

Limit of the Principal's Information

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Abstract

This note characterizes the optimal contract when a principal has unverifiable subjective information that is correlated with an agent's private information. We find that the principal's subjective information cannot alleviate the information asymmetry and, moreover, the second best contract is independent from it if the correlation is low.

Keywords

Adverse Selection, Subjective Information, Correlation

1. Introduction

It is known that the problems arising from asymmetric information can often be mitigated by additional information. Although this paradigm has been applied in many different contexts, it fails to explain many inefficient economic interactions. For example, headquarters often neglect some information regarding subsidiaries' performances. To provide some reasons for such phenomenon, we investigate the role of additional unverifiable information.

Specifically, we consider a model with one principal and one agent, in which the agent produces some goods for the principal. It is assumed that knowledge of the agent's production cost is his private information. In this standard adverse selection setting, in order to learn the actual cost incurred by the agent, the principal must pay rent to the agent; efficiency between the parties is thus distorted. In this note, we extend this standard model to include the case in which the principal learns some subjective information after the parties have signed the contract. This subjective information is correlated with the agent's information, but it is the principal's private information. If the correlated information is verifiable, contracts contingent on this information can replace contracts that are independent of this information. Consequently, a contract that maximizes the principal's utility always utilizes this correlated information so as to punish an agent who behaves untruthfully.

On the other hand, it is not clear whether the principal's subjective information can be instrumental in correcting the problem, since the principal has an incentive to bend that information. Suppose that the principal needs to pay more when the agent's cost is high than when it is low, and he has some information indicating that the agent's cost is high. By revealing this information truthfully, the principal acknowledges that the agent's cost is high, and consequently, the principal has an incentive to misreport this and to state that there is information indicating that the agent's cost is low. As discussed in MacLeod [1], in order to elicit this subjective information truthfully and to utilize it, and thus to correct the agency problem, a monetary transfer from the principal to the agent must be burnt. Thus, the principal's subjective information can be used to advantage to motivate the agent to tell the truth, but there is the disadvantage that this requires money burning.

The purpose of this note is to examine whether the principal's subjective information can always be an instrument to solve the agency problem. We compare the advantage and disadvantage of the principal's subjective information, and find that the principal's subjective information is ignored for the optimal contract if it is weakly correlated with the agent's type.¹ This result is in sharp contrast with existing researches. In the literature on informed principals (Maskin and Tirole [2], [3], Mylovanov and Tröger [4], Cella [5] and Skreta [6]), the principal also has her own private information, but learns it *ex ante*, while intermediate in our model. Of these studies, [5] and [6], are close to this research, considered when the principal's and the agent's types are correlated. In their researches, the optimal contracts always utilize the principal's information.

When the correlation is high, the principal's information is adopted in the optimal contract. However, the principal still cannot extract full surplus from the agent. This contrasts with previous results that show full-surplus extraction when the information is correlated. Riordan and Sappington [7] considered the question of when there will be *ex post* public information correlated with the agent's type. Cremer and McLean [8] and McAfee and Reny [9] considered the situation in which an uninformed principal faces many privately informed agents whose types are correlated.

Usage of subjective information to solve the agency problem has also been studied by Levin [10] and Fuchs [11]. They focused on the use of intertemporal incentives for solving the principal's truth telling, whereas we consider a one-shot relation.

