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General Type-2 Fuzzy Topological Spaces

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Abstract

In this paper, a presented definition of type-2 fuzzy sets and type-2 fuzzy set operation on it was given. The aim of this work was to introduce the concept of general topological spaces were extended in type-2 fuzzy sets with the structural properties such as open sets, closed sets, interior, closure and neighborhoods in topological spaces were extended to general type-2 fuzzy topological spaces and many related theorems are proved.

Keywords

Type-2 Fuzzy Set, Interval Type-2 Fuzzy Topological Space, General Type-2 Fuzzy Topological Spaces, Type-2 Fuzzy Open Sets, Type-2 Fuzzy Closed Sets, Type-2 Fuzzy Interior, Type-2 Fuzzy Closure, Neighborhood of a Type-2 Fuzzy Set

1. Introduction

The fuzzy set theory proposed by Zadeh [1] extended the classical notion of sets and permitted the gradual assessment of membership of elements in a set [2]. After introducing the notion of fuzzy sets and fuzzy set operations, several attempts have been made to develop mathematical structures using fuzzy set theory. In 1968, chang [3] introduced fuzzy topology which provides a natural framework for generalizing many of the concepts of general topology to fuzzy topological spaces and its development can be found in [3]. The concept of a type-2 fuzzy set as extension of the concept of an ordinary fuzzy set (henceforth called a type-1 fuzzy set) in which the membership function falls into a fuzzy set in the interval [0,1], [2] [4]. Many scholars have conducted research on type-2 fuzzy set and their properties, including Mizumoto and Tanaka [5], Mendel [6], Karnik and Mendel [4] and Mendel and John [7]. Type-2 fuzzy sets are called "fuzzy", so, it could be called fuzzy set [6]. In [6] Mendel was introduced the concept of an interval type-2 fuzzy set. Type-2 fuzzy sets have also been widely applied to many fields with two parts general type-2 fuzzy set and interval type-2 fuzzy sets. The interval type-2 fuzzy topological space introduced by [2]. Because the interval type-2 fuzzy set, as a special case of general type-2 fuzzy sets, and general type-2 fuzzy sets may be better that the interval type-2 fuzzy sets to deal with uncertainties and because general type-2 fuzzy sets can obtain more degrees of freedom [8], we introduce general type-2 fuzzy topological spaces. The paper is organized as follows. Section 2 is the preliminary section which recalls definitions and operations to gather with some properties type-2 fuzzy sets. In Section 3, we introduce the definition of general type-2 fuzzy topology and some of its structural properties such as type-2 fuzzy open sets, type-2 fuzzy closed sets, type-2 fuzzy interior, type-2 fuzzy closure and neighborhood of a type-2 fuzzy set are studied.

2. Preliminaries

In this section, we recall the preliminaries of type-2 fuzzy sets, define type-2 fuzzy and some important associated concepts from [7] [9] and throughout this paper, let X be anon empty set and I be closed unit interval, *i.e.*, $I=[0,1]$.

Definition 1 [7] [9]. Let X be a finite and non empty set, which is referred to as the universe a type-2 fuzzy set, denoted by $\tilde{\tilde{A}}$ is characterized by a type-2 memberships function $\mu_{\tilde{\tilde{A}}}(x,u)$, as

$$\mu_{\tilde{\tilde{A}}}: X\times[0,1]\to[0,1]^{J_x}\left(J_x\subseteq[0,1]\right),\text{ where } x\in X \text{ and } u\in J_x,\text{ that is}$$

$$\tilde{\tilde{A}}=\left\{\left((x,u),\mu_{\tilde{\tilde{A}}}(x,u)\right):\text{where } x\in X \text{ and } u\in J_x\subseteq[0,1],\text{ where } 0\le\mu_{\tilde{\tilde{A}}}(x,u)\le 1\right\} \quad (1)$$

$\tilde{\tilde{A}}$ can also be expressed as

$$\begin{aligned}\tilde{\tilde{A}}&=\sum_{x\in X}\sum_{u\in Jx}\mu_{\tilde{\tilde{A}}}(x,u)/(x,u)\\&=\sum_{x\in X}\sum_{u\in Jx}f_x(u)/u/x,\ J_x\subseteq[0,1]\end{aligned} \quad (2)$$

where $f_x(u)=\mu_{\tilde{\tilde{A}}}(x,u)$ an $\sum\sum$ denotes union over all admissible x and u for continuous universes of discourse, $\sum$ is replaced by $\int$. The class of all type-2 fuzzy sets of the universe X denoted by $\tilde{\tilde{\mathbb{F}}}_{T_2}(X)$.

Definition 2 [2] [7]. A vertical slice, denoted $\mu_{\tilde{\tilde{A}}}(x')$, of $\tilde{\tilde{A}}$, is the intersection between the two-dimensional plane whose axes are u and $\mu_{\tilde{\tilde{A}}}(x',u)$ and the three-dimensional type-2membership function $\tilde{\tilde{A}}$, *i.e.*,
$\mu_{\tilde{\tilde{A}}}(x')=\mu_{\tilde{\tilde{A}}}(x=x',u)=\sum_{u\in J_{x'}}f_{x'}(u)/u,\ J_{x'}\subseteq I$ in which $0\le f_{x'}(u)\le 1$. $\tilde{\tilde{A}}$ can also be expressed as follows: $\tilde{\tilde{A}}=\left\{\left(x,\mu_{\tilde{\tilde{A}}}(x)\right):\forall x\in X\right\}$ or as following

$$\begin{aligned}\tilde{\tilde{A}}&=\sum_{x\in X}\sum_{u\in Jx}\mu_{\tilde{\tilde{A}}}(x)/(x)\\&=\sum_{x\in X}\sum_{u\in Jx}f_x(u)/u/x,\ J_x\subseteq[0,1]\end{aligned} \quad (3)$$

The vertical slice, $\mu_{\tilde{\tilde{A}}}(x')$ is also called the secondary membership function, and its domain is called the primary membership of *x*, which is denoted by J_X where $J_X\subseteq I$ for any $x\in X$. The amplitude of a secondary membership function is called the secondary grade.

When configuring any type-2 fuzzy topological structures we must present some special types of type-2 fuzzy sets.

Definition 3 [5] [8]. (*Type-2 fuzzy universe set*).

A type-2 fuzzy universe set, denoted $\tilde{\tilde{X}}$, such that

$$\tilde{\tilde{X}} = \sum_{x\in X} \sum_{u\in[1,1]} 1/u/x \tag{4}$$

Definition 4 [5] [8]. (*Type-2 fuzzy empty set*)

A type-2 fuzzy empty set, denoted $\tilde{\tilde{\varnothing}}$, such that

$$\tilde{\tilde{\varnothing}} = \sum_{x\in X} \sum_{u\in[0,0]} 1/u/x \tag{5}$$

Definition 5 [6]. (*Interval type-2 fuzzy set*).

