



Determining the Radius of the Magnetic Vortex Core of YBCO123 and Bi2212

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Abstract

A plasma has been defined as a quasi-neutral gas of charged particles showing collective behaviour. Plasmas can support waves depending on the local conditions, the presence of external electric and magnetic fields. A characteristic property of plasmas is their ability to transfer momentum and energy via collective motion. An example in this case, is the Langmuir waves where plasma electrons oscillate against a stationary ion background. In a superconductor, two plasma electrons arise one that is made up of normal electrons and the other that is made up of super-electrons. In this study, we consider a system of super-electrons forming a super-particle. The motion of the plasma super-particles around a magnetic vortex core has been studied in the YBCO123 and Bi2212 systems. The results reveal an assemblage of super-particles that contain the magnetic flux within the vortex core of radius

$$r_0 \cong \frac{1}{3} \left(1.728\mathcal{E} \times 10^{-18} \right)^{\frac{1}{2}} \text{ \AA} .$$

Subject Areas

Applied Physics, Industrial Engineering, Mechanical Engineering

Keywords

Plasma, Magnetic Flux, Magnetic Vortex Core, Super-Particles, Diamagnetism

1. Introduction

Ginzburg-Landau (GL) theory gives a good account of interactions between magnetic field and electric field in a type II superconductor [1]. The theory agrees well with the BCS theory. However, the BCS theory is not sufficient to

explain the high energy pairing in the High- T_c superconductors. While the lower and the upper critical fields of a superconductor have been determined using the GL theory, finding the radius of the magnetic vortex core theoretically has been a challenge. Therefore, there is need for a new approach in studying the motion of charged particles in an electromagnetic field, especially in high-superconductor systems. In this study, we consider assemblage of charged particles, each moving independently in prescribed electromagnetic fields which constitute a plasma [2]. The plasma of charged particles responds inertially to electric field perturbations by oscillating at the electron plasma frequency. Externally imposed electric fields will induce perturbations in the plasma that are combinations of the time dependence of the externally imposed field and the electron plasma oscillations. The response of charged particles to the electric field force is limited by the inertial force $ma = m \frac{dv}{dt}$, and is inversely proportional to the mass of the charged particle. Thus, the lighter electrons will give the primary inertial response to an electric field perturbation in a plasma [2]. Consequently, the electrons give the dominant contribution to the plasma polarization. In the Langmuir waves, plasma electrons oscillate against a stationary ion background in a normal state of a material [3]. The force experienced by an i^{th} particle in motion is given by

$$F_i = m \frac{dv_i}{dt} \quad (1)$$

where $v_i = \frac{dx_i}{dt}$ is the velocity of the i^{th} particle. The equation of motion of a charged particle in a magnetic field (\mathbf{B}) due to an electric field (\mathbf{E}) is given by [4],

$$\frac{dv_i}{dt} = \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (2)$$

where $v_i = \frac{dx_i}{dt}$ is the velocity of the i^{th} particle, q is the particle charge and $F_i = q(\mathbf{E} + \mathbf{v}_i \times \mathbf{B})$ is the Lorentz force on the i^{th} particle. The charge and current densities respectively are obtained by integrating the distribution function over velocity space and summing over species

$$\rho(x, t) = \sum_i q_i \int d^3v \cdot f_i(x, v, t) \quad (3)$$

And

$$\mathbf{J}(x, t) = \sum_i q_i \int d^3v \cdot \mathbf{v} f_i(x, v, t) \quad (4)$$

where $f = f(x, v, t)$ is the microscopic distribution function whose time derivative is given by

$$\frac{df}{dt} = \left(\frac{\partial f}{\partial t} + \frac{\partial x_i}{\partial t} \frac{\partial f}{\partial x} + \frac{\partial v_i}{\partial t} \frac{\partial f}{\partial v} \right) \sum_{i=1}^N \delta(x - x_i(t)) \delta(\mathbf{v} - \mathbf{v}_i(t)) \quad (5)$$

here, $v = |\mathbf{v}|$ Putting Equation (2) into Equation (5) yields the Klimontovich equation given as

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{v}} = C(f) \quad (6)$$

This is the plasma kinetic energy. The term $C(f)$ is known as the Coulomb collision operator on the average distribution function $f = f(x, v, t)$. The new form of Equation (6) becomes