2. Model

Consider one principal and one agent, both being risk neutral. The principal procures $q \in \mathfrak{R}^+$ unit of goods from the agent. The agent's type is either H or L , and this is his private information and known *ex ante* (date 0). When the agent's type is $\theta \in \{H, L\}$, the cost of producing q unit of goods is $c_\theta q^2/2$,

¹MacLeod [1] has compared different level of correlation between the principal's evaluation and the agent's signal in the context of the moral hazard problem. In fact, the agent's amount receiving always depends on the principal's evaluation.

where $c_H > c_L$. At this point, the principal only knows that the agent is L type with probability π and H type with probability $1-\pi$; this is common prior knowledge. On date 1, the principal offers a contract to the agent, and the agent accepts it or reject. When the contract is rejected, both parties earn zero profit. Upon acceptance of the contract, the agent produces the goods and the principal learns subjective information $s \in \{h, l\}$ (date 2). When the agent is H type, the probabilities of observing h and l are ρ and $1-\rho$, respectively, where $\rho > 1/2$; when the agent is L type, the probability of observing l and h are ρ and $1-\rho$, respectively. On date 3, the principal reports her observation and makes payment to the agent as specified in the contract.

The payment is denoted by $t_{\hat{\theta}s}$, contingent on unit of goods produced $q_{\hat{\theta}}$, where $\hat{\theta} \in \{L, H\}$, and the report of the principal's subjective information $\hat{s} \in \{h, l\}$. In order to motivate the principal telling truth, some part of the principal's payment must be burnt before the agent receives it. The amount of money to be burnt is denoted by $b_{\hat{\theta}s} \geq 0$. Thus, the contract is represented by $(q_L, t_{Ll}, t_{Lh}, b_{Ll}, b_{Lh})$ and $(q_H, t_{Hh}, t_{Hl}, b_{Hh}, b_{Hl})$. Without loss of generality, the first contract is chosen by the type L agent and the latter is chosen by the type H agent.

Finally, the principal's payoff is represented by unit of goods she received minus her payment,

$$q - t,$$

and the agent payoff is, because he receives the principal's payment minus the money that was burnt,

$$t - b - \frac{c_{\theta}}{2} q^2.$$

It is assumed that the agent is protected by limited liability. Therefore, the amount of money burnt cannot be more than the payment, $t_{\hat{\theta}s} \geq b_{\hat{\theta}s}$.²

3. Main Results

In this section, we analyze the principal's expected utility maximizing problem. First, by backward induction, we consider the principal's incentive problem. Suppose that the agent is type θ . The principal's subjective information is truthfully reported when

$$\begin{aligned} q_{\theta} - t_{\theta l} &\geq q_{\theta} - t_{\theta h}, \\ q_{\theta} - t_{\theta h} &\geq q_{\theta} - t_{\theta l}. \end{aligned}$$

From these constraints immediately yield:

Lemma 1. *When the principal's subjective information is truthfully reported, the contract requires that $t_{Hh} = t_{Hl} \equiv t_H$ and $t_{Ll} = t_{Lh} \equiv t_L$.*

Due to the principal's utility does not satisfy the single crossing property, the transfer cannot be contingent on the principal's observation. This result shows that, solely by a payment by the principal to the agent, the principal's subjective information cannot add any extra incentive for the agent to be self-screening.

²The result is not significantly different even without limited liability.

The principal's information can affect the agent's incentives only through burning money.

Next, we consider the agent's incentive compatible and participation constraints. These are:

$$t_L - \rho b_{Ll} - (1 - \rho)b_{Lh} - \frac{c_L}{2}q_L^2 \geq t_H - \rho b_{Hl} - (1 - \rho)b_{Hh} - \frac{c_L}{2}q_H^2, \quad (\text{AIC-L})$$

$$t_H - \rho b_{Hh} - (1 - \rho)b_{Hl} - \frac{c_H}{2}q_H^2 \geq t_L - \rho b_{Lh} - (1 - \rho)b_{Ll} - \frac{c_H}{2}q_L^2, \quad (\text{AIC-H})$$

$$u_L \equiv t_L - \rho b_{Ll} - (1 - \rho)b_{Lh} - \frac{c_L}{2}q_L^2 \geq 0, \quad (\text{IR-L})$$

$$u_H \equiv t_H - \rho b_{Hh} - (1 - \rho)b_{Hl} - \frac{c_H}{2}q_H^2 \geq 0. \quad (\text{IR-H})$$

The first two constraints, equation (AIC-L) and (AIC-H), are the agent's incentive compatibility constraints, and the constraints (IR-L) and (IR-H) are the agent's participation constraints.