When all the secondary grades of types $\tilde{\tilde{A}}$ are equal to 1, that is $\mu_{\tilde{\tilde{A}}}(x,u)=1$ for all $x\in X$ and for all $u\in J_x\subseteq[0,1]$, $\tilde{\tilde{A}}$ is as an Interval type-2 fuzzy set.

Operation of Types-2 fuzzy sets 6. Consider two type-2 fuzzy sets, $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$, in a universe *X*. Let $\mu_{\tilde{\tilde{A}}}(x)$ and $\mu_{\tilde{\tilde{B}}}(x)$ be the membership grades of these two sets, which are represented for each $x\in X$, $\mu_{\tilde{\tilde{A}}}(x)=\sum_{u\in J_x^u} f_x(u)/u$ and $\mu_{\tilde{\tilde{B}}}(x)=\sum_{w\in J_x^w} g_x(w)/w$, respective, where $u\in J_x^u$, $w\in J_x^w$ indicate the primary memberships of *x* and $f_x(u), g_x(w)\in[0,1]$ indicate the secondary memberships (grades) of *x*. The membership grades for the union, intersection and complement of the type-2 fuzzy sets $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ have been defined as follows [5].

Containment:

$\tilde{\tilde{A}}$ is a subtype-2 fuzzy set of $\tilde{\tilde{B}}$ denoted $\tilde{\tilde{A}}\subseteq\tilde{\tilde{B}}$ if $u\le w$ and $f_x(u)\le g_x(w)$ for every $x\in X$.

Equality:

$\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are type-2 fuzzy sets are equal, denoted $\tilde{\tilde{A}}=\tilde{\tilde{B}}$ if $u=w$ and $f_x(u)=\mu_{\tilde{\tilde{A}}}(x,u)=g_x(w)=\mu_{\tilde{\tilde{B}}}(x,w)$ for every $x\in X$.

Union of two type-2 fuzzy sets:

$$\begin{aligned}\tilde{\tilde{A}}\cup\tilde{\tilde{B}} \Leftrightarrow \mu_{\tilde{\tilde{A}}\cup\tilde{\tilde{B}}}(x) &= \sum_{u\in J_x^u} \sum_{w\in J_x^w} f_x(u)\star g_x(w)/(u\vee w) \\ &\equiv \mu_{\tilde{\tilde{A}}}(x)\sqcup\mu_{\tilde{\tilde{B}}}(x),\quad x\in X\end{aligned} \tag{6}$$

Intersection of two type-2 fuzzy sets:

$$\begin{aligned}\tilde{\tilde{A}}\cap\tilde{\tilde{B}} \Leftrightarrow \mu_{\tilde{\tilde{A}}\cap\tilde{\tilde{B}}}(x) &= \sum_{u\in J_x^u} \sum_{w\in J_x^w} f_x(u)\star g_x(w)/(u\vee w) \\ &\equiv \mu_{\tilde{\tilde{A}}}(x)\sqcap\mu_{\tilde{\tilde{B}}}(x),\quad x\in X\end{aligned} \tag{7}$$

Complement of a type-2 fuzzy set:

$$\sim\tilde{\tilde{A}} = \mu_{\sim\tilde{\tilde{A}}}(x) = \sum_{u\in J_x^u} f_x(u)/(1-u) \equiv \neg\mu_{\tilde{\tilde{A}}}(x),\ x\in X \tag{8}$$

Where $\vee$ represent the max t-conorm and $\star$ represent a t-norm. The summation indicate logical unions. We refer to the operations $\sqcup,\sqcap$ and $\neg$ as join, meet and negation respectively and $\mu_{\tilde{\tilde{A}}\cup\tilde{\tilde{B}}}(x)$, $\mu_{\tilde{\tilde{A}}\cap\tilde{\tilde{B}}}(x)$, $\mu_{\tilde{\tilde{A}}}(x)$ and $\mu_{\tilde{\tilde{B}}}(x)$ are the secondary membership functions and all are type-1 fuzzy sets. If

$\mu_{\tilde{\tilde{A}}}(x)$ and $\mu_{\tilde{\tilde{B}}}(x)$ have continuous domains, then the summations in 3, 4 and 5 are replaced by integrals.

Example 7: Let $X=\{x_1,x_2,x_3\}$ be anon empty set, and let $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are type-2 fuzzy sets over the same universe *X*.

$$\tilde{\tilde{A}}=\{((x_1,0.1),0.3),((x_1,0.5),1),((x_2,0.5),1),((x_2,0.6),0.3),((x_3,0.8),1)\}$$

$$\tilde{\tilde{B}}=\{((x_1,0.1),0.7),((x_1,0.2),1),((x_2,0.6),1),((x_3,0.5),0.6),((x_3,0.9),1)\}$$

$\tilde{\tilde{A}}\cup\tilde{\tilde{B}}$ for $x=x_1$ to get

$$\mu_{\tilde{\tilde{A}}\cup\tilde{\tilde{B}}}(x_1)=\frac{0.3\wedge0.7}{0.1\vee0.1}+\frac{0.3\wedge1}{0.1\vee0.2}+\frac{1\wedge0.7}{0.5\vee0.1}+\frac{1\wedge1}{0.5\vee0.2}$$
$$=\frac{0.3}{0.1}+\frac{0.3}{0.2}+\frac{0.7}{0.5}+\frac{1}{0.5}=\{(0.1,0.3),(0.2,0.3),(0.5,\max\{0.7,1\})\}$$

$\tilde{\tilde{A}}\cup\tilde{\tilde{B}}$ for $x=x_1$, $\{((x_1,0.1),0.3),((x_1,0.2),0.3),((x_1,0.5),1)\}$

$\tilde{\tilde{A}}\cup\tilde{\tilde{B}}$ for $x=x_2$ to get

$$\mu_{\tilde{\tilde{A}}\cup\tilde{\tilde{B}}}(x_2)=\frac{1\wedge1}{0.5\vee0.6}+\frac{0.3\wedge1}{0.6\vee0.6}=\frac{1}{0.6}+\frac{0.3}{0.6}\Rightarrow\{(0.6,\max\{1,0.3\})\}$$

$\tilde{\tilde{A}}\cup\tilde{\tilde{B}}$ for $x=x_2\Rightarrow\{((x_2,0.6),1)\}$

$\tilde{\tilde{A}}\cup\tilde{\tilde{B}}$ for $x=x_3$ to get

$$\mu_{\tilde{\tilde{A}}\cup\tilde{\tilde{B}}}(x_3)=\frac{1\wedge0.6}{0.8\vee0.5}+\frac{1\wedge1}{0.8\vee0.9}=\frac{0.6}{0.8}+\frac{1}{0.9}=\{(0.8,0.6),(0.9,1)\}$$

