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Common Fixed Point Theorems in Metric Space by Altering Distance Function

Vishnu Narayan Mishra[1,2*], Balaji Raghunath Wadkar[3], Ramakant Bhardwaj[4], Idrees A. Khan[5], Basant Singh[3]

[1]Applied Mathematics & Humanities Department, S.V. National Institute of Technology, Gujarat, India
[2]Department of Mathematics, Indira Gandhi National Tribal University, Madhya Pradesh, India
[3]Department of Mathematics, AISECT University, Bhopal, India
[4]Department of Mathematics, TIT Group of Institutes, Anand Nagar, India
[5]Department of Mathematics, Integral University, Lucknow, India
Email: vishnunarayanmishra@gmail.com, wadkar.bhardwaj@gmail.com, rkbhardwaj100@gmail.com, idrees_maths@yahoo.com, wbrlatur@gmail.com

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Abstract

In the present paper, we prove two theorems. In first theorem, we prove fixed point result for self-maps in the metric space under contractive condition of integral type by altering distance. In second result, we prove a unique common fixed point theorem by considering four sub compatible maps under a contractive condition of integral type.

Keywords

Altering Distance Function, Sub Compatible

1. Introduction and preliminaries

In [1], Khan introduced and proved fixed point results by the altering distance in metric space. Aliouche [2] proved common fixed point results in symmetric space for weakly compatible mappings under contractive condition of integral type. In [3], Babu generalized and proved fixed point results using control function. Later Bouhadjera and Godet [4] generalized concept of pair sub compatible maps and proved fixed point results. Also Chaudhari [5] [6], Chugh & Kumar [7], Naidu [8], Sastry *et al.* [9] generalized and proved some fixed point results. Recently in [10] [11], Hosseni used contractive rule of integral type by altering distance and generalized common fixed point results. Many authors proved fixed point results with different techniques in different spaces (see [12]-[17]). In [18] [19] [20] [21], Wadkar *et al.* proved fixed point theorems using the concept of soft metric space. In the present paper, we prove two theorems on fixed point under contraction rule of integral type in metric space by altering distance function, first for self map and second for a pair of sub compatible maps. Our results are motivated by V. R. Hosseni, Neda Hosseni.

Definition 1.1: A function $\psi : R^+ \to R^+ = [0,1)$ is an altering distance functions if ψ is continuous with monotone increasing in all variables and $\psi(x_1, x_2, x_3, \cdots, x_n) = 0$ if $x_1 = x_2 = x_3 = \cdots = x_n = 0$.

The collection of all altering distance is denoted by Ψ_n.

Now let us define a function $m(y)$ by $m(y) = \psi(y, y, y, y, \cdots, y)$ for $y \in [0, \infty)$, clearly $m(y) = 0$ if and only if $y = 0$.

Examples of ψ are $\psi(f_1, f_2, f_3, \cdots, f_n) = \mu \max\{f_1, f_2, f_3, \cdots, f_n\}$, for $\mu > 0$, (1)

$$\psi(f_1, f_2, f_3, \cdots, f_n) = f_n^{a_1} + f_n^{a_2} + f_n^{a_3} + \cdots + f_n^{a_n}, \quad a_1, a_2, \cdots, a_n \ge 1. \tag{2}$$

Definition 1.2: The maps $p, q : E \to E$ of metric space (E, σ) are called as sub compatible if and only if the sequence $\{e_n\}$ in E such that $\lim_{n\to\infty} p_n = \lim_{n\to\infty} q_n = t, t \in E$ and which satisfies $\lim_{n\to\infty} \sigma(pqe_n, qpe_n) = 0$.

Example 1.3: Let $E = [0, \infty)$ we define p & q with metric σ as follows

$$p(e) = e^2 \quad \& \quad q(e) = \begin{cases} e + 6 & \text{if } e \in [4,9] \cup (27, \infty) \\ e^2, & \text{if } e \in [9, 27] \end{cases} \tag{3}$$

Let us define the sequence $\{e_n\}$ in E as $e_n = 3 + \frac{1}{n}$, for $n = 0, 1, 2, \cdots$ then

$$\lim_{n\to\infty} pe_n = \lim_{n\to\infty} e_n^2 = 9 = \lim_{n\to\infty} qe_n = \lim_{n\to\infty} (e_n + 6), \tag{4}$$

$$\text{and} \quad \lim_{n\to\infty} pq(e_n) = \lim_{n\to\infty} p(e_n + 6) = \lim_{n\to\infty} (e_n + 6)^2 = 81 \text{ when } n \to \infty, \tag{5}$$

$$\lim_{n\to\infty} qpe_n = \lim_{n\to\infty} qe_n^2 = (e_n^2)^2 = \lim_{n\to\infty} (e_n)^4 = 81 \text{ when } n \to \infty. \tag{6}$$

Thus, we have $\lim_{n\to\infty} \sigma(pqe_n, qpe_n) = 0$. (7)

Hence maps p and q are sub-compatible.

On the other hand, we have $pe = qe$ if and only if $e = 3$, $pq(3) = p(9) = 81$ and $qp(3) = q(9) = 9 + 6 = 15$.

Then $p(3) = 9 = q(3)$ but $pq(3) = 81 \ne 15 = qp(3)$, hence p and q are not OWC (Oscillatory weakly commuting).

