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Common Fixed Point Theorem for Six Selfmaps of a Complete G-Metric Space

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Abstract

By using weakly compatible conditions of selfmapping pairs, we prove a common fixed point theorem for six mappings in generalized complete metric spaces. An example is provided to support our result.

Keywords

G-Metric Space, Weakly Compatible Mappings, Fixed Point, Associated Sequence of a Point Relative to Six Selfmaps

1. Introduction

The study of fixed point theory has been at the centre of vigorous activity and it has a wide range of applications in applied mathematics and sciences. Over the past two decades, a considerable amount of research work for the development of fixed point theory have executed by several authors.

In 1963, Gahler [1] [2] introduced 2-metric spaces and claimed them as generalizations of metric spaces. But many researchers proved that there was no relation between these two spaces. These considerations led Dhage [3] to initiate a study of general metric spaces called D-metric spaces. As a probable modification to D-metric spaces, Shaban Sedghi, Nabi Shobe and Haiyun Zhou [4] have introduced D^*-metric spaces. In 2006, Zead Mustafa and Brailey Sims [5] initiated G-metric spaces. Several researchers proved many common fixed point theorems on G-metric spaces.

The purpose of this paper is to prove a common fixed point theorem for six weakly compatible selfmaps of a complete G-metric space. Now we recall some basic definitions and results on G-metric space.

2. Preliminaries

We begin with

Definition 2.1: ([5], Definition 3) Let X be a non-empty set and $G: X^3 \to [0, \infty)$ be a function satisfying:

(G1) $G(x, y, z) = 0$ if $x = y = z$.

(G2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$.

(G3) $G(x, x, y) < G(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$.

(G4) $G(x, y, z) = G(\sigma(x, y, z))$ for all $x, y, z \in X$, where $\sigma(x, y, z)$ is a permutation of the set $\{x, y, z\}$.

And

(G5) $G(x, y, z) < G(x, w, w) + G(w, y, z)$ for all $x, y, z, w \in X$.

Then G is called a G-metric on X and the pair (X, G) is called a G-metric Space.

Definition 2.2: ([5], Definition 4) A G-metric Space (X, G) is said to be symmetric if

(G6) $G(x, y, y) = G(x, x, y)$ for all $x, y \in X$.

The example given below is a non-symmetric G-metric space.

Example 2.3: ([5], Example 1): Let $X = \{a, b\}$ Define $G: X^3 \to [0, \infty)$ by $G(a, a, a) = G(b, b, b) = 0;$ $G(a, a, b) = 1,$ $G(a, b, b) = 2$ and extend G to all of X^3 by using (G4).

Then it is easy to verify that (X, G) is a G-metric space. Since $G(a, a, b) \neq G(a, b, b)$, the space (X, G) is non-symmetric, in view of (G6).

Example 2.4: Let (X, d) be a metric space. Define $G_s^d : X^3 \to [0, \infty)$ by $G_s^d(x, y, z) = \frac{1}{3}[d(x, y) + d(y, z) + d(z, x)]$ for $x, y, z \in X$ .Then (X, G_s^d) is a G-metric Space.

Lemma (2.5): ([5], p. 292) If (X, G) is a G-metric space then $G(x, y, y) \leq 2G(y, x, x)$ for all $x, y \in X$.

Definition 2.6: Let (X, G) be a G-metric Space. A sequence $\{x_n\}$ in X is said to be G-convergent if there is a $x_0 \in X$ such that to each $\varepsilon > 0$ there is a natural number N for which $G(x_n, x_n, x_0) < \varepsilon$ for all $n \geq N$.

Lemma 2.7: ([5], Proposition 6) Let (X, G) be a G-metric Space, then for a sequence $\{x_n\} \subseteq X$ and point $x \in X$ the following are equivalent.

(1) $\{x_n\}$ is G- convergent to x.

(2) $d_G(x_n, x) \to 0$ as $n \to \infty$ (that is $\{x_n\}$ converges to x relative to the metric d_G).

(3) $G(x_n, x_n, x) \to 0$ as $n \to \infty$.