$\tilde{\tilde{A}}\cup\tilde{\tilde{B}}$ for $x=x_3$, $\{((x_3,0.8),0.6),((x_3,0.9),1)\}$

$$\tilde{\tilde{A}}\cup\tilde{\tilde{B}}=\{((x_1,0.1),0.3),((x_1,0.2),0.3),((x_1,0.5),1),((x_2,0.6),1),$$
$$((x_3,0.8),0.6),((x_3,0.9),1)\}$$

$\tilde{\tilde{A}}\cap\tilde{\tilde{B}}$ for $x=x_1$ to get

$$\mu_{\tilde{\tilde{A}}\cap\tilde{\tilde{B}}}(x_1)=\frac{0.3\wedge0.7}{0.1\wedge0.1}+\frac{0.3\wedge1}{0.1\wedge0.2}+\frac{1\wedge0.7}{0.5\wedge0.1}+\frac{1\wedge1}{0.5\wedge0.2}$$
$$=\frac{0.3}{0.1}+\frac{0.3}{0.1}+\frac{0.7}{0.1}+\frac{1}{0.2}=\{(0.1,\max\{0.3,0.3,0.7\}),(0.2,1)\}$$

$\tilde{\tilde{A}}\cap\tilde{\tilde{B}}$ for $x=x_1$, $\{((x_1,0.1),0.7),((x_1,0.2),1)\}$

$\tilde{\tilde{A}}\cap\tilde{\tilde{B}}$ for $x=x_2$ to get

$$\mu_{\tilde{\tilde{A}}\cap\tilde{\tilde{B}}}(x_2)=\frac{1\wedge1}{0.5\wedge0.6}+\frac{0.3\wedge1}{0.6\wedge0.6}=\frac{1}{0.5}+\frac{0.3}{0.6}\Rightarrow\{(0.5,1),(0.6,0.3)\}$$

$\tilde{\tilde{A}}\cap\tilde{\tilde{B}}$ for $x=x_2$, $\{((x_2,0.5),1),((x_2,0.6),0.3)\}$

$\tilde{\tilde{A}}\cap\tilde{\tilde{B}}$ for $x=x_3$ to get

$$\mu_{\tilde{\tilde{A}}\cap\tilde{\tilde{B}}}(x_3)=\frac{1\wedge0.6}{0.8\wedge0.5}+\frac{1\wedge1}{0.8\wedge0.9}=\frac{0.6}{0.5}+\frac{1}{0.8}\Rightarrow\{(0.5,0.6),(0.8,1)\}$$

$\tilde{\tilde{A}}\cap\tilde{\tilde{B}}$ for $x=x_3$, $\{((x_3,0.5),0.6),((x_3,0.8),1)\}$

$$\tilde{\tilde{A}} \cap \tilde{\tilde{B}} = \{((x_1,0.1),0.7),((x_1,0.2),1),((x_2,0.5),1),((x_2,0.6),0.3),$$
$$((x_3,0.5),0.6),((x_3,0.8),1)\}$$

The complement of a type-2 fuzzy set $\tilde{\tilde{A}}$ is

$$\sim \tilde{\tilde{A}} = \mu_{\sim\tilde{\tilde{A}}}(x) = \sum_{u \in J_x^u} f_x(u)/(1-u)$$
$$\equiv \neg \mu_{\tilde{\tilde{A}}}(x), \; x \in X$$
$$= \{((x_1,0.9),0.3),((x_1,0.5),1),((x_2,0.5),1),((x_2,0.4),0.3),((x_3,0.2),1)\}.$$

Operations under collection of type-2 fuzzy sets 8: Let $\{\tilde{\tilde{A}}_i : i \in \mathbb{N}\}$ be an arbitrary collection of type-2 fuzzy sets subset of X such that $\mathbb{N}$ is countable set, operation are possible under an arbitrary collection of type-2 fuzzy sets.

1) The union $\cup_{i\in\mathbb{N}} \tilde{\tilde{A}}_i$ is defined as

$$\left[\cup_{i\in\mathcal{N}} \tilde{\tilde{A}}_i\right](x) = \sum_{x\in X} \sum_{u\in J_x^u} \frac{\wedge_{i\in\mathcal{N}} (f_x(u))_i}{\vee_{i\in\mathcal{N}} (u)_i} \tag{9}$$

2) The intersection $\cap_{i\in\mathbb{N}} \tilde{\tilde{A}}_i$ is defined as

$$\left[\cap_{i\in\mathbb{N}} \tilde{\tilde{A}}_i\right](x) = \sum_{x\in X} \sum_{u\in J_x^u} \frac{\wedge_{i\in\mathcal{N}} (f_x(u))_i}{\wedge_{i\in\mathcal{N}} (u)_i} \tag{10}$$

Proposition 9: Let $\{\tilde{\tilde{A}}_i : i \in \mathbb{N}\}$ be an arbitrary collection of type-2 fuzzy sets subset of *X* such that $\mathbb{N}$ is countable set and $\tilde{\tilde{B}}$ be another type-2 fuzzy set of *X*, then

1) $\tilde{\tilde{B}} \cap \left[\cup_{i\in\mathbb{N}} \tilde{\tilde{A}}_i\right] = \cup_{i\in\mathbb{N}} \left(\tilde{\tilde{B}} \cap \tilde{\tilde{A}}_i\right)$.

2) $\tilde{\tilde{B}} \cup \left[\cap_{i\in\mathbb{N}} \tilde{\tilde{A}}_i\right] = \cap_{i\in\mathbb{N}} \left(\tilde{\tilde{B}} \cup \tilde{\tilde{A}}_i\right)$.

3) $1 - \left[\cup_{i\in\mathbb{N}} \tilde{\tilde{A}}_i\right] = \cap_{i\in\mathbb{N}} \left(1 - \tilde{\tilde{A}}_i\right)$.

4) $1 - \left[\cap_{i\in\mathbb{N}} \tilde{\tilde{A}}_i\right] = \cup_{i\in\mathbb{N}} \left(1 - \tilde{\tilde{A}}_i\right)$.

3. General Type-2 Fuzzy Topological Space

In this section we introduced the concept general type-2 fuzzy topology.

Definition 1: Let $\tilde{\tilde{\mathfrak{F}}}$ be the collection of type-2 fuzzy set over *X*; then $\tilde{\tilde{\mathfrak{F}}}$ is said to be general type-2 fuzzy topology on *X* if

1) $\tilde{\tilde{\varnothing}}, \tilde{\tilde{X}} \in \tilde{\tilde{\mathfrak{F}}}$

2) $\tilde{\tilde{A}} \cap \tilde{\tilde{B}} \in \tilde{\tilde{\mathfrak{F}}}$ for any $\tilde{\tilde{A}}, \tilde{\tilde{B}} \in \tilde{\tilde{\mathfrak{F}}}$.

