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Endogenous Ranking in the Two-Sector Urn-Ball Matching Process

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Abstract

This paper contributes to the debate concerning micro-foundations of matching functions in frictional labor markets. The focus is on a particular matching regime, *i.e.*, the so-called urn-ball process. It is shown that in a two-sector economy, even in the presence of heterogeneous workers, the assumption of applicants-ranking may be misleading. Instead, the choice concerning the adoption of either ranking or no-ranking behavior is endogenous and it is affected by both the tightness of the two sectors and the composition of the labor force in terms of skills. Moreover it is proved that exogenous shocks may change the form of the matching function. This result casts additional doubts on the assumption of exogenous matching function often made in empirical works aimed at assessing the effectiveness of policy measures.

Keywords

Matching Function, Urn-Ball Model, Bayesian Nash Equilibrium

1. Introduction

Matching functions represent an important tool that allows labor economists to model employment out-flows and in-flows in the presence of frictional labor markets (see [1] [2] [3]). In theoretical models, the functional form of matching functions has generally been assumed to satisfy some desirable properties such as concavity in the arguments and constant returns to scale. Recently, the issue of their micro-foundation has attracted some researchers' attention. Among others, [4] and [5] highlight that the assumed function should be consistent with labor market behavior of firms and workers. Furthermore, [6] and [7] show that agents' behavior can be affected by labor market policies and institutions so that the matching function turns out to be endogenous.

This work contributes to this issue by modeling a frictional labor market where the form of the matching function is endogenous, *i.e.*, it is the result of optimal behavior of firms and workers. The focus is on a particular matching regime known as urn-ball process analyzed by [8] and [9] among the first and it considers specifically the case of applicant-ranking focusing on the rationale of its microfunadation. The urn-ball process, that nowadays is a popular mechanism among labor economists, has proved to be a convenient instrument to describe the labor market when workers are heterogeneous since it makes possible to specify individuals' exit rate from unemployment as a function of their own characteristics [10]. This study considers the case of a urn-ball process in the presence of heterogeneous workers operating in a perfectly segmented two-sector economy. In particular, following [11], the economy is characterized by skilled (high-tech) and unskilled (low-tech) sector. Agents are heterogeneous and have to decide the sector they want to enter. Once the entry decisions have been taken, the pure matching process starts following the lines set out by standard matching models. Differently from [11], the present paper does not assume *ex-ante* that firms rank amongst the applications they receive. Instead, the ranking decision is left to be determined by agents' optimal actions. Using this framework, it is shown that, although in the presence of heterogeneous workers, the assumption of ranking may seem obvious, there can be standard economic environments where the specific form of the hiring process results from a more complex strategic behavior. The rationale behind this result turns out to be straightforward when the issue of sector *tightness* in terms of labor supply and demand is taken into account along with the *composition* of the labor force in terms of productivity. Indeed, firms set the hiring behavior to maximize their expected payoff which depends on both the productivity of employees and the probability of filling vacancies. Therefore, in the presence of a tight market, the adoption of no-ranking may be suitable for firms as far as it increases the labor supply and, consequently, the firm's expected value.

The main implication of this model is that the resulting form of the urn-ball matching function is endogenous and it is shaped by agents' microeconomic behavior. A corollary of this finding is that exogenous shocks influencing agents' behavior might shape the analytical form of the matching function. This result is undoubtedly important for empirical works aimed at evaluating policies affecting matching process. In particular, [7] argues that random shocks to matching efficiency determine the number of matches formed both directly through the matching technology and indirectly through firms' vacancy-posting behavior. From an empirical point of view, this means that simple OLS regressions between the number of job matches and that of job seekers and vacancies fail to account for that endogeneity and deliver misleading predictions. The present paper shows that the parameter-bias problem arising when estimating matching function elasticities may be even more severe since the entire functional form of the matching function can be affected by exogenous policy measures. This means that the empirical assessment of the effect of labor policy on job matches turns out to be a particularly difficult task requiring appropriate econometric strategies.

The outline of the article is as follows. In Section 2 a brief summary of the existing literature on the urn-ball process is presented. Section 3 sets up the theoretical model and Section 4 evaluates the equilibria discussing the endogeneity of the hiring regime. Conclusions are presented in Section 5.

2. Existing Background

2.1. The Basic Framework

In its simplest version, the urn-ball matching function can be described as follows. The economy is assumed to have homogeneous firms and workers searching for each other in the labor market. There is a coordination failure arising because workers simultaneously apply for jobs not knowing where other workers send their applications. This implies that some vacancies may remain unfilled, while others may get one or more applications. When firms receive more than one application they can choose randomly among applicants. As discussed in details in [8] and [9] this process can be described as an urn-ball process where firms are urns and workers are balls. Hence, by indicating with $a(.)$ the probability that a worker receives a job offer, it can be shown that this follows a Poisson process with:

$$a(\lambda)=\frac{1-\exp^{-\lambda}}{\lambda} \tag{1}$$

where $\lambda=U/V$ indicates the tightness of the labor market given by the ratio between the number of workers looking for a job (U) and available vacant jobs (V). By indicating with M the number of matches in the labor market, the matching function is given by:

$$M=a(\lambda)U. \tag{2}$$

2.2. Extensions

Equations ((1), (2)) have been enriched in several directions. On the one hand, [12] and [13] allow for multiple applications of job seekers. The authors prove that, although with multiple applications it is very likely that every vacancy will get at least one application, still a coordination failure may arise because of competition among firms for single candidates. In fact, an applicant may receive more than one job offer and vacancies may remain unfilled because the chosen candidate is hired away by a competing firm. As a consequence, allowing for more applications per worker may not increase the matching efficiency. On the other hand, [10] [11] and [14] model situations in which workers are heterogeneous in terms of their productivity and unemployment experience. This implies that when firms receive more than one application, they are not indifferent among applicants, hence they do not choose randomly. These authors *assume* that firms rank applicants according to their productivity. In this case, the probability that a worker receives a job offer is a function of his/her characteristics. By indicating with θ the individual productivity and assuming θ distributed according to a continuous and strictly increasing cumulative distribution $\Gamma(\theta)$, whose density function is $\gamma(\theta)$, over a support $\left[\underline{\theta},\overline{\theta}\right]$ where $1 \le \underline{\theta} < \overline{\theta}$ (so $\Gamma(\underline{\theta}) = 0$ and $\Gamma(\overline{\theta}) = 1$), the probability that a θ-type worker receives a job offer can be written as follows:

