

Analyzing If a Graviton Gas Acts Like a Cosmological Vacuum State and “Cosmological” Constant Parameter

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Abstract

If a non-zero graviton mass exists, the question arises if a release of gravitons, possibly as a “Graviton gas” at the onset of inflation could be an initial vacuum state. Pros and cons to this idea are raised, in part based upon Bose gases. The analysis starts with Volovik’s condensed matter treatment of GR, and ends with consequences, which the author sees, if the supposition is true.

Keywords

Graviton Gas, Cosmological Vacuum State

1. Introduction

Volovik’s [1] book as of 2003 has a chapter on how a Bose gas can be used to obtain a vacuum energy. We extrapolate from this idea, and link it to what was done by Glinka [2], as to Wheeler De Witt (WdW) treatment of semi-classical style physics in his boson treatment of a “graviton gas” in order to make a similar analogy to what is done by Park [3], namely his so called version of a temperature sensitive cosmological constant parameter. Then, afterwards, links of how entropy may be connected with an evolution of the resulting cosmological vacuum energy expression, for a graviton gas are explored.

The authors’ beliefs as to if this hypothesis can be tested will be the final part of the manuscript.

2. Review of the Volovik Model for Bose Gases

Volovik [1] derives in page 24 of his manuscript a description of a total vacuum energy via an integral over three dimensional space

$$E_{\text{vac}}(N) = \int d^3r \cdot \varepsilon(n) \quad (1.1)$$

The integrand to be considered is, using a potential defined by $U = \frac{c^2 m}{n}$ as given by Volovik for weakly interacting Bose gas particles, as well as

$$\varepsilon(n) = \frac{1}{2}U \cdot n^2 + \frac{8}{15\pi^2\hbar^3}m^{3/2}U^{5/2}n^{5/2} = \frac{1}{2} \cdot c^2 \cdot \left[n \cdot m + \frac{4}{15} \cdot \left(\frac{m^5}{\hbar^2 \cdot \sqrt{c}} \right) \cdot \frac{1}{n^2} \right] \quad (1.2)$$

For the sake of argument, m , as given above will be called the mass of a graviton, n a numerical count of gravitons in a small region of space, and afterwards, adaptations as to what this expression means in terms of entropy generation will be subsequently raised. A simple graph of the 2nd term of Equation (1.2) with comparatively large m and with $\hbar = c = 1$ has the following qualitative behavior. Namely for

$$E1 = \left[c^2/2 \right] \cdot \left[\frac{4}{15} \cdot \left(\frac{m^5}{\hbar^2 \cdot \sqrt{c}} \right) \cdot \frac{1}{n^2} \right] \quad (1.3)$$

$E1 \neq 0$ when n is very small, and $E1 = 0$ as $n \rightarrow 10^{10}$ at the onset of inflation.

If we view this as having an indication of when the deviation from usual quantum linearity, the implication is that right at the start of the production of n “gravitons” that there is a cut off right at the start of graviton production, *i.e.* the implications for ‘tHooft’s [4] non linearity embedding of quantum systems for gravitons would be in that the conditions for non linear embedding are likely in place as a pre cursor to graviton production. What we are observing is right at the start of the production of gravitons, *i.e.* the moment emergence of graviton states occurs, we have extinguishment of a contribution of classical embedding, but the pre cursor to that would mean graviton production would be initially “framed” by a non linear contribution.

To quantify this, it would be to have $\varepsilon(n) \sim \varepsilon_{\text{Linear}}(n) + E1(n)$ with $E1(n)$ an additional, ‘tHooft [4] style embedding of a usual Q.M. treatment of a spin two particle. In what is stated later about emergence, the author claims that, in analogy to CDW, with emergence of CDW particles, that if there is emergence, that the $E1(n)$ would be equivalent to the degree of “slope” of a emergent “instanton” and/or instanton- anti instanton structure, which is written in CDW as **S-S’**. The statement as to emergence, if it occurs is, in both cosmology and CDW given as below, with the caveat that the slope, with its disappearance, in a thin wall representation is for a purely QM treatment of space time emergent particles. The author asserts that a non zero $E1(n)$ would be given in effect via **Figure 3**, as a non box like **S-S’** pair having ‘tHooft [4] style embedding of emergent QM structure.

An interesting datum to bring up for evaluation. ‘tHooft [4] talked about equivalence classes in his 2002 and 2006 publications. We can then write a wave functional for representing the nucleated states as of **Figure 3** as follows. $\phi_0(x)$ moving from the “floor” of **Figure 3**, as it rises above, is in sync with moving

toward the “thin wall approximation” of minimization of classical contributions to the emergence state ϕ , *i.e.* if **Figure 3** were a rectangular block moving upward, with no contributions other than the block itself moving “upward” it would represent a pure “QM” contribution to emergence. Deviations from this block shape represent a non linear semi classical embedding state, with different, continuum of $\phi_0(x)$ being continuum states and part of ‘tHooft [4] equivalence classes as seen in the CDW wave function below [5]

$$\Psi_{i,f} [\phi(x)]_{\phi=\phi_{ci,f}} = c_{i,f} \cdot \exp \left\{ - \int dx \alpha [\phi_{ci,f}(x) - \phi_0(x)]^2 \right\} \tag{1.4}$$

There exist a “regularization term” we identify with regularization term $E1(n) \neq 0 \rightarrow 0$ which will be seen in Equation (1.5) below, and which has a functional dependence in a fashion which will be derived in the future as $\phi_0(x)$ moves “up” from the “floor” of **Figure 3**. Also, if we are talking about the beginning of inflation, where $\varepsilon(n)$ would be approximately a constant in time, we can, in the neighborhood of Planck time.

$$E_{vac}(N) = \int d^3r \cdot \varepsilon(n) \sim [\text{Vol}]_{\text{Planck}} \cdot \frac{c^2}{2} \cdot \left[n \cdot m + \frac{4}{15} \cdot \left(\frac{m^3}{\hbar^2 \cdot \sqrt{c}} \right) \cdot \frac{1}{n^2} \right] \tag{1.5}$$

Furthermore, if we take density of this initial state, as given by $\rho = E_{vac}(N) / [\text{Vol}]_{\text{Planck}}$ as far as an information density value at the start of inflation, we get that there is initially a situation for which the regularization term does not contribute right at/just after Planck time t_{Planck}

$$\rho = E_{vac}(N) / [\text{Vol}]_{\text{Planck}} \approx \frac{c^2}{2} \cdot [n \cdot m] \tag{1.6}$$

Go to **Appendix A** as far as a description as to how and why $m \equiv m_{\text{graviton}} \neq 0$ in four dimensions. The links to entropy generation, and actual vacuum state values, will be subsequently raised after elucidating the particulars of a modification of Y.J. Ng’s [6] entropy count hypothesis, brought up by Beckwith in several conferences. The point to raise is the following about a graviton gas. *i.e.* if the mass of a graviton is nearly zero, and if the term $\frac{4}{15} \cdot \left(\frac{m^3}{\hbar^2 \cdot \sqrt{c}} \right) \cdot \frac{1}{n^2}$ plays a role, albeit in nearly a nearly non-existent fashion, for tiny graviton mass, then the existence of this second term is in sync with ‘tHooft’s deterministic quantum mechanics. Volovik calls the 2nd term a “regularization term”, and its importance can be seen as a way to quantify the affects of an embedding of initial quantum information within a larger structure, which is highly non linear. Doing so would help us determine if $f \sim f_*$ with f_* an initial frequency which can be picked up in GW/Graviton detectors. We shall now consider how to model emergent structure as given in **Figure 1**, **Figure 2**, and **Figure 3**.

3. Review of Y. J. Ng’s Entropy Hypothesis

As used by Ng [6]

$$Z_N \sim (1/N!) \cdot (V/\tilde{\lambda}^3)^N \tag{1.7}$$

This, according to Ng [6], leads to entropy of the limiting value of, if $S = (\log[Z_N])$ will be modified by having the following done, namely after his use of quantum infinite statistics, as commented upon by Beckwith

$$S \approx N \cdot (\log[V/\lambda^3] + 5/2) \approx N \tag{1.8}$$

Eventually, the author hopes to put on a sound foundation what 'tHooft [4] is doing with respect to. 'tHooft [4] deterministic quantum mechanics and equiva-

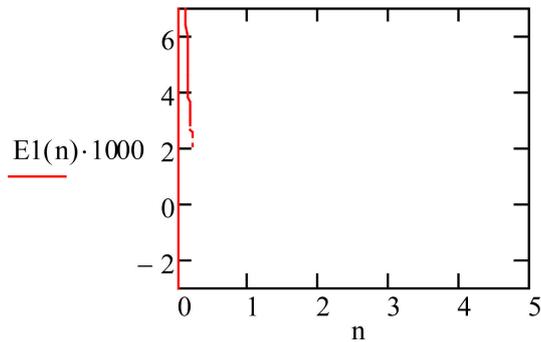


Figure 1. Graph of $E1(n)$ as an additional embedding structure for a 'tHooft style extension of QM. The smaller the mass is, the closer the $E1(n)$ regularization term is to not contributing at all, and *i.e.* its imprint exist before the creation of n “emergent” states. Later on, each state so created will be connected with gravitons.