3) $\cup_{i\in\mathbb{N}} \tilde{\tilde{A}}_i \in \tilde{\tilde{\mathfrak{F}}}$ for any $\tilde{\tilde{A}}_i \in \tilde{\tilde{\mathfrak{F}}}$, $\mathbb{N}$ countable set.

The pair $\left(X, \tilde{\tilde{\mathfrak{F}}}\right)$ is called general type-2 fuzzy topological space over *X*.

Remark 2: Let $\left(X, \tilde{\tilde{\mathfrak{F}}}\right)$ be general type-2 fuzzy topological space over *X*; then the members of $\tilde{\tilde{\mathfrak{F}}}$ are said to be type-2 fuzzy open set in *X* and a type-2 fuzzy set $\tilde{\tilde{A}}$ is said to be a type-2 fuzzy closed set in X, if its complement $\sim \tilde{\tilde{A}} \in \tilde{\tilde{\mathfrak{F}}}$.

Proposition 3: Let $\left(X, \tilde{\tilde{\mathfrak{F}}}\right)$ be general type-2 fuzzy topological space over X then the following conditions hold:

1) $\tilde{\tilde{\varnothing}}, \tilde{\tilde{X}}$ are type-2 fuzzy closed sets.

2) Arbitrary intersection of type-2 fuzzy closed sets is closed sets.

3) Finite union of type-2 fuzzy closed sets is closed sets.

Proof:

1) $\tilde{\tilde{\varnothing}}, \tilde{\tilde{X}}$ are type-2 fuzzy closed sets because they are the complements of the type-2 fuzzy open sets $\tilde{\tilde{\varnothing}}, \tilde{\tilde{X}}$ is respectively.

2) Let $\left\{\tilde{\tilde{A}}_i : i \in \mathbb{N}\right\}$ be an arbitrary collection of type-2 fuzzy closed sets, then

$$\begin{aligned}\left[\cap_{i\in\mathbb{N}} \tilde{\tilde{A}}_i\right](x) &= \sum_{x\in X}\sum_{u\in J_x^u} \frac{\wedge_{i\in\mathcal{N}}\left(f_x(u)\right)_i}{\wedge_{i\in\mathcal{N}}(u)_i} \\ &= \sum_{x\in X}\sum_{u\in J_x^u} \frac{\wedge_{i\in\mathcal{N}}\left(f_x(u)\right)_i}{1-\left(\vee_{i\in\mathcal{N}}(1-u)\right)_i}\left(\text{proposition 2.7 part 3}\right) \\ &= \left[\cup_{i\in\mathbb{N}} \sim \tilde{\tilde{A}}_i\right](x)\end{aligned}$$

since arbitrary union of type-2 fuzzy open sets are open $\left[\cup_{i\in\mathbb{N}} \sim \tilde{\tilde{A}}_i\right](x)$ is an open and $\left[\cap_{i\in\mathbb{N}} \tilde{\tilde{A}}_i\right](x)$ is a type-2 fuzzy closed sets.

3) If $\tilde{\tilde{A}}_i\left(i\in\mathbb{N}\right)$ is type-2 fuzzy closed sets, then $\cup_{i\in\mathbb{N}} \tilde{\tilde{A}}_i$ is a type-2 fuzzy closed set, [finite intersection of type-2 fuzzy open sets are open].

Example 4: Let $X = \{x_1, x_2\}$ and let $\tilde{\tilde{A}}, \tilde{\tilde{\varnothing}}$ and $\tilde{\tilde{X}}$ be three type-2 fuzzy sets in X which are

$$\tilde{\tilde{\varnothing}} = \left(\left(x_1,0\right),1\right),\left(\left(x_2,0\right),1\right),\quad \tilde{\tilde{X}} = \left\{\left(\left(x_1,1\right),1\right),\left(\left(x_2,1\right),1\right)\right\}$$

$$\begin{aligned}\tilde{\tilde{A}} = \{&\left(\left(x_1,0.8\right),1\right),\left(\left(x_1,0.6\right),0.7\right),\left(\left(x_1,0.3\right),0.6\right),\\ &\left(\left(x_2,0.8\right),0.9\right),\left(\left(x_2,0.5\right),1\right),\left(\left(x_2,0.4\right),0.5\right)\}.\end{aligned}$$

$$\tilde{\tilde{\varnothing}} \cup \tilde{\tilde{X}} \text{ for } x_1 : \mu_{\tilde{\tilde{\varnothing}}\cup\tilde{\tilde{X}}}\left(x_1\right) = \frac{1\wedge 1}{0\vee 1} \Rightarrow = (1,1) \Rightarrow = \left\{\left(\left(x_1,1\right),1\right)\right\}.$$

$$\tilde{\tilde{\varnothing}} \cup \tilde{\tilde{X}} \text{ for } x_2 : \mu_{\tilde{\tilde{\varnothing}}\cup\tilde{\tilde{X}}}\left(x_2\right) = \frac{1\wedge 1}{0\vee 1} \Rightarrow = (1,1) \Rightarrow = \left\{\left(\left(x_2,1\right),1\right)\right\}.$$

$$\tilde{\tilde{\varnothing}} \cup \tilde{\tilde{X}} = \left\{\left(\left(x_1,1\right),1\right),\left(\left(x_2,1\right),1\right)\right\} = \tilde{\tilde{X}}$$

$$\tilde{\tilde{\varnothing}} \cap \tilde{\tilde{X}} \text{ for } x_1 : \mu_{\tilde{\tilde{\varnothing}}\cap\tilde{\tilde{X}}}\left(x_1\right) = \frac{1\wedge 1}{0\wedge 1} \Rightarrow = (0,1) \Rightarrow = \left\{\left(\left(x_1,0\right),1\right)\right\}.$$

$$\tilde{\tilde{\varnothing}} \cap \tilde{\tilde{X}} \text{ for } x_2 : \mu_{\tilde{\tilde{\varnothing}}\cap\tilde{\tilde{X}}}\left(x_2\right) = \frac{1\wedge 1}{0\wedge 1} \Rightarrow = (0,1) \Rightarrow = \left\{\left(\left(x_2,0\right),1\right)\right\}.$$

$$\tilde{\tilde{\varnothing}} \cap \tilde{\tilde{X}} = \left\{\left(\left(x_1,0\right),1\right),\left(\left(x_2,0\right),1\right)\right\} = \tilde{\tilde{\varnothing}}$$