2. Main Result

Theorem 2.1: Let us consider the mappings $U, V : E \to E$ of complete metric space (E, σ) be such that for all $c, d \in E$

$$\begin{aligned} \int_0^{\phi_1(\sigma(Uc,Vd))} \eta(y)\,\mathrm{d}y \le {} & \int_0^{\psi_1\left(\sigma(c,d),\sigma(Uc,,c),\sigma(Vd,d),\frac{1}{2}\{\sigma(Vd,c)+\sigma(Uc,d)\},\frac{1}{2}\{\sigma(c,d)+\sigma(Uc,c)\}\right)} \eta(y)\,\mathrm{d}y \\ & - \int_0^{\psi_2\left(\sigma(c,d),\sigma(Uc,,c),\sigma(Vd,d),\frac{1}{2}\{\sigma(Vd,c)+\sigma(Uc,d)\},\frac{1}{2}\{\sigma(c,d)+\sigma(Uc,c)\}\right)} \eta(y)\,\mathrm{d}y, \end{aligned} \tag{8}$$

where $\psi_1, \psi_2 \in \Psi_5$ with $\phi_1 = \psi(e, e, e, e, e)$, $e \in [0, \infty)$ and Lebesgue-integrable mapping $\eta : R^+ \to R^+$, which is positive, sum able, and for each $\in > 0, \int_0^{\in} \eta(y)\,\mathrm{d}y > 0$, then there exist a unique common fixed point in E for U

and V.

Proof: Consider arbitrary point e_0 of E, for $n = 1, 2, 3, \cdots$ we have

$$e_{2n+1} = Ue_{2n}$$

$$\text{and} \quad e_{2n+2} = Ve_{2n+1}.$$

Let $r_n = \sigma\left(e_n, e_{n+1}\right)$ (9)

Substituting $c = e_{2n}$ and $d = e_{2n+1}$ in Equation (8), then for all $n = 1, 2, 3, \cdots$ we have

$$\int_0^{\phi_1\left(\sigma\left(Ue_{2n}, Ve_{2n+1}\right)\right)} \eta\left(y\right)\mathrm{d}y \le \int_0^{\phi_1\left(\sigma\left(e_{2n+1}, e_{2n+2}\right)\right)} \eta\left(y\right)\mathrm{d}y$$

$$\le \int_0^{\psi_1\left(\sigma\left(e_{2n}, e_{2n+1}\right), \sigma\left(Ue_{2n}, e_{2n}\right), \sigma\left(Ve_{2n+1}, e_{2n+1}\right), \frac{1}{2}\left\{\sigma\left(Ve_{2n+1}, e_{2n}\right) + \sigma\left(Ue_{2n}, e_{2n+1}\right)\right\}, \frac{1}{2}\left\{\sigma\left(e_{2n}, e_{2n+1}\right) + \sigma\left(Ue_{2n}, e_{2n}\right)\right\}\right)} \eta\left(y\right)\mathrm{d}y$$

$$- \int_0^{\psi_2\left(\sigma\left(e_{2n}, e_{2n+1}\right), \sigma\left(Ue_{2n}, e_{2n}\right), \sigma\left(Ve_{2n+1}, e_{2n+1}\right), \frac{1}{2}\left\{\sigma\left(Ve_{2n+1}, e_{2n}\right) + \sigma\left(Ue_{2n}, e_{2n+1}\right)\right\}, \frac{1}{2}\left\{\sigma\left(e_{2n}, e_{2n+1}\right) + \sigma\left(Ue_{2n}, e_{2n}\right)\right\}\right)} \eta\left(y\right)\mathrm{d}y$$

$$\le \int_0^{\psi_1\left(\sigma\left(e_{2n}, e_{2n+1}\right), \sigma\left(e_{2n+1}, e_{2n}\right), \sigma\left(e_{2n+2}, e_{2n+1}\right), \frac{1}{2}\left\{\sigma\left(e_{2n+2}, e_{2n}\right) + \sigma\left(e_{2n+1}, e_{2n+1}\right)\right\}, \frac{1}{2}\left\{\sigma\left(e_{2n}, e_{2n+1}\right) + \sigma\left(e_{2n+1}, e_{2n}\right)\right\}\right)} \eta\left(y\right)\mathrm{d}y$$

$$- \int_0^{\psi_2\left(\sigma\left(e_{2n}, e_{2n+1}\right), \sigma\left(e_{2n+1}, e_{2n}\right), \sigma\left(e_{2n+2}, e_{2n+1}\right), \frac{1}{2}\left\{\sigma\left(e_{2n+2}, e_{2n}\right) + \sigma\left(e_{2n+1}, e_{2n+1}\right)\right\}, \frac{1}{2}\left\{\sigma\left(e_{2n}, e_{2n+1}\right) + \sigma\left(e_{2n+1}, e_{2n}\right)\right\}\right)} \eta\left(y\right)\mathrm{d}y$$

Using Equation (9) for all $n = 1, 2, 3, \cdots$ we get

$$\begin{aligned} \int_0^{\phi_1\left(r_{2n+1}\right)} \eta\left(y\right)\mathrm{d}y &\le \int_0^{\psi_1\left(r_{2n}, r_{2n}, r_{2n+1} \frac{1}{2}\left\{r_{2n+1} + r_{2n} + 0\right\}, \frac{1}{2}\left\{r_{2n} + r_{2n}\right\}\right)} \eta\left(y\right)\mathrm{d}y \\ &\quad - \int_0^{\psi_2\left(r_{2n}, r_{2n}, r_{2n+1} \frac{1}{2}\left\{r_{2n+1} + r_{2n} + 0\right\}, \frac{1}{2}\left\{r_{2n} + r_{2n}\right\}\right)} \eta\left(y\right)\mathrm{d}y \end{aligned} \tag{10}$$

As $r_{2n+1} > r_{2n}$ implies that $r_{2n+1} + r_{2n} \le 2r_{2n+1}$, so we have

$$\int_0^{\phi_1\left(r_{2n+1}\right)} \eta\left(y\right)\mathrm{d}y \le \int_0^{\psi_1\left(r_{2n}, r_{2n}, r_{2n+1} r_{2n+1}, r_{2n}\right)} \eta\left(y\right)\mathrm{d}y = \int_0^{\phi_1\left(r_{2n+1}\right)} \eta\left(y\right)\mathrm{d}y \tag{11}$$