CDW and its Solitons

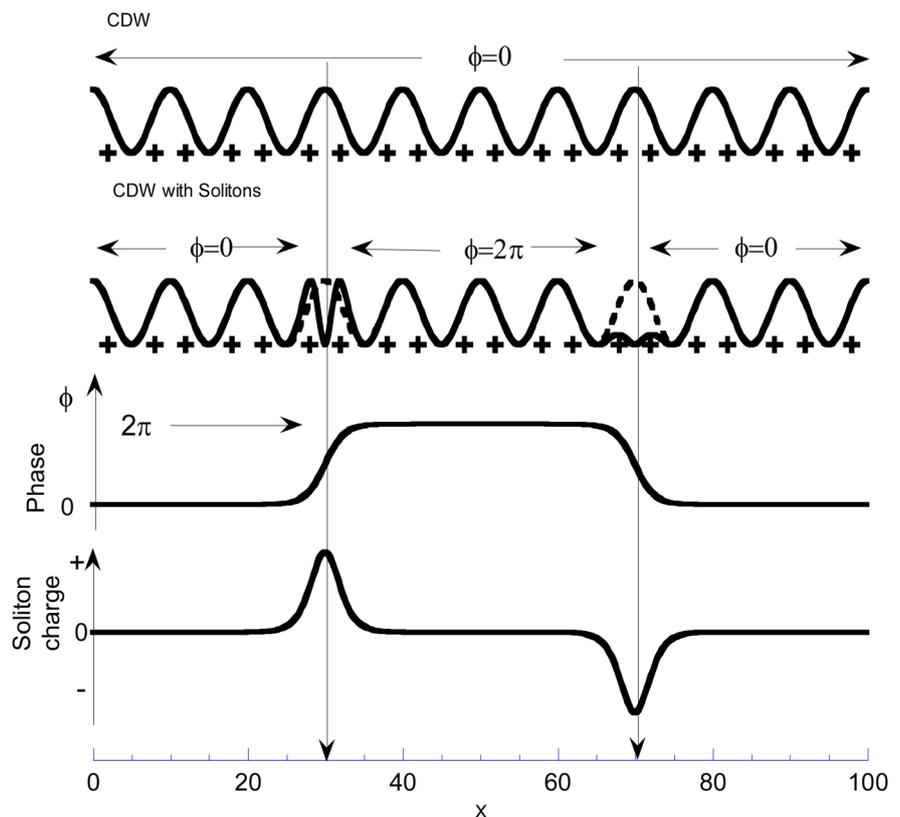


Figure 2. Eventual emergent structure, in terms of kink- anti kinks in space time [5].

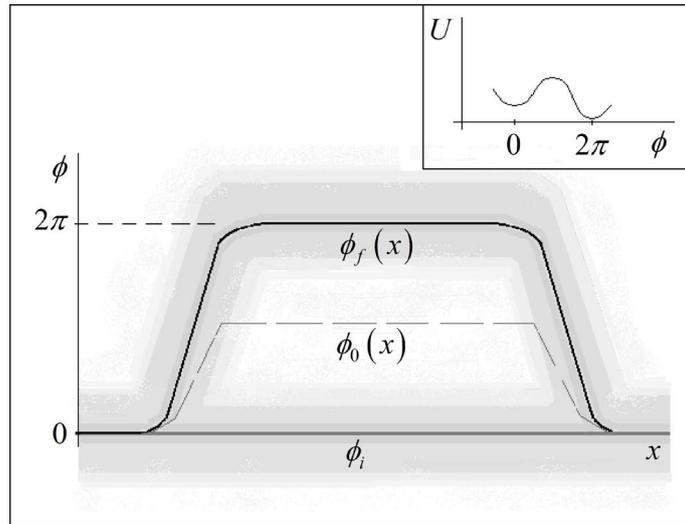


Figure 3. Sloped walls correspond to $E1(n) \neq 0$, with $E1(n) \neq 0 \rightarrow 0$ being purely QM effects for representation of emergent structure. $\phi_0(x)$ Rising with increased slope the smaller $\phi_0(x)$ is as representing how quantum structure becomes dominant for a (soliton-anti soliton) S-S' pair the further the a S-S' emerges and develops in space time [5].

lence classes embedding quantum particle structures. Our supposition is that the sample space, V is extraordinarily small, putting an emphasis upon λ being quite small, leading to high frequency behavior for the resulting generated N . For extremely small volumes for nucleation of a particle, in initial space, this leads to looking at an inter relationship between a term for initial entropy, of the order of 10^{10} , and if the following expression for detectable frequency, with f_* = initial frequency $\sim 1/\lambda$, a_* an initial scale factor, and a_0 today's scale factor behavior, as given by Buoanno [7] is true.

$$f \equiv f_* \cdot [a_*/a_0] \tag{1.9}$$

As written up by Buoanno [7], even if initial frequencies are enormous, the present day frequencies should be, tops of the order of 100 Hz for initial gravitational waves, *i.e.* the factor, $[a_*/a_0]$ would be almost non-existent. On the other hand, if the embedding structure containing the initial vacuum energy formation has an initially undisturbed character, with minimum breakage of an instanton formation of composite particles, then the frequency would be, instead closer to $f \sim f_*$ with f_* an initial frequency $\sim 1/\lambda$. We assert that the embedding structure of initial space time would be important to determining if $f \sim f_*$ is a datum we can extract, and observe.

4. Conditions to Test for Experimentally to Determine if $f \sim f_*$ Exist in the Present Era

As an example we consider a first order phase transition in the early universe. This can lead to a period of turbulent motion in the broken phase fluid, giving rise to a GW signal. Using the results from Durrer [8].

“If turbulence is generated in the early universe during a first order phase

transition, as discussed in the introduction, one has the formation of a cascade of eddies. The largest ones have a period comparable to the time duration of the turbulence itself (of the phase transition). According to Equation (16), these eddies generate GWs which inherit their wavenumber. Smaller eddies instead have much higher frequencies, and one might at first think that they imprint their frequency on the GW spectrum. However, since they are generated by a cascade from the larger eddies, they are correlated and cannot be considered as individual sources of GWs.” We have serious doubts about that last sentence.

Also brought up are GWs produced by the neutrino anisotropic stresses, which generate a turbulent phase. These would be weaker than E and M contributions to anisotropic stresses. For the record as stated in Kojima’s [9] article

Another more familiar example of extra anisotropic stress is that of a primordial magnetic field (PMF). The amplitude of the energy density $B^2/8\pi$ and magnetic anisotropic stress of the PMF again both scale as radiation density $\propto a^{-4}$. We doubt that such anisotropic stress would be pertinent to HFGW production. Our supposition is that relic graviton production, not just eddies, as speculated by Durrer also play a role as far as detection, Durrer’s [8] write up exclusively focuses upon eddies, and turbulence in initial GW production.

Wei-Tou Ni [10] in has a very direct statement that *DECIGO [11] and Big Bang Observer [12] look for GWs in the higher frequency range, which may give $f \sim f_*$ measurements, especially if f_* is not low frequency. Ni also writes, for stochastic backgrounds, that “The minimum detectable intensity of a stochastic GW background”*

$$h_0 \Omega_{GW \min}(f) \sim \text{const.} \times f^3 S_n(f) \tag{1.10}$$

i.e. Equation (1.9), and the primary difficulty is in accommodating $S_n(f)$ in a sensible fashion. Where $S_n(f)$ is in part analyzed by data brought up by M. Maggiore, [11]. Having said that, then the issue is, are relic conditions for gravitons and GW are linked to entropy, and an initial entropy value of $\sim 10^{10}$. Before saying this, we need to consider the role degrees of freedom, g_* is in the initial phases of inflation.