$$\begin{aligned}\tilde{\tilde{\varnothing}} \cup \tilde{\tilde{A}} \text{ for } x_1 : \mu_{\tilde{\tilde{\varnothing}}\cup\tilde{\tilde{A}}}\left(x_1\right) &= \frac{1\wedge 1}{0\vee 0.8} + \frac{1\wedge 0.7}{0\vee 0.6} + \frac{1\wedge 0.6}{0\vee 0.3} \\ &= \left\{\left(\left(x_1,0.8\right),1\right),\left(\left(x_1,0.6\right),0.7\right),\left(\left(x_1,0.3\right),0.6\right)\right\}\end{aligned}$$

$$\tilde{\tilde{\varnothing}} \cup \tilde{\tilde{A}} \text{ for } x_2 : \mu_{\tilde{\tilde{\varnothing}} \cup \tilde{\tilde{A}}}(x_2) = \frac{1 \wedge 0.9}{0 \vee 0.8} + \frac{1 \wedge 1}{0 \vee 0.5} + \frac{1 \wedge 0.5}{0 \vee 0.4}$$
$$= \{((x_2, 0.8), 0.9), ((x_2, 0.5), 1), ((x_2, 0.4), 0.5)\}$$

$$\tilde{\tilde{\varnothing}} \cup \tilde{\tilde{A}} = \{((x_1, 0.8), 1), ((x_1, 0.6), 0.7), ((x_1, 0.3), 0.6),$$
$$((x_2, 0.8), 0.9), ((x_2, 0.5), 1), ((x_2, 0.4), 0.5)\} = \tilde{\tilde{A}}$$

$$\tilde{\tilde{\varnothing}} \cap \tilde{\tilde{A}} \text{ for } x_1 : \mu_{\tilde{\tilde{\varnothing}} \cap \tilde{\tilde{A}}}(x_1) = \frac{1 \wedge 1}{0 \wedge 0.8} + \frac{1 \wedge 0.7}{0 \wedge 0.6} + \frac{1 \wedge 0.6}{0 \wedge 0.3} = \frac{1}{0} + \frac{0.7}{0} + \frac{0.6}{0}$$
$$= (0, \max\{1, 0.7, 0.6\}) \Rightarrow \{((x_1, 0), 1)\},$$

$$\tilde{\tilde{\varnothing}} \cap \tilde{\tilde{A}} \text{ for } x_2 : \mu_{\tilde{\tilde{\varnothing}} \cap \tilde{\tilde{A}}}(x_2) = \frac{1 \wedge 0.9}{0 \wedge 0.8} + \frac{1 \wedge 1}{0 \wedge 0.5} + \frac{1 \wedge 0.5}{0 \wedge 0.4} = \frac{0.9}{0} + \frac{1}{0} + \frac{0.5}{0}$$
$$= (0, \max\{0.9, 1, 0.5\}) \Rightarrow \{((x_2, 0), 1)\},$$

$$\tilde{\tilde{\varnothing}} \cap \tilde{\tilde{A}} = \{((x_1, 0), 1), ((x_2, 0), 1)\} = \tilde{\tilde{\varnothing}}$$

$$\tilde{\tilde{A}} \cup \tilde{\tilde{X}} \text{ for } x_1 : \mu_{\tilde{\tilde{A}} \cup \tilde{\tilde{X}}}(x_1) = \frac{1 \wedge 1}{1 \vee 0.8} + \frac{1 \wedge 0.7}{1 \vee 0.6} + \frac{1 \wedge 0.6}{1 \vee 0.3} = \frac{1}{1} + \frac{0.7}{1} + \frac{0.6}{1}$$
$$= (1, \max\{1, 0.7, 0.6\}) \Rightarrow \{((x_1, 1), 1)\},$$

$$\tilde{\tilde{A}} \cup \tilde{\tilde{X}} \text{ for } x_2 : \mu_{\tilde{\tilde{A}} \cup \tilde{\tilde{X}}}(x_2) = \frac{1 \wedge 0.9}{1 \vee 0.8} + \frac{1 \wedge 1}{1 \vee 0.5} + \frac{1 \wedge 0.5}{1 \vee 0.4} = \frac{0.9}{1} + \frac{1}{1} + \frac{0.5}{1}$$
$$= (1, \max\{1, 0.9, 0.5\}) \Rightarrow \{((x_2, 1), 1)\}$$

$$\tilde{\tilde{A}} \cup \tilde{\tilde{X}} = \tilde{\tilde{X}}$$

$$\tilde{\tilde{A}} \cap \tilde{\tilde{X}} \text{ for } x_1 : \mu_{\tilde{\tilde{A}} \cap \tilde{\tilde{X}}}(x_1) = \frac{1 \wedge 1}{1 \wedge 0.8} + \frac{1 \wedge 0.7}{1 \wedge 0.6} + \frac{1 \wedge 0.6}{1 \wedge 0.3} = \frac{1}{0.8} + \frac{0.7}{0.6} + \frac{0.6}{0.3}$$
$$= \{((x_1, 0.8), 1), ((x_1, 0.6), 0.7), ((x_1, 0.3), 0.6)\}$$

$$\tilde{\tilde{A}} \cap \tilde{\tilde{X}} = \{((x_1, 0.8), 1), ((x_1, 0.6), 0.7), ((x_1, 0.3), 0.6),$$
$$((x_2, 0.8), 0.9), ((x_2, 0.5), 1), ((x_2, 0.4), 0.5)\} = \tilde{\tilde{A}}$$

Then $\tilde{\tilde{\mathfrak{F}}} = \left\{ \tilde{\tilde{X}}, \tilde{\tilde{\varnothing}}, \tilde{\tilde{A}} \right\}$ is general type-2 fuzzy topologies defined on X and the pair $\left(X, \tilde{\tilde{\mathfrak{F}}} \right)$ is called general type-2 fuzzy topological space over X, every member of $\tilde{\tilde{\mathfrak{F}}}$ is called type-2 fuzzy open sets.

Theorem 5: Let $\left\{ \tilde{\tilde{\mathfrak{F}}}_r : r \in \mathbb{R} \right\}$ be a family of all general type-2 fuzzy topologies on X; then $\cap_{r \in \mathbb{R}} \tilde{\tilde{\mathfrak{F}}}_r$ is general type-2 fuzzy topologies on X.

proof: we must prove three conditions of topologies,

1) $\tilde{\tilde{\varnothing}}, \tilde{\tilde{X}} \in \left\{ \tilde{\tilde{\mathfrak{F}}}_r : r \in \mathbb{R} \right\} \Rightarrow \tilde{\tilde{\varnothing}}, \tilde{\tilde{X}} \in \cap_{r \in \mathbb{R}} \tilde{\tilde{\mathfrak{F}}}_r$.