$$a(\lambda,\theta) = \exp^{-[1-\Gamma(\theta)]\lambda} \tag{3}$$

where the probability of receiving the offer increases with individual ability ($\frac{\partial a(\lambda,\theta)}{\partial\theta} > 0$) and if $\theta = \overline{\theta}$ then $a(\lambda,\theta)$ has a unit value since $\overline{\theta}$-types get any job they apply for. By integrating $a(\lambda,\theta)$ over $\left[\underline{\theta},\overline{\theta}\right]$, it is possible to obtain the unconditional probability of being hired, called $\mathbf{a}(\lambda)$. Therefore the matching function can be written as:

$$M = \mathbf{a}(\lambda)U. \tag{4}$$

[11] and [14] present analytical derivations of the previous expressions. [10] set out the conditions under which Equation (3) applies in a continuous time setting—as that presented in this paper—giving rising to a steady-state unemployment equilibrium.

The present paper shows that in a two-sector model the choice between Equations ((1) and (3)) should be solved endogenously. It is proved that both specifications can be consistent with a profit maximizing behavior conditional upon labor market institutions. Furthermore, it is discussed that some policies may induce a switch from (1) to (3) and vice-versa, implying instability of the matching function.

3. The Model

3.1. Overview

Consider an economy characterized by a *continuum* of risk-neutral individuals and firms matching in the labor market following the lines set out by Diamond-Mortensen-Pissarides. Before entering the job-market, it is assumed that these agents have to make a choice concerning the sector they want to enter. According to [11], there are two sectors in this economy: Skilled (high-tech) and unskilled (low-tech) sector. The skilled sector is characterized by workers who invested in human capital and by firms with (costly) high technology. Conversely, no particular investment is required to firms and workers in order to enter the unskilled sector. The mass and the distribution of agents, defined in due course, remain constant over time. As discussed in details in the next paragraph, individuals are assumed to be heterogeneous with respect to their pre-university individual skills which determine their productivity on the job and affect the cost of entering the skilled sector.[1] From now on, these individual

[1]This assumption is well supported by the existing empirical evidence. Among others, [15] show that innate talent, family background, and social environment represent elements that might shape schooling results promoting cognitive and noncognitive ability and they have long-run effects in terms of labor market outcomes.

characteristics are simply defined as ability. On the demand side, each firm can post a limited number of vacancies, normalized to 1, and it decides the sector where posting the vacancy on the basis of a technological choice. In particular, a firm can choose to operate either within the high- or the low-technological sector. In order to simplify notation, from now on this paper refers to skilled *versus* unskilled choice for both firms and individuals. However the reader should keep in mind that individuals make an educational choice while firms take a technological decision. Once the educational/technological choices have been made, the pure matching-process starts. As in [11], the two sectors are assumed to be perfectly segmented, *i.e.*, skilled and unskilled workers can be matched only with high-tech and low-tech firms respectively.

3.2. Individuals

Consider a *continuum* of individuals of mass 1. According to the notation introduced in Section 2, individuals are characterized by heterogeneous individual ability θ. $\Gamma(\theta)$ and $\gamma(\theta)$ are the c.d.f. and p.d.f. respectively and both are assumed to be stationary over time. Indicate with $e=\{s,us\}$ the educational choice made by individuals in order to maximize their expected discounted utility (*s* stands for skilled while *us* stands for unskilled). For the sake of simplicity individuals are assumed to have no income if unemployed (no unemployment benefits). As a consequence, once the educational choice has been made, in each instant of time the individual's utility function $W(e)$ is given by:

$$W(e)=\begin{cases} 0 & \text{if unemployed} \\ w_{us} & \text{if unskilled and employed} \\ w_s & \text{if skilled and employed} \end{cases} \tag{5}$$

where w_{us} and w_s indicate wage for employed unskilled and skilled workers respectively. The cost of acquiring education *us* is normalized to zero while, when individuals decide to acquire education *s*, on top of monetary costs, they have to sustain a cost $c(\theta)>0$ related to their individual ability with $\frac{\partial c}{\partial \theta}<0$. Monetary costs are assumed to be the same for all individuals, while the effort required to achieve a degree qualification is determined by personal ability. From now on, $\left|\frac{\partial c}{\partial \theta}\right|$ indicates a measure of the *selectivity* of the higher education sector. In words, the more the cost of education rises when ability decreases the more selective may be considered the higher education sector. It will be shown that the selectivity of the higher education system shapes the tightness of the two sectors and affects firms' optimal behavior in terms of ranking.

3.3. Firms

Consider a *continuum* of firms of mass 1. Indicate with $T=\{s,us\}$ firm's investment in skilled and unskilled vacancies respectively. The cost of entering the *s* sector is given by $\delta > 0$. The cost of entering the *us* sector is normalized to zero.[2] Firms are assumed to be heterogeneous with respect to the cost they have to sustain in order to enter the *s* sector. In fact, in the growth theory literature, the cost of advanced technology has been considered typically related to the actual firm's technological endowment. The closer is a firm to the technological frontier the lower is the cost it needs to sustain in order to update its technology (see [17]). In the present model, firms are assumed to be distributed according to a continuous and strictly increasing cumulative distribution $\Phi(\delta)$ whose density function is $\phi(\delta)$, over a support $\left[\underline{\delta}, \overline{\delta}\right]$ where $0 < \underline{\delta} < \overline{\delta}$ (so $\Phi(\underline{\delta}) = 0$ and $\Phi(\overline{\delta}) = 1$). $\Phi(.)$ and $\phi(.)$ are stationary over time.

