

Adaptive Control for a Class of Systems with Output Deadzone Nonlinearity

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Abstract

This paper presents a continuous-time adaptive control scheme for systems with uncertain non-symmetrical deadzone nonlinearity located at the output of a plant. An adaptive inverse function is developed and used in conjunction with a robust adaptive controller to reduce the effect of deadzone nonlinearity. The deadzone inverse function is also implemented in continuous time, and an adaptive update law is designed to estimate the deadzone parameters. The adaptive output deadzone inverse controller is smoothly differentiable and is combined with a robust adaptive nonlinear controller to ensure robustness and boundedness of all the states of the system as well as the output signal. The mismatch between the ideal deadzone inverse function and our proposed implantation is treated as a disturbance that can be upper bounded by a polynomial in the system states. The overall stability of the closed-loop system is proven by using Lyapunov method, and simulations confirm the efficacy of the control methodology.

Keywords

Adaptive Inverse Control, Output Deadzone, Hard Nonlinearity

1. Introduction

The problem of deadzone nonlinearity has been addressed by many researches with great success by utilizing adaptive control methods to eliminate the undesirable effects on the output of a plant [1]-[5]. Demonstrated in

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Figure 1 is the effect of deadzone on the output of a plant for a pure sinusoidal input trajectory. The majority of earlier investigations to this problem focus on the problem where the nonlinearity is located at the input of the plant as an actuator problem [1] [2]. In an actuator deadzone, the control effort is within the span of the nonlinearity which makes it somewhat easier to reduce or eliminate its deleterious effects before it enters the dynamics of the system to be controlled. As a matter of fact, several papers present a two structure control schemes that can be designed to handle deadzone as well as other requirements for plant performance criteria [3]. On the other hand, output deadzone, which is physically inherent in some sensors that measure output signals of a plant, is a more complicated problem. The control effort has to eliminate the deleterious effect of the deadzone nonlinearity whilst going through the complicated dynamics of the plant. Therefore, whatever added control requirements enforced on the designer due to disturbances or noise affecting the plant, will further complicated the task. One of the earliest investigations of output nonlinearities such as deadzone was presented by [4]. Their proposed methodology was based on output matching control which involved the design of an adaptive deadzone inverse used to reshape the input reference trajectory to negate the effect of the deadzone. The parameters of the deadzone were adaptively estimated by designing an error function utilizing the output to observe plants states. The implementation was quiet complex in design and implemented in discrete time. In [5], an output feedback design was analysed for robustness and was developed using input to state stability (ISS) small gain tools. The combination of observer and controller design was proved to be essential when handling output nonlinearities. An adaptive compensation scheme without constructing a dead-zone inverse was presented in [6]. The proposed adaptive method requires only the information of bounds of the deadzone slopes and treats the time-varying input coefficient as a system uncertainty. The new control scheme ensures bounded-error trajectory tracking and assures the boundedness of all the signals in the adaptive closed loop. Tian Ping *et al.* utilized the integral-type Lyapunov function to design an adaptive compensation term for the upper bound of the residual and optimal approximation error as well as the dead-zone disturbance [7]. It was demonstrated that the closed-loop control system was semi-globally uniformly bounded. In [8], an inverse deadzone function was incorporated in control system driven from a mathematical model of a deadzone in pneumatic servo valves. Tests were performed out using controllers with and without dead zone compensation to comparison validated the efficacy of the method. In [9], a somewhat earlier work was presented in discrete time which successfully achieved reduction

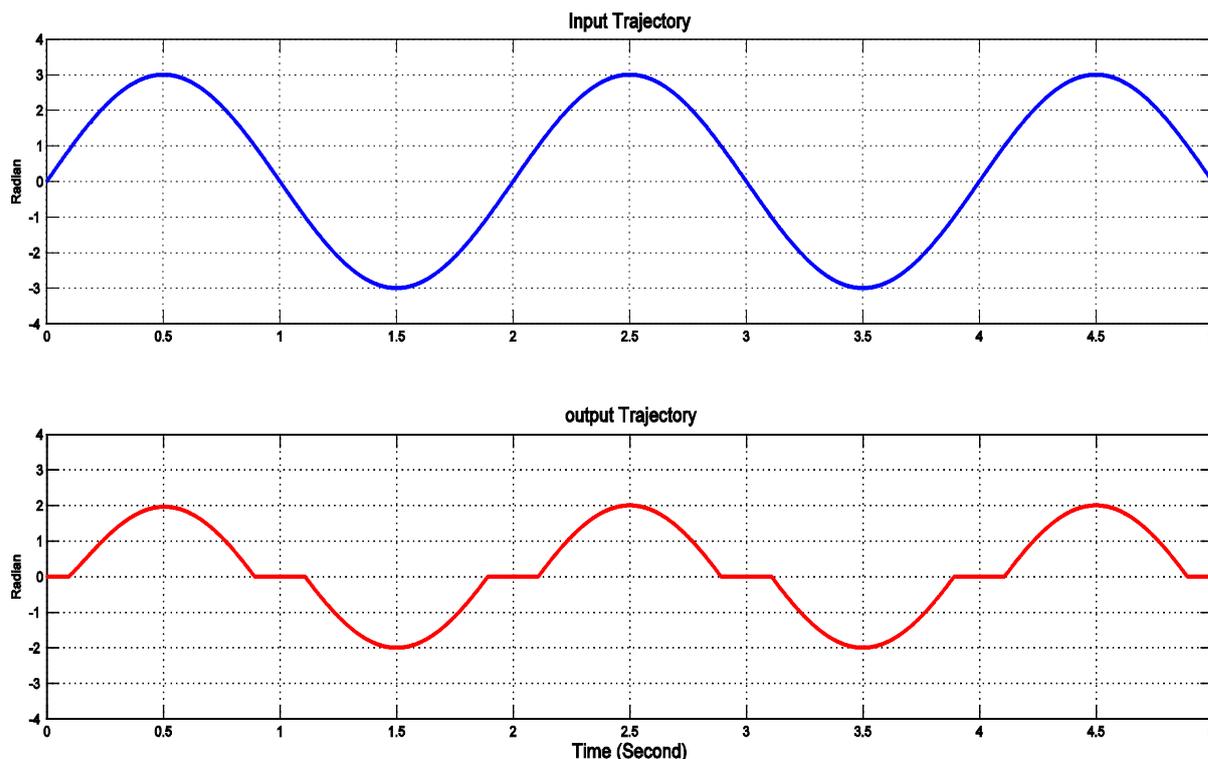


Figure 1. The distortion effect of output deadzone nonlinearity on a sinusoidal of signal.

of the tracking error in plants with output deadzone nonlinearity while ensuring the global boundedness stability. The paper presented by Jing Zhoua *et al.* introduced a smooth approximation to the deadzone model which allowed them to employ back stepping technique [10]. In their approach, no knowledge was assumed of the uncertainty's and the deadzone's parameters. It is shown that the proposed controller not only can guarantee global stability, but also can achieve excellent transient performance. It is worthwhile to note that other non-classical control methods, such as fuzzy logic or neural network, have been presented by several researchers to reduce the effect of a deadzone nonlinearity [11]-[14]. For example, Wallace and Max used an adaptive fuzzy controller for nonlinear systems subject to dead-zone input. The boundedness of all closed-loop signals and the convergence properties of the tracking error are proven using Lyapunov stability theory and Barbalat's lemma [15].