Now by monotone increase of ψ_1 in all variables and using the property that $\psi_2\left(r_{2n}, r_{2n}, r_{2n+1}, r_{2n+1}, r_{2n}\right) \ne 0$ whenever $r_{2n+1} \ne 0$, we get a contradiction *i.e.* r_{2n+1} not greater than r_{2n}. Hence we have $r_{2n+1} \le r_{2n}$, for

$$n = 0, 1, 2, 3, \cdots \tag{12}$$

Substituting $c = e_{2n-1}, d = e_{2n}$ in Equation (8) we have

$$\int_0^{\phi_1\left(r_{2n}\right)} \eta\left(y\right)\mathrm{d}y \le \int_0^{\psi_1\left(r_{2n-1}, r_{2n-1}, r_{2n}, r_{2n}, r_{2n-1}\right)} \eta\left(y\right)\mathrm{d}y - \int_0^{\psi_2\left(r_{2n-1}, r_{2n-1}, r_{2n}, r_{2n}, r_{2n-1}\right)} \eta\left(y\right)\mathrm{d}y \tag{13}$$

By using (12) we consider

$$r_{2n+2} \le r_{2n+1} \tag{14}$$

From (10) and (12) we obtain

$$r_{n+1} \le r_n \tag{15}$$

From (8) & (11) for all $n = 1, 2, 3, \cdots$, we have

$$\int_0^{\phi_1\left(r_{n+1}\right)} \eta\left(y\right)\mathrm{d}y \le \int_0^{\psi_1\left(r_n\right)} \eta\left(y\right)\mathrm{d}y - \int_0^{\psi_2\left(r_n\right)} \eta\left(y\right)\mathrm{d}y$$

then

$$\int_0^{\phi_2\left(r_{n+1}\right)} \eta\left(y\right)\mathrm{d}y \le \int_0^{\phi_1\left(r_n\right)} \eta\left(y\right)\mathrm{d}y - \int_0^{\phi_1\left(r_{n+1}\right)} \eta\left(y\right)\mathrm{d}y$$

Taking summation in above equation we obtain

$$\sum_{0}^{\infty}\int_{0}^{\phi_2(r_{n+1})}\eta(y)\,\mathrm{d}y \le \int_{0}^{\phi_1(r_0)}\eta(y)\,\mathrm{d}y < \infty,$$

which implies $\phi_2(r) \to 0$ as $n \to \infty$. (16)

Now from (13) sequence $\{r_n\}$ is convergent and as $n \to \infty$, $r_n \to r$. We know that ϕ is continuous and from Equation (14) we obtain $\phi_2(r) = 0$ which implies that $r = 0$, *i.e.* as

$$n \to \infty, \quad r = \sigma(e_{n+1}, e_n) \to 0. \tag{17}$$

We now show that the sequence $\{e_n\}$ is a Cauchy sequence in E. Keeping in mind Equation (15) it is require to show that $\{e_{2s}\}_{s=1}^{\infty} \subset \{e_n\}$ is a Cauchy sequence. If $\{e_{2s}\}_{s=1}^{\infty}$ is not a Cauchy sequence of natural number $\{2m(k)\},\{2n(k)\}$ such that $n(k) > m(k)$, $\sigma(e_{2m(k)}, e_{2n(k)}) \ge \in$

$$\sigma(e_{2m(k)}, e_{2n(k)-1}) < \in \tag{18}$$

Hence using (16)

$$\begin{aligned}\in &< \sigma(e_{2m(k)}, e_{2n(k)})\\ &\le \sigma(e_{2m(k)}, e_{2n(k)-1}) + \sigma(e_{2n(k)}, e_{2n(k)-1})\\ &< \in + \sigma(e_{2n(k)}, e_{2n(k)-1}).\end{aligned}$$

Taking $k \to \infty$ in the inequality above & by result of Equation (15), we arrive at

$$\lim_{n\to\infty}\sigma(e_{2m(k)}, e_{2n(k)}) = \in. \tag{19}$$

For all $k = 1, 2, 3, \cdots$

$$\sigma(e_{2n(k)+1}, e_{2m(k)}) \le \sigma(e_{2n(k)+1}, e_{2n(k)}) + \sigma(e_{2n(k)}, e_{2m(k)}) \tag{20}$$

Also for $k = 1, 2, 3, \cdots$

$$\sigma(e_{2n(k)}, e_{2m(k)}) \le \sigma(e_{2n(k)}, e_{2n(k)+1}) + \sigma(e_{2n(k)+1}, e_{2m(k)}). \tag{21}$$

Making $k \to \infty$ in (18) & (19) respectively by using (15) & (17) we have

$$\lim_{k\to\infty}\sigma(e_{2n(k)+1}, e_{2m(k)}) \le \in$$

$$\text{and} \quad \in \le \lim_{k\to\infty}\sigma(e_{2n(k)+1}, e_{2m(k)})$$

Therefore, $\lim_{k\to\infty}\sigma(e_{2n(k)+1}, e_{2m(k)}) = \in$, for $k = 1, 2, 3, \cdots$ (22)

$$\sigma(e_{2n(k)}, e_{2m(k)-1}) \le \sigma(e_{2n(k)}, e_{2m(k)}) + \sigma(e_{2m(k)}, e_{2m(k)-1}),$$

$$\sigma(e_{2n(k)}, e_{2m(k)}) \le \sigma(e_{2n(k)}, e_{2m(k)-1}) + \sigma(e_{2m(k)-1}, e_{2m(k)}).$$