Following [18], the production function is given by:

$$y = y(e, T, \theta) = \begin{cases} \overline{y} & \text{if } T = us \text{ and } e = us \\ \theta \overline{y} & \text{if } T = s \text{ and } e = us \end{cases} \tag{6}$$

where $\overline{y} > 0$ is a constant. Relation (6) indicates that there is homogeneity in the unskilled sector, *i.e.*, when individuals work in the *us* sector they produce an output $\overline{y}$ independently on their ability. Conversely, skilled technologies are complementary only to skilled workers and the intensity of such complementarity is given by individual's ability θ. In fact, in Equation (6) *skill-ability complementary* technology has been assumed. This conjecture regarding the centrality of the positive interaction between technologies and ability is largely consistent with the empirical evidence.[3] Finally, Q indicates the cost of maintaining a vacancy $\forall T$, and it is assumed that in the steady-state vacancies yield zero profit (free-entry condition).[4] Once the technological decision has been made, in each instant of time each firm realizes a profit $\Pi(T)$ given by:

$$\Pi(T) = \begin{cases} -Q & \text{if unfilled vacancy } \forall T \\ \overline{y} - w_{us} - Q & \text{if filled } us \text{ vacancy} \\ \theta \overline{y} - w_s - Q & \text{if filled } s \text{ vacancy.} \end{cases} \tag{7}$$

3.4. Interaction Process and Bellman Equations

The interaction process evaluated in this paper consists in the following stages. At the first stage, individuals and firms conditional on their own type (ability and distance to the frontier) simultaneously decide the sector they want to enter, *i.e.*, they choose between skilled and unskilled sectors. Also at this stage, firms set out the ranking behavior they want to adopt. Once the educational/technological choices have been made and the hiring process has been established,

[2]This assumption may easily be justified by thinking that in order to enter the graduate sector; firms are required to have costly technological endowment that should be used by engineers, doctors, investors, etc.; while low-skills complementary machines are typically less costly. See [16] on this argument.

[3]Among others, [19] find that the education premium in the US over the period 1979-1993 is the result of an increase in demand for innate ability or other unobserved characteristics of more educated workers.

[4]We could assume $Q_s \neq Q_{us}$. However, by assuming $Q_s = Q_{us} = Q$ we simplify the notation and, because of free-entry condition, this does not affect our main results.

individuals and firms enter the labor market as unemployed and with unfilled vacancies respectively, and then the matching process starts. Finally, when a match is realized, standard individual Nash-bargaining axiomatic solution is applied.

In order to solve the model, a backward procedure is adopted. Firstly, the actual expected value functions for individuals and firms are evaluated using a standard dynamic programming method; secondly, by using the obtained results the Bayesian Nash Equilibrium (BNE) of the simultaneous game in which agents decide, conditional upon their own type, educational level and technological contents to maximize their expected steady-state payoffs is established. Then, the hiring regime characterizing the BNE is set out.

3.5. The Frictional Labor Market

3.5.1. The Matching Functions

Indicate with E_e the employment level per educational groups ($e=\{s,us\}$) and with M_e the number of matches per educational level. The (exogenous) quitting rate is indicated by $b>0$. By indicating with U_e the number of unemployed workers with education e and with V_T the number of posted vacancies per sector *T*, the urn-ball matching function can be written as follows:

$$M_e=\mathbf{a}_e(\lambda_e)U_e. \tag{8}$$

where $\mathbf{a}_e(\lambda_e)$ is the unconditional probability that an individual with education e is employed, expressed as a function of the tightness of the e sector with $\lambda_e=U_e/V_e$. Crucially, it is assumed that the functional form of $\mathbf{a}_e(\lambda_e)$ is endogenous. In particular, indicate with $a_e(.)$ the probability that an individual with ability θ and education e receives a job offer. This probability is given by

$$a_e(\lambda_e,\theta)=\begin{cases}\exp^{-[1-\Gamma(\theta)]\lambda_e} & \text{if ranking}\\ \dfrac{1-\exp^{-\lambda_e}}{\lambda_e} & \text{if no-ranking}\end{cases} \tag{9}$$

In the first line of Equation (9) the probability of receiving a job offer increases along with individual ability (as in Equation (3)) while, when no-ranking applies all workers have the same job finding rate and this is equal to the average arrival rate of jobs to workers (as in Equation (1)). Consider the s sector. By integrating $a_s(\lambda_s,\theta)$ over $[\theta^*,\bar{\theta}]$, whose lower bound θ^* is the threshold-ability of individuals in the s sector (it is determined in the BNE), it is possible to indicate the unconditional probability of being hired in a s position, $\mathbf{a}_s(\lambda_s)$, as follows:

$$\mathbf{a}_s(\lambda_s)=\int_{\theta^*}^{\bar{\theta}}\gamma(\theta)a_s(\lambda_s,\theta)\mathrm{d}\theta. \tag{10}$$

Mutatis mutandis, in the us sector the unconditional probability of being hired is given by:

$$\mathbf{a}_{us}(\lambda_{us})=\int_{\underline{\theta}}^{\theta^*}\gamma(\theta)a_{us}(\lambda_{us},\theta)\mathrm{d}\theta. \tag{11}$$