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x} \cdot \mathbf{v}f + \frac{\partial}{\partial v} \cdot \left[\frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \right] f = C(f) \quad (7)$$

For a plasma of particles, Coulomb collision effects over short time scales are negligible. Thus, for such plasma processes,

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x} \cdot \mathbf{v}f + \frac{\partial}{\partial v} \cdot \left[\frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \right] f = 0 \quad (8)$$

The solution to this equation is

$$f = f(c_1, c_2, c_3, \dots, c_N) \quad (9)$$

where c_i is the constant of motion. The distribution is kinetically stable when the energy ϵ is the constant of motion and the equilibrium constant of distribution depends on it. Fluid moments are obtained by integrating low order powers of the product of velocity and the distribution function over velocity space in the laboratory frame, that is

$$\int d^3v v^i \cdot f : i = 0, 1, 2, \quad (10)$$

Theoretical Formulation

For a material in a superconducting state, two particles arise: normal particles that consist of normal electrons and the super-particles made up of super-electrons. The motion of the plasma super-particles around a magnetic vortex core contains the magnetic flux within the vortex core.

Some of the velocity moments of the distribution function f are listed below

Density of states:

$$N = \int d^3v f \quad (11)$$

Flow velocity:

$$V = \frac{1}{N} \int d^3v v \cdot f \quad (12)$$

Average Kinetic energy (meV):

$$\mathcal{E} = N \left(\int d^3v \frac{mv_r^2}{3} f \right) = \frac{mv_r^2}{2} \quad (13)$$

Pressure (NM^{-2}):

$$p = \int d^3v \frac{mv_r^2}{3} f = N\mathcal{E} \quad (14)$$

Pressure Tensor (NM^{-2}):

$$\mathbf{P} = \int (d^3v) m \mathbf{v} \cdot \mathbf{v}_r f = p \mathbf{I} + \boldsymbol{\pi} = p + \boldsymbol{\pi} \tag{15}$$

A system of interacting super-particles consists of a number (n) of electrons each carrying a charge e . The total charge of the super-particle will be denoted by Q . Physical properties of plasma such as energy can be generated from the velocity moment of the plasma kinetic equation of the super-particles that is expressed as

$$\int d^3v g(\mathbf{v}) \left\{ \frac{\partial f}{\partial t} + \frac{\partial}{\partial x} \cdot \mathbf{v} f + \frac{\partial}{\partial \mathbf{v}} \cdot \left[\frac{Q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \right] f \right\} = 0 \tag{16}$$

where, $g(\mathbf{v})$ is the velocity function for the particles. To find the density moment, $g = 1$ is used while for the momentum moment, $g = m\mathbf{v}$ is used as a velocity function. The momentum equation for the plasma super-particles is

$$\int d^3v m \mathbf{v} \left\{ \frac{\partial f}{\partial t} + \frac{\partial}{\partial x} \cdot \mathbf{v} f + \frac{\partial}{\partial \mathbf{v}} \cdot \left[\frac{Q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \right] f \right\} = 0 \tag{17}$$

Each part is tackled separately using the commutation property before combining the solutions:

$$\int d^3v m \mathbf{v} \frac{\partial f}{\partial t} = \frac{\partial}{\partial t} \int d^3v f m \mathbf{v} = mN \frac{d\mathbf{v}}{dt} \tag{18}$$

$$\int d^3v m \mathbf{v} \frac{\partial}{\partial x} \cdot \mathbf{v} f = \frac{\partial}{\partial x} \int d^3v f m \mathbf{v} \mathbf{v} = \nabla \cdot (p + \boldsymbol{\pi}) \tag{19}$$

$$\begin{aligned} & \int d^3v m \mathbf{v} \frac{\partial}{\partial \mathbf{v}} \cdot \left[\frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \right] f \\ &= \frac{d}{d\mathbf{v}} \int d^3v f \left[m \mathbf{v} \frac{Q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \right] = -[Nq(\mathbf{E} + \mathbf{v} \times \mathbf{B})] \end{aligned} \tag{20}$$

Substituting the solutions back to Equation (17) we have

$$mN \frac{\partial \mathbf{v}}{\partial t} = NQ(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \nabla \cdot p - \nabla \cdot \boldsymbol{\pi} \tag{21}$$