Motivated by the success in producing successful results in handling input deadzone, we present an extended method to reduce the errors caused by output deadzone nonlinearity. The proposed method relies on the premise that by pre-shaping the input trajectory to mimic an inverse form of the deadzone nonlinearity, the combined effect will reduce if not completely eliminating the effect of output deadzone.

In this paper, a new continuous time robust adaptive output deadzone inverse controller (RAODI) is used in conjunction with a conventional model reference adaptive control to counter the distortions cause by output deadzone. The ideal deadzone inverse controller is approximated by an infinitely differentiable implementation to insure asymptotic tracking and minimized error generation. The overall stability of the system under the proposed scheme will be proven analytically and demonstrated by simulation to a practical application. The structure of the paper starts with a brief presentation of the dynamics of an output deadzone nonlinearity that defines various parameters and its effect on the output of a system are presented in Section 2. Meanwhile, the proposed control methodology is presented and its analytical proof using the Lyapunov argument is shown in Section 3. Consequently, an illustrative example of a model reference adaptive control scheme combined with the inverse control method is presented and followed by simulation results in Section 4.

2. The dynamics of Output Deadzone Nonlinearity

A common representation of a non-symmetrical deadzone nonlinearity, shown in **Figure 1**, can be described as follows

$$DZ(y) = \begin{cases} m(x - d_r), & \text{if } x > d_r \\ 0, & \text{if } -d_l < x < d_r \\ m(x + d_l), & \text{if } x < -d_l \end{cases} \quad (1)$$

where $DZ(y)$ denotes the output of deadzone function, $x(t)$ the output of a plant, m is the slope of the lines, $(d_r - d_l)$ is the width of the deadzone distance, and $u(t)$ is the input of the plant block as shown in **Figure 2**. Although the width of the deadzone spacing is assumed not to be exactly known, an upper bounds on it is given by

$$|d_r - d_l| \leq d_M \quad (2)$$

where d_M is a positive scalar. Output deadzone may also be written as

$$DZ(y) = x - sat_d(x) \quad (3)$$

where $sat_d(u)$ represents a non-symmetrical saturation function given by

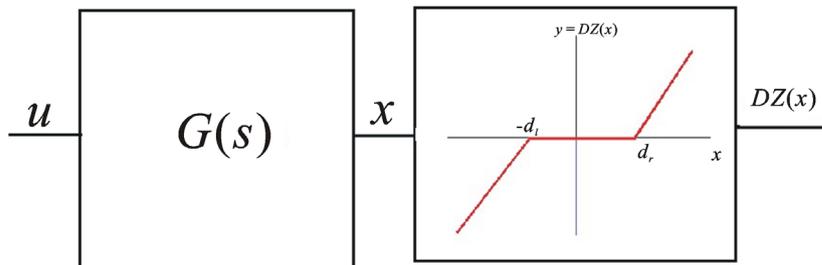


Figure 2. Non-symmetric deadzone nonlinearity as a function of a plant output signal.

$$sat_d(x) = \begin{cases} d_r, & \text{if } x > d_r \\ x, & \text{if } d_l < x < d_r \\ -d_l, & \text{if } x < -d_l \end{cases} \quad (4)$$

By defining a logical switching operator

$$\chi_r = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

$$\chi_l = \begin{cases} 1 & \text{if } x < 0 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

Then, the dynamics of the non-symmetrical deadzone presented in (3) can be rewritten as follows

$$y = DZ(x) = x(t) - \chi_l d_l - \chi_r d_r = x(t) - d^T \bar{\chi} \quad (7)$$

where $x(t)$ is the. Meanwhile, thelogical indicators, $\bar{\chi} = [\chi_r \chi_l]$ can be implemented by utilizing the definition of a sign function given as

$$\text{sgn}(x_d) = \begin{cases} 1 & x_d > 0 \\ -1 & x_d \leq 0 \end{cases} \quad (8)$$

To obtain a smoothly differentiable implementation of (8), we replace it with a

$$\text{sgn}(x_d) \approx \tanh(k_s \cdot x_d). \quad (9)$$

with $k_s > 0$ appropriately selected with high value for fast switching applications.

Hence, rewriting Equation (5) and Equation (6) as

$$\chi_r = \frac{1 + \tanh(k_s \cdot x_d)}{2} \quad (10)$$

$$\chi_l = \frac{1 - \tanh(k_s \cdot x_d)}{2} = (1 - \chi_r). \quad (11)$$

To proceed with the design of the compensator the following assumptions are required:

(A1) The deadzone parameters $d_r > 0$ and $-d_l < 0$.

(A2) The deadzone parameters d_r and d_l are bounded as follows:

$$d_l \in [d_l \text{ min}, d_l \text{ max}] \quad \text{and} \quad d_r \in [d_r \text{ min}, d_r \text{ max}].$$

(A3) Without any loss of generality the slope of the deadzone m is positive and is set to 1.

Assumption (A1) and (A2) are the actual physical attributes of a real industrial deadzone and is adopted in [16]. Therefore, the saturation function given by (4) is physically bounded

$$\|sat(x)\| = \|d^T \bar{\chi}\| \leq d_M. \quad (12)$$

3. Robust Adaptive Controller Design

Considering the following nonlinear systems with input deadzone nonlinearity described as

$$\begin{aligned} \dot{x} &= Ax + f(x) + B\{(u) + \psi(x)\} \\ y &= DZ(x) \end{aligned} \quad (13)$$

where the matrices A and B are given by

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & 1 & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}.$$

Meanwhile, the unmeasurable disturbances represented as $\psi(x)$ and $f(x)$ are assumed to be bounded by a known p^{th} order polynomial in the states [17]:

$$\|\psi(x)\| \leq \sum_{k=0}^p \zeta_k \|x\|^k \quad (14a)$$

$$\|f(x)\| \leq \sum_{k=0}^p \zeta_k \|x\|^k. \quad (14b)$$

The desired reference model is given by

$$\dot{x}_d = Ax_d + B\{Kx_d + r\}, \quad (15)$$

where $K \in R^{1 \times n}$ and r is a reference signal. By reshaping the desired reference model in a way to produce a deadzone inversed version of it will reduce the effect of the deadzone. Tracking the reshaped copy of the reference model will force the output of the deadzone nonlinearity to track the original desired reference signal. The adaptive output deadzone inverse compensator can be deduced from (7) as

$$x_d^* = \widehat{DI}(x_d) = x_d + \chi_l \hat{d}_l + \chi_r \hat{d}_r = x_d + \hat{d}^T \bar{\chi}, \quad (16)$$

where $\hat{d}^T = [\hat{d}_r \hat{d}_l]$ is the adaptively estimated values of the exact deadzone spacing $d = [d_r^* d_l^*]$. The adaptive inverse dynamics may be determined by differentiating (16) as follows

$$\begin{aligned} x_{1d}^* &= x_d + \hat{d}^T \bar{\chi} \\ x_{2d}^* &= \dot{x}_{1d}^* = \dot{x}_d + \hat{d}^T \dot{\bar{\chi}} + \dot{\hat{d}}^T \bar{\chi} \\ x_{3d}^* &= \ddot{x}_{1d}^* = \ddot{x}_d + \ddot{\hat{d}}^T \bar{\chi} + 2\dot{\hat{d}}^T \dot{\bar{\chi}} + \hat{d}^T \ddot{\bar{\chi}} \\ &\vdots \\ x_{nd}^* &= x_d^{*(n)} = x_d^{(n)} + \sum_{k=0}^n \binom{n}{k} \cdot \hat{d}^{T(k)} \cdot \bar{\chi}^{(n-k)} \end{aligned}$$