Now, it is useful to describe the urn-ball process from firms' perspective. The probability that a T firm hires a θ-type individual, indicated with $\alpha_T(.)$, can be written as follows:

$$\alpha_T(\lambda_e,\theta)=\begin{cases}\exp^{-([1-\Gamma(\theta)]\lambda_e)}\lambda_e\gamma(\theta) & \text{if ranking}\\ (1-\exp^{-\lambda_e})\gamma(\theta) & \text{if no-ranking}\end{cases} \tag{12}$$

The first line of Equation (12) contains the probability that a T firm does not meet any applicant of ability greater than θ times the probability of matching a worker with ability θ. In the no-ranking case, the probability of hiring a θ-type contains the probability that the firm receives an application times the probability that this application is from an individual with ability θ. Consider the case of a s firm. When integrating $\alpha_s(\lambda_s,\theta)$ over $[\theta^*,\bar{\theta}]$ the unconditional probability that a s vacancy is filled is obtained and it can be defined as follows:

$$\boldsymbol{\alpha}_s(\lambda_s)=\int_{\theta^*}^{\bar{\theta}}\gamma(\theta)\alpha_s(\lambda_s,\theta)\,\mathrm{d}\theta. \tag{13}$$

Mutatis mutandis, for a *us* firm the probability of filling a vacancy is given by:

$$\boldsymbol{\alpha}_{us}(\lambda_{us})=\int_{\underline{\theta}}^{\theta^*}\gamma(\theta)\alpha_{us}(\lambda_{us},\theta)\,\mathrm{d}\theta. \tag{14}$$

Having fixed this formalism, it is crucial to point out that the pure matching process can be solved as a function of the parameters $a_e(\lambda_e,\theta)$, $\mathbf{a}_e(\lambda_e)$, $\alpha_T(\lambda_e,\theta)$, and $\boldsymbol{\alpha}_T(\lambda_e)$. Put differently, given the sequential structure of the interaction process, it is possible to solve the matching part of the model by not imposing either ranking or no-ranking behavior. Then, by using the obtained payoffs in terms of wages and profits, the educational/technological choices are established. Simultaneously, firms' behavior in terms of hiring process is set out.

3.5.2. The Value Functions

The notation for actual expected values is set in Table 1. By indicating with $r>0$ the intertemporal interest rate, the value functions can be written as follows.

- Undergraduate individuals:

$$rV_{us}^E=w_{us}-b\left(V_{us}^E-V_{us}^U\right) \tag{15}$$

$$rV_{us}^U=a_{us}(\lambda_{us},\theta)\left(V_{us}^E-V_{us}^U\right). \tag{16}$$

- Graduate individuals:

Table 1. Notation for actual expected values.

Firms	Individuals
V_s^F $\Rightarrow$ filled s position	V_{us}^E $\Rightarrow$ empl. *us* individual
V_s^V $\Rightarrow$ vacant s position	V_{us}^U $\Rightarrow$ unempl. *us* individual
V_{us}^F $\Rightarrow$ filled *us* position	V_s^E $\Rightarrow$ empl. s individual
V_{us}^V $\Rightarrow$ vacant *us* position	V_s^U $\Rightarrow$ unempl. s individual

$$rV_s^E = w_s - b\left(V_s^E - V_s^U\right) \tag{17}$$

$$rV_s^U = a_s\left(\lambda_s, \theta\right)\left(V_s^E - V_s^U\right). \tag{18}$$

- Firms with unskilled job-positions:

$$rV_{us}^F = \bar{y} - w_{us} - Q - b\left(V_{us}^F - V_{us}^V\right) \tag{19}$$

$$rV_{us}^V = -Q + \alpha_{us}\left(\lambda_{us}, \theta\right)\left(V_{us}^F - V_{us}^V\right). \tag{20}$$

- Firms with skilled job-positions:

$$rV_s^F = \theta\bar{y} - w_s - Q - b\left(V_s^F - V_s^V\right) \tag{21}$$

$$rV_s^V = -Q + \alpha_s\left(\lambda_s, \theta\right)\left(V_s^F - V_s^V\right). \tag{22}$$

Notice that relations above represent standard value functions for two-sector matching models.

4. The Equilibria

4.1. Equilibrium Wages

In order to set the equilibrium of the model, it is crucial to solve the last stage of the interaction process, *i.e.*, to establish the payoffs resulting from the matching process in the two sectors. Since individual Nash-bargaining solution is applied, when a match is realized the generated surpluses for firm and worker must be equal conditional upon agents' characteristics and labor market opportunities. Formally:

$$V_{us}^E - V_{us}^U = V_{us}^F - V_{us}^V \tag{23}$$

$$V_s^E - V_s^U = V_s^F - V_s^V \tag{24}$$

By combining the relative value functions, the following wage expressions for unskilled and skilled individuals are obtained:

$$w_{us} = \frac{\bar{y}\left[r + b + a_{us}(.)\right]}{a_{us}(.) + \alpha_{us}(.) + 2b + 2r}. \tag{25}$$

$$w_s = \frac{\theta\bar{y}\left[r + b + a_s(.)\right]}{\alpha_s(.) + a_s(.) + 2r + 2b}. \tag{26}$$

As expected-given the perfect segmentation between the two sectors wage equations are similar to those of standard matching models. Moreover, since skilled workers' ability is reveled once the match is realized, in this sector the wage is expressed as a function of θ. Now it is possible to proceed backward to determine the sector-choice for firms and individuals.

