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Hall Effect on Peristaltic Flow of Third Order Fluid in a Porous Medium with Heat and Mass Transfer

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Abstract

We investigated the influence of hall, heat and mass transfer on the peristaltic flow of MHD third order fluid under long-wavelength and low Reynolds number approximation. The governing equations are solved analytically with the appropriate boundary conditions by using perturbation technique. The formula of velocity with temperature and concentration is obtained analytically as a function of the physical parameters of the problem.

Keywords

Peristaltic Flow, Third Order Fluid, Hall Effect, Heat, Mass Transfer

1. Introduction

Many fluids in biological system are transported by peristalsis. The word peristalsis stems from the Greek word peristaltikos, which means clasping and compressing. Physically, it means the mechanism for pumping fluid in a tube by means of a moving contractile ring around the tube, which pushes the material onward. The need for peristaltic pumping may arise in circumstances where it is desirable to avoid using any internal moving parts such as pistons to be one of the main mechanisms of fluid transport in a biological system. The application of peristaltic motion as a mean of transporting fluid has aroused interested in engineering fields. Latham [1] was probably the first to study the mechanism of peristaltic pumping in his M. S. Thesis. Several researches have analyzed the phenomenon of peristaltic transport under various assumptions. Haroun [2] studied the effect of a third-order fluid on the peristaltic transport in an asymmetric channel. In his study, the wavelength of the peristaltic waves is assumed to be large compared to the varying channel width, whereas the wave amplitudes need not be small compared to the varying channel width. Eldabe *et al*. [3] analyzed the incompressible flow of electrically con-

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ducting biviscosity fluid through an axisymmetric nonuniform tube with a sinusoidal wave under the considerations of long wavelength and low Reynolds number.

In the last years, several simple flow problems of classical hydrodynamics have received new attention in the more general context magnetohydrodynamics (MHD). The study of the motion of non-Newtonian fluids in the presense of the magnetic field has applications in many devices such as magneto hydrodynamic (MHD) power generator, MHD pumps, bioengineering devices and accelerators. Also it has been established that the biological systems are greatly affected by the application of the external magnetic field. Moreover, the MHD flow of a fluid in a channel with elastic, rhythmically contracting walls (peristaltic flow) is of interest in connection with certain problems of the movement of conductive physiological fluids. Some recent investigations made to discuss the mechanism of MHD include the works. Hayat *et al*. [4] studied the peristaltic transport of a third order fluid under the effect of a magnetic field. Srinivas and Kothandapani [5] have studied the influence of heat and mass transfer on MHD peristaltic flow through a porous space with compliant walls. Another important aspect in MHD is related to Hall effect. Such effect cannot be overlooked when flow subject to high magnetic field is considered. Siddiqui *et al*. [6] studied effects of Hall current and heat transfer on MHD flow of a Burgers fluid due to a pull of eccentric rotating disks. Hall effects on peristaltic flow of a Maxwell fluid in a porous medium have been studied by Hayat *et al*. [7] studied effects of Hall current and heat transfer on rotating flow of a second grade fluid through a porous medium. Khalid Nowar [8] studied Peristaltic Flow of a Nanofluid under the effect of Hall Current and Porous Medium.

The study of the influence of mass and heat transfer on non-Newtonian fluids has become important in the last few years. This importance is due to number of industrial processes. Examples are food processing, biochemical operations and transport in polymers, biomedical engineering; micro fabrication technologies etc., besides these biological tissues with heat transfer involve modes like heat conduction in tissues, heat convection by blood flow through the pores of tissue and radiation heat transfer between surface and its environment. Motivated by such facts, the peristaltic flow with heat transfer has been explored. El-Dabe *et al*. [9] studied magnetohydrodynamic flow and heat transfer for a peristaltic motion of carreau fluid through a porous medium. El-Dabe *et al*. [10] studied Peristaltic Motion of Non-Newtonian Fluid with Heat and Mass Transfer through a Porous Medium in Channel under Uniform Magnetic Field. El-Dabe *et al*. [11] analyzed the Magnetohydrodynamic Peristaltic motion with heat and mass transfer of a Jeffery fluid in a tube through porous medium.

With the above discussion in mind, we propose to study the peristaltic motion of non-Newtonian fluid through a porous medium in the channel under the effect of magnetic field. A third order non-Newtonian constitutive model is employed for the transport fluid. The effects of hall, body temperature and concentration are taken into consideration. The governing equations of motion, energy, and concentration have been reduced under the assumption of long wavelength. The reduced equations are then solved analytically via perturbation method. The physical behaviors of emerging parameters are discussed through graphs.