2) Let $\left\{ \tilde{\tilde{A}}_i : i \in \mathbb{N} \right\} \subseteq \cap_{r \in \mathbb{R}} \tilde{\tilde{\mathfrak{F}}}_r$, then $\tilde{\tilde{A}}_i \in \tilde{\tilde{\mathfrak{F}}}_r$ for all $i \in \mathbb{N}$ so thus $\cup_{i \in \mathbb{N}} \tilde{\tilde{A}}_i \in \cap_{r \in \mathbb{R}} \tilde{\tilde{\mathfrak{F}}}_r$.

3) Let $\tilde{\tilde{A}}, \tilde{\tilde{B}} \in \cap_{r \in \mathbb{R}} \tilde{\tilde{\mathfrak{F}}}_r$, then $\tilde{\tilde{A}}, \tilde{\tilde{B}} \in \tilde{\tilde{\mathfrak{F}}}_r$ and because $\tilde{\tilde{\mathfrak{F}}}_r$ are all general type-2

fuzzy topologies $\tilde{\tilde{A}} \cap \tilde{\tilde{B}} \in \tilde{\tilde{\mathfrak{F}}}_r$ for all $r \in \mathbb{R}$, so $\tilde{\tilde{A}} \cap \tilde{\tilde{B}} \in \cap_{r\in\mathbb{R}} \tilde{\tilde{\mathfrak{F}}}_r$.

Remark 6: Let $\left(X, \tilde{\tilde{\mathfrak{F}}}_1\right)$ *and* $\left(X, \tilde{\tilde{\mathfrak{F}}}_2\right)$ be two general type-2 fuzzy topological spaces over the same universe X then $\left(X, \tilde{\tilde{\mathfrak{F}}}_1 \cup \tilde{\tilde{\mathfrak{F}}}_2\right)$ need not be general type-2 fuzzy topological space over *X*, we can see that in example 3.7.

Example 7: Let $X = \{x_1, x_2\}$ and $\tilde{\tilde{\mathfrak{F}}}_1 = \left\{\tilde{\tilde{X}}, \tilde{\tilde{\varnothing}}, \tilde{\tilde{A}}\right\}$, $\tilde{\tilde{\mathfrak{F}}}_2 = \left\{\tilde{\tilde{X}}, \tilde{\tilde{\varnothing}}, \tilde{\tilde{B}}\right\}$ be two general type-2fuzzy topologies defined on X where $\tilde{\tilde{A}}, \tilde{\tilde{B}}, \tilde{\tilde{\varnothing}}$ and $\tilde{\tilde{X}}$ defined as follows: $\tilde{\tilde{\varnothing}} = \{((x_1,0),1),((x_2,0),1)\}$,

$$\tilde{\tilde{X}} = \{((x_1,1),1),((x_2,1),1)\}$$

$$\tilde{\tilde{A}} = \{((x_1,0.8),1),((x_1,0.6),0.7),((x_1,0.3),0.6),$$
$$((x_2,0.8),0.9),((x_2,0.5),1),((x_2,0.4),0.5)\}.$$

$$\tilde{\tilde{B}} = \{((x_1,0.5),1),((x_1,0.6),0.2),((x_2,0.3),0.7),((x_2,0.9),1)\}.$$

Let $\tilde{\tilde{\mathfrak{F}}}_1 \cup \tilde{\tilde{\mathfrak{F}}}_2 = \left\{\tilde{\tilde{\varnothing}}, \tilde{\tilde{X}}, \tilde{\tilde{A}}, \tilde{\tilde{B}}\right\}$ so $\left(X, \tilde{\tilde{\mathfrak{F}}}_1 \cup \tilde{\tilde{\mathfrak{F}}}_2\right)$ is not general type-2 fuzzy topological space over X since $\tilde{\tilde{A}} \cap \tilde{\tilde{B}} \notin \tilde{\tilde{\mathfrak{F}}}_1 \cup \tilde{\tilde{\mathfrak{F}}}_2$.

Definition 8: Let $\left(X, \tilde{\tilde{\mathfrak{F}}}\right)$ be general type-2 fuzzy topological space over X and let $\tilde{\tilde{A}}$ be type-2 fuzzy set over *X*. Then the type-2 fuzzy interior of $\tilde{\tilde{A}}$, denoted by $\operatorname{int}\left(\tilde{\tilde{A}}\right)$, is defined as the union of all type-2 fuzzy open sets contained in $\tilde{\tilde{A}}$. That is,

$\operatorname{int}\left(\tilde{\tilde{A}}\right) = \cup\left\{\tilde{\tilde{G}}_i : \tilde{\tilde{G}}_i \text{ type-2 fuzzy open sets in } X, \tilde{\tilde{G}}_i \subseteq \tilde{\tilde{A}}, i \in \mathbb{N}\right\}$, $\operatorname{int}\left(\tilde{\tilde{A}}\right)$ is the largest type-2 fuzzy open set contained in $\tilde{\tilde{A}}$.

Theorem 9: Let $\left(X, \tilde{\tilde{\mathfrak{F}}}\right)$ be general type-2 fuzzy topological space over *X*, and let $\tilde{\tilde{A}}, \tilde{\tilde{B}}$ be two type-2 fuzzy sets in *X*. Then

1) $\operatorname{int}\left(\tilde{\tilde{\varnothing}}\right) = \tilde{\tilde{\varnothing}}$ and $\operatorname{int}\left(\tilde{\tilde{X}}\right) = \tilde{\tilde{X}}$.

2) $\operatorname{int}\left(\tilde{\tilde{A}}\right) \subseteq \tilde{\tilde{A}}$.

3) $\tilde{\tilde{A}}$ is type-2 fuzzy open set if and only if $\operatorname{int}\left(\tilde{\tilde{A}}\right) = \tilde{\tilde{A}}$.

4) $\operatorname{int}\left(\operatorname{int}\left(\tilde{\tilde{A}}\right)\right) = \operatorname{int}\left(\tilde{\tilde{A}}\right)$.

5) $\tilde{\tilde{A}} \subseteq \tilde{\tilde{B}} \to \operatorname{int}\left(\tilde{\tilde{A}}\right) \subseteq \operatorname{int}\left(\tilde{\tilde{B}}\right)$.

6) $\operatorname{int}\left(\tilde{\tilde{A}} \cap \tilde{\tilde{B}}\right) = \operatorname{int}\left(\tilde{\tilde{A}}\right) \cap \operatorname{int}\left(\tilde{\tilde{B}}\right)$.

Proof:

1) $\operatorname{int}\left(\tilde{\tilde{A}}\right) = \cup\left\{\tilde{\tilde{G}}_i : \tilde{\tilde{G}}_i \text{ type-2 fuzzy open sets in } X, \tilde{\tilde{G}}_i \subseteq \tilde{\tilde{A}}, i \in \mathbb{N}\right\}$, $\tilde{\tilde{\varnothing}}$ is type-2 fuzzy open set in $\tilde{\tilde{\mathfrak{F}}}$ and $\tilde{\tilde{\varnothing}} \subseteq \tilde{\tilde{\varnothing}} \Rightarrow \operatorname{int}\left(\tilde{\tilde{\varnothing}}\right) = \tilde{\tilde{\varnothing}}$.