4.2. The Entry Game

Individuals and firms have to decide, conditional on their ability and distance to the frontier, the level of education and the technology they want to acquire respectively. In order to solve the game, it is assumed that agents ground their decisions considering the parameters $a_{us}(.)$, $a_s(.)$, $\alpha_{us}(.)$, and $\alpha_s(.)$ as if they were at their steady-state values. Put differently, agents choose their strategy in order to maximize the payoffs they obtain in the steady-state.[5] Once they make their choice, they enter labor market(s) as unemployed individuals and as firms with unfilled vacancies and then the matching process starts. The interaction process is Bayesian since each agent knows his own type (ability/distance to the frontier) and just the distribution of types of player to whom he may be matched. Since individual's ability is revealed only when a match is realized, $E\left[V_s^V \mid \theta\right]$, *i.e.*, the expected payoff of a *s* firm that matches a *s* worker, need to be evaluated. Notice that this interaction process considers *pure strategies* of firms and individuals that are best responses to each other, conditional on the type of player. As a consequence, the evaluation of the BNE gives the shares of individuals and firms that acquire higher education and invest in skilled positions respectively and it provides a measure of the relative tightness of the two sectors in steady-state.

Proposition 1. *It exists a unique BNE in which only individuals with ability* $\theta \geq \theta^*$ *set* $e = s$ *and only firms with* $\delta \leq \delta^*$ *set* $T = s$.

Proof. Consider the firm's choice. Indicate with κ the probability (it is a density) that the individual sets $e = s$. In this case, a firm invests in *s* position only if:

$$\delta \leq \kappa E[V_s^V \mid \theta] - V_{us}^V. \tag{27}$$

Given the assumption on the monotonicity of $\Phi(.)$, it is possible to indicate with δ^* the cutoff level of distance to the frontier for which relation (27) is satisfied. Now, indicate with ω the probability that a firm set $T = s$ and consider the individual's educational choice. Setting $e = s$ is optimal for an individual only if:

$$c(\theta) \leq \omega\left(V_s^U + V_{us}^U\right) - V_{us}^U. \tag{28}$$

Given the assumption on the monotonicity of $\Gamma(.)$ and given that $\frac{\partial c}{\partial \theta} < 0$, it is possible to indicate with θ^* the cutoff ability level for which relation (28) is satisfied. Hence, the following pair characterizes the BNE:

$$\begin{cases} \kappa = 1 - \Gamma\left(\theta^*\right) \\ \omega = \Phi\left(\delta^*\right). \end{cases} \tag{29}$$

Intuitively, a firm invests in a *s* position only if the associated expected payoff is greater than that associated with a *us* position. Crucially, this depends on the distribution of θ within individuals that decide to acquire education *s*, on the relative markets' tightness, and on firm's distance to the technological frontier (Equation (27)). At the same time, worker's decision of investing in education *s*

[5]This assumption allows for the identification of a unique BNE and it is similar to the assumption made by [10] in order to discuss the existence of a steady-state in a dynamic urn-ball process with ranking, *i.e.*, the economy should operate always around its hypothetical steady-state.

is a function of the number of firms that decide to create s positions and of his own ability (Equation (28)). Relation (29) contains the shares that are best response to each other and these can be considered as the shares of agents that represent the only steady-state of the interaction process.[6]

4.3. Endogenous Hiring Process

4.3.1. Analysis of the BNE

In order to simplify the discussion concerning the hiring process adopted by firms, it is worthwhile to undertake an in-depth analysis of the BNE established in the previous paragraph. This investigation is particularly useful since it allows for the identification of two different types of BNE each of them consistent only with a specific hiring regime. Moreover, this analysis is important since it considerably eases the assessment of the effect that exogenous shocks may have on the form of the matching function discussed in paragraph 4.4.

As already pointed out, the BNE gives a measure of the tightness of the two sectors. By focusing on the cutoff level δ^*, *i.e.*, the one that satisfies relation (27) as an equality it is possible to describe the BNE. In fact, since the greater δ^* the larger the share of s firms in the considered economy, δ^* approximates the share $\phi(\delta^*)$ of firms creating skilled-complementary positions. To evaluate δ^* relation (27) has to be spelled out. By combining Equations ((20) and (22)) it is possible to write the cutoff level δ^* in relation (27) as follows:

$$\delta^*(\theta^*) = \Gamma(\theta^*)\frac{Q}{r} + \frac{\left[1-\Gamma(\theta^*)\right]\boldsymbol{\alpha}_s(.)\bar{y}}{rA}\left[E\left[\theta \mid \theta \geq \theta^*\right]C - \frac{A}{F}\right] - \frac{\alpha_{us}(.)\bar{y}}{rD} \tag{30}$$

where A, B, C, D, and F summarize strictly positive constants.[7] Relation (30) gives the *best response function* in terms of share of firms investing in skilled positions. Since the best response δ^* is evaluated when the share of skilled workers is $\Gamma(\theta^*)$, Equation (30) represents the intersection of the best responses and, as a consequence, it describes the BNE of the game. Notice that in Equation (30)

$$E\left[\theta \mid \theta \geq \theta^*\right] = \frac{\int_{\theta^*}^{\bar{\theta}} \theta\gamma(\theta)\mathrm{d}\theta}{1-\Gamma(\theta^*)} \tag{31}$$

and

$$\boldsymbol{\alpha}_s(\lambda_s) = \int_{\theta^*}^{\bar{\theta}} \gamma(\theta)\alpha_s(\lambda_s,\theta)\mathrm{d}\theta.$$

Before turning to the discussion of ranking behavior, it is useful to evaluate how the share δ^* changes *in equilibrium* as θ^* changes. By differentiating Equation (30) with respect to θ^* using the Leibniz rule for differentiation of definite integrals it results that:

[6]See [20] p. 38-39 on the interpretation of BNE as steady-state equilibria.