Taking $k \to \infty$ in the above two inequalities and using (15) & (17) we obtain

$$\lim_{n\to\infty}\sigma(e_{2n(k)}, e_{2m(k)-1}) = \in. \tag{23}$$

Putting $c = e_{2n(k)}, d = e_{2m(k)-1}$ in (8), for all $k = 1, 2, 3, \cdots$, we obtain

$$\int_0^{\phi_1 \sigma\left(e_{2n(k)+1}, e_{2m(k)}\right)} \eta(y)\,\mathrm{d}y = \int_0^{\phi_1 \sigma\left(Ue_{2n(k)}, Ve_{2m(k)-1}\right)} \eta(y)\,\mathrm{d}y$$

$$\le \int_0^{\psi_1\left(\sigma\left(e_{2n}, e_{2m(k)-1}\right), \sigma\left(e_{2n(k)+1}, e_{2n}\right), \sigma\left(e_{2m(k)}, e_{2m(k)-1}\right), \frac{1}{2}\left\{\sigma\left(e_{2m(k)}, e_{2n(k)}\right) + \sigma\left(e_{2n(k)+1}, e_{2m(k)-1}\right)\right\}, \frac{1}{2}\left\{\sigma\left(e_{2n(k)}, e_{2m(k)-1}\right) + \sigma\left(e_{2n(k)+1}, e_{2n}(k)\right)\right\}\right)} \eta(y)\,\mathrm{d}y$$

$$- \int_0^{\psi_2\left(\sigma\left(e_{2n}, e_{2m(k)-1}\right), \sigma\left(e_{2n(k)+1}, e_{2n}\right), \sigma\left(e_{2m(k)}, e_{2m(k)-1}\right), \frac{1}{2}\left\{\sigma\left(e_{2m(k)}, e_{2n(k)}\right) + \sigma\left(e_{2n(k)+1}, e_{2m(k)-1}\right)\right\}, \frac{1}{2}\left\{\sigma\left(e_{2n(k)}, e_{2m(k)-1}\right) + \sigma\left(e_{2n(k)+1}, e_{2n}(k)\right)\right\}\right)} \eta(y)\,\mathrm{d}y$$

Now in above inequality if we take $k \to \infty$ and by using results of (15), (20) & (21) we get

$$\int_0^{\phi_1(\in)} \eta(y)\,\mathrm{d}y \le \int_0^{\psi_1\left(\in, 0, 0, \in, \frac{1}{2}\in\right)} \eta(y)\,\mathrm{d}y - \int_0^{\psi_2\left(\in, 0, 0, \in, \frac{1}{2}\in\right)} \eta(y)\,\mathrm{d}y.$$

Then $\phi_1(\in) \le \psi_1\left(\in, 0, 0, \in, \frac{1}{2}\in\right) - \psi_2\left(\in, 0, 0, \in, \frac{1}{2}\in\right) = \phi_1(\in)$.

This is due to monotone increasing fact of ψ_1 in its variable and by using property of ψ_2 that $\psi_2(y_1, y_2, y_3, y_4, y_5) = 0,$ if and only if $y_1 = y_2 = y_3 = y_4 = y_5 = 0$.

From the above inequality we get a contradiction. So that $\in = 0$. This establishes convergent sequence in (E, σ).

Let $e_n \to z$ as $n \to \infty$. (24)

Substituting $c = e_{2n}$, $d = z$ in (8) for all $n = 1, 2, 3, \cdots$

$$\int_0^{\phi_1\left(\sigma\left(e_{2n+1}, Vz\right)\right)} \eta(y)\,\mathrm{d}y$$

$$\le \int_0^{\psi_1\left(\sigma\left(e_{2n}, z\right), \sigma\left(e_{2n+1}, e_{2n}\right), \sigma(Vz, z), \frac{1}{2}\left\{\sigma\left(Vz, e_{2n}\right) + \sigma\left(e_{2n+1}, z\right)\right\}, \frac{1}{2}\left\{\sigma\left(e_{2n}, z\right) + \sigma\left(e_{2n+1}, e_{2n}\right)\right\}\right)} \eta(y)\,\mathrm{d}y$$

$$- \int_0^{\psi_2\left(\sigma\left(e_{2n}, z\right), \sigma\left(e_{2n+1}, e_{2n}\right), \sigma(Vz, z), \frac{1}{2}\left\{\sigma\left(Vz, e_{2n}\right) + \sigma\left(e_{2n+1}, z\right)\right\}, \frac{1}{2}\left\{\sigma\left(e_{2n}, z\right) + \sigma\left(e_{2n+1}, e_{2n}\right)\right\}\right)} \eta(y)\,\mathrm{d}y$$

Taking limit n tends to infinity in the above inequality and using continuity of ψ_1 and ψ_2 and Equations (15), (22) we get

$$\int_0^{\phi_1(\sigma(z, Vz))} \eta(y)\,\mathrm{d}y$$

$$\le \int_0^{\psi_1\left(\sigma(z, z), \sigma(z, z), \sigma(Vz, z), \frac{1}{2}\{\sigma(Vz, z)\}, 0\right)} \eta(y)\,\mathrm{d}y - \int_0^{\psi_2\left(\sigma(z, z), \sigma(z, z), \sigma(Vz, z), \frac{1}{2}\{\sigma(Vz, z)\}, 0\right)} \eta(y)\,\mathrm{d}y$$

$$\le \int_0^{\psi_1\left(0, 0, \sigma(Vz, z), \frac{1}{2}\{\sigma(Vz, z)\}, 0\right)} \eta(y)\,\mathrm{d}y - \int_0^{\psi_2\left(0, 0, \sigma(Vz, z), \frac{1}{2}\{\sigma(Vz, z)\}, 0\right)} \eta(y)\,\mathrm{d}y$$