Consequently, we can utilize (15) to construct the inverse deadzone model reference as

$$\dot{x}_d^* = Ax_d^* + B \left\{ K \cdot \left(x_d + \sum_{k=0}^n \binom{n}{k} \cdot \hat{d}^{T(k)} \cdot \bar{\chi}^{(n-k)} \right) + r \right\}. \quad (17)$$

Hence, the states tracking error dynamics $\tilde{x} = x - x_d^*$ may be written as follows

$$\dot{\tilde{x}} = A\tilde{x} + B \left\{ u + \psi(\tilde{x}) - K \cdot \left(x_d - \sum_{k=0}^n \binom{n}{k} \cdot \hat{d}^{T(k)} \cdot \bar{\chi}^{(n-k)} \right) - r \right\}, \quad (18)$$

where r is the desired reference signal. Equation (18) is written compactly as

$$\dot{\tilde{x}} = A\tilde{x} + B \{ u + \psi(\tilde{x}) - Kx_d^* - r \}. \quad (19)$$

where dynamics of x_d^* are given by (17).

By defining the output tracking error $\epsilon(t) = y - x_d$ an adaptive update law for \hat{d}^T can be written as

$$\dot{\hat{d}} = -\sigma \epsilon(t) \bar{\chi} \quad (20)$$

Once again, by ensuring that the plant states $x(t)$ tracking $x_d^*(t)$ will cause

$$y(t) = DZ(x_d^*(t)) = DZ(\widehat{DI}(x_d(t))) = x_d + \epsilon(t) \quad (21)$$

where $\epsilon(t)$ is the output mismatch error caused by the difference between the exact deadzone parameter and the estimated one is expressed as $\tilde{d} = d^* - \hat{d}$. To parameterize $\epsilon(t)$, we utilize Equation (7) to get

$$\epsilon(t) = y(t) - y^*(t) = x_d(t) - d^T \bar{\chi} - x_d(t) + \hat{d}^T \bar{\chi} \quad (22)$$

or simply written as

$$\epsilon(t) = \tilde{d}^T \bar{\chi}. \quad (23)$$

where \tilde{d}^T the deadzone parameters estimation error is

$$\tilde{d}^T = \begin{bmatrix} d_r^* - \hat{d}_r \\ d_l^* - \hat{d}_l \end{bmatrix}, \quad (24)$$

Therefore, the deadzone effect noted by the term $d^T \bar{\chi}$ in (7) can be cancelled by simply ensuring that the system's states vector $x(t)$ track the inverse dynamics of the desired trajectory $x_d(t)$. To achieve proper tracking and global bounded stability of the overall system, we propose the following RAODI controller:

$$u_d(t) = -\alpha B^T P \tilde{x} - \hat{\beta} B^T P \tilde{x} + K x_d^* + r \quad (25)$$

where $\alpha > 0$, $\tilde{x} = x - x_d^*$, and P is the positive definite symmetric solution of the Algebraic Riccati equation (ARE). Moreover, the adaptation law for $\hat{\beta}$ is given by

$$\dot{\hat{\beta}} = \Gamma \|B^T P \tilde{x}\|, \quad \Gamma > 0. \quad (26)$$

The properties of the controller (25) are stated in the following theorem:

Theorem. For the plant described by (13) with input deadzone (1), and the RAODI control law (25) along with the adaptive update laws (22) and (26) will ensure the closed-loop stability and boundedness of tracking error, hence reducing the effects of deadzone on the control law driving the system dynamics and ensures bounded output tracking.

Proof. Using the following positive definite control Lyapunov function

$$V = \tilde{x}^T P \tilde{x} + \frac{\Gamma^{-1}}{2} \tilde{\beta}^2 + \frac{\sigma^{-1}}{2} \tilde{d}^2 \quad (27)$$

Differentiating along the trajectories of the system and substituting for the closed loop dynamics given by (19) yields

$$\begin{aligned} \dot{V} &= \dot{\tilde{x}}^T P \tilde{x} + \tilde{x}^T P \dot{\tilde{x}} + \Gamma^{-1} \tilde{\beta} \dot{\tilde{\beta}} + \sigma^{-1} \tilde{d} \dot{\tilde{d}} \\ &= \left(A \tilde{x} + B \{u + \psi(x) - K x_d^* - r\} \right)^T P \tilde{x} \\ &\quad + \tilde{x}^T P \left(A \tilde{x} + B \{u + \psi(x) - K x_d^* - r\} \right) + \Gamma^{-1} \tilde{\beta} \dot{\tilde{\beta}} + \sigma^{-1} \tilde{d} \dot{\tilde{d}} \end{aligned} \quad (28)$$

Applying the robust controller given in (25) into (28) gives

$$\begin{aligned} \dot{V} &= \left(A \tilde{x} + B \left\{ -\alpha B^T P \tilde{x} - \hat{\beta} B^T P \tilde{x} + \psi(x) \right\} \right)^T P \tilde{x} \\ &\quad + \tilde{x}^T P \left(A \tilde{x} + B \left\{ -\alpha B^T P \tilde{x} - \hat{\beta} B^T P \tilde{x} + \psi(x) \right\} \right) + \Gamma^{-1} \tilde{\beta} \dot{\tilde{\beta}} + \sigma^{-1} \tilde{d} \dot{\tilde{d}} \end{aligned} \quad (29)$$

Collecting terms and simplifying

$$\dot{V} = \tilde{x}^T \left(A^T P + PA \right) \tilde{x} + 2 \tilde{x}^T P B \left\{ \left\{ -\alpha B^T P \tilde{x} - \hat{\beta} B^T P \tilde{x} + \psi(x) \right\} \right\} + \Gamma^{-1} \tilde{\beta} \dot{\tilde{\beta}} + \sigma^{-1} \tilde{d} \dot{\tilde{d}} \quad (30)$$

$$\dot{V} = \tilde{x}^T \left(A^T P + PA - 2\alpha P B B^T P \right) \tilde{x} + 2 \tilde{x}^T P B \left\{ \left\{ -\hat{\beta} B^T P \tilde{x} + \psi(x) \right\} \right\} + \Gamma^{-1} \tilde{\beta} \dot{\tilde{\beta}} + \sigma^{-1} \tilde{d} \dot{\tilde{d}}$$

$$\dot{V} = \tilde{x}^T \left(A^T P + PA - 2\alpha P B B^T P \right) \tilde{x} - \hat{\beta} B^T P \tilde{x} \tilde{x}^T P B + 2 \tilde{x}^T P B \left(\psi(x) \right) + \Gamma^{-1} \tilde{\beta} \dot{\tilde{\beta}} + \sigma^{-1} \tilde{d} \dot{\tilde{d}} \quad (31)$$