[7] $A=(r+b)\left[2r+2b+a_g(.)\right]$; $B=\left[2r+2b+\alpha_g(.)+a_g(.)\right]$; $C=(r+b)$; $D=\left(a_{ug}(.)+\alpha_{ug}(.)+2b+2r\right)$; $F=\left[2r+2b+\alpha_g(.)+a_g(.)\right]\left[2r+2b+\alpha_g(.)+a_{ug}(.)\right]$.

$$\frac{\partial \delta^*}{\partial \theta^*}=\frac{1}{r}\left(\underbrace{\gamma\left(\theta^*\right)\left[Q+\frac{\boldsymbol{\alpha}_s\left(\lambda_s\right)\bar{y}C}{A\left[1-\Gamma\left(\theta^*\right)\right]}\right]\left[\int_{\theta^*}^{\bar{\theta}}\theta\gamma\left(\theta\right)\mathrm{d}\theta-\theta^*\right]}_{>0}\right.$$
$$+\underbrace{\frac{\bar{y}}{A}\left[\frac{A}{F}\left(\gamma\left(\theta^*\right)\boldsymbol{\alpha}_s\left(\lambda_s\right)+\alpha_s\left(\lambda_s,\theta^*\right)\left[1-\Gamma\left(\theta^*\right)\right]\right)\right]}_{>0} \tag{32}$$
$$\left.\underbrace{-\frac{\bar{y}}{A}\left[E\left[\theta\mid\theta>\theta^*\right]C\left(\gamma\left(\theta^*\right)\boldsymbol{\alpha}_s\left(\lambda_s\right)+\alpha_s\left(\lambda_s,\theta^*\right)\left[1-\Gamma\left(\theta^*\right)\right]\right)\right]}_{<0}\right).$$

Relation (32) indicates how a variation in the best response in terms of share of skilled workers (θ^*) affects *in equilibrium* the share of firms investing in skilled positions. The first two lines indicate that firms' expectation positively depends on θ^*: The higher the cutoff ability level, the higher is the expected productivity of skilled workers and this induces a *composition effect* which fosters firms' investment in skilled jobs. Conversely, the bottom line of Equation (32) shows the negative effect that a rise in θ^* has on firms' expectation: In this case, as the cutoff point θ^* rises, the probability of filling a vacancy reduces, inducing a *tightness effect* that limits the creation of skilled-complementary positions. Assuming satisfied second order conditions, it is possible to indicate with θ^{**} the share of skilled workers that *ceteris paribus* maximizes firms' investments in skilled positions, *i.e.*, the share of skilled workers balancing tightness and composition effects:

$$\left.\frac{\partial \delta^*}{\partial \theta^*}\right|_{\theta^*=\theta^{**}}=0. \tag{33}$$

It is important to note that only the appropriate *selectivity* level $\left|\frac{\partial c}{\partial \theta}\right|$ can ensure that θ^{**} is actually achieved in equilibrium. If this is the case, the resulting steady-state allows for a perfect balance between tightness and composition effects ($\theta^*=\theta^{**}$).

4.3.2. The Hiring Process

In this paragraph, it is shown that the particular case where $\theta^*=\theta^{**}$ defined in Equation (33) separates two different types of BNE. Then, it is proved that these two types of equilibria are characterized by different (optimal-)ranking behavior. Consider the case wherein $\theta^*>\theta^{**}$. In words, few individuals have access to the *s* sector and this constrains the creation of skilled complementary jobs. In this case, a reduction in the selectivity level of the higher education sector ($\left|\frac{\partial c}{\partial \theta}\right|\downarrow$) would induce a rise in the share of skilled workers ($\theta^*\downarrow$) that in turn induces an increase in the share of firms investing in skilled positions. Put differently, when $\theta^*>\theta^{**}$ the labor market is characterized by a tightness-related scenario. Now, consider the case where $\theta^* < \theta^{**}$. These equilibria hide a *composition problem* within the *s* sector: A large number of individuals acquire education *s* implying a low expected productivity of the skilled labor force. This brakes the creation of skilled jobs. In this case an increase in the selectivity level of the higher education sector ($\left|\frac{\partial c}{\partial \theta}\right| \uparrow$) induces a reduction in the share of skilled workers ($\theta^* \uparrow$) and this generates an increase in the share of firms investing in skilled positions. It is possible to prove that the hiring process adopted in the skilled sector depends on the particular scenario faced by firms, *i.e.*, it depends on whether firms are in the presence of *tightness*- or *composition*-related situations.

Proposition 2. *a*) *In tightness-related equilibria s firms maximize their actual expected value by adopting a no-ranking behavior. b*) *In composition-related scenarios us firms' maximize their actual expected value by applying ranking amongst applicants. c*) *In all scenarios us firms rank applicants.*

Proof. Part a). Consider *s* firms. Consider a BNE characterized by a *tightness*-related scenario and, by contradiction, assume that the application of ranking among applicants represents an optimal choice for *s* firms, *i.e.*, it maximizes the expected value for a *s* firm in the steady-state. In the presence of ranking, an individual who is at-the-margin, *i.e.*, he has ability just below θ^* ($\theta \approxeq \theta^*$) decides not to acquire education *s*. In the Appendix it is shown that the value of V_s^U under no-ranking is greater than the value it takes under ranking when $\theta \approxeq \theta^*$. This implies that the individual at-the-margin would choose to become skilled under the no-ranking case, hence θ^* would decrease if firms decide to switch from the ranking to the no-ranking case. By definition of *tightness*-related equilibria, the reduction of θ^* raises *ex-ante* the expected value of all firms investing in *s* positions, therefore all firms find convenient to adopt the no-ranking behavior and this leads to a contradiction. Notwithstanding, firms may still apply a dynamic inconsistent behavior deciding to apply ranking *ex-post*, *i.e.*, once matches are realized. Since an infinitely repeated setting is considered and agents care about their future payoffs ($r > 0$), by applying the standard folk-theorem it would be possible to set a threshold level of the inter-temporal discount rate *r* under which firms do not deviate from a no-ranking strategy in order not to lose those skilled workers at-the-margin in the future.