(4) $G(x_n, x, x) \to 0$ as $n \to \infty$.

(5) $G(x_m, x_n, x) \to 0$ as $m, n \to \infty$.

Definition 2.8: ([5], Definition 8) Let (X, G) be a G-metric space, then a sequence $\{x_n\} \subseteq X$ is said to be G-Cauchy if for each $\varepsilon > 0$, there exists a natural number N such that $G(x_n, x_m, x_l) < \varepsilon$ for all $n, m, l \geq N$.

Note that every G-convergent sequence in a G-metric space (X, G) is G-Cauchy.

Definition 2.9: ([5], Definition 9) A G-metric space (X, G) is said to be G-complete if every G-Cauchy sequence in (X, G) is G-convergent in (X, G).

Gerald Jungck [6] initiated the notion of weakly compatible mappings, as a generalization of commuting maps. We now give the definition of weakly compatibility in a G-metric space.

Definition 2.10: [7] Suppose f and g are selfmaps of a G-metric space (X,G). The pair (f,g) is said to be weakly compatible if $G(fgx, gfx, gfx)=0$ whenever $G(fx, gx, gx)=0$.

3. Main Theorem

Theorem 3.1: Suppose f,g,h,p,Q and R are six selfmaps of a complete G-metric space (X,G) satisfying the following conditions.

(3.1.1) $fg(X)\subseteq R(X)$ and $hp(X)\subseteq Q(X)$,

(3.1.2)

$$G(hpx, fgy, fgy)\le \alpha G(Rx,Qy,Qy)+\beta\left[G(Rx,hpx,hpx)+G(Qy, fgy, fgy)\right] + \gamma\left[G(Rx, fgy, fgy)+G(hpx,Qy,Qy)\right]$$

for all $x,y\in X$ and α,β,γ are non-negative real numbers such that $\alpha+2\beta+2\gamma<1$,

(3.1.3) one of $R(X),Q(X)$ is closed sub subset of X,

(3.1.4) (fg,Q) and (hp,R) are weakly compatible pairs,

(3.1.5) The pairs $(h,p),(h,R),(f,g)$, and (f,Q) are commuting.

Then f,g,h,p,Q and R have a unique common fixed point in X.

Proof: Let $x_0\in X$ be an arbitrary point. Since $fg(X)\subseteq R(X)$ and $hp(X)\subseteq Q(X)$ there exists $x_1,x_2\in X$ such that $hpx_0=Qx_1$ and $fgx_1=Rx_2$ again there exists $x_3,x_4\in X$ such that $hpx_2=Qx_3$ and $fgx_3=Rx_4$, continuing in the same manner for each $n\ge 0$, we obtain a sequence $\{x_n\}$ in X such that

$$y_{2n}=hpx_{2n}=Qx_{2n+1},\quad y_{2n+1}=fgx_{2n+1}=Rx_{2n+2}\quad \text{for } n\ge 0. \tag{3.1.6}$$

From condition (3.1.2), we have

$$\begin{aligned}G(y_{2n},y_{2n+1},y_{2n+1}) &= G(hpx_{2n}, fgx_{2n+1}, fgx_{2n+1})\\ &\le \alpha G(Rx_{2n},Qx_{2n+1},Qx_{2n+1})+\beta\left[G(Rx_{2n},hpx_{2n},hpx_{2n})+G(Qx_{2n+1}, fgx_{2n+1}, fgx_{2n+1})\right]\\ &\quad+\gamma\left[G(Rx_{2n}, fgx_{2n+1}, fgx_{2n+1})+G(hpx_{2n},Qx_{2n+1},Qx_{2n+1})\right]\\ &= \alpha G(y_{2n-1},y_{2n},y_{2n})+\beta\left[G(y_{2n-1},y_{2n},y_{2n})+G(y_{2n},y_{2n+1},y_{2n+1})\right]\\ &\quad+\gamma\left[G(y_{2n-1},y_{2n+1},y_{2n+1})+G(y_{2n},y_{2n},y_{2n})\right]\\ &\le (\alpha+\beta+\gamma)G(y_{2n-1},y_{2n},y_{2n})+(\beta+\gamma)G(y_{2n},y_{2n+1},y_{2n+1}).\end{aligned}$$