The first term can be simplified by solving the Algebraic Reccati Equation given by

$$A^T P + PA - 2\alpha P B B^T P = -Q \quad (32)$$

which gives

$$\dot{V} = -\tilde{x}^T Q \tilde{x} - \hat{\beta} \|\tilde{x}^T P B\|^2 + 2\tilde{x}^T P B (\psi(x)) + \Gamma^{-1} \tilde{\beta} \dot{\hat{\beta}} + \sigma^{-1} \tilde{d}^T \dot{\hat{d}} \quad (33)$$

Replacing the adaptation law (23) and replacing $\hat{\beta} = \tilde{\beta} + \beta^*$ in (31) yields

$$\dot{V} = -\tilde{x}^T Q \tilde{x} - (\tilde{\beta} + \beta^*) \|\tilde{x}^T P B\|^2 + 2\tilde{x}^T P B (\psi(x)) + \tilde{\beta} \|\tilde{x}^T P B\|^2 + \sigma^{-1} \tilde{d}^T \dot{\hat{d}} \quad (34)$$

$$\dot{V} = -\tilde{x}^T Q \tilde{x} - \beta^* \|\tilde{x}^T P B\|^2 + 2\tilde{x}^T P B \psi(x) + \sigma^{-1} \tilde{d}^T \dot{\hat{d}} \quad (35)$$

Substituting the adaptive update law (7) $\dot{\hat{d}} = -\sigma \epsilon(t) \bar{\chi}$ makes the fourth term in

$$\dot{V} = -\tilde{x}^T Q \tilde{x} - \beta^* \|\tilde{x}^T P B\|^2 + 2\tilde{x}^T P B \psi(x) - \tilde{d}^T \epsilon(t) \bar{\chi} \quad (36)$$

Utilizing Equation (23) for output tracking error $\epsilon(t) = \tilde{d}^T \bar{\chi}$.

$$\dot{V} = -\tilde{x}^T Q \tilde{x} - \beta^* \|\tilde{x}^T P B\|^2 + 2\tilde{x}^T P B \psi(x) - (\tilde{d}^T \bar{\chi})^2 \quad (37)$$

Renders the last term negative. For the third term, we utilize the general inequality $2ab \leq a^2 + b^2$ the third term in (37) can be bounded as

$$\|2\tilde{x}^T P B \psi(x)\| \leq \varsigma \|\tilde{x}^T P B\| + \varsigma^{-1} \|\tilde{x}\| \quad (38)$$

Applying this bound to (37)

$$\dot{V} \leq -(\lambda_{\min}(Q) - \varsigma^{-1} \gamma) \|\tilde{x}\| - \beta^* \|\tilde{x}^T P B\|^2 - (\tilde{d}^T \bar{\chi})^2 \quad (39)$$

By choosing the degree of freedom ς satisfying the condition $\varsigma < \frac{\lambda_{\min}(Q)}{\gamma}$ and choosing β^* to be greater than ς ensures all terms of \dot{V} negative.

4. Illustrative Example & Simulations

To illustrate the efficacy of the proposed compensator a second order sinusoidal desired reference model is selected for tracking. Simulations of the system in (22) under the adaptive control law (23) and (24) have been performed for a sinusoidal reference trajectory given by $x_d(t) = 3 \sin(\pi t)$ represented by a second order model. The actual plant is also chosen to be a second order system simulating a rotational gear with deadzone resulting from the spacing between its meshing teeth.

$$\begin{aligned} \ddot{\theta}_m + k_1 \dot{\theta}_m + k_2 \theta_m &= u(t) \\ \theta_l &= DZ(\theta_m) = \theta_m - sat(\theta_m), \end{aligned} \quad (40)$$

where $[\theta_m \ \dot{\theta}_m]^T$ represent the driving motor angle and velocity respectively; $[k_1 k_2]^T$ represent the viscous friction and the electromotive force constant; and θ_l represents the output load angle. By defining the state vector $[x_1 x_2]^T$ to represent $[\theta_m \ \dot{\theta}_m]$, then the system under investigation can be represented in space state form as

$$\begin{aligned} \dot{x} &= Ax + B \{k^T x + u(t)\} \\ y &= DZ(x) = x - sat(x). \end{aligned} \quad (41)$$

where the matrices A and B along with the gain k are given by

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, k = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} \quad (42)$$

Meanwhile, the desired reference model to be tracked at the output for the overall system may be rewritten as

$$\begin{aligned} \dot{x}_d &= Ax_d + B\{\gamma k^T x_d - 3\pi^2 \sin(\pi t)\} \\ y &= x_d, \end{aligned} \tag{43}$$

where $\gamma > 0$ used to insure the stability of the desired tracked model. In the case of meshing gears, the deadzone spacing parameter can be easily predetermined and measured. The reference point is chosen to be at the center of the deadzone spacing. Hence, define $d^* = d_r^* = -d_l^* = 1$ with \hat{d} being the adaptation that estimates its value as given by Equation (20). Therefore, the adaptive deadzone inverse trajectory written as follows

$$x_d^* = \widehat{DI}(x_d) = x_d + \hat{d}\bar{x}. \tag{44}$$

The proposed controller is given by

$$u_d(t) = -\alpha B^T P\tilde{x} - \hat{\beta} B^T P\tilde{x} + Kx_d^* + r \tag{45}$$

where the first term is the conventional PD-controller, the second term is the robust adaptive controller, and the third term is the adaptive deadzone inverse one.

$$\dot{\hat{\beta}} = \Gamma \|B^T P\tilde{x}\|, \Gamma > 0. \tag{46}$$

Meanwhile, the initial value of \hat{d} is set to be zero and no prior knowledge of its values is needed. The exact value of the simulated deadzone parameter is set to $d^* = 1$. For all other simulated parameters refer to **Table 1**.

Figure 3 shows the output trajectory $y_o = \theta_l$ for the system under RAODI control is presented and is compared to the trajectory tracking of the system under adaptive without the inverse (in dotted blue), and a PD-controller (dashed red). The system performance is shown with the black solid line while the performance of a regular PD controller is shown in dotted red line. Clearly, the output of the system under RAODI outperforms the system with a conventional PD controller. The deadzone spacing effect is practically eliminated and the tracking error is held to a small negligible amount.

The improvement in reducing the effect of output deadzone on the output signal is demonstrated in **Figure 4** where the error $(y_o - x_d) = \theta_l - \theta_d$ is plotted in solid line as apposed to the same error for the system under a PD controller plotted in dotted red line. In addition, in **Figure 4**, the **dashed blue line** reflects the output tracking error for the system without the use of inverse deadzone modifier. The error without the deadzone inverter is much larger than the improved performance due to RAODI controller. The system state $x_1 = \theta_m(t)$ tracking performance (solid) versus the deadzone inverted trajectory $x_{1d} = \theta_d$ for the system under RAODI control is presented in **Figure 5**, with **Figure 6** demonstrating the state tracking error $\epsilon(t) = \theta_l - \theta_d$ for the system under the proposed control scheme. The second state $x_2 = \omega(t)$ tracking performance and its error $\epsilon_2 = \omega - \omega_d$ are presented in **Figure 7** and **Figure 8**, respectively. In addition, **Figure 9** and **Figure 10** show the evolution of the adaptations $\hat{\beta}$ and \hat{d} confirming their bounded stability. Meanwhile, the adaptive controller effort $u_d(t)$ is shown in **Figure 11**.