5. Difficulty in Visualizing What g_* Is in the Initial Phases of Inflation

Secondly, we look for a way to link initial energy states, which may be pertinent to entropy, in a way which permits an increase in entropy from about 10^{10} at the start of the big bang to about 10^{90} to 10^{100} today. One such way to conflate entropy with an initial cosmological constant may be of some help, *i.e.* if $V_4|_{\text{Threshold-volume-for-quantum-effects}} \sim (10^{-4} \text{ cm})^3$ or smaller, *i.e.* in between the threshold value, and the cube of Planck length, one may be able to look at coming up with an initial value for a cosmological constant as given by Λ_{Max} as given by [12]

$$\frac{\Lambda_{\text{Max}} V_4}{8\pi G} \sim T^{00} V_4 \equiv \rho \cdot V_4 = E_{\text{total}} \tag{1.11}$$

We assert here, that Equation (1.10) is the same order of magnitude as Equation (1.4). To get this, we also look at how to get a suitable Λ_{Max} value. Then making the following identification of total energy with entropy via looking at Λ_{Max} models, *i.e.* consider Park’s model of a cosmological “constant” parameter scaled via background temperature [3]

$$\Lambda_{\text{Max}} \sim c_2 \cdot T^{\beta} \tag{1.12}$$

A linkage between energy and entropy, as seen in the construction, looking at what Kolb [13] put in, *i.e.*

$$\rho = \rho_{\text{radiation}} = (3/4) \cdot \left[\frac{45}{2\pi^2 g_*} \right]^{1/3} \cdot S^{4/3} \cdot r^{-4} \tag{1.13}$$

Here, the idea would be, to make the following equivalence, namely look at,

$$\left[\left[\frac{\Lambda_{\text{Max}} r^4}{8\pi G} \right] \cdot (4/3) \cdot \left[\frac{2\pi^2 g_*}{45} \right]^{1/3} \right]^{3/4} \sim S_{\text{initial}} \tag{1.14}$$

Note that in the case that quantum effects become highly significant, that the contribution as given by $V_4|_{\text{Threshold-volume-for-quantum-effects}} \sim (10^{-4} \text{ cm})^3$ and potentially much smaller, as in the threshold of Planck’s length, going down to possibly as low as $4.22419 \times 10^{-105} \text{ m}^3 = 4.22419 \times 10^{-96} \text{ cm}^3$ leads us to conclude that even with very high temperatures, as an input into the initial entropy, that $S_{\text{initial}} \approx 10^{10}$ is very reasonable. Note though that Kolb and Turner [13], however, have that g_* is at most about 120, whereas the author, in conversation with H. De La Vega [14], in 2009 indicated that even the exotic theories of g_* have an upper limit of about 1200, and that it is difficult to visualize what g_* is in the initial phases of inflation.

De La Vega [14] stated in Como Italy, that he, as a conservative cosmologist, viewed defining g_* in the initial phases of inflation as impossible. So, then the following formulation of density fluctuations would have to be looked at directly

$$\frac{\left[\frac{\Delta E / l_p^3}{H_{\text{early}}^2 / l_p^2} \right]}{\left[\frac{\Delta S}{H_{\text{early}}^2 / l_p^2} \right]} \approx \frac{\Delta S}{H_{\text{early}}^2 / l_p^2} \sim \frac{l_p^2 \cdot \Delta S}{H_{\text{early}}^2} \sim \frac{\delta \rho}{\rho} \tag{1.15}$$

where we will put in a candidate for the ΔS for initial conditions, and then use that as far as answering questions as far as formulating an answer as far as entropy fluctuations, and candidates for density fluctuations, as well as early values of the Hubble parameter. Having such a relatively small value of $l_p^2 \propto [1.616 \times 10^{-35} \text{ meters}]^2$ as placed with $\Delta S \sim 10^{10}$

$$10^{-4} - 10^{-5} \sim \frac{l_p^2 \cdot \Delta S}{H_{\text{early}}^2} \tag{1.16}$$

This will lead to comparatively low values for H_{early}^2 which will be linked to the behavior of a cosmological “constant” parameter value, which subsequently changes in value later, *i.e.*, Equation (1.17) will be for a configuration just before the onset of the big bang itself. Also one can directly write

$$H_{\text{early}}^2 \sim \left[\Lambda_{\text{Cosmological}} \cdot l_p^2 / 8\pi G \right] \tag{1.17}$$

And, also,

$$\frac{l_p^2 \cdot \Delta S}{H_{\text{Early}}^2} \approx \frac{8\pi G \cdot \Delta S}{\Lambda_{\text{Cosmological}}} \sim 10^{-4} - 10^{-5} \tag{1.18}$$

An initially

$$f_{\text{Peak}} \cong 10^{-8} \cdot [\beta/H_*] \cdot [T_*/16 \text{ GeV}] \cdot [g_*/100]^{1/6} \text{ Hz} \tag{1.19}$$

By conventional cosmological theory, limits of g_* are at the upper limit of 100 - 120, at most, according to Kolb and Turner [13] (1991). $T_* \sim 10^2 \text{ GeV}$ is specified for nucleation of a bubble, as a generator of GW. Early universe models with $g_* \sim 1000$ or so are not in the realm of observational science, yet, according to Hector De La Vega [14] (2009) in personal communications with the author,) at the Colmo, Italy astroparticle physics school, ISAPP, Furthermore, the range of accessible frequencies as given by Equation (1.19) is in sync with

$$h_0^2 \Omega_{\text{gw}}(f) \sim 10^{-10} \tag{1.20}$$

for peak frequencies with values of 10 MHz. The net affect of such thinking is to proclaim that all relic GW are inaccessible. If one looks at Figure, $h_0^2 \Omega_{\text{GW}} > 10^{-6}$ for frequencies as high as up to 10^6 Hertz, this counters what was declared by Turner and Wilzenk [15] (1990): that inflation will terminate with observable frequencies in the range of 100 or so Hertz. The problem is though, that after several years of LIGO, no one has observed such a GW signal from the early universe, from black holes, or any other source, yet. About the only way one may be able to observe a signal for GW and/or gravitons may be to consider how to obtain a numerical count of gravitons and/or neutrinos for

$$h_0^2 \Omega_{\text{gw}}(f) \cong \frac{3.6}{2} \cdot \left[\frac{n_f [\text{graviton}] + n_f [\text{neutrino}]}{10^{37}} \right] \cdot \left(\frac{\langle f \rangle}{1 \text{ kHz}} \right)^4 \tag{1.21}$$

And this leads to the question of how to account for a possible mass/information content to the graviton.

6. Break Down of Quark—Gluon Models for Generation of Entropy

It gets worse if one is asserting that there is, in any case, a quark gluon route to determine the role of entropy. To begin this analysis, let us look at what goes wrong in models of the early universe. The assertion made is that this is due to the quark—Gluon model of plasmas having major “counting algorithm” breaks with non counting algorithm conditions, *i.e.* when plasma physics conditions BEFORE the advent of the Quark gluon plasma existed. Here are some questions which need to be asked.

1) Is QGP strongly coupled or not? Note: Strong coupling is a natural explanation for the small (viscosity) Analogy to the RHIC: J/y survives DE confinement phase transition

2) What is the nature of viscosity in the early universe? What is the standard story? (Hint: AdS-CFT correspondence models). Question 2 comes up since

$$\frac{\eta}{s} = \frac{1}{4\pi} \tag{1.22}$$

typically holds for liquid helium and most bosonic matter. However, this relation breaks down. At the beginning of the big bang. As follows *i.e.* if Gauss-Bonnet gravity is assumed, in order to still keep causality, one needs

$$\lambda_{BG} \leq \frac{9}{100}$$

This even if one writes for a viscosity over entropy ratio the following

$$\frac{\eta}{s} \equiv \frac{1}{4\pi} \cdot [1 - 4\lambda_{GB}] \leq \frac{1}{4\pi} \tag{1.23}$$

A careful researcher may ask why this is so important. If a causal discontinuity as indicated means the $\frac{\eta}{s}$ ratio is $\approx \frac{1}{4\pi} \cdot \frac{33}{50}$, or less in value, it puts major restrictions upon viscosity, as well as entropy. A drop in viscosity, which can lead to major deviations from $\frac{1}{4\pi}$ in typical models may be due to more collisions.