Part b). Consider *s* firms. Consider a BNE characterized by a *composition*-related scenario and, by contradiction, assume that the application of no-ranking among applicants represents an optimal choice for *s* firms. By replicating *mutatis mutandis* the argument made above and using the result presented in the Appendix, it is easy to show that θ^* increases if firms decide to switch from the no-ranking to the ranking case. By definition of *composition*-related equilibria, an increase of θ^* raises *ex-ante* the expected value of all firms investing in *s* positions, therefore all firms find convenient to adopt the ranking behavior and this leads to a contradiction.

Part c). Consider *us* firms. In this sector, since there is no *composition* effect, firms only care about the share of *us* workers in the labor market in order to rise the probability of filling their vacancies. As a consequence *us* firms decide their matching regime to attract as many *us* workers as they can. This implies that independently on the specific scenario generated by the institutional setting the adoption of ranking represents an optimal action for *us* firms to retain in their sector those individuals near to θ^*.

4.4. Discussion

The intuition behind Proposition 2 is straightforward. Whether the selectivity level of the higher education sector limits the availability of skilled workers, firms find optimal not to add additional screening since this practice would lower the expected value of education for individuals at-the-margin, leading to a reduction in the number of skilled workers and to a worsening of the tightness problem. Conversely, the ranking process represents an optimal choice whenever firms face *composition*-related problems since it discourages individuals at-the-margin to enter the skilled sector. Notwithstanding, some arguments are required at this stage since the result of no-ranking among skilled workers may seem, at a first sight, counterfactual. In this respect, it should be remarked that in this paper a unique level of selectivity of the higher education sector has been modeled and it has been shown that firms' ranking decisions are conditioned on it. In the presence of heterogeneous selectivity levels, *i.e.*, in the presence of heterogeneous universities, *ceteris paribus* firms would ground their ranking decision by conditioning on the institution-specific selectivity. Hence, in this case we would observe both ranking and no-ranking behavior. This result is perfectly in line with the existing empirical evidence reporting that employment probability of skilled workers seems to be affected by the characteristics of the attended university in terms of admission's requirements (among others see [21] and [22]).

At this stage, in order to have a complete picture of the model's results, firms' behavior in the *us* sector needs to be discussed. The characteristics of the *us* sector in terms of ranking are perfectly in line with the main message of this work: When tightness issues are taken into account, the presence of ranking could not be easily determined *ex-ante* by relying only on the presence of heterogeneous workers' productivity. Ranking may be applied even if firms operate in a sector characterized by homogeneous workers simply because this hiring regime maximizes the availability of workers in this sector and, consequently, the probability of filling a vacancy.

A final point that needs to be remarked concerns the relevance that the presented results may have for empirical works. Indeed, the analysis presented in paragraphs 4.3.1 and 4.3.2 allows for an immediate assessment of this issue. In particular, it is easy to figure out that a change of the selectivity of the university system may induce a switch from the *tightness* to the *composition* scenario whenever θ^* moves from the RHS to the LHS of θ^{**}. This implies that an exogenous variation of the selectivity level of higher education system may induce a switch in the matching regime going from the no-ranking to the ranking case. Hence, the functional form of the matching function changes too, by relying on the top line instead of the bottom line of Equation (9). This consideration implies that the matching technology changes with exogenous policies and rises concerns about the validity of policy evaluations employing exogenous matching functions. Results of models with exogenous matching regimes could be biased if modelers do not take into account that the matching technology itself may also change with the policy.

5. Conclusions

This study enters the debate concerning the endogeneity of matching functions by focusing on a particular matching regime known as urn-ball process. In this case, either ranking or no-ranking behavior may be adopted by firms when choosing among multiple applications. It is argued that the choice of the correct modeling strategy is not an obvious one and it does not only depend on workers' heterogeneity in terms of productivity. Using a simple continuous time two-sector matching model with endogenous technological and educational choice, it has been shown that the specific form of the matching process depends on the characteristics of the labor market. In particular, when the two sectors compete to attract workers, firms evaluate their optimal actions in the light of the tightness of the sector in which they operate. Overall, the study highlights the relevance that endogenous matching process may have in order to correctly capture labor market dynamics and agents' behavior. This has important implications also for empirical works aimed at evaluating policy measures and their effect on workers' employability since the properties usually imposed on exogenous matching functions are justified on the basis of agents' microbehavior. Indeed it has been shown that the specific form of the matching process can be affected by firms' behavior resulting from the specific institutional setting. As policies are targeted to change agents' choices, these may very well also affect the properties of the matching technology. These aspects should be taken into account by policy evaluators in order to avoid misleading predictions on the effect of policy measures.

Finally we remark that the present study considers only a specific matching regime and this represents the main limitation of this research. Further studies are required in order to evaluate the stability of other matching regimes and to assess the empirical relevance of the question at hand.

References

[1] Diamond, P. (1982) Aggregate Demand Management in Search Equilibrium. *Journal of Political Economy*, **90**, 881-894. https://doi.org/10.1086/261099

[2] Mortensen, D.T. (1989) The Persistence and Indeterminacy of Unemployment in Search Equilibrium. *Scandinavian Journal of Economics*, **91**, 347-370. https://doi.org/10.2307/3440116

[3] Pissarides, C.A. (1987) Search, Wage Bargains and Cycles. *Review of Economic Studies*, **54**, 473-483. https://doi.org/10.2307/2297570

[4] Stevens, M. (2007) New Microfoundations for the Aggregate Matching Function. *International Economic Review*, **48**, 847-868. https://doi.org/10.1111/j.1468-2354.2007.00447.x

[5] Brown, A.J.G., Merkl, C. and Snower, D. (2009) An Incentive Theory of Matching. IZA Discussion Paper No. 4145.

[6] Neugart, M. (2004) Endogenous Matching Functions: An Agent-Based Computational Approach. *Advances in Complex Systems*, **7**, 187-202. https://doi.org/10.1142/S0219525904000147

[7] Borowczyk-Martins, D., Jolivet, G. and Postel-Vinay, F. (2011) Accounting for Endogenous Search Behavior in Matching Function Estimation. University of Bristol, Mimeo.