Therefore

$$(1-\beta-\gamma)G(y_{2n},y_{2n+1},y_{2n+1})\le(\alpha+\beta+\gamma)G(y_{2n-1},y_{2n},y_{2n})$$

$$G(y_{2n},y_{2n+1},y_{2n+1})\le\frac{(\alpha+\beta+\gamma)}{(1-\beta-\gamma)}G(y_{2n-1},y_{2n},y_{2n}) \tag{3.1.7}$$

$$G(y_{2n},y_{2n+1},y_{2n+1})\le kG(y_{2n-1},y_{2n},y_{2n})$$

where $k=\dfrac{(\alpha+\beta+\gamma)}{(1-\beta-\gamma)}<1$.

Similarly, we can show that

$$G(y_{2n+1}, y_{2n+2}, y_{2n+2}) \le kG(y_{2n}, y_{2n+1}, y_{2n+1}). \tag{3.18}$$

From (3.1.7) and (3.1.8) we have

$$G(y_n, y_{n+1}, y_{n+1}) \le kG(y_{n-1}, y_n, y_n) \le \cdots \le k^n G(y_0, y_1, y_1).$$

Now for every $n, m \in N$ such that $m > n$ we have

$$\begin{aligned} G(y_n, y_m, y_m) &\le G(y_n, y_{n+1}, y_{n+1}) + G(y_{n+1}, y_{n+2}, y_{n+2}) + \cdots + G(y_{m-1}, y_m, y_m) \\ &\le k^n G(y_0, y_1, y_1) + k^{n+1} G(y_0, y_1, y_1) + \cdots + k^{m-1} G(y_0, y_1, y_1) \\ &\le k^n \left(1 + k + k^2 + \cdots + k^{m-n+1}\right) G(y_0, y_1, y_1) \\ &\le k^n \frac{\left(1 - k^{m-n}\right)}{1-k} G(hx_0, hx_1, hx_1) \to 0 \text{ as } n \to \infty. \end{aligned}$$

Since $k < 1$.

Therefore, $\{y_n\}$ is a Cauchy sequence in X. Since X is a complete G-metric space, then there exists a point $z \in X$ such that

$$\lim_{n\to\infty} hpx_{2n} = \lim_{n\to\infty} Qx_{2n+1} = \lim_{n\to\infty} fgx_{2n+1} = \lim_{n\to\infty} Rx_{2n+2} = z. \tag{3.1.9}$$

If $R(X)$ is a closed subset of X, then there exists a point $u \in X$ such that $z = Ru$.

Now from (3.1.2), we have

$$\begin{aligned} G(hpu, fgx_{2n+1}, fgx_{2n+1}) \le{}& \alpha G(Ru, Qx_{2n+1}, Qx_{2n+1}) + \beta\left[G(Ru, hpu, hpu) + G(Qx_{2n+1}, fgx_{2n+1}, fgx_{2n+1})\right] \\ &+ \gamma\left[G(Ru, fgx_{2n+1}, fgx_{2n+1}) + G(hpu, Qx_{2n+1}, Qx_{2n+1})\right]. \end{aligned} \tag{3.1.10}$$

Letting $n \to \infty$ in (3.1.10) and by the continuity of G we have

$$\begin{aligned} G(hpu, z, z) &\le \alpha G(z, z, z) + \beta\left[G(z, hpu, hpu) + G(z, z, z)\right] \\ &\quad + \gamma\left[G(z, z, z) + G(hpu, z, z)\right] \\ &\le (2\beta + \gamma) G(hpu, z, z), \end{aligned}$$

which leads to a contradiction as $2\beta + \gamma < 1$.

Hence $G(hpu, z, z) = 0$, which implies $hpu = z$.

Therefore,

$$hpu = Ru = z. \tag{3.1.11}$$

Now since $hp(X) \subseteq Q(X)$ then there exists a point $v \in X$ such that $z = Qv$.