2. Mathematical Analysis

Consider a two-dimensional channel of uniform thickness $2a$, filled with incompressible homogeneous electrically conducting non-Newtonian third order fluid through a porous medium with heat and mass transfer. The channels walls are considered and flexible the vertical displacements for the upper and lower walls are H and $-H$, see **Figure 1**, where H is defined by

$$H^*\left(X^*,t^*\right)=a+b\sin\frac{2\pi}{\lambda}\left(X^*-ct^*\right), \tag{1}$$

where In the above equation b is the wave amplitude, λ is the wave length and t^* is the time. A uniform magnetic field with magnetic flux density vector $B=\left(0,0,B_0\right)$ is applied, neglecting the induced magnetic field under the assumption that the magnetic Reynolds number is small, the expression for the current density $\underline{J}$ including the Hall effect and neglecting ion-slip and thermoelectric effects is given by

$$\underline{J}=\sigma\left[\underline{E}+\underline{V}^*\wedge\underline{B}-\frac{1}{en_e}\left(\underline{J}\wedge\underline{B}\right)\right], \tag{2}$$

where σ is the electric conductivity of the fluid is, $\underline{V}^*$ is the velocity vector, It is also assumed that $\underline{E}=0$

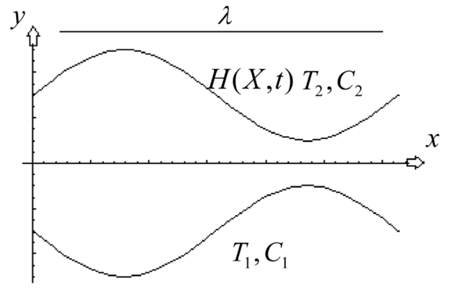

Figure 1. Sketch of the problem.

(since there is no applied polarization voltage), $m = \frac{\sigma B_0}{e n_e}$ is the Hall parameter, e is the electric charge and n_e is the number of density of electron. The constitutive equation for the non-Newtonian third order fluid can be written as in [4].

Consider

$$\tau^* = -P^* I + S^*, \tag{3}$$

$$S^* = \mu A_1^* + \alpha_1 A_2^* + \alpha_2 A_1^{*2} + \beta\left(tr\, A_1^{*2}\right) A_1^*. \tag{4}$$

Here $\underline{\tau}^*$ is the extra stress tensor, $-P^* I$ is the indeterminate part of the stress due to the constraint of incompressibility and A_n^* are the Rivlin-Ericksen tensors, defined by

$$\begin{aligned} A_1^* &= \left(grad\, \underline{V}^*\right) + \left(grad\, \underline{V}^*\right)^{\mathrm{T}}, \\ A_n^* &= \frac{\mathrm{d}A_{n-1}^*}{\mathrm{d}t^*} + A_{n-1}^*\left(grad\, \underline{V}^*\right) + \left(grad\, \underline{V}^*\right)^{\mathrm{T}} A_{n-1}^*, \quad n > 1, \end{aligned} \tag{5}$$

where grad denotes the gradient operator, $\frac{\mathrm{d}}{\mathrm{d}t^*}$ the material time derivative, μ is the coefficient of shear viscosity, the normal stress coefficients α_1 and α_2, and the coefficient β,

$$\mu \ge 0,\ \alpha_1 \ge 0,\ \alpha_2 \ge 0,\ \beta \ge 0, \quad \left|\alpha_1 + \alpha_2\right| \le \sqrt{24\mu\beta} \tag{6}$$

The fundamental equations governing this model together with the generalized Ohm's law taking the effects of Hall currents and Maxwell's equations into account are

$$\nabla \cdot \underline{V}^* = 0, \tag{7}$$

$$\rho \frac{\mathrm{d}\underline{V}^*}{\mathrm{d}t^*} = -\nabla P^* + \nabla \cdot \underline{\tau}^* + \underline{J} \wedge \underline{B}, \tag{8}$$

$$\rho c_p \left[\frac{\partial T}{\partial t^*} + \left(\underline{V}^* \cdot \nabla\right) T\right] = \kappa \nabla^2 T + \Phi + \nabla \cdot q_r, \tag{9}$$

$$\left[\frac{\partial C}{\partial t^*} + \left(\underline{V}^* \cdot \nabla\right) C\right] = D\nabla^2 C + \frac{D\kappa_T}{T_m}\nabla^2 T - L_c\left(C - C_2\right). \tag{10}$$

where ρ is the density of the fluid is, P^* is the pressure, c_p is the specific heat capacity at constant pressure, T is the temperature, κ is the thermal conductivity, Φ is the dissipation function, q_r is the radiative heat flux, C is the concentration of the fluid, D is the coefficient of mass diffusivity, κ_T is the thermal diffusion ratio, T_m is the mean fluid temperature and L_c is the reaction rate constant.