To determine the perpendicular guiding center (radius) drifts, the perpendicular flow responses are obtained by taking the cross product of the momentum equation with the magnetic field B . Thus, Equation (21) becomes

$$mN \frac{\partial}{\partial t} (\mathbf{B} \times \mathbf{v}) = NQ(\mathbf{B} \times \mathbf{E} + \mathbf{B} \times \mathbf{v} \times \mathbf{B}) - \mathbf{B} \times \nabla \cdot p \tag{22}$$

Performing the product yields

$$mN \frac{\partial}{\partial t} (\mathbf{B} \times \mathbf{v}) - NQ(\mathbf{B} \times \mathbf{E}) + NQ(\mathbf{B} \cdot \mathbf{V}) \mathbf{B} + \mathbf{B} \times \nabla \cdot p = 0 \tag{23}$$

Dividing Equation (23) by NqB^2 we have

$$\frac{m}{QB^2} \frac{\partial}{\partial t} (\mathbf{B} \times \mathbf{v}) + \frac{1}{B^2} (\mathbf{B} \times \mathbf{E}) + \frac{1}{B^2} (\mathbf{B} \cdot \mathbf{V}) \mathbf{B} + \frac{1}{NQB^2} (\mathbf{B} \times \nabla \cdot p) = 0 \tag{24}$$

Equation (24) gives the total flow velocity of the system

$$\mathbf{V} = \mathbf{V}_{\parallel} + \epsilon \mathbf{V}_{\wedge} + \epsilon^2 \mathbf{V}_{\perp} \tag{25}$$

The ϵ indicates the ordering of the various flow components while

$$\mathbf{V}_{\parallel} = V_{\parallel} \hat{\mathbf{b}} = \frac{1}{B^2} (\mathbf{B} \cdot \mathbf{V}) \mathbf{B} \tag{26}$$

$$\mathbf{V}_{\wedge} = \frac{1}{B^2} (\mathbf{E} \times \mathbf{B}) + \frac{1}{NQB^2} (\mathbf{B} \times \nabla \cdot p) \tag{27}$$

$$\mathbf{V}_{\perp} = \frac{m}{qB^2} \frac{\partial}{\partial t} (\mathbf{B} \times \mathbf{v}) \tag{28}$$

In this case, \mathbf{V}_{\wedge} is the cross flow velocity, \mathbf{V}_{\parallel} is the parallel flow velocity and \mathbf{V}_{\perp} is the perpendicular flow velocity. The directions, here, are relative to the direction of the magnetic flux, which is in the z -direction. Of importance, here, is the cross flow velocity given as

$$\mathbf{V}_{\wedge} = \frac{1}{B^2} (\mathbf{E} \times \mathbf{B}) + \frac{1}{NQB^2} (\mathbf{B} \times \nabla p) \tag{29}$$

where,

$$\mathbf{V}_d = \frac{1}{NQB^2} (\mathbf{B} \times \nabla p) \tag{30}$$

$$\mathbf{V}_E = \frac{1}{B^2} (\mathbf{E} \times \mathbf{B}) \tag{31}$$

\mathbf{V}_d represents the *diamagnetic* flow velocity while \mathbf{V}_E represents the interaction between the fields. This study revolves around the *diamagnetic* flow velocity, \mathbf{V}_d that describes the motion of super-electrons while exerting pressure p on the magnetic flux. This pressure contains the flux along cylindrical vortices. The quantity \mathbf{B} is the *diamagnetisation* of the super-particles. To work out the various components of the $\nabla \mathbf{B}$ tensor, a local Cartesian coordinate system with coordinates \hat{e}_x , \hat{e}_y and \hat{e}_z is used such that \hat{e}_z is aligned along $\hat{\mathbf{b}}$ at $\hat{\mathbf{b}}$ and \hat{e}_x pointing towards the centre of the vortex core and perpendicular to $\hat{\mathbf{b}}$. The y -axis of the local co-ordinate, $\hat{e}_y = \hat{\mathbf{b}} \times \hat{e}_x$ becomes tangential to the surface of the vortex core (interface between the flux and the plasma-super-particles' flow). Thus, the unit vectors characterizing this local coordinate system will be \hat{e}_x , $\hat{e}_z = \hat{\mathbf{b}}$ and $\hat{e}_y = \hat{\mathbf{b}} \times \hat{e}_x$. Equation (31) can be evaluated as follows:

$$\mathbf{V}_d = \frac{1}{NQB^2} (\mathbf{B} \times \nabla p) \tag{32}$$

Note that ∇p is one-dimensional in the \hat{e}_x -direction *i.e.* towards the centre of the cylindrical vortex.