If $(Vz, z) \ne 0$ then monotone increasing ψ_1 and ψ_2 are monotone increasing and $\psi_2(y_1, y_2, y_3, y_4, y_5) = 0,$ if and only if $y_1 = y_2 = y_3 = y_4 = y_5 = 0$, we obtain

$$\int_0^{\phi_1(\sigma(z, Vz))} \eta(y)\,\mathrm{d}y \le \int_0^{\phi_1(\sigma(z, Vz))} \eta(y)\,\mathrm{d}y.$$

This contradiction, hence we obtain $(Vz, z) = 0.$ (25)

In similar way we prove that $z = Uz.$ Hence $z = Uz = Vz.$ (26)

Hence (25) & (26) shows that z is a common fixed point of U and V.

Theorem 2.2: Let (E, σ) be a complete metric space and p, q, U and V be four mappings from E to itself such that

$$\int_0^{\phi_1(\sigma(ps,qt))}\eta(y)\,\mathrm{d}y$$
$$\le \int_0^{\psi_1\left(\sigma(Us,Vt),\sigma(Us,qt),\sigma(ps,Vt),\sigma(Us,ps),\sigma(Vt,qt),\frac{1}{2}\{\sigma(qt,Us)+\sigma(ps,Vt)\},\frac{1}{2}\{\sigma(Us,Vt)+\sigma(ps,Us)\}\right)}\eta(y)\,\mathrm{d}y \tag{27}$$
$$-\int_0^{\psi_2\left(\sigma(Us,Vt),\sigma(Us,qt),\sigma(ps,Vt),\sigma(Us,ps),\sigma(Vt,qt),\frac{1}{2}\{\sigma(qt,Us)+\sigma(ps,Vt)\},\frac{1}{2}\{\sigma(Us,Vt)+\sigma(ps,Us)\}\right)}\eta(y)\,\mathrm{d}y,$$

for all $s,t\in E$, where $\psi_1,\psi_2\in\Psi_7$, $\phi_1=\psi(e,e,e,e,e,e,e)$, for $e\in[0,\infty)$.

i: One of the four mappings *p*, *q*, *U* and *V* is continuous.

ii: (*p*, *U*) & (*q*, *V*) are sub compatible.

iii: The pairs $p(s)\subseteq V(s)$ and $q(s)\subseteq U(s)$.

iv: Where $\eta:R^+\to R^+$ is Lebesgue-integrable mappings, which is sum able, non negative and such that for each $\in>0,\int_0^{\in}\eta(y)dy>0$.

Then *p*, *q*, *U* and *V* have a unique common fixed point in *E*.

Proof: Consider arbitrary point $e_0\in E$, we construct the sequence $\{e_n\}$ and $\{w_n\}$ in *E* such that

$$pe_{2n}=Ve_{2n+1}=w_{2n}\ \text{ and }\ qe_{2n+1}=Ue_{2n+2}=w_{n+1},\quad n=0,1,2,\cdots$$

Let $r_n=\sigma(w_n,w_{n+1})$, Substitution $s=e_{2n}$ and $t=e_{2n+1}$ in (27) we have

$$\int_0^{\phi_1(\sigma(pe_{2n},qe_{2n+1}))}\eta(y)\,\mathrm{d}y=\int_0^{\phi_1(\sigma(e_{2n+1},e_{2n+2}))}\eta(y)\,\mathrm{d}y$$
$$\le\int_0^{\substack{\psi_1(\sigma(Ue_{2n},Ve_{2n+1}),\sigma(Ue_{2n},qe_{2n+1}),\sigma(pe_{2n+1},Ve_{2n+1}),\sigma(Ue_{2n},pe_{2n}),\sigma(Ve_{2n+1},qe_{2n+1}),\\ \frac{1}{2}\{\sigma(qe_{2n+1},Ue_{2n})+\sigma(pe_{2n},Ve_{2n+1})\},\frac{1}{2}\{\sigma(Ue_{2n},Ve_{2n+1})+\sigma(pe_{2n},Ue_{2n})\})}}\eta(y)\,\mathrm{d}y$$
$$-\int_0^{\substack{\psi_2(\sigma(Ue_{2n},Ve_{2n+1}),\sigma(Ue_{2n},qe_{2n+1}),\sigma(pe_{2n+1},Ve_{2n+1}),\sigma(Ue_{2n},pe_{2n}),\sigma(Ve_{2n+1},qe_{2n+1}),\\ \frac{1}{2}\{\sigma(qe_{2n+1},Ue_{2n})+\sigma(pe_{2n},Ve_{2n+1})\},\frac{1}{2}\{\sigma(Ue_{2n},Ve_{2n+1})+\sigma(pe_{2n},Ue_{2n})\})}}\eta(y)\,\mathrm{d}y$$

$$\int_0^{\phi(\sigma(w_{2n},w_{2n+1}))}\eta(y)\,\mathrm{d}y$$
$$\le\int_0^{\substack{\psi_1(\sigma(w_{2n-1},w_{2n}),\sigma(w_{2n-1},w_{2n+1}),\sigma(w_{2n+1},w_{2n}),\sigma(w_{2n-1},w_{2n}),\sigma(w_{2n},w_{2n+1}),\\ \frac{1}{2}\{\sigma(w_{2n+1},w_{2n})+\sigma(w_{2n},w_{2n})\},\frac{1}{2}\sigma(w_{2n-1},w_{2n})+\sigma(w_{2n},w_{2n-1}))}}\eta(y)\,\mathrm{d}y$$
$$-\int_0^{\substack{\psi_2(\sigma(w_{2n-1},w_{2n}),\sigma(w_{2n-1},w_{2n+1}),\sigma(w_{2n+1},w_{2n}),\sigma(w_{2n-1},w_{2n}),\sigma(w_{2n},w_{2n+1}),\\ \frac{1}{2}\{\sigma(w_{2n+1},w_{2n})+\sigma(w_{2n},w_{2n})\},\frac{1}{2}\sigma(w_{2n-1},w_{2n})+\sigma(w_{2n},w_{2n-1}))}}\eta(y)\,\mathrm{d}y$$