Then, more collisions due to WHAT physical process? Recall the argument put up earlier, *i.e.* the reference to causal discontinuity in four dimensions, and a restriction of information flow to a fifth dimension at the onset of the big bang/ transition from a prior universe? That process of a collision increase may be inherent in the restriction to a fifth dimension, just before the big bang singularity, in four dimensions, of information flow. In fact, it very well be true, that initially, during the process of restriction to a 5th dimension, right before the big bang, that $\left| \frac{\eta}{s} \approx \varepsilon^+ \right| \ll \frac{1}{4\pi}$. Either the viscosity drops nearly to zero, or else the entropy density may, partly due to restriction in geometric “sizing” may become effectively nearly infinite. It is due to the following qualifications put in about Quark – Gluon plasmas which will be put up, here. Namely, more collisions imply less viscosity. More Deflections ALSO implies less viscosity. Finally, the more momentum transport is prevented, the less the viscosity value becomes. Say that a physics researcher is looking at viscosity due to turbulent fields. Also, perturbative calculated viscosities: due to collisions. This has been known as *Anomalous Viscosity* in plasma physics, (this is going nowhere, from pre-big bang to big bang cosmology). **Appendix B** gives some more details as far as the

So happens that RHIC models for viscosity assume

$$\frac{1}{\eta} \approx \frac{1}{\eta_A} + \frac{1}{\eta_C} \tag{1.24}$$

As Akazawa [16] noted in an RHIC study, equation 1.80 above makes sense if one has stable temperature T , so that

$$\frac{\eta_A}{s} = \bar{c}_0 \cdot \left(\frac{T}{g^2 |\nabla u|} \right)^{\frac{2n-1}{2n+1}} \Leftrightarrow \frac{\eta_C}{s} = \text{Constant} \tag{1.25}$$

If the temperature T wildly varies, as it does at the onset of the big bang, this

breaks down completely. This development is FRANKLY Mission impossible: AND why we need a different argument for entropy, *i.e.* Even for the RHIC, and in computational models of the viscosity for closed geometries—what goes wrong in computational models

- Viscous Stress is *NOT* \propto shear
- Nonlinear response: impossible to obtain on lattice (computationally speaking)
- Bottom line: we DO NOT have a way to even define SHEAR in the vicinity of big bang!!!!

i.e. the quark gluon stage of production of entropy, and its connections to early universe conditions may lead to undefined conditions which, *i.e.* like shear in the beginning of the universe, cannot be explained. *i.e.* what does viscosity mean in the neighborhood of time where 10^{-44} s < time < 10^{-35} s ?

7. Inter Relationship between Graviton Mass m_g and the Problem of a Sufficient Number of Bits of \hbar from a Prior Universe, to Preserve Continuity between Fundamental Constants from a Prior to the Present Universe?

V.A. Rubakov and, P.G. Tinyakov [17] gives that there is, with regards to the halo of sub structures in the local Milky Way galaxy an amplitude factor for gravitational waves of

$$\langle h_{ij} \rangle \sim 10^{-10} \times \left[\frac{2 \times 10^{-4} \text{ Hz}}{m_{\text{graviton}}} \right] \tag{1.26}$$

If we use LISA values for the Pulsar Gravitational wave frequencies, this may mean that the massive graviton is ruled out. On the other hand

$$\sqrt{\frac{M}{2.8 M_{\text{solar-mass}}}} \cdot \sqrt{\frac{90 \text{ km}}{R}} \approx 10^8 - 10^{10} \text{ leads to looking at, if}$$

$$\langle h_{ij} \rangle \sim h \sim 10^{-5} \cdot \left[\frac{15 \text{ Mpc}}{r} \right]^{1/2} \cdot \left[\frac{M}{2.8 \cdot M_{\text{solar-mass}}} \right]^{-1/2} \approx 10^{-30} \tag{1.27}$$

If the radius is of the order of $r \geq 10$ billion light-years ~ 4300 Mpc *or much greater, so then we have, as an example*

$$\langle h_{ij} \rangle \sim 10^{-10} \cdot \left[\frac{2 \times 10^{-4} \text{ Hz}}{m_{\text{graviton}}} \right] \approx 5.9 \times 10^{-7} \cdot \left[\frac{M}{2.8 \cdot M_{\text{solar-mass}}} \right]^{1/2}, \text{ so then one is getting}$$

$$\left[\frac{10^{-7} \text{ Hz}}{m_{\text{graviton}}} \right] \approx \left[\frac{5.9}{\sqrt{5.6}} \right] \cdot \sqrt{\frac{M}{M_{\text{solar-mass}}}} \tag{1.28}$$

This Equation (1.28) is in units where $\hbar = c = 1$.

If $10^{-60} - 10^{-65}$ grams per graviton, and 1 electron volt is in rest mass, so 1.6×10^{-33} grams \Rightarrow gram = 6.25×10^{32} eV. Then [18]

$$\left[\frac{10^{-7} \text{ Hz}}{m_{\text{graviton}}} \right] \equiv \left[\frac{10^{-7} \text{ Hz} \cdot [6.582 \times 10^{-15} \text{ eV} \cdot \text{s}]}{[10^{-60} \text{ grams} \equiv 6.25 \times 10^{-28} \text{ eV}] \cdot [2.99 \times 10^9 \text{ meter/sec}]^2} \right] \sim \frac{10^{-22}}{10^{-9}} \sim 10^{-13} \tag{1.29}$$

Then, exist

$$M \sim 10^{-26} M_{\text{solar-mass}} \approx 1.99 \times 10^{33-26} \equiv 1.99 \times 10^7 \text{ grams.} \tag{1.30}$$

If each photon, as stated above is 3.68×10^{-48} grams per photon [19], then

$$M \sim 5.44 \times 10^{54} \text{ Initially transmitted photons.} \tag{1.31}$$

Furthermore, if there are, today for a back ground CMBR temperature of 2.7 degrees Kelvin 5×10^8 photons/cubic – meter , with a wave length specified as $\lambda_{\text{max}} \approx 1 \text{ cm}$. This is for a numerical density of photons per cubic meter given by

$$n_{\text{photon}} := \frac{\sigma \cdot (T)^4 \cdot (\lambda_{\text{max}})}{h \cdot c^2} \tag{1.32}$$

As a rough rule of thumb, if, as given by Weinberg [20] (1972) that early quantum effects, for quantum gravity take place at a temperature $T \approx 10^{33}$ Kelvin, then, if there was that temperature for a cubic meter of space, the numerical density would be, roughly 10^{132} times greater than what it is today. Forget it. So what we have to do is to consider a much smaller volume area. If the radii of the volume area is $r \equiv 4 \times 10^{-35}$ meters $\equiv l_p = \text{Planck – length}$, then we have to work with a de facto initial volume $\approx 64 \times 10^{-105} \sim 10^{-103} \text{ (meters)}^3$. *i.e.* the numerical value for the number of photons at $T \approx 10^{33}$, if we have a per unit volume area based upon Planck length, instead of meters, cubed is $10^{29} \times (5 \times 10^8) \approx 5 \times 10^{37}$ photons for a cubic area with sides $r \equiv 4 \times 10^{-35}$ meters $\equiv l_p$ at $T|_{\text{quantum-effects}} \approx 10^{33}$ Kelvin However, $M \sim 5.44 \times 10^{54}$ initially transmitted photons! Either the minimum distance, *i.e.* the grid is larger, or $T|_{\text{quantum-effects}} \gg 10^{33}$ Kelvin

8. Finally: What Can be Stated about

$$h_o \Omega_{GW} \times \min(f) \sim \text{const.} \times f^3 S_n(f) ?$$

We assert that at a minimum, we can write, the following. Namely that to begin a reasonable inquiry, that

$$\begin{aligned} h_o \Omega_{GW} \times \min(f) &\sim \text{const.} \times f^3 S_n(f) \sim h_o^2 f \times \Omega_{gw}(f) \\ &\cong 3.6 \cdot \left[\frac{n_f [\text{graviton}]}{10^{37}} \right] \cdot \left(\frac{f}{\text{kHz}} \right)^4 \cdot f \end{aligned} \tag{1.33}$$

If one has that $h_o^2 \times \Omega_{gw}(f) \sim 10^{-6} - 10^{-10}$, the above effect is to put restrictions upon stochastic treatments of $S_n(f)$ for frequencies at or above 10^6 Hertz. Note here that $S_n(f)$ spectral density is, in some cases allowing for substitution of the spectral density function via the sort of arguments given in **Appendix B** below.

9. Conclusion. A Graviton Gas Inevitably Has Semi Classical Features. Cosmological Constant Parameter Initially May Be Accounted for via Graviton Release Initially?