Then we have by (3.1.2)

$$\begin{aligned} G(hpu, fgv, fgv) \le{}& \alpha G(Ru, Qv, Qv) + \beta\left[G(Ru, hpu, hpu) + G(Qv, fgv, fgv)\right] \\ &+ \gamma\left[G(Ru, fgv, fgv) + G(hpu, Qv, Qv)\right] \end{aligned} \tag{3.1.12}$$

$$\begin{aligned} G(z, fgv, fgv) &\le \alpha G(z, z, z) + \beta\left[G(z, z, z) + G(z, fgv, fgv)\right] \\ &\quad + \gamma\left[G(z, fgv, fgv) + G(z, z, z)\right] \\ &\le (\beta + \gamma) G(z, fgv, fgv), \end{aligned}$$

which leads to a contradiction, since $\beta + \gamma < 1$. Hence $fgv = z$.

Therefore,

$$fgv = Qv = z. \tag{3.1.13}$$

From (3.1.11) and (3.1.13) we have $Ru = hpu = fgv = Qv = z.$

Since the pair (fg, Q) is weakly compatible then $fgQv = Qfgv$ which gives $fgz = Qz.$

Now (3.1.2) we have

$$
\begin{aligned}
G(z, fgz, fgz) &= G(hpu, fgz, fgz) \\
&\le \alpha G(Ru, Qz, Qz) + \beta\left[G(Ru, hpu, hpu) + G(Qz, fgz, fgz)\right] \\
&\quad + \gamma\left[G(Ru, fgz, fgz) + G(hpu, Qz, Qz)\right] \\
&= \alpha G(z, fgz, fgz) + \beta\left[G(z, z, z) + G(fgz, fgz, fgz)\right] \\
&\quad + \gamma\left[G(z, fgz, fgz) + G(z, fgz, fgz)\right] \\
&= (\alpha + 2\gamma) G(z, fgz, fgz)
\end{aligned}
$$

which is a contradiction, since $\alpha + 2\gamma < 1.$ Hence $G(z, fgz, fgz) = 0$ thus $fgz = z.$

Showing that z is a common fixed point of fg and $Q.$

Since the pair (hp, R) is weakly compatible then $hpRu = Rhpu$ which gives $hpz = Rz.$

Then we have by (3.1.2)

$$
\begin{aligned}
G(hpz, z, z) &= G(hpz, fgz, fgz) \\
&\le \alpha G(Rz, Qz, Qz) + \beta\left[G(Rz, hpz, hpz) + G(Qz, fgz, fgz)\right] \\
&\quad + \gamma\left[G(Rz, fgz, fgz) + G(hpz, Qz, Qz)\right] \\
&= \alpha G(hpz, z, z) + \beta\left[G(hpz, hpz, hpz) + G(z, z, z)\right] \\
&\quad + \gamma\left[G(hpz, z, z) + G(hpz, z, z)\right] \\
&= (\alpha + 2\gamma) G(hpz, z, z),
\end{aligned}
$$

which is a contradiction, since $\alpha + 2\gamma < 1$. Hence $G(hpz, z, z) = 0$ thus $hpz = z.$

Showing that z is a common fixed point of hp and $R.$

Therefore, z is a common fixed point of fg , hp , R and $Q.$

By commuting conditions of the pairs in (3.1.5), we have

$$fz = f(fgz) = f(gfz) = fg(fz), \quad fz = f(Qz) = Q(fz).$$

And

$$hz = h(hpz) = h(phz) = hp(hz), \qquad hz = h(Rz) = R(hz).$$

From (3.1.2)

$$
\begin{aligned}
G(z, fz, fz) &= G(hpz, fgfz, fgfz) \\
&\le \alpha G(Rz, Qfz, Qfz) + \beta\left[G(Rz, hpz, hpz) + G(Qfz, fgfz, fgfz)\right] \\
&\quad + \gamma\left[G(Rz, fgfz, fgfz) + G(hpz, Qfz, Qfz)\right] \\
&= \alpha G(z, fz, fz) + \beta\left[G(z, z, z) + G(fz, fz, fz)\right] \\
&\quad + \gamma\left[G(z, fz, fz) + G(z, fz, fz)\right] \\
&= (\alpha + 2\gamma) G(z, fz, fz).
\end{aligned}
$$

Since $\alpha + 2\gamma < 1,$ we have $G(z, fz, fz) = 0$ thus $fz = z.$

Also $gz = gfz = fgz = z.$

Therefore, we have $fz = gz = Rz = fgz = z.$

Similarly, we have $hz = pz = Qz = hpz = z$.