$$\int_0^{\phi_1(r_{2n})}\eta(y)\,\mathrm{d}y\le\int_0^{\psi_1\left(r_{2n-1},r_{2n-1}+r_{2n},r_{2n},r_{2n-1},r_{2n},\frac{1}{2}\{r_{2n}\},\frac{1}{2}\{r_{2n-1}+r_{2n-1}\}\right)}\eta(y)\,\mathrm{d}y$$
$$-\int_0^{\psi_2\left(r_{2n-1},r_{2n-1}+r_{2n},r_{2n},r_{2n-1},r_{2n},\frac{1}{2}\{r_{2n}\},\frac{1}{2}\{r_{2n-1}+r_{2n-1}\}\right)}\eta(y)\,\mathrm{d}y$$

If $r_{2n+1}\le r_{2n}$ then $r_{2n+1}+r_{2n}\le 2r_{2n}$ and

$$\int_0^{\phi_1(r_{2n})}\eta(y)\,\mathrm{d}y\le\int_0^{\psi_1\left(r_{2n-1},2r_{2n},r_{2n},r_{2n-1},r_{2n},\frac{1}{2}\{r_{2n}\},r_{2n-1}\right)}\eta(y)\,\mathrm{d}y$$
$$-\int_0^{\psi_2\left(r_{2n-1},2r_{2n},r_{2n},r_{2n-1},r_{2n},\frac{1}{2}\{r_{2n}\},r_{2n-1}\right)}\eta(y)\,\mathrm{d}y \tag{28}$$
$$<\int_0^{\phi_1(r_{2n})}\eta(y)\,\mathrm{d}y.$$

Thus we arrive at a contradiction. Hence $r_{2n}\le r_{2n-1}$, similarly by substituting $s=r_{2n+2},t=r_{2n+1}$ in (27) we can prove that, $r_{2n+1}\le r_{2n}$, for $n=0,1,2,\cdots$. Thus $r_{n+1}\le r_n$, for $n=0,1,2,\cdots$. Hence the sequence $\{r_n\}$ is sequence of positive real numbers, which is decreasing and converges to $r\in R$.

Let $m=\lim_{n\to\infty}\frac{1}{2}d(w_n,w_{n+2})$. Taking $n\to\infty$ in (27) we have

$$\int_0^{\phi_1(r_{2n})}\eta(y)\,\mathrm{d}y \le \int_0^{\psi_1(r,r,r,r,r,r,r)}\eta(y)\,\mathrm{d}y - \int_0^{\psi_2(r,r,r,r,r,r,m)}\eta(y)\,\mathrm{d}y$$
$$\le \int_0^{\phi_1(r)}\eta(y)\,\mathrm{d}y - \int_0^{\psi_2(r,r,r,r,r,r,m)}\eta(y)\,\mathrm{d}y.$$

Thus $\psi_2(r,r,r,r,r,r,m)=0$ So that $r=m=0$.

Hence $\lim_{n\to\infty} d(y_n, y_{n+1})=0$ (29)

In view of (29), to prove sequence $\{w_n\}$ is a Cauchy sequence it is sufficient to prove the subsequence $\{w_{2n}\}$ of sequence $\{w_n\}$ is a Cauchy sequence. If $\{w_{2n}\}$ is not a Cauchy sequence there exist $\in > 0$ & sequence of natural numbers $\{2m(k)\}$ & $\{2n(k)\}$ which are monotone increasing such that $n(k) > m(k)$.

$$\sigma\left(w_{2m(k)}, w_{2n(k)}\right) \ge \in \ \ \& \ \ \sigma\left(w_{2m(k)}, w_{2n(k)-2}\right) < \in. \tag{30}$$

Then from (29) we have

$$\begin{aligned} \in &< \sigma\left(w_{2m(k)}, w_{2n(k)}\right) \\ &\le \sigma\left(w_{2m(k)}, w_{2n(k)-2}\right) + \sigma\left(w_{2n(k)-1}, w_{2n(k)-2}\right) + \sigma\left(w_{2n(k)-1}, w_{2n(k)}\right) \\ &< \in + \sigma\left(w_{2n(k)-1}, w_{2n(k)-2}\right) + \sigma\left(w_{2n(k)-1}, w_{2n(k)}\right). \end{aligned} \tag{31}$$

Taking $k\to\infty$ and using (29) we have

$$\lim_{n\to\infty}\sigma\left(w_{2m(k)}, w_{2n(k)}\right) = \in. \tag{32}$$

Taking $k\to\infty$ using (29) & (30) in

$$\left|\sigma\left(w_{2m(k)}, w_{2n(k)+1}\right) - \sigma\left(w_{2m(k)}, w_{2n(k)}\right)\right| \le \sigma\left(w_{2n(k)}, w_{2n(k)+1}\right). \tag{33}$$

We get $\lim_{n\to\infty}\sigma\left(w_{2n(k)+1}, w_{2m(k)}\right) = \in.$ (34)

Letting $k\to\infty$ and from Equations (29) & (30) in

$$\left|\sigma\left(w_{2m(k)-1}, w_{2n(k)}\right) - \sigma\left(w_{2m(k)}, w_{2n(k)}\right)\right| \le \sigma\left(w_{2m(k)}, w_{2m(k)-1}\right).$$