The author is fully aware of how Durrer [8] and others use turbulence in early universe conditions, as a way, at the time of the electro weak transition to ac-

count for relic graviton production. The electro weak transition, as noted by Rubakov [21], and others [22] is a candidate for computing the gravity waves induced by anisotropic stresses of stochastic primordial magnetic fields, *i.e.* a specified magnetic field in the onset of early universe conditions. The author suggests that earlier generation, requiring increased sensitivity of GW detectors, perhaps of $h \sim 10^{-24} - 10^{-25}$ may be necessary as to be able to reach higher frequency GW created by graviton production at the onset of inflation. Note that L. Grishchuk [23], in 2007 specified relic GW production as up to 10 GHz which is far in excess of the values Durrer and others proposal. Indeed, Durrer, Marozzi, and Rinaldi [24] are convinced that any relic conditions for GW must be much lower, with no relic GW observable as they specify it on alleged practical grounds. If one is unable to obtain detector sensitivities of the order of $h \sim 10^{-24} - 10^{-25}$ in the foreseeable future, Durrer, Marozzi, and Rinaldi [24] may be right by default. It is worth noting though that physics should be considering if relic GW occurs at all, and the author, and L. Grishchuk [23] have presented mechanisms which may account for their existence in regions of space time evolution well before the electro weak transition, and not necessarily due to conditions linked to anisotropic stress of magnetic fields.

The authors supposition is, in line with what has been presented in the above, that graviton production and early universe entropy production of the order of $S \sim 10^{10}$ in initial Planck time $t \sim t_{\text{Planck}} \propto 10^{-43}$ seconds may be crucial in formation of an initial graviton gas, which may act like an initial cosmological parameter. The supposition inevitably would be part of the problem of. confirming if

confirming if $\frac{8\pi G \cdot [\Delta S_{\text{initial}} \sim 10^{10}]}{[\Lambda_{\text{Cosmological}} \sim c_2 T_{\text{Planck}}^4]} \sim 10^{-4} - 10^{-5}$ is possible. Here, Planck tem-

perature $T_{\text{Planck}} = 1.416785(71) \times 10^{32}$ Kelvin, and the issue would be, if this is true, of giving sufficient reasons for having a scaling argument from initial condition, as specified, of confirming if an analytical proof, backed up by measurements confirms

$$\Lambda_{\text{Today}} \sim c_2 \left[\frac{T_{\text{Today}}^4}{(2.75 \text{ Kelvin})^4} \right] \sim [10^4 - 10^5] \cdot 8\pi G \cdot \left[\frac{\Delta S_{\text{initial}}}{T_{\text{Planck}}^4} \right] \quad (1.34)$$

$$\approx 10^{-35} \text{ s}^{-2}$$

or 10^{-47} GeV^4 , or 10^{-29} g/cm^3 or about 10^{-120} in reduced Planck units.

I.e. what value of $\Delta S_{\text{initial}}$ is really needed, so as to obtain 10^{-120} today?

If falsifiable experimental measurements for Equation (1.34) may be obtained, the next step would be perhaps in confirming what degree of information exchange such a scaling may imply. The information exchange from a prior to a present universe would be modeled on the template of what $\Delta S_{\text{initial}}$ would be required, and of what dimensional embedding is needed to do so. Furthermore, what is obtained should be reconciled with an additional constraint which will be put in the next page.

Note that Corda [25] has modeled adiabatically-amplified zero-point fluctuations processes in order to show how the standard inflationary scenario for the early universe can provide a distinctive spectrum of relic gravitational waves. De

Laurentis, and Capozziello [26] (2009) have further extended this idea to give a qualified estimate of GW from relic conditions which will be re produced here. Begin with De Laurentis's idea of a gravitational wave spectrum

$$\Omega_{sgw} = \frac{16}{9} \cdot \left(\frac{\rho_{dS}}{\rho_{Planck}} \right) / (1 + z_{eq}) \xrightarrow{f \rightarrow \text{low-value}} f^{-2} \Leftrightarrow f|_{\text{present=era}} > (1 + z_{eq})^{1/2} \cdot H_0 \quad (1.35)$$

H_0 is today's Hubble parameter, while f is GW frequency, and z_{eq} is the red shift value of when the universe became matter dominated, *i.e.* red shift $z = 1.55$ with an estimated age of 3.5 Giga year, or larger, would be a good starting point, *i.e.* this is for larger than 3.5 Giga years for when matter domination became most prominent, *i.e.* the further back z_{eq} goes the larger the upper bound for frequency f . The upper range for f appears to be about 100 Hertz. Needless to state, though, if z_{eq} drifted to a value of $z_{eq} \sim 10$ then the upper bound to $f \sim 1000$ Hertz. And, we suggest that $f > 1000$ Hz, if $z_{eq} \sim 10$ is set higher, *i.e.* $z_{eq} \sim 100$, which should be investigated.

We at the close refer the readers to **Appendix C** for crucial considerations as to the emergence of gravitational astronomy as this relates to a summary as to how to confirm the models so referenced in this paper, as to work by Corda, and the LIGO GW team which is of potentially revolutionary import as far as observational astronomy confirming these ideas so presented.

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Appendix A: Looking at Situations When the Mass of a Graviton is not Zero

A1: Linkage of DM to gravitons and gravitational waves?

Let us state that the object of early universe GW astronomy would be to begin with confirmation of whether or not relic GW were obtainable, and then from there to ascertain is there is linkage which can be made to DM production ... Durrer, Massimiliano Rinaldi [24] (2009), state that there would be probably negligible for this case (practically non-existent) graviton production in cosmological eras after the big bang.. In fact, they state that they investigate the creation of massless particles in a Universe which transits from a radiation- dominated era to any other (via an) expansion law. “We calculate in detail the generation of gravitons during the transition to a matter dominated era. We show that the resulting gravitons generated in the standard radiation/matter transition are negligible” This indicated to the author, Beckwith that it is appropriate to look at the onset of relic GW/Graviton production. One of the way to delineating the evolution of GW is the super adiabatic approximation, done for when $k^2 \ll |a''/a|$ as given by M. Giovannini [27] (page 138), when $\mu_k \equiv a \cdot h_k$ is a solution to

$$\mu_k'' + \left[k^2 - \frac{a''}{a} \right] \mu_k = 0. \tag{A.1}$$

Which to first order when $k^2 \ll |a''/a|$ leads to a GW solution

$$h_k(\tau) \cong A_k + B_k \cdot \int_0^\tau \frac{dx}{a(x)} \tag{A.2}$$

This will be contrasted with a very similar evolution equation for gravitons, of (*i.e.* KK gravitons in higher dimensions)

$$h'' - \left[4k^2 + \frac{m^2}{a^2(z)} \right] h \cong 0 \tag{A.3}$$

One of the models of linkage between gravitons, and DM is the KK graviton, *i.e.* as a DM candidate. KK gravitons. Note that usual Randall Sundrum brane theory has a production rate of $\Gamma \sim T^6 / M_{\text{Planck}}^2$ as the number of Kaluza Klein gravitons per unit time per unit volume Note this production rate is for a formula assuming mass for which $T > M_{\text{p}}$ and that we are assuming that the temperature $T \sim T_*$. Furthermore, we also are looking at total production rate of KK gravitons of the form

$$\frac{dn}{dt} \sim \frac{T^6}{M_{\text{Planck}}^2} \cdot (T \cdot R)^d \sim T^4 \cdot \left(\frac{T}{M_X} \right)^{2+d} \tag{A.4}$$

where R is the assumed higher dimension ‘size’ and, d is the number of dimensions above 4, and typically we obtain $T \gg 1/R$. *I.e.* we can typically assume tiny higher dimensional ‘dimensions’, very high temperatures, and also a wave length for the resulting KK graviton for a DM candidate looking like

$$\lambda_{\text{KK-Graviton}} \sim T^{-1} \tag{A.5}$$

If KK gravitons have the same wavelength as DM, this will support Jack Ng’s treatment of DM. All that needs to put this on firmer ground will be to make a de facto linkage of KK Gravitons, as a DM candidate, and more traditional treatments of gravitons, which would assume a steady drop in temperature from $T \sim T^*$, to eventually much lower temperature scales. Note that in a time interval based as proportional to the inverse of the Hubble parameter, we have the total numerical density of KK gravitons (on a brane?) as $n(T) \sim T^2 M_{\text{Planck}}^* \cdot (T/M)^{2+d}$, where $M_{\text{Planck}}^* \sim 10^{18}$ GeV give or take an order of magnitude. This number density $n(T)$ needs to be fully reconciled to $\lambda_{\text{KK-Graviton}} \sim T^{-1}$ and can be conflated with the dimensionality ‘radius’ value $R \sim 10^{\frac{32}{d}} \times 10^{-17}$ centimeters for dimensions above 4 space time GR values, with this value of R being unmanageable for $d < 2$. V.A. Rubakov [21] and others also (2002) makes the claim of the KK graviton obeying the general Yukawa style potential

$$V(r) = -\frac{G_4}{r} \cdot \left(1 + \frac{\text{const}}{k^2 r^2}\right) \tag{A.6}$$