$$\frac{1}{NQB^2} (\mathbf{B} \times \nabla p) = \frac{1}{NQB} \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 0 & 0 & B \\ \frac{dp}{dx} & 0 & 0 \end{vmatrix} = \frac{1}{NQB} \left(\frac{dp}{dx} \right) \hat{e}_y \tag{33}$$

Recall that $p \equiv N\varepsilon$, so that $\frac{\varepsilon}{p} = 1$, where ε is the kinetic energy of the super-particles, at absolute zero temperature and E_k is the excitation energy of the quasi particles. Therefore, equation becomes

$$V_d = \frac{\varepsilon}{\left(\frac{Q}{e}\right)_B} \left(\frac{1}{p} \frac{dp}{dx}\right) \hat{e}_y \tag{34}$$

The quantity $\frac{1}{L_p} = -\frac{1}{p} \frac{dp}{dx}$ is known as the pressure-gradient scale length and is proportional to the radius, r of the vortex core, *i.e.*

$$r = \frac{1}{L_p} = -\frac{1}{p} \frac{dp}{dx} \tag{35}$$

The vector \hat{e}_y shows that plasma superfluid moves tangentially to the interface as the flux concentrates about the centre of the vortex. These interactions form a cylindrical vortex core of radius $r = \frac{1}{L_p}$ at diamagnetisation B and flux

H . The diamagnetisation B is proportional to the $\mu_0 \left(\frac{H_{c2} + H_0}{H}\right) e^{-\left(\frac{H_{c1}}{H}\right)}$. Therefore,

$$B = \eta \mu_0 \left(\frac{H_{c2} + H_0}{H}\right) e^{-\left(\frac{H_{c1}}{H}\right)} \tag{36}$$

where η a constant that depends on the material of the superconductor is, H is the applied external field, H_{c1} is the lower critical field, H_{c2} is the upper critical field, μ_0 is the permeability of free space and $\frac{H}{H_{c2}} = b$ is the reduced magnetic field [5]. Considering all these cases, Equation (34) becomes

$$V_d = -\frac{1}{\eta \mu_0 \left(\frac{H_{c2} + H_0}{H}\right)} \left(\frac{\varepsilon}{\left(\frac{Q}{e}\right)r}\right) e^{-\left(\frac{H_{c1}}{H}\right)} \tag{37}$$

When the external magnetic field H is increased, the resulting magnetic flux (ϕ) mounts more pressure on the superelectrons and weakens the diamagnetisation. This results into an increase in the radius, r , of the core. At $H = H_{c2}$, the supe-particles are so weak that the magnetic flux permeates the whole spectrum of the super-particles causing the superconductivity of the material to breakdown and $r \rightarrow \infty$. It is important to note that the values of ε are in meV. Consequently,

$$V_d = \frac{1}{\eta \left(\frac{H_{c2} + H_0}{H}\right)} \left(\frac{\varepsilon}{\mu_0 \left(\frac{Q}{e}\right)r}\right) e^{-\left(\frac{H_{c1}}{H}\right)} \tag{38}$$

At absolute zero temperature, the thermal velocity of the super electrons is zero and therefore, Equation (38) relies on the acoustic velocity. Hence, for a super-electron at absolute zero temperature,

$$V_d = \sqrt{\frac{\mathcal{E}}{m_p}} \quad (39)$$

Hence, using Equations (39) into (38) we get

$$Q = -\frac{1}{\eta \left(\frac{H_{c2} + H_0}{H} \right)} \left(\frac{\mathcal{E}}{\left(\sqrt{\frac{\mathcal{E}}{m_p}} \right) \mu_0 r} e^{-\left(\frac{H_{c1}}{H} \right)} \right) e \quad (40)$$

From Equation (40), it should be noted that for a three-electron system,

$$\mu_0 \sqrt{\frac{\mathcal{E}}{3m_e}} = 12.57 \times 10^{-7} \sqrt{\frac{\mathcal{E}}{27.3 \times 10^{-31}}} = (7.608 \times 10^8) \sqrt{\mathcal{E}} \quad (41)$$