We get $\lim_{k\to\infty}\sigma\left(w_{2m(k)}, w_{2m(k)-1}\right) = \in.$ (35)

Putting $s = x_{2m(k)}, t = x_{2n(k)-1}$ in (27), for all $k = 1,2,3,\cdots$ we obtain

$$\begin{aligned}
&\int_0^{\phi_1\left(px_{2m(k)},qx_{2n(k)-1}\right)}\eta(y)\,\mathrm{d}y \\
&\le \int_0^{\substack{\psi_1\left(\sigma\left(Ux_{2m(k)},Vx_{2m(k)-1}\right),\sigma\left(Ux_{2m(k)},qx_{2n(k)-1}\right),\sigma\left(px_{2m(k)},Vx_{2n(k)-1}\right),\sigma\left(Ux_{2m(k)},px_{2m(k)}\right),\sigma\left(Vx_{2n(k)-1},qx_{2n(k)-1}\right) \\ \frac{1}{2}\left\{\sigma\left(qx_{2n(k)-1},Ux_{2m(k)}\right)+\sigma\left(px_{2m(k)},Vx_{2n(k)-1}\right)\right\},\frac{1}{2}\left\{\sigma\left(Ux_{2m(k)},Vx_{2n(k)-1}\right)+\sigma\left(px_{2m(k)},Ux_{2m(k)}\right)\right\}\Big)}}\eta(y)\,\mathrm{d}y \\
&\quad - \int_0^{\substack{\psi_2\left(\sigma\left(Ux_{2m(k)},Vx_{2m(k)-1}\right),\sigma\left(Ux_{2m(k)},qx_{2n(k)-1}\right),\sigma\left(px_{2m(k)},Vx_{2n(k)-1}\right),\sigma\left(Ux_{2m(k)},px_{2m(k)}\right),\sigma\left(Vx_{2n(k)-1},qx_{2n(k)-1}\right) \\ \frac{1}{2}\left\{\sigma\left(qx_{2n(k)-1},Ux_{2m(k)}\right)+\sigma\left(px_{2m(k)},Vx_{2n(k)-1}\right)\right\},\frac{1}{2}\left\{\sigma\left(Ux_{2m(k)},Vx_{2n(k)-1}\right)+\sigma\left(px_{2m(k)},Ux_{2m(k)}\right)\right\}\Big)}}\eta(y)\,\mathrm{d}y \\
&\le \int_0^{\substack{\psi_1\left(\sigma\left(w_{2m(k)-1},w_{2n(k)-2}\right),\sigma\left(w_{2m(k)-1},w_{2n(k)-1}\right),\sigma\left(w_{2m(k)},w_{2n(k)-2}\right),\sigma\left(w_{2m(k)-1},w_{2m(k)}\right),\sigma\left(w_{2m(k)-2},w_{2n(k)-1}\right), \\ \frac{1}{2}\left\{\sigma\left(w_{2n(k)-1},w_{2m(k)-1}\right)+\sigma\left(w_{2m(k)},w_{2n(k)-2}\right)\right\},\frac{1}{2}\left\{\sigma\left(w_{2m(k)-1},w_{2n(k)-2}\right)+\sigma\left(w_{2m(k)},w_{2n(k)-1}\right)\right\}\Big)}}\eta(y)\,\mathrm{d}y \\
&\quad - \int_0^{\substack{\psi_2\left(\sigma\left(w_{2m(k)-1},w_{2n(k)-2}\right),\sigma\left(w_{2m(k)-1},w_{2n(k)-1}\right),\sigma\left(w_{2m(k)},w_{2n(k)-2}\right),\sigma\left(w_{2m(k)-1},w_{2m(k)}\right),\sigma\left(w_{2m(k)-2},w_{2n(k)-1}\right), \\ \frac{1}{2}\left\{\sigma\left(w_{2n(k)-1},w_{2m(k)-1}\right)+\sigma\left(w_{2m(k)},w_{2n(k)-2}\right)\right\},\frac{1}{2}\left\{\sigma\left(w_{2m(k)-1},w_{2n(k)-2}\right)+\sigma\left(w_{2m(k)},w_{2n(k)-1}\right)\right\}\Big)}}\eta(y)\,\mathrm{d}y
\end{aligned}$$

Taking $k \to \infty$ & using (29), (30), (32), (33) & (35) we get

$$\int_0^{\phi_1(\epsilon)} \eta(y)\mathrm{d}y \le \int_0^{\psi_1\left(\epsilon,\epsilon,\epsilon,0,0,\frac{1}{2}[\epsilon+\epsilon],\frac{1}{2}\epsilon\right)} \eta(y)\mathrm{d}y - \int_0^{\psi_2\left(\epsilon,\epsilon,\epsilon,0,0,\frac{1}{2}[\epsilon+\epsilon],\frac{1}{2}\epsilon\right)} \eta(y)\mathrm{d}y$$
$$< \int_0^{\psi_1(\epsilon,\epsilon,\epsilon,\epsilon,\epsilon,\epsilon,\epsilon)} \eta(y)\mathrm{d}y = \int_0^{\phi_1(\epsilon)} \eta(y)\mathrm{d}y.$$

This is contradiction. Hence $\{w_{2n}\}$ is a Cauchy sequence and is convergent. Since E is complete there exist $z \in E$ such that as $n \to \infty$ we have $w_n \to z$.

Case I: Assume that U is continuous then $Upe_{2n} \to Uz$, $U^2 e_{2n} \to Uz$. Since (p, U) is sub compatible, we have $pUe_{2n} \to Uz$.