Therefore, z is a common fixed point of f, g, h, p, Q and R.

The proof is similar in case if $Q(X)$ is a closed subset of X.

We now prove the uniqueness of the common fixed point.

If possible, assume that w is another common fixed point of f, g, h, p, Q and R.

By condition (3.1.2) we have

$$\begin{aligned} G(z,w,w) &= G(hpz, fgw, fgw) \\ &\le \alpha G(Rz,Qw,Qw) + \beta\left[G(Rz,hpz,hpz) + G(Qw, fgw, fgw)\right] \\ &\quad + \gamma\left[G(Rz, fgw, fgw) + G(hpz,Qw,Qw)\right] \\ &= \alpha G(z,w,w) + \beta\left[G(z,z,z) + G(w,w,w)\right] + \gamma\left[G(z,w,w)) + G(z,w,w)\right] \\ &= (\alpha + 2\gamma)G(z,w,w), \end{aligned}$$

which is a contradiction, since $\alpha + 2\gamma < 1$.

Hence $G(z,w,w) = 0$ which gives $z = w$.

Therefore, z is a unique common fixed point of f, g, h, p, Q and R.

As an example, we have the following.

3.1. Example

Let $X = [0,1]$ with $G(x, y, z) = |x-y| + |y-z| + |z-x|$ for $x, y, z \in X$. Then G is a G-metric on X.

Define

$$f : X \to X,\ g : X \to X,\ h : X \to X,\ p : X \to X,\ Q : X \to X,\ R : X \to X$$

by

$$fx = hx = \frac{x+1}{3},\ \forall x \in X,$$

$$gx = px = \frac{3x+1}{5},\ \forall x \in X,$$

$$Qx = Rx = x,\ \forall x \in X.$$

$$fgx = f\left(\frac{3x+1}{5}\right) = \frac{x+2}{5},\ hpx = h\left(\frac{3x+1}{5}\right) = \frac{x+2}{5},$$

$$fgX = \left[\frac{2}{5},\frac{3}{5}\right],\ hpX = \left[\frac{2}{5},\frac{3}{5}\right],\ RX = [0,1],\ QX = [0,1]$$

$$fgX \subseteq RX,\ hpX \subseteq QX.$$

Proving the condition (3.1.1) of the Theorem (3.1).

RX and QX are closed subsets of X. Proving the condition (3.1.3) of the Theorem (3.1).

Since $fg\left(\frac{1}{2}\right) = \frac{1}{2}$ and $Q\left(\frac{1}{2}\right) = \frac{1}{2}$ then $fgQ\left(\frac{1}{2}\right) = Qfg\left(\frac{1}{2}\right)$, showing that the pair (fg, Q) is weakly compatible.

Also, the pair (hp, R) is weakly compatible.

Proving the condition (3.1.4) of the Theorem (3.1).

$$hp(x) = \frac{x+2}{5} = ph(x),\ hR(x) = h(x) = Rh(x),$$
$$fg(x) = \frac{x+2}{5} = gf(x),\ fQ(x) = f(x) = Qf(x),$$

showing that (h,R), (f,Q), (h,p) and (f,g) are commuting pairs.

Proving the condition (3.1.5) of the Theorem (3.1).

Now we prove the condition (3.1.2) of the Theorem (3.1).

On taking $\alpha = \frac{1}{10},\ \beta = \frac{1}{8},\ \gamma = \frac{1}{12}$ then $\alpha + 2\beta + 2\gamma = \frac{31}{60} < 1$.