By using Rosselant approximation we have

$$q_Y = \frac{4\sigma_0}{3K_0}\frac{\partial T^4}{\partial Y}. \tag{11}$$

where σ_0 is the Stefan Boltizman constant and K_0 is the mean absorption coefficient. We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_2, and neglecting higher order terms, we get

$$T^4 \approx 4T_2^3 T - 3T_2^4.$$

The equations governing the two-dimensional motion of this model (7)-(10)

$$\frac{\partial U^*}{\partial X^*} + \frac{\partial V^*}{\partial Y^*} = 0, \tag{12}$$

$$\rho\left(\frac{\partial}{\partial t^*} + U^*\frac{\partial}{\partial X^*} + V^*\frac{\partial}{\partial Y^*}\right)U^* = -\frac{\partial P^*}{\partial X^*} + \frac{\partial S^*_{X^*X^*}}{\partial X^*} + \frac{\partial S^*_{X^*Y^*}}{\partial Y^*} - \frac{\mu}{k}U^* + \frac{\sigma B_0^2}{1+m^2}\left(mV^* - U^*\right), \tag{13}$$

$$\rho\left(\frac{\partial}{\partial t^*} + U^*\frac{\partial}{\partial X^*} + V^*\frac{\partial}{\partial Y^*}\right)V^* = -\frac{\partial P^*}{\partial Y^*} + \frac{\partial S^*_{X^*Y^*}}{\partial X^*} + \frac{\partial S^*_{Y^*Y^*}}{\partial Y^*} - \frac{\mu}{k}V^* - \frac{\sigma B_0^2}{1+m^2}\left(mU^* + V^*\right), \tag{14}$$

where

$$\begin{aligned} S^*_{X^*X^*} &= 2\mu U^*_{X^*} + \alpha_1\left(2U_{tX} + 2UU_{XX} + 2VU^*_{X^*Y^*} + 4U^{*2}_{X^*} + 2V^*_{X^*}U^*_{Y^*} + 2U^{*2}_{Y^*}\right) \\ &+\alpha_2\left(4U^{*2}_{X^*} + 2V^*_{X^*}U^*_{Y^*} + U^{*2}_{Y^*} + V^{*2}_{X^*}\right) + 2\beta\left(U^{*3}_{X^*} + 2U^*_{X^*}V^{*2}_{X^*} + 2U^*_{X^*}U^{*2}_{Y^*} + 2V^*_{X^*}U^*_{X^*}U^*_{Y^*} + 4U^*_{X^*}V^{*2}_{Y^*}\right), \end{aligned} \tag{15}$$

$$\begin{aligned} S^*_{X^*Y^*} &= \mu\left(U^*_{Y^*} + V^*_{X^*}\right) + \alpha_1(V^*_{X^*t} + U^*_{tY^*} + UU^*_{X^*X^*} + UV^*_{X^*X^*} + VU^*_{Y^*Y^*} + VV^*_{X^*Y^*} + 2U^*_{X^*}U^*_{Y^*} + 2V^*_{X^*}V^*_{Y^*} \\ &+\alpha_2\left(2U^*_{X^*}U^*_{Y^*} + 2V^*_{X^*}V^*_{Y^*}\right) + 2\beta\left(U^{*2}_{X^*}U^*_{Y^*} + 2U^{*3}_{Y^*} + 6V^{*2}_{X^*}U^*_{Y^*} + 4V^{*2}_{Y^*}U^*_{Y^*} + 6V^*_{X^*}U^{*2}_{Y^*} + V^*_{X^*}U^{*2}_{X^*} + 2V^{*3}_{X^*} + 4V^{*2}_{Y^*}V^*_{X^*}\right), \end{aligned} \tag{16}$$