As well as being related to an overall wave functional which can be derived from a line element

$$dS^2 \equiv \left[a^2(z) \cdot \eta_{uv} + h_{uv}(x, z) \right] \cdot dx^u dx^v + dz^2 \tag{A.7}$$

With $h'' - \left[4k^2 + \frac{m^2}{a^2(z)} \right] h \equiv 0$ (suppressing the u, v coefficients). This evolution equation for the KK gravitons is very similar to work done by Baumann, Daniel, Ichiki, Kiyotomo, Steinhardt, Paul J. Takahashi, Keitaro [28] (2007) with similar assumptions, with the result that KK gravitons are a linear combination of Bessel functions. Note that one has for gravitons.

$$h \equiv h_m(z \rightarrow 0) = \text{const} \cdot \sqrt{\frac{m}{k}} \tag{A.8}$$

Ruth Gregory, Valery A. Rubakov and Sergei M. Sibiryakov [29] (2000) make the additional claim that for large z (the higher dimensions get significant) that there are marked oscillatory behaviors, *i.e.* Rapid oscillations as one goes into the space for branes for massive graviton expansion.

$$h \equiv h_m(z \neq 0) \approx \text{const} \cdot \sqrt{a(z)} \cdot \sin\left(\frac{m}{k} \cdot \text{esp}(kz) + \varphi_m\right) \tag{A.9}$$

This is similar to what Baumann, Ichiki, Steinhardt, and Takahashi [28] (2007) for GW, in a relic setting, with the one difference being that the representation for a graviton is in the z (additional dimension) space, as opposed to what Bauman *et al.* [28] did for their evolution of GW, with an emphasis upon generation in overall GR space time.. Furthermore, the equation given in

$h'' - \left[4k^2 + \frac{m^2}{a^2(z)} \right] h \equiv 0$ for massive graviton evolution as KK gravitons along

dS branes is similar to evolution of GW in more standard cosmology that the

author, Beckwith, thinks that the main challenge in clarifying this picture will be in defining the relationship of dS geometry, in overall Randall Sundrum brane world to that of standard 4 space. We need though, now to look at whether or not higher dimensions are even relevant to GR itself.

A2: How DM would be influenced by gravitons, in 4 dimensions

We will also discuss the inter relationship of structure of DM, with challenges to Gaussianity. The formula as given by

$$\delta \equiv - \left[\frac{3}{2} \cdot \Omega_m \cdot H^2 \right]^{-1} \cdot \nabla^2 \Phi \tag{A.10}$$

Will be gone into. The variation, so alluded to which we will link to a statement about the relative contribution of Gaussianity, via looking at the gravitational potential

$$\Phi \equiv \Phi_L + f_{NL} \cdot \left[\Phi_L^2 - \langle \Phi_L^2 \rangle \right] + g_{NL} \cdot \Phi_L^3 \tag{A.11}$$

Here the expression f_{NL} = variations from Gaussianity, while the statements as to what contributes, or does not contribute will be stated in our presentation. Furthermore, $\Phi_L \equiv$ is a linear Gaussian potential, and the overall gravitational potential is altered by inputs from the term, presented, f_{NL} . The author discussed inputs into variations from Gaussianity, which were admittedly done from a highly theoretical perspective with Sabino Matarre [30] on July 10, with his contributions to non Gaussianity being constricted to a reported range of $-4 < f_{NL} < 80$, as given to Matarre [30], by Senatore, *et al* [31], 2009. The author, Beckwith, prefers a narrower range along the lines of $5 < f_{NL} < 20$. Needless to state, though, dealing with what we can and cannot measure, what is ascertained as far as DM, via a density profile variation needs to have it reconciled with DM detection values

$$\sigma_{DM-detection} \leq 3 \times 10^{-8} \text{ pb (pico barns)} \tag{A.12}$$

It is note worthy to note that the question of DM/KK gravitons, and also the mass of the graviton not only has relevance to whether or not, higher dimensions are necessary/advisable in space time models, but also may be relevant to if massive gravitons may solve/partly fulfill the DE puzzle. To wit, \KK gravitons would have a combined sum of Bessel equations as a wave functional representation. In fact V. A Rubakov [21] (2002) writes that KK graviton representation as, after using the following normalization $\int \frac{dz}{a(z)} \cdot [h_m(z) \cdot h_{\tilde{m}}(z)] \equiv \delta(m - \tilde{m})$,

where J_1, J_2, N_1, N_2 are different forms of Bessel functions, to obtain the KK graviton/ DM candidate representation along RS dS brane world

$$h_m(z) = \sqrt{m/k} \cdot \frac{J_1(m/k) \cdot N_2([m/k] \cdot \exp(k \cdot z)) - N_1(m/k) \cdot J_2([m/k] \cdot \exp(k \cdot z))}{\sqrt{[J_1(m/k)]^2 + [N_1(m/k)]^2}} \tag{A.13}$$

This allegedly is for KK gravitons having an order of TeV magnitude mass $M_z \sim k$ (*i.e.* for mass values at 0.5 TeV to above a TeV in value) on a negative tension RS brane. What would be useful would be managing to relate this KK

graviton, which is moving with a speed proportional to H^{-1} with regards to the negative tension brane with $h \equiv h_m(z \rightarrow 0) = \text{const} \cdot \sqrt{\frac{m}{k}}$ as a possible initial starting value for the KK graviton mass, before the KK graviton, as a ‘massive’ graviton moves with velocity H^{-1} along the RS dS brane. If so, and if $h \equiv h_m(z \rightarrow 0) = \text{const} \cdot \sqrt{\frac{m}{k}}$ represents an initial state, then one may relate the mass of the KK graviton, moving at high speed, with the initial rest mass of the graviton, which in four space in a rest mass configuration would have a mass many times lower in value, *i.e.* of at least $m_{\text{graviton}}(4\text{-Dim GR}) \sim 10^{-48} \text{ eV}$, as opposed to $M_X \sim M_{\text{KK-Graviton}} \sim .5 \times 10^9 \text{ eV}$. Whatever the range of the graviton mass, it may be a way to make sense of what was presented by Dubovsky, Flauger, Starobinsky, and Thackev [32] (2009) who argue for graviton mass using CMBR measurements, of up to $m_{\text{graviton}}(4\text{-Dim GR}) \sim 10^{-20} \text{ eV}$. This can be conflated with M. Alves, O. Miranda, and J de Araujo’s [33] results arguing that non zero graviton mass may lead to acceleration of our present universe, in a manner usually conflated with DE, *i.e.* their graviton mass would be about $m_{\text{graviton}}(4\text{-Dim GR}) \sim 10^{-48} \times 10^{-5} \text{ eV} \sim 10^{-65} \text{ grams}$, leading to a possible explanation for when the universe accelerated, *i.e.* the de-acceleration parameter, due to changes in the scale factor, written as

Appendix B. Next Generation GW Detectors

The following section is to improve upon the range of GW detected, as can be presented below. We use **Figure 4** as given explicitly below

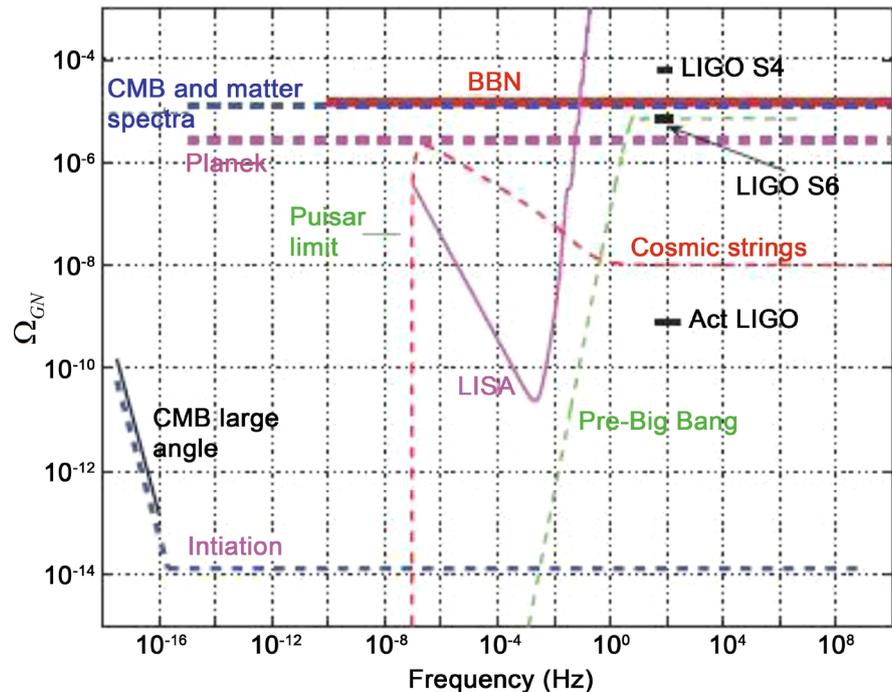


Figure 4. This figure from B. P. Abbott *et al.* [34] (2009) shows the relation between Ω_g and frequency.

