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Infinite Sets of Related *b*-wARH Pairs

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Abstract

Let $b \geq 2$ be a numeration base. A *b*-weak additive Ramanujan-Hardy (or *b*-wARH) number N is a non-negative integer for which there exists at least one non-negative integer A, such that the sum of A and the sum of base b digits of N, added to the reversal of the sum, give N. We say that a pair of such numbers are related of degrees $d \geq 0$ if their difference is d. We show for all numeration bases an infinity of degrees d for which there exists an infinity of pairs of *b*-wARH numbers related of degree d.

Keywords

Palindrome, Integer Number Theory, Numeration Base

1. Introduction

Let $b \geq 2$ be a numeration base. In Niţică [1], motivated by some properties of the *taxicab number*, 1729, we introduced the class of *b-additive Ramanujan-Hardy (or b-*ARH*) numbers*. It consists of non-negative integers N for which there exist at least an integer $M \geq 1$ such that the product of M and the sum of base b digits of N, added to the reversal of the product, give N. Many examples of *b*-ARH numbers can be found in [1] [2]. In [3], we introduced the class of *b-weak-additive Ramanujan-Hardy (or b-w*ARH*) numbers*. It consists of non-negative integers N for which there exist at least an integer $A \geq 0$, such that the sum of A and the sum of base b digits of N, added to the reversal of the sum, give N. It is shown in [3] that the class of *b-w*ARH numbers contains the class of *b*-ARH numbers. Moreover, the class of *b-w*ARH numbers contains all numerical palindromes with an even number of digits or with an odd number of digits and the middle digit even.

We say that a pair of *b-w*ARH numbers are related of degree $d \geq 0$ if their difference is d. Our main result shows, for all numeration base $b \geq 2$ an infinity of degrees *d* for which there exists an infinity of pairs of *b*-*w*ARH numbers related of degree *d*. Our main result leaves open the case when $b = 10$ and $d = 2$, which is of strong particular interest and for which Table 1 in [3] suggests a positive answer. This case is solved by following example.

Example 1. The palindromes $9^{\wedge k}$ and $10^{\wedge k-2}1, k \geq 1$ are a pair of 10-*w*ARH numbers separated of degree 2.

2. The Statement of the Main Result

Let $s_b(N)$ denote the sum of base *b* digits of integer *N*. If *x* is a string of digits, let $(x)^{\wedge k}$ denote the base 10 integer obtained by repeating *x* *k*-times. Let $[x]_b$ denote the value of the string *x* in base *b*. If *N* is an integer, let $N^{\mathcal{R}}$ denote the reversal of *N*, that is, the number obtained from *N* writing its digits in reverse order. The operation of taking the reversal is dependent on the base. In the definition of a *b*-ARH *number* or a *b*-*w*ARH *number N* we take the reversal of the base *b* representation of $s_b(N)\cdot M$, respectively $s_b(N)+A$. The following Theorem is our main result.

Theorem 2. *For all numeration bases* $b \geq 2$ *there exists an infinity of degrees* $d \geq 0$ *for which there exists an infinity of pairs of b-w*ARH *numbers related of degree d.*

Theorem 2 is proved in Section 3. The following Theorem is ([2], Theorem 1) and it is a crucial ingredient in the proof of our main result, Theorem 2.

Theorem 3. *Let* $\alpha \geq 1$ *integer,* $b \geq \alpha + 1$ *integer, and* $k = (1+\alpha)^l, l \geq 0$. *Assume* $b \equiv 2+\alpha \pmod{2+2\alpha}$. *Define* $N_k = \left[(1\alpha)^{\wedge k}\right]_b$. *Then there exists* $M \geq 0$ *integer such that*

$$s_b(N_k)\cdot M = \left(s_b(N_k)\cdot M\right)^R = \frac{N_k}{2}.$$

In particular, the numbers $N_k, k \geq 1$, *are b-*ARH *numbers and consequently also b-w*ARH *numbers.*

***Remark* 4.** The particular case $b = 10$, $\alpha = 2$, of Theorem 2, which gives $N_k = (12)^{3^l}$, is also covered by ([1], Example 10). Theorem 3 does not give any information if $b = 2$.

3. Proof of Theorem 2

Proof. If $b \geq 3$ Theorem 3 can be applied to $\alpha = b-2$. This gives the *b*-*w*ARH numbers $N_k = \left[(1\alpha)^{\wedge k}\right]_b$ for $k = (1+\alpha)^l, l \geq 0$. Consider now the degrees $d_q = \left[1\left(b^2-4b+3\right)^{\wedge q}\right]_b, q \geq 1$.

Using that $[1\alpha]_b + \left[1\left(b^2-4b+3\right)\right]_b = [1\alpha]_b$, the following computation, in which the right hand side is a palindrome with an even number of digits, shows that the numbers N_k and $\left[(1\alpha)^{\wedge k-q}\right]\left[1(\alpha 1)^{\wedge q}\right]_b$ form a pair of *b*-*w* ARH numbers separated of degree d_q.

$$\left[(1\alpha)^{\wedge k}\right]_b + \left[1\left(b^2-4b+3\right)^{\wedge q}\right]_b = \left[(1\alpha)^{\wedge k-q}\right]\left[1(\alpha 1)^{\wedge q}\right]_b$$

Assuming $k \ge q$, this finishes the proof of the theorem if $b \ge 3$. Assume now $b = 2$. Consider the degrees $d_{k,q} = \left[1^{\wedge k} 0^{\wedge q}\right]_2, k \ge 1, q \ge 1$. Let S be a string of length q with 0 and 1 digits. The following computation shows that the palindromes $\left[S10^{\wedge k}1S^R\right]_2$ and $\left[S(1)^{\wedge k+2} S^R\right]_2$ form a pair of 2-wARH numbers separated of degree $d_{k,q}$.

$$\left[S10^{\wedge k}1S^R\right]_2 + \left[1^{\wedge k}0^{\wedge q}\right]_2 = \left[S(1)^{\wedge k+2} S^R\right]_2.$$

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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