and

$$\begin{aligned} S^*_{Y^*Y^*} &= 2\mu V^*_{Y^*} + \alpha_1\left(2V^*_{tY^*} + 2UV^*_{X^*Y^*} + 2VV^*_{Y^*Y^*} + 2U^{*2}_{Y^*} + 2V^*_{X^*}U^*_{Y^*} + 4V^{*2}_{Y^*}\right) \\ &+\alpha_2\left(U^{*2}_{Y^*} + 2V^*_{X^*}U^*_{Y^*} + 4V^{*2}_{Y^*} + V^{*2}_{X^*}\right) + 2\beta\left(4V^{*3}_{Y^*} + UV^*_{Y^*} + 2V^{*2}_{X^*}V_Y + 2V^*_{X^*}U^*_{X^*}U^*_{Y^*} + 4U^{*2}_{Y^*}V^*_{Y^*}\right), \end{aligned} \tag{17}$$

where $\left(U^*, V^*\right)$ is the velocity components in fixed frame of reference $\left(X^*, Y^*\right)$

The dissipation function Φ can be written as follows

$$\Phi = \tau^*_{ij}\frac{\partial V^*_i}{\partial X^*_j}, \tag{18}$$

$$\Phi = S^*_{X^*X^*}\frac{\partial U^*}{\partial X^*} + S^*_{X^*Y^*}\left(\frac{\partial V^*}{\partial X^*} + \frac{\partial U^*}{\partial Y^*}\right) + S^*_{Y^*Y^*}\frac{\partial V^*}{\partial Y^*} \tag{19}$$

$$\begin{aligned} &\rho c_p\left[\frac{\partial T}{\partial t^*} + U^*\frac{\partial T}{\partial X^*} + V^*\frac{\partial T}{\partial Y^*}\right] \\ &= \kappa\left(\frac{\partial^2 T}{\partial X^{*2}} + \frac{\partial^2 T}{\partial Y^{*2}}\right) + S^*_{X^*X^*}\frac{\partial U^*}{\partial X^*} + S^*_{X^*Y^*}\left(\frac{\partial V^*}{\partial X^*} + \frac{\partial U^*}{\partial Y^*}\right) + S^*_{Y^*Y^*}\frac{\partial V^*}{\partial Y^*} + \frac{16\sigma_0}{3K_0}T_2^3\frac{\partial^2 T}{\partial Y^{*2}}, \end{aligned} \tag{20}$$

$$\left[\frac{\partial C}{\partial t^*} + U^*\frac{\partial C}{\partial X^*} + V^*\frac{\partial C}{\partial Y^*}\right] = D\left(\frac{\partial^2 C}{\partial X^{*2}} + \frac{\partial^2 C}{\partial Y^{*2}}\right) + \frac{D\kappa_T}{T_m}\left(\frac{\partial^2 T}{\partial X^{*2}} + \frac{\partial^2 T}{\partial Y^{*2}}\right) - L_c\left(C - C_2\right), \tag{21}$$

The appropriate boundary conditions taken as follows:

$$\begin{aligned} &U^* = 0, T = T_2, C = C_2 \quad \text{at } Y^* = H^*, \\ &U^* = 0, T = T_1, C = C_1 \quad \text{at } Y^* = -H^*, \end{aligned} \tag{22}$$

Consider a wave frame $\left(x^*, y^*\right)$ which moving with speed c. Coordinates and velocity components in wave frame are related by the following transformations

$$x^* = X^* - ct,\ y^* = Y^*,\ u^* = U^* - c,\ v^* = V^*,\ p^*\left(x^*, y^*\right) = P^*\left(X^*, Y^*, t\right) \tag{23}$$

In which $\left(u^*, v^*\right)$ are components of the velocity in the moving coordinates system.
Then, the system of Equations (12)-(22) can be written as:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0, \tag{24}$$

$$\rho\left(u^*\frac{\partial}{\partial x^*} + v^*\frac{\partial}{\partial y^*}\right)u^* = -\frac{\partial p^*}{\partial x^*} + \frac{\partial s^*_{x^*x^*}}{\partial x^*} + \frac{\partial s^*_{x^*y^*}}{\partial y^*} - \frac{\mu}{k}u^* + \frac{\sigma B_0^2}{1+m^2}\left(mv^* - u^*\right), \tag{25}$$

$$\rho\left(u^*\frac{\partial}{\partial x^*} + v^*\frac{\partial}{\partial y^*}\right)v^* = -\frac{\partial P}{\partial y^*} + \frac{\partial s^*_{x^*y^*}}{\partial x^*} + \frac{\partial s^*_{y^*y^*}}{\partial y^*} - \frac{\mu}{k}v^* - \frac{\sigma B_0^2}{1+m^2}\left(mu^* + v^*\right), \tag{26}$$