Now to prove $\operatorname{int}\left(\tilde{\tilde{X}}\right)=\tilde{\tilde{X}}$,

$\operatorname{int}\left(\tilde{\tilde{X}}\right)=\cup\left\{\tilde{\tilde{G}}_i:\tilde{\tilde{G}}_i \text{ type-2 fuzzy open sets in } X,\tilde{\tilde{G}}_i\subseteq\tilde{\tilde{X}},i\in\mathbb{N}\right\}$, $\tilde{\tilde{X}}$ is type-2 fuzzy open set in $\tilde{\tilde{\mathfrak{F}}}$ and $\tilde{\tilde{X}}\subseteq\tilde{\tilde{X}}\Rightarrow\operatorname{int}\left(\tilde{\tilde{X}}\right)=\tilde{\tilde{X}}$.

2) To prove $\operatorname{int}\left(\tilde{\tilde{A}}\right)\subseteq\tilde{\tilde{A}}$, since

$\operatorname{int}\left(\tilde{\tilde{A}}\right)=\cup\left\{\tilde{\tilde{G}}_i:\tilde{\tilde{G}}_i \text{ type-2 fuzzy open sets in } X,\tilde{\tilde{G}}_i\subseteq\tilde{\tilde{A}},i\in\mathbb{N}\right\}$, such that $\tilde{\tilde{G}}_i\subseteq\tilde{\tilde{A}}$ that is $\tilde{\tilde{A}}$ is type-2 membership function $\mu_{\tilde{\tilde{A}}}(x,u)$ where $x\in X$ and $u\in J_X\subseteq[0,1]$ less than a type-2 membership function $\mu_{\tilde{\tilde{G}}_i}(x,u)$ where $x\in X$ and $w\in J_X\subseteq[0,1]$ such that $w\le u$ and $\mu_{\tilde{\tilde{G}}_i}(x,u)\le\mu_{\tilde{\tilde{A}}}(x,u)$, $\sup\left\{\mu_{\tilde{\tilde{G}}_i}(x,u)\le\mu_{\tilde{\tilde{A}}}(x,u),w\le u\right\}$ hence $\cup\tilde{\tilde{G}}_i\subseteq\tilde{\tilde{A}}\Rightarrow\cup\tilde{\tilde{G}}_i\subseteq\operatorname{int}\left(\tilde{\tilde{A}}\right)$, therefore $\operatorname{int}\left(\tilde{\tilde{A}}\right)\subseteq\tilde{\tilde{A}}$.

3) If $\tilde{\tilde{A}}$ is type-2 fuzzy open set, then $\tilde{\tilde{A}}\subseteq\operatorname{int}\left(\tilde{\tilde{A}}\right)$, but $\operatorname{int}\left(\tilde{\tilde{A}}\right)\subseteq\tilde{\tilde{A}}$ from part (2), hence $\operatorname{int}\left(\tilde{\tilde{A}}\right)=\tilde{\tilde{A}}$.

4) $\operatorname{int}\left(\tilde{\tilde{A}}\right)$ is a type-2 fuzzy open set and from part (3) we have $\operatorname{int}\left(\operatorname{int}\left(\tilde{\tilde{A}}\right)\right)=\operatorname{int}\left(\tilde{\tilde{A}}\right)$

5) If $\tilde{\tilde{A}}\subseteq\tilde{\tilde{B}}$ and from part(2) $\operatorname{int}\left(\tilde{\tilde{A}}\right)\subseteq\tilde{\tilde{A}}$, $\operatorname{int}\left(\tilde{\tilde{B}}\right)\subseteq\tilde{\tilde{B}}$, then $\operatorname{int}\left(\tilde{\tilde{A}}\right)\subseteq\tilde{\tilde{A}}\subseteq\tilde{\tilde{B}}$. Therefore $\operatorname{int}\left(\tilde{\tilde{A}}\right)\subseteq\tilde{\tilde{B}}$ and $\operatorname{int}\left(\tilde{\tilde{A}}\right)$ is a type-2 fuzzy open set contained in $\tilde{\tilde{B}}$, so $\operatorname{int}\left(\tilde{\tilde{A}}\right)\subseteq\operatorname{int}\left(\tilde{\tilde{B}}\right)$.

6) Because $\left(\tilde{\tilde{A}}\cap\tilde{\tilde{B}}\right)\subseteq\tilde{\tilde{A}}$ and $\left(\tilde{\tilde{A}}\cap\tilde{\tilde{B}}\right)\subseteq\tilde{\tilde{B}}$, from part (5) $\operatorname{int}\left(\tilde{\tilde{A}}\cap\tilde{\tilde{B}}\right)\subseteq\operatorname{int}\left(\tilde{\tilde{A}}\right)$ and $\operatorname{int}\left(\tilde{\tilde{A}}\cap\tilde{\tilde{B}}\right)\subseteq\operatorname{int}\left(\tilde{\tilde{B}}\right)$, thus $\operatorname{int}\left(\tilde{\tilde{A}}\cap\tilde{\tilde{B}}\right)\subseteq\operatorname{int}\left(\tilde{\tilde{A}}\right)\cap\operatorname{int}\left(\tilde{\tilde{B}}\right)$, since $\operatorname{int}\left(\tilde{\tilde{A}}\cap\tilde{\tilde{B}}\right)\subseteq\tilde{\tilde{A}}\cap\tilde{\tilde{B}}$, so $\operatorname{int}\left(\operatorname{int}\left(\tilde{\tilde{A}}\right)\right)\cap\operatorname{int}\left(\tilde{\tilde{B}}\right)\subseteq\left(\tilde{\tilde{A}}\cap\tilde{\tilde{B}}\right)$ from part(5) but $\operatorname{int}\left(\tilde{\tilde{A}}\right)\cap\operatorname{int}\left(\tilde{\tilde{B}}\right)$ is a type-2 fuzzy open sets then $\operatorname{int}\left(\operatorname{int}\left(\tilde{\tilde{A}}\right)\right)\cap\operatorname{int}\left(\tilde{\tilde{B}}\right)\subseteq\operatorname{int}\left(\tilde{\tilde{A}}\cap\tilde{\tilde{B}}\right)$ from part*(3)*.Hence $\operatorname{int}\left(\tilde{\tilde{A}}\cap\tilde{\tilde{B}}\right)=\operatorname{int}\left(\tilde{\tilde{A}}\right)\cap\operatorname{int}\left(\tilde{\tilde{B}}\right)$.