Now $G(hpx, fgy, fgy) = 2|hpx - fgy| = \frac{2}{5}|x-y|$

$$G(Rx, Qy, Qy) = 2|Rx - Qy| = 2|x-y|,$$
$$G(Rx, hpx, hpx) = 2|Rx - hpx| = \frac{4}{5}|2x-1|,$$
$$G(Qy, fgy, fgy) = 2|fgy - Qy| = \frac{4}{5}|1-2y|,$$
$$G(Rx, fgy, fgy) = 2|Rx - fgy| = \frac{2}{5}|5x-y-2|,$$
$$G(hpx, Qy, Qy) = 2|hpx - Qy| = \frac{2}{5}|x+2-5y|$$

$$\begin{aligned}
&\alpha G(Rx, Qy, Qy) + \beta\left[G(Rx, hpx, hpx) + G(Qy, fgy, fgy)\right] \\
&+\gamma\left[G(Rx, fgy, fgy) + G(hpx, Qy, Qy)\right] \\
&= 2\alpha|x-y| + \frac{4}{5}\beta\left(|2x-1| + |1-2y|\right) + \frac{2}{5}\gamma\left(|5x-y-2| + |x-5y-2|\right) \\
&\geq 2\alpha|x-y| + \frac{4}{5}\beta|2x-2y| + \frac{2}{5}\gamma|6x-6y| \\
&= \left(2\alpha + \frac{8\beta}{5} + \frac{12}{5}\gamma\right)|x-y| \\
&= \frac{3}{5}|x-y| \geq \frac{2}{5}|x-y| = G(fgx, hpy, hpy).
\end{aligned}$$

Therefore,

$$\begin{aligned}
G(hpx, fgy, fgy) \leq{}& \alpha G(Rx, Qy, Qy) + \beta\left[G(Rx, hpx, hpx) + G(Qy, fgy, fgy)\right] \\
&+ \gamma\left[G(Rx, fgy, fgy) + G(hpx, Qy, Qy)\right].
\end{aligned}$$

Proving the condition (3.1.2) of the Theorem (3.1).

Hence all the conditions of the Theorem (3.1) are satisfied.

Therefore, $\frac{1}{2}$ is a unique common fixed point of f, g, h, p, Q and R.

3.2. Corollary

Suppose f, p, Q and R are four selfmaps of a complete G-metric space (X, G) satisfying the following conditions:

(3.1.1) $f(X) \subseteq R(X)$ and $p(X) \subseteq Q(X)$,

(3.1.2)
$$\begin{aligned}
G(px, fy, fy) \leq{}& \alpha G(Rx, Qy, Qy) + \beta\left[G(Rx, px, px) + G(Qy, fy, fy)\right] \\
&+ \gamma\left[G(Rx, fy, fy) + G(px, Qy, Qy)\right]
\end{aligned}$$

for all $x, y \in X$ and α, β, γ are non-negative real numbers such that $\alpha + 2\beta + 2\gamma < 1$,

(3.1.3) One of $R(X), Q(X)$ is closed sub subset of *X*,

(3.1.4) (p, R) and (f, Q) are weakly compatible pairs,

Then f, p, Q and R have a unique common fixed point in X.

Proof: Follows from the Theorem (3.1) if $g = h = I$ the identity map.

3.3. Corollary

Suppose f, p and R are three selfmaps of a complete G-metric space (X, G) satisfying the following conditions:

(3.1.1) $f(X) \subseteq R(X)$ and $p(X) \subseteq R(X)$,

(3.1.2)
$$G(px, fy, fy) \le \alpha G(Rx, Ry, Ry) + \beta\left[G(Rx, px, px) + G(Ry, fy, fy)\right] + \gamma\left[G(Rx, fy, fy) + G(px, Ry, Ry)\right]$$

for all $x, y \in X$ and α, β, γ are non-negative real numbers such that $\alpha + 2\beta + 2\gamma < 1$,

(3.1.3) $R(X)$ is closed sub subset of *X*,

(3.1.4) (p, R) and (f, R) are weakly compatible pairs.

Then f, p and R have a unique common fixed point in X.

Proof: Follows from the Theorem (3.1) if $g = h = I$ the identity map, and $Q = R$.

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