where

$$\begin{aligned} s^*_{x^*x^*} &= 2\mu u^*_{x^*} + \alpha_1\left(2u^*u^*_{x^*x^*} + 2v^*u^*_{x^*y^*} + 4u^{*2}_{x^*} + 2v^*_{x^*}u^*_{y^*} + 2u^{*2}_{y^*}\right) + \alpha_2\left(4u^{*2}_{x^*} + 2v^*_{x^*}u^*_{y^*} + u^{*2}_{y^*} + v^{*2}_{x^*}\right) \\ &+ 2\beta\left(u^{*3}_{x^*} + 2u^*_{x^*}v^{*2}_{x^*} + 2u^*_{x^*}u^{*2}_{y^*} + 2v^*_{x^*}u^*_{x^*}u^*_{y^*} + 4u^*_{x^*}v^{*2}_{y^*}\right), \end{aligned} \tag{27}$$

$$\begin{aligned} s^*_{x^*y^*} &= \mu\left(u^*_{y^*} + v^*_{x^*}\right) + \alpha_1\left(u^*u^*_{x^*x^*} + u^*v^*_{x^*x^*} + v^*u^*_{y^*y^*} + v^*v^*_{x^*y^*} + 2u^*_{x^*}u^*_{y^*} + 2v^*_{x^*}v^*_{y^*}\right) + \alpha_2\left(2u^*_{x^*}u^*_{y^*} + 2v^*_{x^*}v^*_{y^*}\right) \\ &+ 2\beta\left(u^{*2}_{x^*}u^*_{y^*} + 2u^{*3}_{y^*} + 6v^{*2}_{x^*}u^*_{y^*} + 4v^{*2}_{y^*}u^*_{y^*} + 6v^*_{x^*}u^{*2}_{y^*} + v^*_{x^*}u^{*2}_{x^*} + 2v^{*3}_{x^*} + 4v^{*2}_{y^*}v^*_{x^*}\right), \end{aligned} \tag{28}$$

And

$$\begin{aligned} s^*_{y^*y^*} &= 2\mu v^*_{y^*} + \alpha_1\left(2u^*v^*_{x^*y^*} + 2v^*v^*_{y^*y^*} + 2u^{*2}_{y^*} + 2v^*_{x^*}u^*_{y^*} + 4v^{*2}_{y^*}\right) + \alpha_2\left(u^{*2}_{y^*} + 2v^*_{x^*}u^*_{y^*} + 4v^{*2}_{y^*} + v^{*2}_{x^*}\right) \\ &+ 2\beta\left(4v^{*3}_{y^*} + u^{*2}_{x^*}v^*_{y^*} + 2v^{*2}_{x^*}v^*_{y^*} + 2v^*_{x^*}u^*_{x^*}u^*_{y^*} + 4u^{*2}_{y^*}v^*_{y^*}\right), \end{aligned} \tag{29}$$

$$\rho c_p\left[u^*\frac{\partial T}{\partial x^*} + v^*\frac{\partial T}{\partial y^*}\right] = \kappa\left(\frac{\partial^2 T}{\partial x^{*2}} + \frac{\partial^2 T}{\partial y^{*2}}\right) + s^*_{x^*x^*}\frac{\partial u^*}{\partial x^*} + s^*_{x^*y^*}\left(\frac{\partial v^*}{\partial x^*} + \frac{\partial u^*}{\partial y^*}\right) + s^*_{y^*y^*}\frac{\partial v^*}{\partial y^*} + \frac{16\sigma_0}{3K_0}T_2^3\frac{\partial^2 T}{\partial y^{*2}}, \tag{30}$$

$$\left[u^*\frac{\partial C}{\partial x^*} + v^*\frac{\partial C}{\partial y^*}\right] = D\left(\frac{\partial^2 C}{\partial x^{*2}} + \frac{\partial^2 C}{\partial y^{*2}}\right) + \frac{D\kappa_T}{T_m}\left(\frac{\partial^2 T}{\partial x^{*2}} + \frac{\partial^2 T}{\partial y^{*2}}\right) - L_c\left(C - C_2\right), \tag{31}$$

The boundary conditions become:

$$\begin{aligned} &u^* = -c,\ T = T_2,\ C = C_2 \quad \text{at } y^* = H, \\ &u^* = -c, T = T_1,\ C = C_1 \quad \text{at } y^* = -H, \end{aligned} \tag{32}$$