Substituting 41 into 40, we have

$$Q = -\frac{1}{\eta \left(\frac{H_{c2} + H_0}{H} \right)} \left(\frac{\sqrt{\mathcal{E}} e^{-\left(\frac{H_{c1}}{H} \right)} \times 1.314 \times 10^{-9}}{r} \right) e \quad (42)$$

If we make an assumption that the boundaries of the vortex core are clear when $B = B_0$ and $H = H_0$. Therefore, the radius $r = r_0$ is experimentally determined at this point and

$$B = B_0 = \eta \mu_0 \left(\frac{H_{c2} + H_0}{H_0} \right) e^{-\left(\frac{H_{c1}}{H_0} \right)} \quad (43)$$

At $B = B_0 = \mu_0 e^{-\left(\frac{H_{c1}}{H_0} \right)}$, $r = r_0$, $H = H_0$ and $\eta = \left(\frac{H_0}{H_{c2} + H_0} \right)$. Consequently,

at $B = B_0$, Equation (42) becomes

$$\frac{Q}{e} = -\left(\frac{1}{r_0} \right) \left(1.728 \mathcal{E} \times 10^{-18} \right)^{\frac{1}{2}} e^{-\left(\frac{H_{c1}}{H_0} \right)} \quad (44)$$

2. Results and Discussion

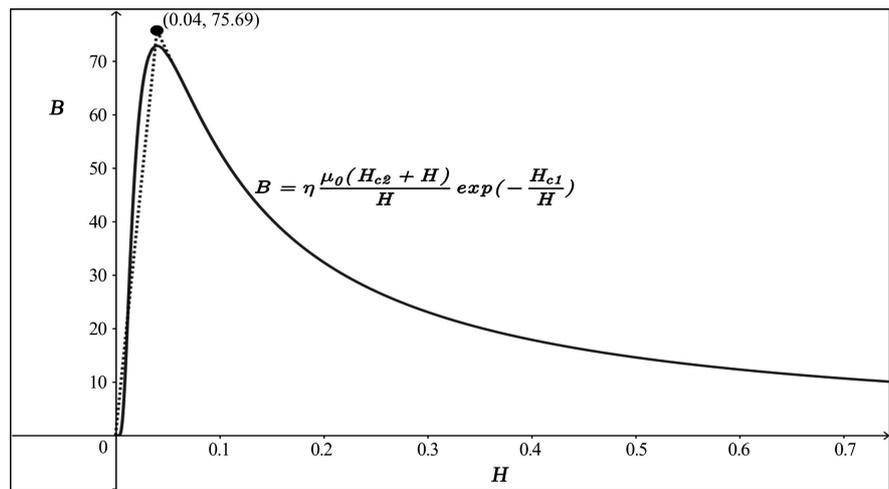
The average kinetic energy (\mathcal{E}) of super-particle consisting of three super-electrons in a YBCO123 system, by calculation, is found to be 24 meV while that of Bi2212 is 25.1 meV. Mass of the model $m_p = 3m_e$. Equations (36) and (43) has been used to generate the graphs of *diamagnetisation*, B as a function of the external applied magnetic field H . **Figure 1** shows a graph of *diamagnetisation*, B as a function of the external applied magnetic field H over varied ranges of the applied external magnetic field for both YBCO123 and Bi2212 systems.

In **Figure 1(a)**, the Continuous line represents the theoretical results while the