Besides the above constraints, the principal faces limited liability (LL)

$$t_H \geq b_{Hl}, b_{Hh}, \quad t_L \geq b_{Ll}, b_{Lh}, \quad (\text{LL})$$

and non-negativity constraints:

$$b_{Ll}, b_{Lh}, b_{Hh}, b_{Hl} \geq 0.$$

Under these constraints, the principal maximizes her expected utility:

$$V(\rho) \equiv \max_{q,t,b} \pi(q_L - t_L) + (1 - \pi)(q_H - t_H)$$

Although the principal's expected utility does not contain amount of money burnt, burning money is also a cost for the principal, because she needs to pay more to the agent to satisfy the participation constraints when money is burnt. Hence, the principal burns money only if it returns some benefit.

When the agent's and principal's choices are consistent, the agent's type is likely revealed honestly. In this case, there is no need to burn money.

Lemma 2. *Money is not burnt when the agent's and principal's choices are consistent: $b_{Ll} = b_{Hh} = 0$.*

Proof. Suppose that $b_{Ll} > 0$. This is not optimal, because by reducing b_{Ll} by a small amount $\Delta > 0$ and reducing t_L by $(1 - \rho)\Delta$, we can increase the principal's payoff without changing constraint (AIC-H) and while relaxing constraint (AIC-L) and (IR-L), which is a contradiction. Similarly, $b_{Hh} > 0$ is not optimal, because we can always reduce b_{Hh} by a small amount $\Delta > 0$ and reduce t_H by $(1 - \rho)\Delta$, without changing constraint (AIC-L) and while relaxing constraint (AIC-H) and (IR-H), and thus increase the principal's payoff. \square

As in the standard adverse selection problem, we consider only the downward incentive problem, that is, the one in which there is an incentive for an efficient agent (type L) to claim to be inefficient (type H). Next lemma shows when this approach is valid.

Lemma 3. *Suppose that the constraint (AIC-L) is binding. The constraint (AIC-H) holds when $q_L \geq q_H$.*

Proof. By substituting the binding constraint (AIC-L) into the constraint (AIC-H), we can obtain

$$(2\rho - 1)(b_{Hl} + b_{Lh}) \geq -\frac{c_H - c_L}{2}(q_L^2 - q_H^2).$$

Because $\rho > 1/2$, $b_{Lh}, b_{Hl} \geq 0$ and $c_H > c_L$, the inequality holds when $q_L \geq q_H$.

In following analysis, we ignore the constraint (AIC-H) for the moment and, later, check whether the solutions satisfy $q_L \geq q_H$. Because we are only considering the downward incentive problem, when the agent is revealed to be type L , the principal does not need to be anxious for the agent's misconduct.

Lemma 4. *Suppose that the constraint (AIC-H) is not binding. Money is not burnt when the agent is revealed to be type L , $b_{Lh} = 0$.*

Proof. Without the constraint (AIC-H), the other constraints are relaxed when b_{Lh} is small. Hence, $b_{Lh} = 0$. □

At this point, the remaining question is b_{Hl} , how to punish an agent who claims to be type H , when the principal has contradictory information l . The result varies depending on accuracy of the principal's information.

In next proposition, we consider when the principal's information is inaccurate. In this case, money is not burned because cost of burning money overwhelms benefit.