The relation between Ω_g and the spectrum $h(\nu_g, \tau)$ is often expressed as written by L. P. Grishchuk, (2001) [35], as

$$\Omega_g \approx \frac{\pi^2}{3} \left(\frac{\nu}{\nu_H} \right)^2 h^2(\nu, \tau), \tag{B.1}$$

The curve of the pre-big-bang models shows that Ω_g of the relic GWs is almost constant $\sim 6.9 \times 10^{-6}$ from 10 Hz to 10^{10} Hz. Ω_g of the cosmic string models is about 10^{-8} in the region 1 Hz to 10^{10} Hz; its peak value region is about $10^{-7} - 10^{-6}$ Hz. The reason for this section is to deal with the statement made by Buoanno [7] (2006) that the following limit is verbatim, and cannot be improved upon if one looks at BBN, the following upper bound should be considered:

$$h_0^2 \Omega_{gw}(f) \leq 4.8 \times 10^{-9} \cdot (f/f_*)^2 \tag{B.2}$$

Here, Buoanno [7] is using $f > f_* = 4.4 \times 10^{-9}$ Hz, and a reference from Kosowky, Mack, and Kahnishvili [36] (2002) as well as Jenet *et al.* [37] (2006). Using this upper bound, if one insist upon assuming, as Buoanno [7] (2006) does, that the frequency today depends upon the relation

$$f \equiv f_* \cdot [a_*/a_0] \tag{B.3}$$

The problem in this is that the ratio $[a_*/a_0] \ll 1$, assumes that a_0 is “today’s” scale factor. In fact, using this estimate, Buoanno [7] comes up with a peak frequency value for relic/early universe values of the electroweak era-generated GW graviton production of

$$f_{\text{Peak}} \cong 10^{-8} \cdot [\beta/H_*] \cdot [T_*/16 \text{ GeV}] \cdot [g_*/100]^{1/6} \text{ Hz} \tag{B.4}$$

By conventional cosmological theory, limits of g_* as given by Kolb and Turner [13] (1991) are at the upper limit of 100 - 120. In addition according to Kolb and Turner [13] (1991), $T_* \sim 10^2$ GeV is specified for nucleation of a bubble, as a generator of GW. Early universe models with $g_* \sim 1000$ or so are not in the realm of observational science, yet, according to Hector De La Vega [14] (2009) in personal communications with the author, at the Colmo, Italy astroparticle physics school, ISAPP. All the assumptions above lead to a de facto limit of $h_0^2 \Omega_{gw}(f) \sim 10^{-10}$, which is what Dr. Fangyu Li [38] disputes: The following notes are also in response to a referee quote which Fangyu answered the following query, which is reproduced

Quote:

“The most serious is that a background strain $h \sim 10^{-30}$ at 10 GHz corresponds to a Ω_g (total) $\sim 10^{-3}$ which violates the baryon nuclei-synthesis epoch limit for either GWs or EMWs. Ω_g (Total) needs to be smaller than 10^{-5} otherwise the cosmological Helium/hydrogen abundance in the universe would be strongly affected ...”

The answer, which the author copied from Dr. Li, *i.e.*, If $\nu = 10$ GHz, $h = 10^{-31}$, then Dr. Li claims

$$\Omega_g = 8.3 \times 10^{-7} < \Omega_{g \text{ max}} \tag{B.5}$$

The following is Dr. Fangyu Li’s argument as given to the author in personal

notes:

- 1) LIGO and our coupling electromagnetic system [39] [40] in the free space are different detecting schemes for GWs. LIGO detects shrinking and extension of interferometer legs, this is a displacement effect. The CEMS detects the perturbative photon fluxes, this is a parameter perturbation effect of the EM fields. Although their sensitivities all are limited by relative quantum limits, concrete mechanisms of the quantum limits are quite different.
- 2) The minimal detectable amplitude of LIGO depends on [41]

$$h_{\min} \sim \frac{\lambda}{Lb\sqrt{N\tau}} \quad (\text{B.6})$$

where L is the interferometer length. Because detecting band of LIGO is limited in ~ 1 Hz - 1000 Hz, this is a very strong constraint for h_{\min} . Thus, h_{\min} of LIGO is about $\sim 10^{-23}$ - 10^{-24} in this band.

- 3) The minimal detectable amplitude of cavity depends on arguments similar to the ones brought up in reference [42] as well as the following formulation

$$h_{\min} \sim \frac{1}{Q} \sqrt{\frac{\mu_0 \hbar \omega_e}{B^2 V}}, \quad (\text{B.7})$$

For the constant-amplitude HFGWs, and

$$h_{\min} \sim \sqrt{\frac{1}{Q}} \sqrt{\frac{\mu_0 \hbar \omega_e}{B^2 V}}, \quad (\text{B.8})$$

for the stochastic relic HFGWs and considerations which were given to the author in a discussion he had with Dr. Weiss of MIT [15].

Because Q factor of superconducting cavity in the low-temperature condition can reach up to $\sim 10^{10}$ - 10^{12} , if we assume $Q = 10^{11}$, $\nu_g = \nu_e = 2.9$ GHz, $B = 3$ T (coupling static magnetic field to the cavity), $V = 1$ m³, then

$$h_{\min} \sim 10^{-27}, \quad (\text{B.9})$$

for the constant-amplitude HFGW.

and

$$h_{\min} \sim 10^{-21} - 10^{-22} \quad (\text{B.10})$$

for the stochastic relic HFGW.

- 4) The CEMS [40] is that

The minimal detectable amplitude h depends on the relative standard quantum limit (SQL) (G.V. Stephenson 2008, 2009), [42]

$$h_{\min} \sim \sqrt{\frac{1}{Q}} \sqrt{\frac{\hbar \omega_e}{\mathcal{E}}}, \quad (\text{B.11})$$

for the stochastic relic HFGW, \mathcal{E} is the total EM energy of the system. For the typical parameters: $B = 3$ T, $L = 6$ m. $V = L\Delta S = 2$ m³ $\tau = 3 \times 10^5$ s signal accumulation time, $P = 10$ W (the power of Gaussian Beam-GB) $\nu_g = \nu_e = 2.9$ GHz, even if the fractal membranes are absent (using natural decay rate of the GB in the radial direction), then equivalent Q factor (Notice, here Q factor is different

from cavity's Q factor) can reach up to 10^{31} , then

$$h_{\min} \sim 10^{-30} - 10^{-31}. \tag{B.12}$$

If we use fractal membranes, even if a conservative estimation, we have

$$h_{\min} \sim 10^{-32} - 10^{-33}. \tag{B.13}$$

Equation (B.11) is similar to Equation (B.6) and Equation (B.7). An important difference is that $\tau = Q/\omega$ in the cavity case, while there is no limitation of the maximum accumulation time of the signal in the CEMS, but only minimal accumulation time of the signal. Thus, the sensitivity in the CEMS is the photon signal limited, not quantum noise limited.

5) LIGO and our scheme have quite different detecting mechanisms (the displacement effect and the EM parameter perturbation effect) and detecting bands (~ 1 Hz - 1000 Hz and 1 GHz \sim 10 GHz), their comparison should not be only the amplitude of GWs, but also the energy flux of GWs. In fact, the energy flux of any weak GW is proportional to $h^2\nu_g^2$. Thus, the CEMS with sensitivity $h = 10^{-30}$, $\nu_g = 10$ GHz and the LIGO with sensitivity $h = 10^{-22}$, $\nu_g = 100$ Hz correspond to the GWs of the same energy flux density. This means that the EM detection schemes with the sensitivity of $h = 10^{-30}$, (or better) $\nu_g \sim 1$ GHz-10 GHz in the future should not be surprise.