[8] Butters, G. (1977) Equilibrium Distribution of Sales and Advertising Prices. *The Review of Economic Studies*, **44**, 465-491. https://doi.org/10.2307/2296902

[9] Hall, R.E. (1979) A Theory of the Natural Unemployment Rate and the Duration of Employment. *Journal of Monetary Economics*, **5**, 153-169. https://doi.org/10.1016/0304-3932(79)90001-1

[10] Blanchard, O.L. and Diamond, P. (1994) Ranking, Unemployment Duration, and Wages. *Review of Economic Studies*, **61**, 417-434. https://doi.org/10.2307/2297897

[11] Gavrel, F. (2009) Technical Skill Bias as a Response of Firms to Unemployment: A Matching Model with Applicant Ranking and Endogenous Skill Requirements. *Labor Economics*, **16**, 304-310. https://doi.org/10.1016/j.labeco.2008.11.003

[12] Albrecht, J.W., Gautier, P.A. and Vroman, S.B. (2006) Matching with Multiple Applications. *Economics Letters*, **78**, 67-70.

[13] Albrecht, J.W., Gautier, P.A., Tan, S. and Vroman, S.B. (2004) Matching with Multiple Applications Revisited. *Economics Letters*, **84**, 311-314. https://doi.org/10.1016/j.econlet.2003.12.024

[14] Moen, E.R. (1999) Education, Ranking, and Competition for Jobs. *Journal of Labor Economics*, **17**, 694-723. https://doi.org/10.1086/209936

[15] Carneiro, P. and Heckman, J.J. (2002) The Evidence on Credit Constraints in Post-Secondary Schooling. *Economic Journal*, **112**, 705-734. https://doi.org/10.3386/w9055

[16] Mokyr, J. (1996) Evolution and Technological Change: A New Metaphor for Economic History? In: Fox, R., Ed., *Technological Change*, Harwood Publishers, London, 63-83.

[17] Acemoglu, D., Aghion, P. and Zilibotti, F. (2006) Distance to the Frontier, Selection and Economic Growth. *Journal of the European Economic Association*, **4**, 37-74. https://doi.org/10.1162/jeea.2006.4.1.37

[18] Acemoglu, D. (1997) Training and Innovation in an Imperfect Labor Market. *Review of Economic Studies*, **64**, 445-464. https://doi.org/10.2307/2971723

[19] Bartel, A.P. and Sicherman, N. (1999) Technological Change and Wages: An Inter-Industry Analysis. *Journal of Political Economy*, **57**, 285-325. https://doi.org/10.1086/250061

[20] Osborne, M. and Rubinstein, A. (1994) A Course in Game Theory. The MIT Press, Cambridge.

[21] Hendel, I., Shapiro, J. and Willen, P. (2005) Educational Opportunity and Income

Inequality. *Journal of Public Economics*, **89**, 841-870. https://doi.org/10.1016/j.jpubeco.2004.11.004

[22] Ordine, P. and Rose, G. (2011) Inefficient Self-Selection into Education and Wage Inequality. *Economics of Education Review*, **30**, 582-597. https://doi.org/10.1016/j.econedurev.2011.03.007

Appendix

Proof that V_s^U increases if *s* firms switch from ranking to no-ranking when $\theta \approxeq \theta^* > \theta^{**}$

In Equation (18) notice that V_s^U is an increasing function of $a_s(\lambda_s,\theta)$. As a consequence, I need to show that *ceteris paribus* the value of $a_s(\lambda_s,\theta)$ in the no-ranking case is greater than its value under the ranking scenario when $\theta \approxeq \theta^*$. Consider the following normalization of individual ability ranking among skilled workers, such that when $e = s$ then $\theta \equiv \chi$ and $\chi \in [\chi^*, \bar{\chi}]$ with $\chi^* = 0$ and $\bar{\chi} = 1$. The Poisson process with ranking gives us the probability that an individual with ability $\chi' \approxeq \chi^*$ is employed in a right position with:

$$\exp^{-\lambda_s(1-\chi')} = \exp^{-\lambda_s(1-\chi^*)} = \exp^{-\lambda_s} \tag{34}$$

Consider now the possibility that all individuals $\chi \in [\chi^*, \bar{\chi}]$ are treated as if they were the same individual (no-ranking). This case is equal to a situation in which in χ^* there is a mass point whose share is $1-\chi^*$. In this case the probability of being employed in a right position for an individual $\chi' \approxeq \chi^*$ is equal to that of all other χ-types and it is given by:

$$\left(1-\exp^{-\lambda_s(1-\chi)}\right)\left[\frac{1-\exp^{-\lambda_s(1-\chi^*)}}{1-\exp^{-\lambda_s}}\frac{1}{(1-\chi^*)}\right] = \frac{1-\exp^{-\lambda_s(1-\chi)}}{\lambda_s} \tag{35}$$

where the terms in square brackets represent the correction for the Poisson probability in the presence of a mass point (see p. 716 in Moen, 1999). Here I prove that Equation (34) is always less than Equation (35). By contradiction assume that (34) ≥ (35). Hence:

$$\exp^{-\lambda_s} \geq \frac{1-\exp^{-\lambda_s(1-\chi)}}{\lambda_s} \tag{36}$$

By taking logs of both sides in the relation above and by applying a first-order Taylor series approximation I have that:

$$-\exp^{-\lambda_s(1-\chi)} \leq \log \lambda_s - \lambda_s \tag{37}$$

It can be easily checked that the RHS of relation (37) is less than -1 $\forall \lambda_s > 0$. Hence, the LHS must be less than -1 too, which implies that $-\lambda_s(1-\chi) > 0$. Since $\lambda_s > 0$, and $(1-\chi) \geq 0$ I have a contradiction. Q.E.D.