature of gas-liquid mixture at the end of the first area, (˚C)	20
T_{atm} atmospheric temperature (˚C)	20
a is resistance factor (s^{-1})	0.04
g is the free fall acceleration (m/s^2)	9.8
R is the inner radius of the pipeline (m)	0.1524
Q is consumption (kg/s)	17.78
l_1 is the length of the first area of the pipeline, (m)	110
l_2 is the length of the second area of the pipeline, (m)	1000
l_3 is the length of the third area of the pipeline, (m)	107
P_1 is mixture pressure at the entrance (Pa)	1.6×10^6

$$P_2 = 1.849 \times 10^6 \text{ Pa}; \;\; P_3 = 1.847 \times 10^6 \text{ Pa}; \;\; P_4 = 1.05 \times 10^6 \text{ Pa}.$$

4. Conclusion

Based on the carried out investigations, we obtain analytic expression allowing to determine pressure change by the length of the pipeline with regard to thermal dependence of rheological properties of gas-liquid mixture that happens as a result of heat exchange process. The results of numerical calculations at practical values of the system parameters show that a change in the rheological properties of a liquid as a result of a heat exchange process with the environment has a significant effect on pressure loss. In this case, due to a change in the rheological properties of the liquid, the pressure loss during its movement in the pipeline increases by about 25% compared with the movement of the liquid without changing them.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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