Proposition 1. *When $\rho \leq (1 + \pi)^{-1} \equiv \rho^*$, the second-best contract is "Contract 0," in which money is not burnt, and the principal's subjective information is not utilized:*

$$\begin{aligned} (q_L^*, t_L^*, b_{Ll}^*, b_{Lh}^*) &= \left(\frac{1}{c_L}, \frac{1}{2c_L} + \frac{c_H - c_L}{2} \left(\frac{1 - \pi}{c_H - \pi c_L} \right)^2, 0, 0 \right), \\ (q_H^*, t_H^*, b_{Hh}^*, b_{Hl}^*) &= \left(\frac{1 - \pi}{c_H - \pi c_L}, \frac{c_H}{2} \left(\frac{1 - \pi}{c_H - \pi c_L} \right)^2, 0, 0 \right). \end{aligned}$$

Under this contract, the agent's utility is

$$u_L^* = \frac{c_H - c_L}{2} \left(\frac{1 - \pi}{c_H - \pi c_L} \right)^2 > u_H^* = 0.$$

Proof. Write λ_L , δ_L , and δ_H as the Lagrange multipliers for constraints (AIC-L), (IR-L), and (IR-H), respectively. Also, let γ_{Hl} be the multiplier for $b_{Hl} \geq 0$, and let ζ_{Hl} be the multiplier for $t_H \geq b_{Hl}$.

First, assume that the constraint (IR-L) is not binding. From the first-order conditions, we can obtain $\lambda_L = \pi$ and $\delta_H = 1$. The first-order condition for b_{Hl} is $\lambda_L \rho - \delta_H (1 - \rho) + \gamma_{Hl} - \zeta_{Hl} = 0$. Hence, when $\lambda_L \rho - \delta_H (1 - \rho) \leq 0$, which is when $\rho \leq (1 + \pi)^{-1}$, $b_{Hl} = 0$ is optimal. When $b_{Hl} = 0$, the optimal contract can be obtained by using a well-known method, and the constraint (IR-L) is not binding. Also, because $q_L^* > q_H^*$, the constraint (AIC-H) is satisfied from Lemma 3, and $b_{Lh}^* = 0$ is satisfied from Lemma 4. □

Burning money to increase b_{Hl} has two effects. By increasing b_{Hl} by one

unit, the constraint (AIC-L) is relaxed, while the constraint (IR-H) is tightened. From the former effect, the principal's payoff is increased by $\pi\rho$, and it is reduced by $1-\rho$ from the latter effect. Money is not burned when the disadvantage overwhelms the benefit, that is, when $\rho \leq \rho^*$.

On the other hand, when $\rho > \rho^*$, burning money is desirable for the principal. She set b_{HI} as large as possible until either of the constraints (IR-L) or (LL) may be binding.

Proposition 2. *When $\rho > \rho^*$, the second-best contract is "Contracts S." The terms of contract and associating the agent's utility are as follows.*

- When $c_H(c_L + c_H)^{-1} \equiv \hat{\rho} > \rho$,

$$\begin{aligned} (q_L^*, t_L^*, b_{Ll}^*, b_{Lh}^*) &= \left(\frac{1}{c_L}, \frac{1}{2c_L} + \frac{(1-\rho)c_H - \rho c_L}{2\rho} \left\{ \frac{(1-\pi)\rho}{(1-\pi\rho)c_H - \pi\rho c_L} \right\}^2, 0, 0 \right), \\ (q_H^*, t_H^*, b_{Hh}^*, b_{HI}^*) &= \left(\frac{(1-\pi)\rho}{(1-\pi\rho)c_H - \pi\rho c_L}, \frac{c_H}{2\rho} \left\{ \frac{(1-\pi)\rho}{(1-\pi\rho)c_H - \pi\rho c_L} \right\}^2, 0, \frac{c_H}{2\rho} \left\{ \frac{(1-\pi)\rho}{(1-\pi\rho)c_H - \pi\rho c_L} \right\}^2 \right). \end{aligned}$$

The agent's utility is

$$u_L^* = \frac{(1-\rho)c_H - \rho c_L}{2\rho} \left\{ \frac{(1-\pi)\rho}{(1-\pi\rho)c_H - \pi\rho c_L} \right\}^2 > u_H^* = 0.$$