Table 1. Parameters utilized in the example.

Systems Physical Attributes			
	Parameter	Value	Unit
1	k_p	40	Gain Constant
2	k_v	13	Gain Constant
3	γ	100	Gain Constant
4	d_r^*	1.0	radian
5	d_l^*	-1.0	radian
6	α	1.0	N.m/rad
7	Γ	100	Gains
8	J	1	$\frac{V}{\text{rad}} \cdot \text{s}^{-2}$
9	ω_d	π	rad/s

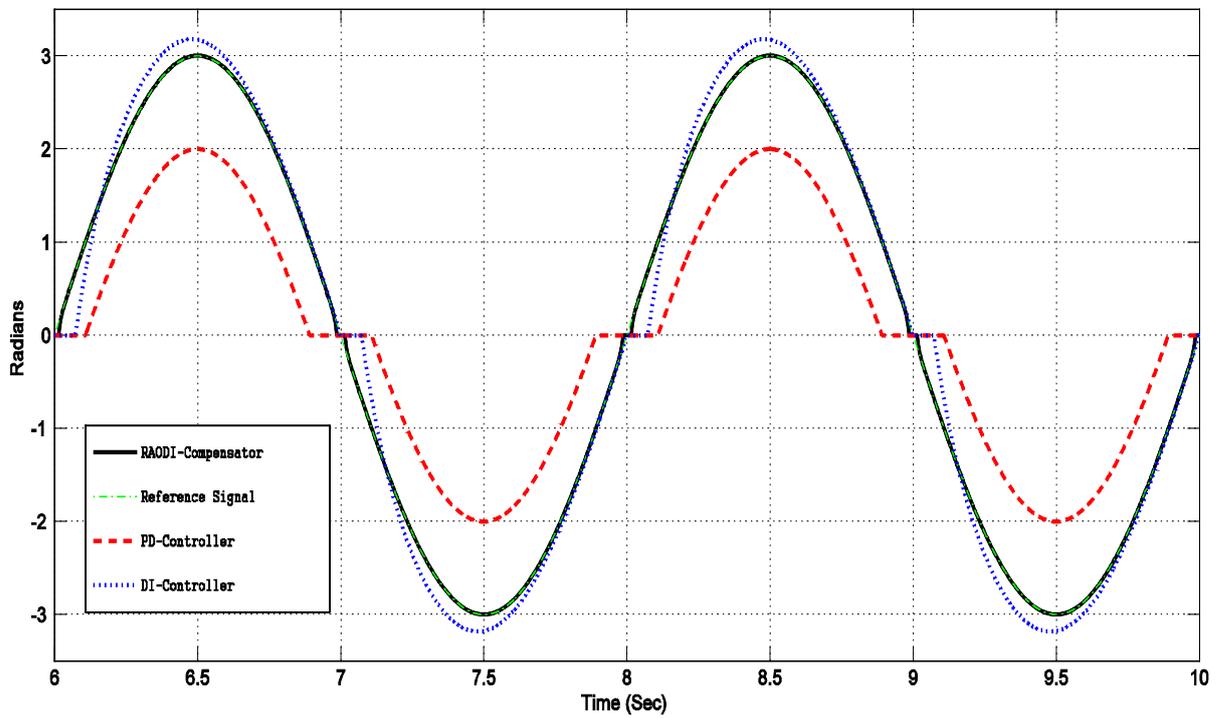


Figure 3. The output trajectory y_o (black-solid) for the system under RAODI control vs. the performance of an adaptive controller (blue-dotted), and a PD-controller (red-dashed).

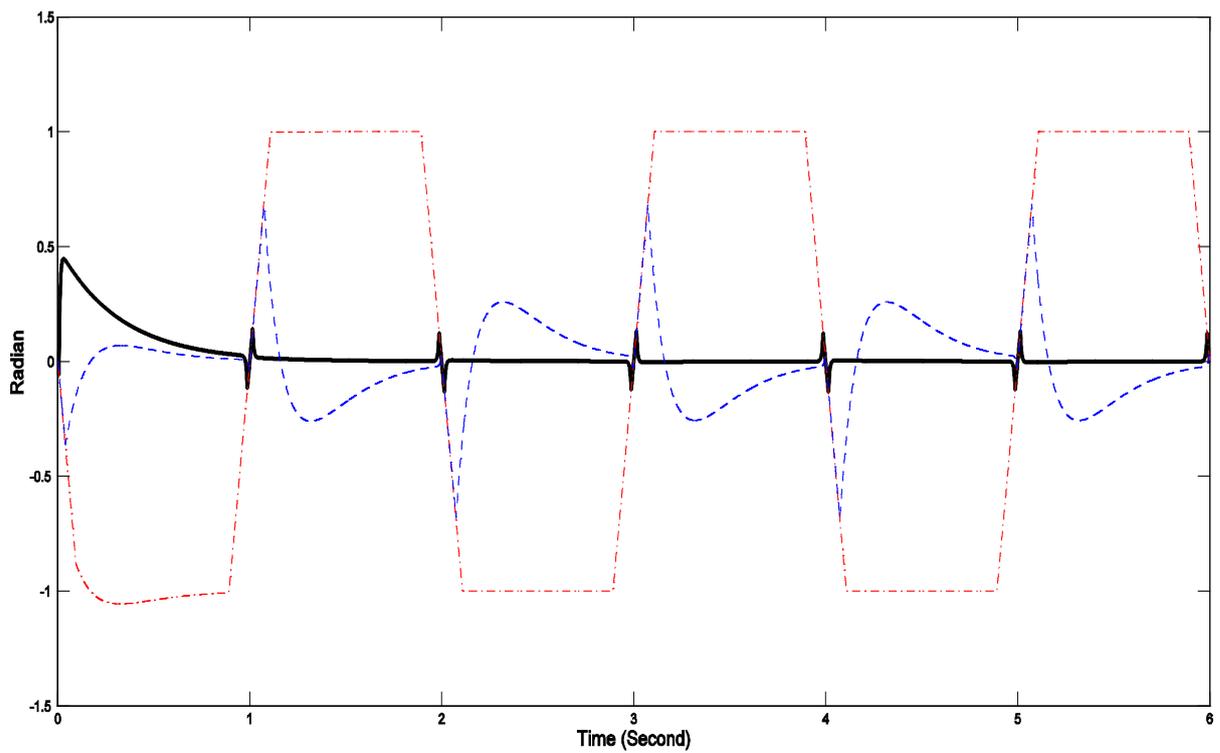


Figure 4. The output tracking error y_o (solid) for the system under RAODI vs. the tracking error of the system under a PD-controller (red-dashed). The dashed blue line reflects the output tracking error for the system without the use of inverse deadzone modifier.