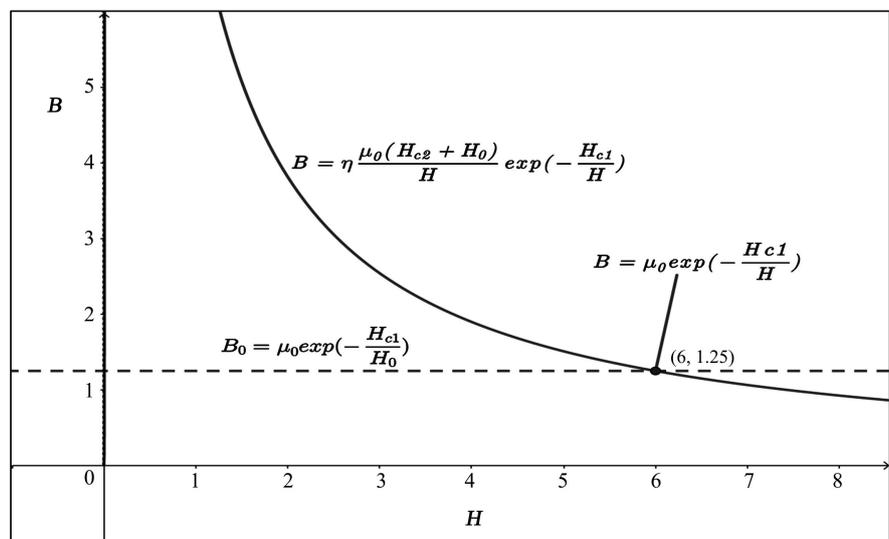
dotted line represents the experimental results. Similar shapes have been obtained through experimental procedures for Type II superconductors. In **Figure 1(a)**, the peak for both the experimental and theoretical occur at the lower critical field. Analysis of **Figure 1(b)** shows some interesting feature that emanates from the assumption in Equation (43). At $B = B_0$, the theoretical estimation puts the external applied field, at which the vortex core boundaries are clear, at 6T which agrees fully with the experimental procedure that gives 6T. The curve finally ends while it is very close the x-axis where the upper critical field of 120T. **Table 1** shows the estimated values of the ratio $\left(-\frac{Q}{e}\right)$ from Equation (44) based on experimental values of r_0 .

The results from **Table 1** have shown that

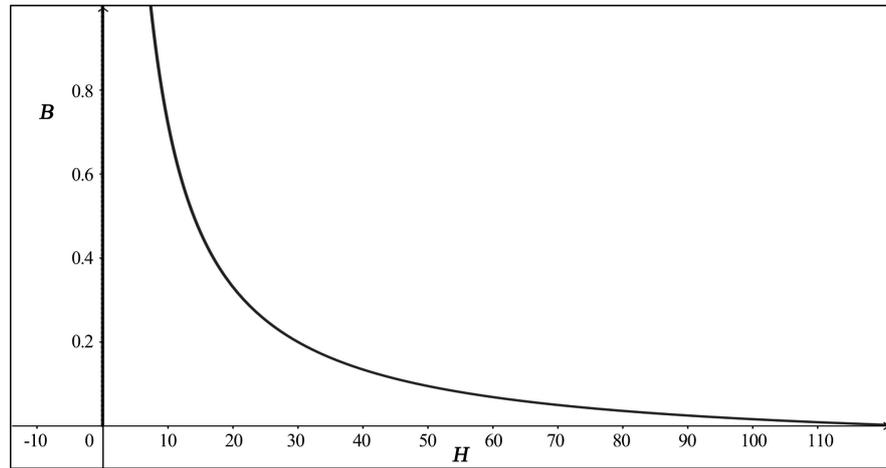
$$\left(\frac{1}{r_0}\right) \left(1.728\mathcal{E} \times 10^{-18}\right)^{\frac{1}{2}} e^{\left(\frac{H_{c1}}{H_0}\right)} \approx 3 \tag{45}$$



(a)



(b)



(c)

Figure 1. Graph of diamagnetisation as a function of applied external magnetic field over the range (a) $0 \leq H < 0.6 \text{ T}$ (b) $1 \text{ T} < H < 8 \text{ T}$ (c) $0 \leq H \leq H_{c2}$.

Table 1. Estimated values of $\left(-\frac{Q}{e}\right)$.

Superconductor	r_0 (Å)	$\left(\frac{1}{r_0}\right) (1.728\mathcal{E} \times 10^{-18})^{\frac{1}{2}} e^{\left(\frac{H_{c1}}{H_0}\right)}$
YBCO123	20 [6]	3.2
Bi2212	22 [7]	3.1

At $H = H_0 \gg H_{c1}$, $e^{-\left(\frac{H_{c1}}{H_0}\right)} \cong 1$. Thus, r_0 can be estimated from the table as

$$r_0 = \frac{1}{3} \left(1.728\mathcal{E} \times 10^{-18}\right)^{\frac{1}{2}} \text{ \AA} \tag{46}$$

Equation (46) shows the dependence of the vortex core radius, r_0 , on the kinetic energy and the *diamagnetisation* of the super-electrons. Increase in the kinetic energy increases the centrifugal force on the super-particles and a consequent increase in the radius of the vortex core.

3. Conclusion

It has been shown through the working that the radius of the magnetic vortex core, when the boundary is clear, increases with the kinetic energy (\mathcal{E}) of the super-particles.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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