- When $\rho \geq \hat{\rho}$,

$$\begin{aligned} (q_L^*, t_L^*, b_{Ll}^*, b_{Lh}^*) &= \left(\frac{1}{c_L}, \frac{1}{2c_L}, 0, 0 \right), \\ (q_H^*, t_H^*, b_{Hh}^*, b_{HI}^*) &= \left(\frac{2\rho-1}{\rho c_H - (1-\rho)c_L}, \frac{2\rho-1}{2\{\rho c_H - (1-\rho)c_L\}}, 0, \frac{c_H - c_L}{2(2\rho-1)} \left\{ \frac{2\rho-1}{\rho c_H - (1-\rho)c_L} \right\}^2 \right), \end{aligned}$$

and the agent's utility is $u_L^* = u_H^* = 0$.

Proof. Suppose $\rho > \rho^*$. The constraint (IR-L) is either binding or not. If it is not binding, b_{HI} should be maximized from the above analysis. Consequently, $b_{HI} = t_H$. The second-best contract can be obtained from substituting the binding constraints (AIC-L) and (IR-H) and $b_{HI} = t_H$ into the principal's expected utility and then maximizing it. This second best contract satisfies the constraint (IR-L) when $\rho < \hat{\rho}$. Also, because $q_L^* > q_H^*$ when $\rho < c_H(c_L + \pi c_H)^{-1}$, the constraint (AIC-H) is satisfied.

If the constraint (IR-L) is binding, the second-best contract can be obtained by maximizing the principal's expected utility subject to the binding constraints (AIC-L), (IR-H) and (IR-L). The contract satisfies $t_H \geq b_{HI}$ when $\rho \geq \hat{\rho}$. Note that $q_L^* > q_H^*$, and the constraint (AIC-H) is satisfied. \square

The remaining part of the section considers the principals expected utility. When $\rho \leq \rho^*$, the second best contract is Contract 0 and the principal's utility is independent from ρ . When $\rho > \rho^*$, her expected utility is

$$V(\rho) = \begin{cases} \frac{\pi(1-\pi\rho)c_H + (1-2\pi)\rho c_L}{2c_L\{(1-\pi\rho)c_H - \pi\rho c_L\}} & \text{if } \rho < \hat{\rho} \\ \frac{\pi\rho(c_H - c_L) + (2\rho - 1)c_L}{2c_L\{\rho c_H - (1-\rho)c_L\}} & \text{if } \rho \geq \hat{\rho}. \end{cases}$$

Comparative statics on the principal's utility is summarized as follows:

Corollary. *The principal's payoff is strictly increasing in ρ when $\rho > \rho^*$ and is constant otherwise. It attains the first best iff $\rho = 1$.*

When $\rho \geq \hat{\rho}$, the agent does not receive any rent. Even this case, the principal cannot fully extract the surplus. This is because the contract mistakenly burns the agent's money and spoils the total surplus unless $\rho = 1$. The agency problem remains unless the principal's subjective information correlates perfectly with the agent's type.

4. Conclusions

We have investigated if the principal can utilize subjective information to solve the adverse selection problem. The result is that the principal cannot take advantage of knowing subjective information if it is not well correlated with the agent's type, because the disadvantage of burning money overwhelms the benefit from motivating the agent to tell the truth. Even when the correlation is high, the principal cannot fully extract the surplus from the agent.

An important practical implication of our results is that supervision by the principal by herself can be fail to solve the agency problem, and alternative form of organization is necessary. Indeed, third parties are often hired as supervisor.³

We have conducted the analysis under the assumption that each of the agent and the principal had two types. Albeit simplicity, it would be desirable to solve the model allowing them for multiple types. Given the well-known difficulties associated with mechanism design problems with multidimensional types, this extension is not only interesting but also apt to be nontrivial.

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³Incentive of the third parties can be another problem, but it is beyond the scope of this research.

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