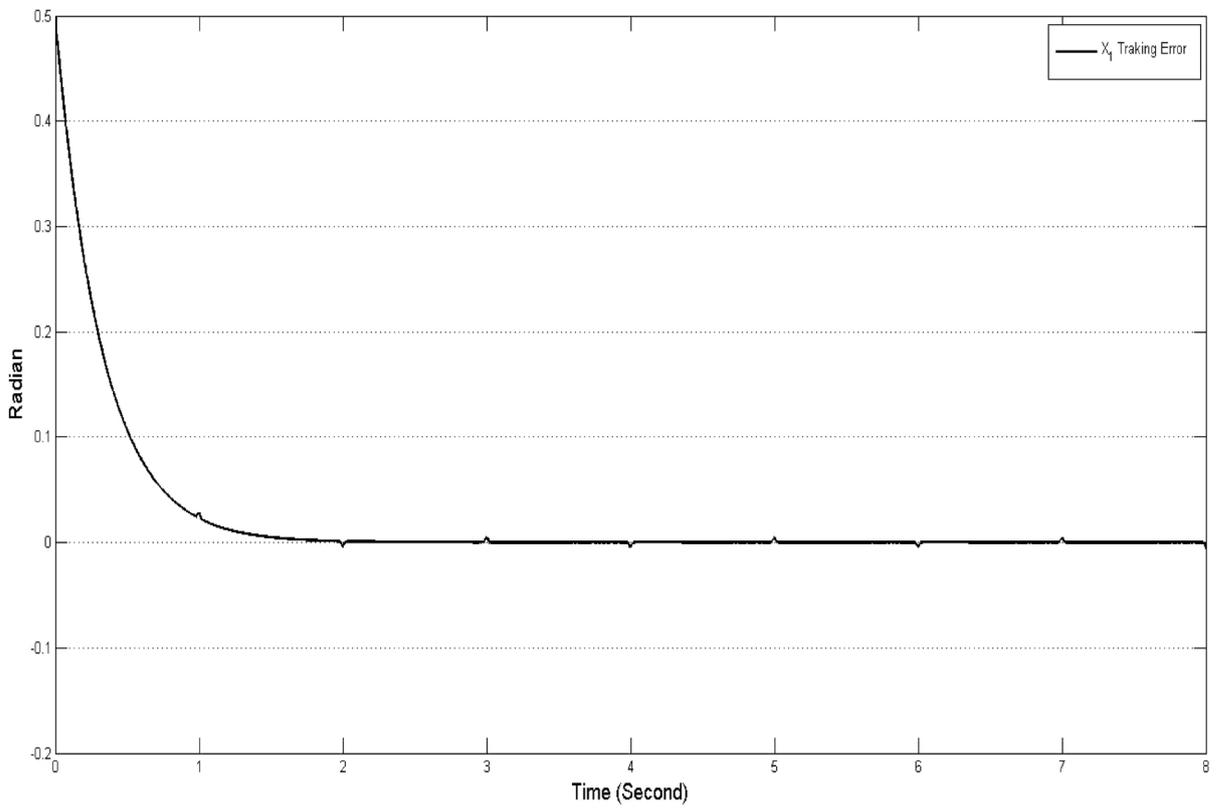


Figure 5. The system state $x_1 = \theta(t)$ tracking performance (solid) versus the deadzone inverted trajectory $x_{1,d} = \theta_d$ for the system under RAODI control (red-dashed).

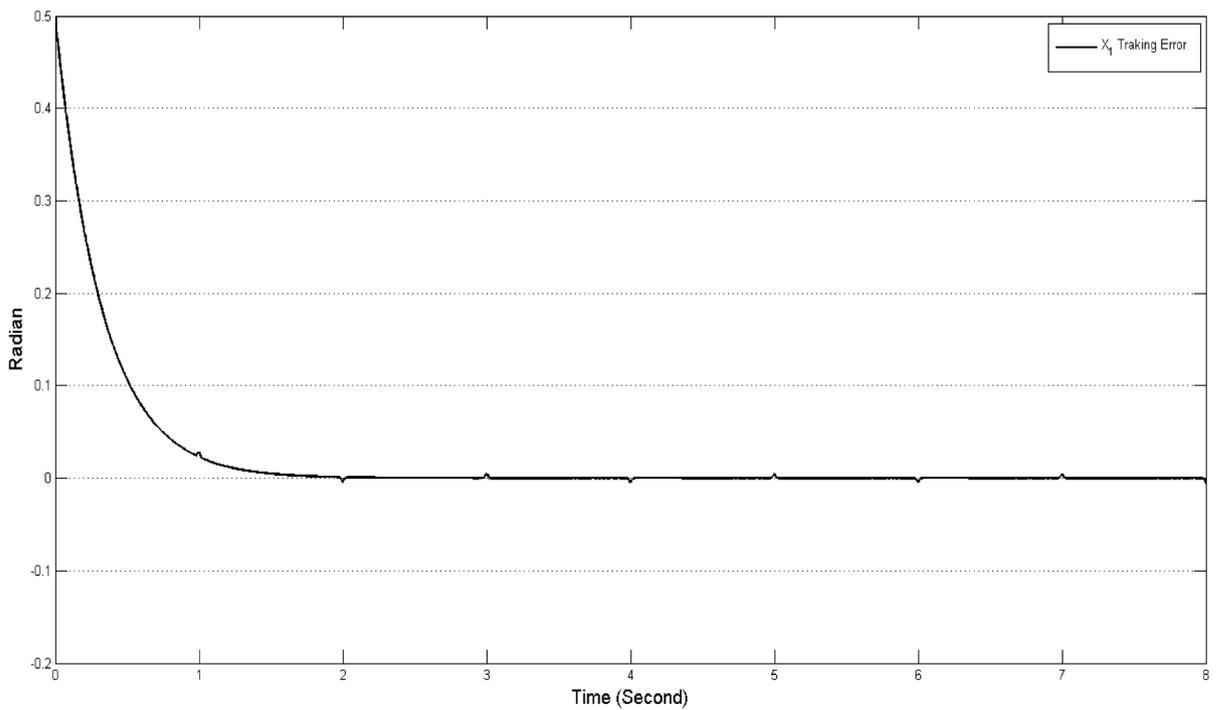


Figure 6. The state tracking error $\epsilon(t) = x_1 - x_{1,d}$ for the system under RAODI control.