Definition 10: Let $\left(X,\tilde{\tilde{\mathfrak{F}}}\right)$ be general type-2 fuzzy topological space over $\tilde{\tilde{X}}$ and let $\tilde{\tilde{A}}$ be type-2 fuzzy set over *X*. Then the type-2 fuzzy closure of $\tilde{\tilde{A}}$, denoted by $cl\left(\tilde{\tilde{A}}\right)$, is defined as the intersection of all type-2 fuzzy closed sets containing $\tilde{\tilde{A}}$. That is

$$cl\left(\tilde{\tilde{A}}\right)=\cap\left\{\tilde{\tilde{M}}_i:\tilde{\tilde{M}}_i \text{ type-2 fuzzy closed sets in } X,\tilde{\tilde{A}}\subseteq\tilde{\tilde{M}}_i,i\in\mathbb{N}\right\},$$

$cl\left(\tilde{\tilde{A}}\right)$ is the smallest type-2 fuzzy closed set containing $\tilde{\tilde{A}}$.

Theorem 11: Let $\left(X,\tilde{\tilde{\mathfrak{F}}}\right)$ be general type-2 fuzzy topological space over *X*, and let $\tilde{\tilde{A}},\tilde{\tilde{B}}$ be two type-2 fuzzy sets in *X*. Then

1) $cl\left(\tilde{\tilde{\varnothing}}\right)=\tilde{\tilde{\varnothing}}$ and $cl\left(\tilde{\tilde{X}}\right)=\tilde{\tilde{X}}$.

2) $\tilde{\tilde{A}}\subseteq cl\left(\tilde{\tilde{A}}\right)$.

3) $\tilde{\tilde{A}}$ is type-2 fuzzy closed set if and only if $cl\left(\tilde{\tilde{A}}\right)=\tilde{\tilde{A}}$.

4) $cl\left(cl\left(\tilde{\tilde{A}}\right)\right)=cl\left(\tilde{\tilde{A}}\right)$.

5) $\tilde{\tilde{A}}\subseteq\tilde{\tilde{B}}\to cl\left(\tilde{\tilde{A}}\right)\subseteq cl\left(\tilde{\tilde{B}}\right)$.

6) $cl\left(\tilde{\tilde{A}}\cap\tilde{\tilde{B}}\right)=cl\left(\tilde{\tilde{A}}\right)\cap cl\left(\tilde{\tilde{B}}\right)$.

Proof: The proof this theorem similar to the proof of theorem 3.7.

Definition 12: Let $\left(X,\tilde{\tilde{\mathfrak{F}}}\right)$ be a general type-2 fuzzy topological space over X and $\tilde{\tilde{N}}\subseteq\tilde{\tilde{\mathfrak{F}}}$. Then is said to be a neighborhood or nbhd for short, of a type-2 fuzzy set $\tilde{\tilde{A}}$ if there exist a type-2 fuzzy open set $\tilde{\tilde{W}}$ such that $\tilde{\tilde{A}}\subseteq\tilde{\tilde{W}}\subseteq\tilde{\tilde{N}}$.

Proposition 13: A type-2 fuzzy set $\tilde{\tilde{A}}$ is open if and only if for each type-2 fuzzy set $\tilde{\tilde{B}}$ contained in $\tilde{\tilde{A}}$, $\tilde{\tilde{A}}$ is a neighborhood of $\tilde{\tilde{B}}$.

Proof: If $\tilde{\tilde{A}}$ is open and $\tilde{\tilde{B}}\subseteq\tilde{\tilde{A}}$ then $\tilde{\tilde{A}}$ is a neighborhood of $\tilde{\tilde{B}}$. Conversely, since $\tilde{\tilde{A}}\subseteq\tilde{\tilde{A}}$, there exists a type-2 fuzzy open set $\tilde{\tilde{W}}$ such that $\tilde{\tilde{A}}\subseteq\tilde{\tilde{W}}\subseteq\tilde{\tilde{A}}$. Hence $\tilde{\tilde{A}}=\tilde{\tilde{W}}$ and $\tilde{\tilde{A}}$ is open.

Definition 14: Let $\left(X,\tilde{\tilde{\mathfrak{F}}}\right)$ be a general type-2 fuzzy topological space over X and $\tilde{\tilde{\mathfrak{B}}}$ be a subfamily of $\tilde{\tilde{\mathfrak{F}}}$. If every member of $\tilde{\tilde{\mathfrak{F}}}$ can be written as the type-2 fuzzy union of some members of $\tilde{\tilde{\mathfrak{B}}}$, then $\tilde{\tilde{\mathfrak{B}}}$ is called a type-2 fuzzy base for the general type-2 fuzzy topology $\tilde{\tilde{\mathfrak{F}}}$. We can see that if $\tilde{\tilde{\mathfrak{B}}}$ be type-2 fuzzy base for $\tilde{\tilde{\mathfrak{F}}}$ then $\tilde{\tilde{\mathfrak{F}}}$ equals the collection of type-2 fuzzy unions of elements of $\tilde{\tilde{\mathfrak{B}}}$.

Definition 15: Let $\left(X,\tilde{\tilde{\mathfrak{F}}}\right)$ and $\left(Y,\tilde{\tilde{\mathfrak{S}}}\right)$ be two general type-2 fuzzy topological space.The general type-2 fuzzy topological space Y is called a subspace of the general type-2 fuzzy topological space X if $Y\subseteq X$ and the open subsets of Y are precisely of the form $\tilde{\tilde{\mathfrak{F}}}_{\tilde{\tilde{Y}}}=\left\{\tilde{\tilde{\mathcal{Y}}}=\tilde{\tilde{Y}}\cap\tilde{\tilde{\mathcal{X}}}:\tilde{\tilde{\mathcal{X}}}\in\tilde{\tilde{\mathfrak{F}}}\right\}$. Here we may say that each open subset $\tilde{\tilde{\mathcal{Y}}}$ of Y is the restriction to $\tilde{\tilde{\mathcal{Y}}}$ of an open subset $\tilde{\tilde{\mathcal{X}}}$ of *X*. That is, $\left(Y,\tilde{\tilde{\mathfrak{S}}}\right)$ is called a subspace of $\left(X,\tilde{\tilde{\mathfrak{F}}}\right)$ if the type-2 fuzzy open sets of Y are the type-2 fuzzy intersection of open sets of X with $\tilde{\tilde{\mathcal{Y}}}$.

4. Conclusion

The main purpose of this paper is to introduce a new concept in fuzzy set theory, namely that of general type-2 fuzzy topological space. On the other hand, type-2 fuzzy set is a kind of abstract theory of mathematics. First, we present definition and properties of this set before introducing definition of general type-2 fuzzy topological space with the structural properties such as open sets, closed sets, interior, closure and neighborhoods in general type-2 fuzzy set topological spaces and some definitions of a type-2 fuzzy base and subspace of general type-2 fuzzy sets.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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