Step I: Substituting $s = Ue_{2n}, t = e_{2n+1}$ in (27), we have

$$\int_0^{\phi_1 \sigma(pUe_{2n}, qe_{2n+1})} \eta(y)\mathrm{d}y$$
$$\le \int_0^{\psi_1\left(\sigma(U^2e_{2n},Ve_{2n+1}),\sigma(U^2e_{2n},qe_{2n+1}),\sigma(pUe_{2n},Ve_{2n+1}),\sigma(U^2e_{2n},pUe_{2n}),\sigma(Ve_{2n+1},qe_{2n+1}),\frac{1}{2}\{\sigma(qe_{2n+1},U^2e_{2n})+\sigma(pUe_{2n},Ve_{2n+1})\},\frac{1}{2}\{\sigma(U^2e_{2n},Ve_{2n+1})+\sigma(pUe_{2n},U^2e_{2n})\}\right)} \eta(y)\mathrm{d}y$$
$$- \int_0^{\psi_2\left(\sigma(U^2e_{2n},Ve_{2n+1}),\sigma(U^2e_{2n},qe_{2n+1}),\sigma(pUe_{2n},Ve_{2n+1}),\sigma(U^2e_{2n},pUe_{2n}),\sigma(Ve_{2n+1},qe_{2n+1}),\frac{1}{2}\{\sigma(qe_{2n+1},U^2e_{2n})+\sigma(pUe_{2n},Ve_{2n+1})\},\frac{1}{2}\{\sigma(U^2e_{2n},Ve_{2n+1})+\sigma(pUe_{2n},U^2e_{2n})\}\right)} \eta(y)\mathrm{d}y,$$

$$\int_0^{\phi_1(\sigma(Uz,z))} \eta(y)\mathrm{d}y$$
$$\le \int_0^{\psi_1\left(\sigma(Uz,z),\sigma(Uz,z),\sigma(Uz,z),\sigma(Uz,Uz),\sigma(z,z),\frac{1}{2}\{\sigma(z,Uz)+\sigma(Uz,z)\},\frac{1}{2}\{\sigma(Uz,z)+\sigma(Uz,Uz)\}\right)} \eta(y)\mathrm{d}y$$
$$- \int_0^{\psi_2\left(\sigma(Uz,z),\sigma(Uz,z),\sigma(Uz,z),\sigma(Uz,Uz),\sigma(z,z),\frac{1}{2}\{\sigma(z,Uz)+\sigma(Uz,z)\},\frac{1}{2}\{\sigma(Uz,z)+\sigma(Uz,Uz)\}\right)} \eta(y)\mathrm{d}y$$
$$\le \int_0^{\psi_1\left(\sigma(Uz,z),\sigma(Uz,z),\sigma(Uz,z),0,0,\sigma(Uz,z),\frac{1}{2}\{\sigma(Uz,z)\}\right)} \eta(y)\mathrm{d}y$$
$$\le \int_0^{\phi_1(\sigma(Uz,z))} \eta(y)\mathrm{d}y.$$

It is contradiction if $Uz \ne z$. Hence $Uz = z$.

Step II: Substituting $s = z, t = e_{2n+1}$ in (27) and taking limit as n tends to infinity we get $pz = z$.

Step III: We know that $z = pz \in p(e) \subseteq V(e)$ then there exist $u \in E$ such that $z = Vu$. Substituting $s = e_{2n}, t = u$ in (27) we get $qz = z$. Hence $qz = z = Vz$ and $qVu = Vqu$, which gives $qz = Vz$.

Step IV: Substituting $s = z, t = z$ in (27) we have $qz = z$ so that $q(z) = z = Vz$. Hence p, q, U & V have a common fixed point z in E.

Case II: Assume that U is continuous then $p^2 e_{2n} \to pz$, $pUe_{2n} \to pz$. Similarly we can prove that z is common fixed point of p, q, U & V. When q or V is continuous, then the uniqueness of common fixed point follows easily from (27).

Example: Let $E = [0,1]$ with the usual metric $\sigma(s,t) = \frac{1}{2}|s - t|$. Define $p, q, U, V : E \to E$ such that $ps = \frac{s}{4}$, $qt = \frac{t}{4}$, $Us = s$, $Vt = t$.

Let $\psi_1(y_1, y_2, y_3, y_4, y_5, y_6, y_7) = \max(y_1, y_2, y_3, y_4, y_5, y_6, y_7)$, $\varphi(y) = 2y$, $\psi_2 = \frac{1}{4}\psi_1$ then $\psi_1(y) = y, \forall y \in [0, \infty)$

$$\left|\frac{s}{4}-\frac{t}{4}\right|^2 \le \frac{1}{4}\max\left\{\sigma(s,t),\sigma\left(s,\frac{t}{4}\right),\sigma\left(\frac{s}{4},t\right),\sigma\left(s,\frac{s}{4}\right),\sigma\left(t,\frac{t}{4}\right),\right.$$
$$\left.\frac{1}{2}\left\{\sigma\left(\frac{t}{4},s\right)+\sigma\left(\frac{s}{4},t\right)\right\},\frac{1}{2}\left\{\sigma(s,t)+\sigma\left(\frac{s}{4},s\right)\right\}\right\}$$

For all $s,t \in E$, it follows that the condition (27).

Let $\{e_n\}$ be a sequence in E such that $pe_n \to z$ & $Ue_n \to z$ for some z in E. Then $z = 0$, $\sigma(pUe_n, Upe_n) \to 0$. Hence $\{p,U\}$ is sub compatible. We have common fixed point in E.

3. Conclusion

In this paper, we proved the fixed point theorem for four sub compatible maps under a contractive condition of integral type. These results can be extended to any directions and can also be extended to fixed point theory of non-expansive multi-valued mappings.

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