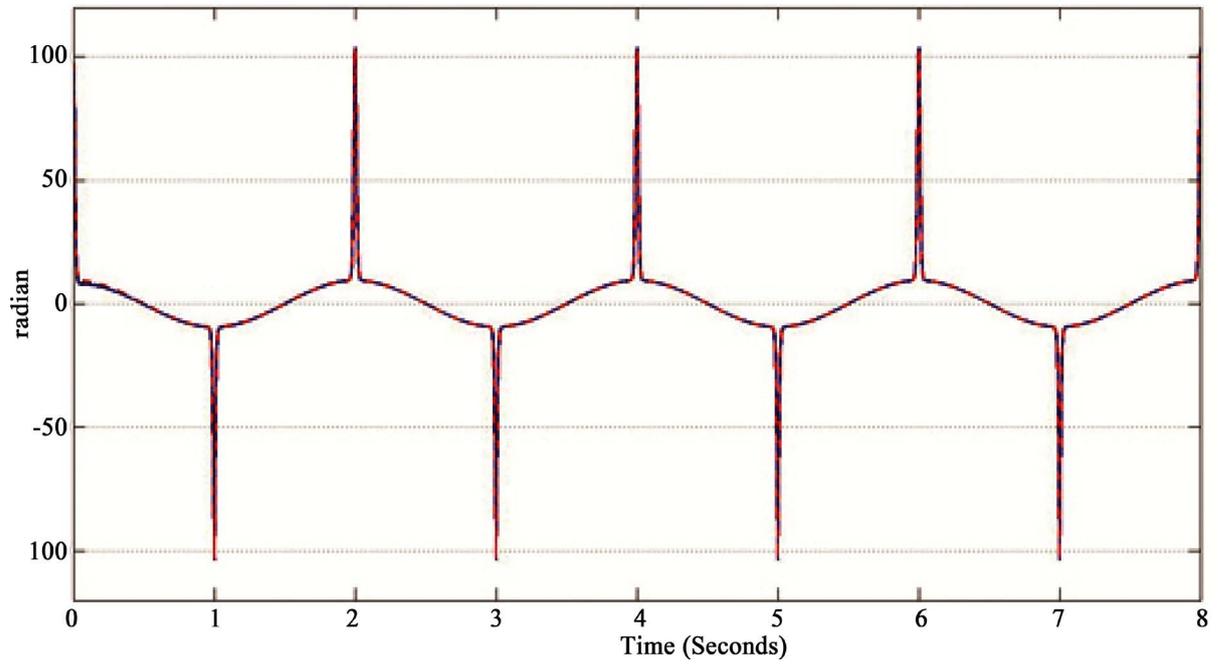


Figure 7. The system state $x_2 = \omega(t)$ tracking performance (solid) versus the inverted deadzone trajectory $x_{2d} = \omega_d$ for the system under RAODI control (red-dashed).

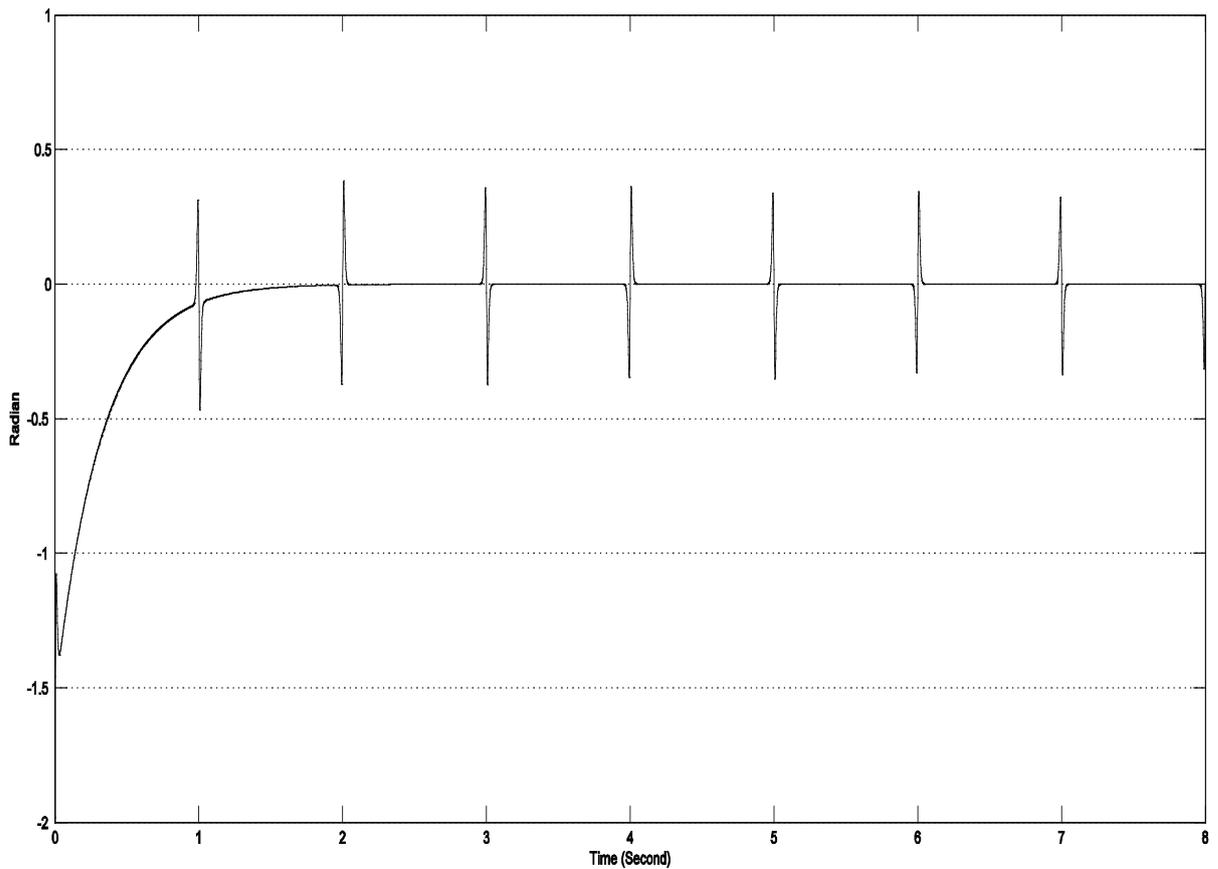


Figure 8. The second state error $e_2 = x_2 - x_{2d}$ for the system under RAODI control.