The SQL is a basic limitation. Any useful means and advanced models might give better sensitivity, but there is no change of order of magnitude in the SQL range. For example, if we use squeezed quantum states for a concrete detector, then the sensitivity would be improved 2 - 3 times than when the squeezed quantum state is absent in the detector, but it cannot improve one order of magnitude or more According to more accepted by the general astrophysics community values as told to the author by Dr. Weiss [41], the estimate, for the upper limit of Ω_g F on relic GWs should be smaller than 10^{-5} , while recent data analysis (B.P. Abbott *et al.*, (2009)) [34] shows the upper limit of Ω_g , as in **Figure 4** should be 6.9×10^{-6} . By using such parameters, Dr. Li estimates the spectrum $h(\nu_g, \tau)$ and the RMS amplitude h_{rms} . The relation between Ω_g and the spectrum $h(\nu_g, \tau)$ is often expressed as (L. P. Grishchuk) [35],

$$\Omega_g \approx \frac{\pi^2}{3} \left(\frac{\nu}{\nu_H} \right)^2 h^2(\nu, \tau), \tag{B.14}$$

so

$$h(\nu, \tau) \approx \frac{\sqrt{3\Omega_g} \nu_H}{\pi \nu}, \tag{B.15}$$

where $\nu_H = H_0 \sim 2 \times 10^{-18}$ Hz, the present value of the Hubble frequency. From Equation (B.14) and Equation (3.15)), we have

(a)

$$\text{If } \nu = 10 \text{ GHz, } h = 10^{-30}, \text{ then } \Omega_g = 8.3 \times 10^{-5}, \tag{B.16}$$

$$\text{If } \nu = 10 \text{ GHz, } h = 10^{-31}, \text{ then } \Omega_g = 8.3 \times 10^{-7} < \Omega_{g \max}, \tag{B.17}$$

(b)

$$\nu = 5 \text{ GHz}, H = 10^{-31}, \text{ then } \Omega_g = 2.1 \times 10^{-7} < \Omega_{g \text{ max}}, \tag{B.18}$$

$$\text{If } \nu = 5 \text{ GHz}, \Omega_g = \Omega_{g \text{ max}} = 6.9 \times 10^{-5}, \text{ then } h = 5.7 \times 10^{-31} \tag{B.19}$$

Such values of $\nu = 5 \text{ GHz}, \Omega_g = \Omega_{g \text{ max}} = 6.9 \times 10^{-5}$, would be essential to ascertain the possibility of detection of GW from relic conditions, whereas Ω_g , as data collected and binned to be summed over different frequencies as given by $\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \rightarrow \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(f)$ with the integral $\int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(f) \equiv$ numerical summed up value, weighted of binned $\Omega_{gw}(f)$ data sets to make the following identification [18].

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(f) \tag{B.20}$$

Furthermore, the numerical summed up value of binned $\Omega_{gw}(f)$ data sets, in each frequency f value is [18]

$$h_0^2 \Omega_{gw}(f) \cong \frac{3.6}{2} \cdot \left[\frac{n_f [\text{graviton}] + n_f [\text{neutrino}]}{10^{37}} \right] \cdot \left(\frac{\langle f \rangle}{1 \text{ kHz}} \right)^4 \tag{B.21}$$

Equation (1.23) is for a very narrow range of frequencies, that to first approximation, make a linkage between an integral representation of Ω_g and $h_0^2 \Omega_{gw}(f)$. Note also that Dr. Li suggests, as an optimal upper frequency to investigate, $\nu_g = 2.9 \text{ GHz}$ (see below, suggestion 1–3), $\Delta \nu = 3 \text{ KHz}$, then

$$h \approx \frac{\sqrt{3\Omega_g} \nu_H}{\pi \nu_g} \approx 1.0 \times 10^{-30}, \tag{B.22}$$

and

$$h_{rms} = \sqrt{\langle h^2 \rangle} \approx h \left[\frac{\Delta \nu}{\nu_g} \right]^{\frac{1}{2}} \approx 1.02 \times 10^{-33} \tag{B.23}$$

Thus an obvious gap still exists between the theoretical estimation and detecting reality, but there are large rooms to advance and improve the CEMS. These are upper values of the spectrum, and should be considered as preliminary. Needed in this mix of calculations would be a way to ascertain a set of input values for $n_f [\text{graviton}], n_f [\text{neutrino}]$ into a formula for $h_0^2 \Omega_{gw}(f)$. The objective is to get a set of measurements to confirm if possible the utility of using, experimentally (in order to ascertain, experimentally, a relationship between gravitational wave energy density, and numerical count of gravitons at a given frequency f) the numerical count of up to a value of having [18]

$$h_0^2 \Omega_{gw}(f) \cong \frac{3.6}{2} \cdot \left[\frac{n_f [\text{graviton}] + n_f [\text{neutrino}]}{10^{37}} \right] \cdot \left(\frac{\langle f \rangle}{1 \text{ kHz}} \right)^4. \text{ If there is roughly a}$$

1-1 correspondence between gravitons and neutrinos (highly unlikely), then

$$h_0^2 \Omega_{gw}(f) \sim 3.6 \cdot \left[\frac{n_f [\text{graviton}]}{10^{37}} \right] \cdot \left(\frac{\langle f \rangle}{1 \text{ kHz}} \right)^4. \text{ [28] counting the number of gravi-}$$

tons per cell space should also consider what Buoanno [7] wrote, for Les Houches if one looks at BBN, the following upper bound should be considered:

$$h_0^2 \Omega_{gw}(f) \leq 4.8 \times 10^{-9} \cdot (f/f_*)^2 \tag{B.24}$$

Here, Buoanno [7] is using $f > f_* = 4.4 \times 10^{-9}$ Hz, does, that the frequency today depends upon the relation

$$f \equiv f_* \cdot [a_*/a_0] \tag{B.25}$$

The problem in this is that the ratio $[a_*/a_0] \ll 1$, assumes that a_0 is “today’s” scale factor. In fact, using this estimate, Buoanno [7] comes up with a peak frequency value for relic/early universe values of the electroweak era-generated GW graviton production of

$$f_{\text{Peak}} \cong 10^{-8} \cdot [\beta/H_*] \cdot [T_*/16 \text{ GeV}] \cdot [g_*/100]^{1/6} \text{ Hz} \tag{B.26}$$

By conventional cosmological theory, limits of g_* are at the upper limit of 100 - 120, at most, according to Kolb and Turner [13] (1991). $T_* \sim 10^2$ GeV is specified for nucleation of a bubble, as a generator of GW. Early universe models with $g_* \sim 1000$ or so are not in the realm of observational science, yet, according to Hector De La Vega [14] (2009) in personal communications with the author, at the Colmo, Italy astroparticle physics school, ISAPP, Furthermore, the range of accessible frequencies as given by Equation (B.26) is in sync with $h_0^2 \Omega_{gw}(f) \sim 10^{-10}$ for peak frequencies with values of 10 MHz. The net affect of such thinking is to rule out examining early universe gravitons as measurable and to state as a way of to rule out being able to measure relic GW and gravitons, via the premise that all relic GW are inaccessible. If one looks at **Figure 4**, $\Omega_{GW} > 10^{-6}$ for frequencies as high as up to 10^6 Hertz, this counters what was declared by Turner and Wilzenk [43] (1990): that inflation will terminate with observable frequencies in the range of 100 or so Hertz. The problem is though, that after several years of LIGO, no one has observed such a GW signal from the early universe, from black holes, or any other source, yet. About the only way one may be able to observe a signal for GW and/or gravitons may be to consider how to obtain a numerical count of gravitons and/or neutrinos for [41]

$$h_0^2 \Omega_{gw}(f) \cong \frac{3.6}{2} \cdot \left[\frac{n_f [\text{graviton}] + n_f [\text{neutrino}]}{10^{37}} \right] \cdot \left(\frac{\langle f \rangle}{1 \text{ kHz}} \right)^4$$
 And this leads to the question of how to account for a possible mass/ information content to the graviton.

Appendix C. Crucially Important Developments as of 2016 Which Impact the Observability of Some of the Phenomena Discussed in This Document

Abbot *et al.*, in [43] outlined the crucial physics of gravitational waves, and this should be a way of either falsifying or confirmation of the essential details of Equation (B.26) of **Appendix B**. *I.e.* nothing should contradict the basics of GW predictions as given in [43]. In addition, it is important to note that not only is

Equation (B.26) to be confirmed or to be falsified, but that the details of [43] plus other work should be used to confirm and get falsifiable criterion to establish if General relativity is the final gravitational theory for Gravitation, but if we have to consider Scalar-Tensor gravity as is gone into great detail in [44] by Corda. The details in **Appendix A** and **Appendix B** could prove decisively important as to this matter. Finally, a subsequent analysis of the event GW150914 in [45] put in a limit of 10 to the 13 kilometers as far as a lower bound to gravitational physics, and by extension affected massive gravity theories significantly. [45] is also linked to Equation (B.26) of **Appendix B** and is of decisive theoretical import too.



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