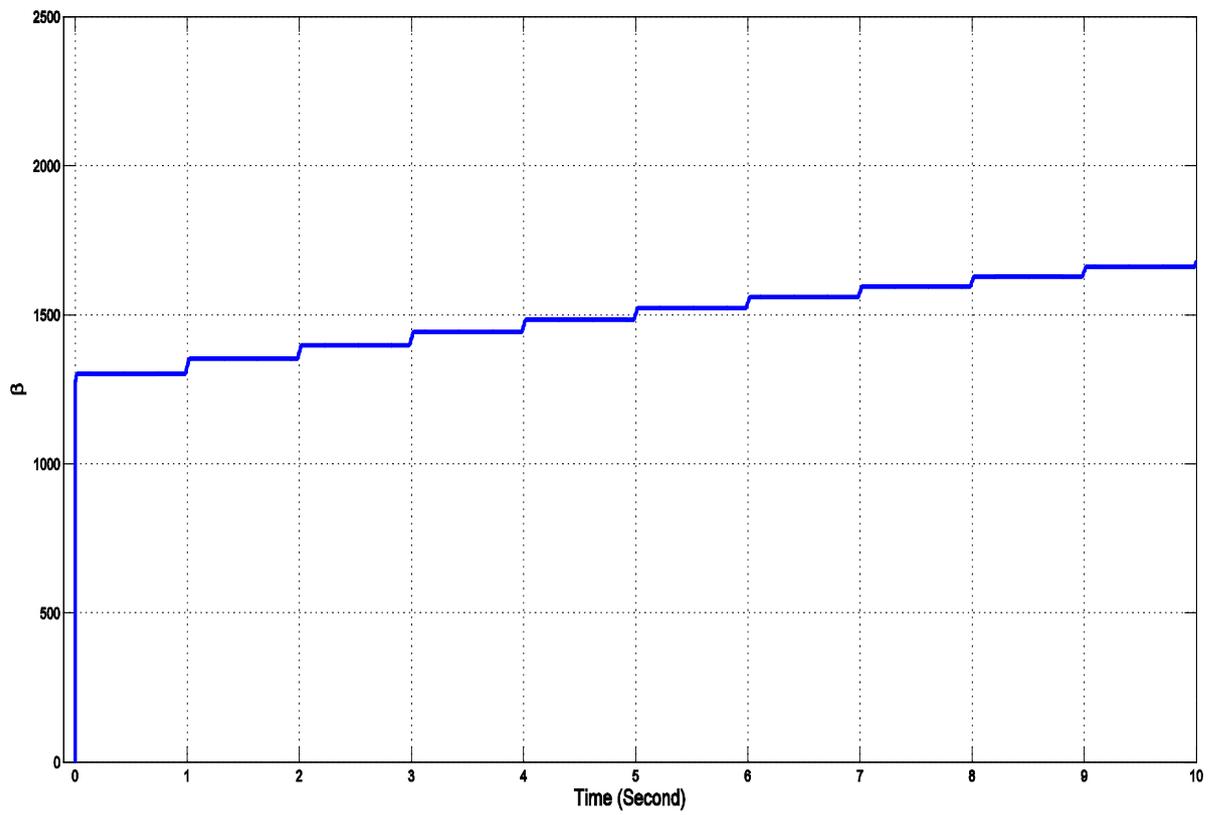


Figure 9. The evolution of the robust adaptation $\hat{\beta}$.

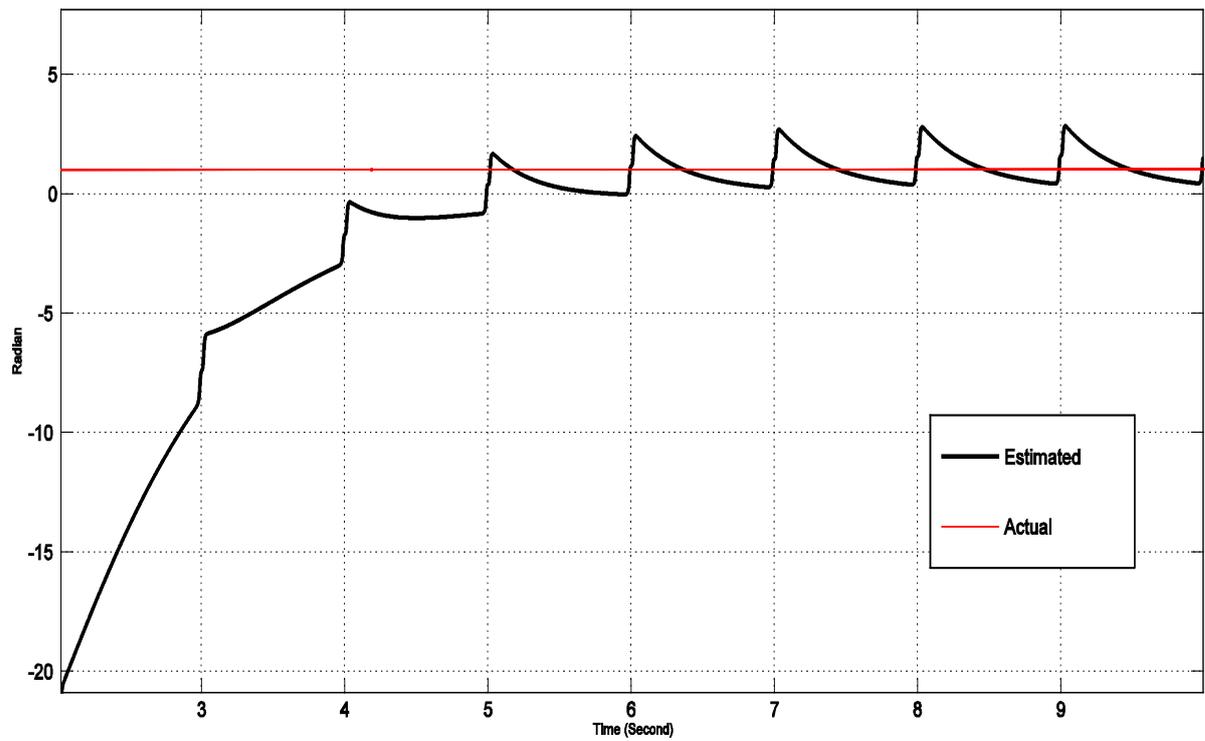


Figure 10. The evolution of the adaptation \hat{d} estimating the actual $d^* = 1.0$ radian.

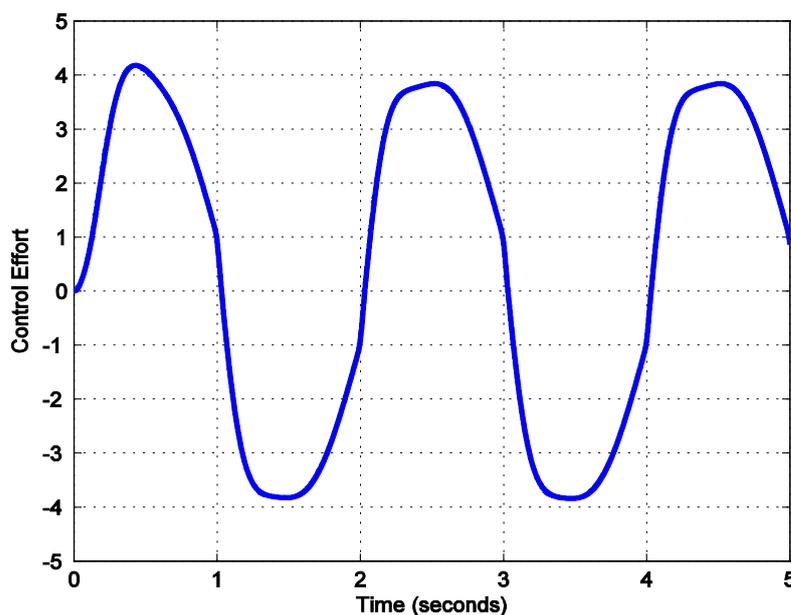


Figure 11. Evolution of the control.

5. Conclusion

In this paper, an adaptive inverse deadzone controller is compared with a robust adaptive controller for systems with output deadzone nonlinearity. Both controllers have been shown to effectively stabilize a second order system, and achieve bounded input bounded output (BIBO) tracking. The proposed deadzone inverse controller has greatly improved the performance of the system over the robust controller. The deadzone inverse controller was implemented in continuous time and was used to modify a desired model reference to mimic an inverse deadzone trajectory. The RAODI is smoothly differentiable and can easily be combined with any of the advanced control methodologies. The stability of the closed-loop system has been proven by using Lyapunov arguments and simulations results confirm the efficacy of the control methodology.

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