We introduce the following non-dimensional quantities:

$$\begin{aligned} &x = \frac{x^*}{\lambda},\quad y = \frac{y^*}{a},\quad u = \frac{u^*}{c},\quad v = \frac{v^*}{c\delta},\quad p = \frac{a^2 p^*}{\lambda\mu c}, \\ &\delta = \frac{2\pi a}{\lambda},\ R_e = \frac{\rho c a}{\mu},\ h = \frac{H}{a},\ S = \frac{a}{\mu c}S^*,\ \theta = \frac{T - T_2}{T_1 - T_2},\ \phi = \frac{C - C_2}{C_1 - C_2}, \\ &D_a = \frac{k}{a^2},\ M = \sqrt{\frac{\sigma}{\mu}}B_0 a,\ \lambda_1 = \frac{\alpha_1 c}{\mu a},\ \lambda_2 = \frac{\alpha_2 c}{\mu a},\ \Gamma = \frac{\beta c^2}{\mu a^2}, \\ &P_r = \frac{\mu c_p}{\kappa},\quad E_c = \frac{c^2}{c_p\left(T_1 - T_0\right)},\quad R_n = \frac{\mu c_p K_0}{4\sigma_0 T_2^3},\quad S_c = \frac{\mu}{\rho D}, \\ &S_r = \frac{\rho D K_T\left(T_1 - T_0\right)}{T_m\mu\left(C_1 - C_0\right)},\quad R_C = \frac{\rho a^2 L_c\left(C_1 - C_2\right)}{\mu}. \end{aligned} \tag{33}$$

where the non-dimensional wave number δ, the Reynolds number R_e, the material coefficients are (λ_1, λ_2), Deborh number is Γ, Darcy number is D_a, M is the Hartman number, P_r is the Prandtl number, E_c is the Eckert number, R_n is the Radiation parameter, S_c is the Schmidt number, S_r is the Soret number and R_C is the Chemical reaction parameter.

Substituting (33) into Equations (24)-(32) we obtain the following non-dimensional equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{34}$$

$$\delta \operatorname{Re}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)u=-\frac{\partial p}{\partial x}+\frac{\partial s_{xx}}{\partial x}+\delta\frac{\partial s_{xy}}{\partial y}-\frac{1}{D_a}(u+1)+\frac{M}{1+m^2}\left(m\delta v-(u+1)\right), \tag{35}$$

$$\delta^3 \operatorname{Re}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)v=-\frac{\partial p}{\partial y}+\delta^2\frac{\partial s_{xy}}{\partial x}+\delta\frac{\partial s_{yy}}{\partial y}-\frac{\delta^2}{D_a}v-\frac{\delta M}{1+m^2}\left(m(u+1)+\delta v\right), \tag{36}$$

where

$$\begin{aligned} s_{xx} &= 2\delta u_x+\lambda_1\left(2\delta^2 uu_{xx}+2\delta^3 vu_{xy}+4\delta^2 u_x^2+2\delta v_x u_y+2\delta^3 v_x^2\right)+\lambda_2\left(4\delta^2 u_x^2+2\delta^2 v_x u_y+u_y^2+\delta^4 v_x^2\right)\\ &\quad +2\Gamma\left(\delta^3 u_x^3+2\delta^5 u_x v_x^2+2\delta u_x u_y^2+2\delta^3 v_x u_x u_y+4\delta u_x v_y^2\right), \end{aligned} \tag{37}$$

$$\begin{aligned} s_{xy} &= \left(u_y+\delta^2 v_x\right)+\lambda_1\left(\delta^2 uu_{xx}+\delta uv_{xx}+\delta vu_{yy}+\delta^3 vv_{xy}+2\delta u_x u_y+2\delta^3 v_x v_y\right)+\lambda_2\left(2\delta^2 u_x u_y+2\delta^3 v_x v_y\right)\\ &\quad +2\Gamma\left(\delta^2 u_x^2 u_y+2u_y^3+6\delta^4 v_x^2 u_y+4\delta^2 v_y^2 u_y+6\delta^2 v_x u_y^2+\delta^4 v_x u_x^2+2\delta^3 v_x^3+4\delta^3 v_y^2 v_x\right), \end{aligned} \tag{38}$$

and

$$\begin{aligned} s_{yy} &= 2\delta v_y+\lambda_1\left(2\delta^2 uv_{xy}+2\delta^2 vv_{yy}+2u_y^2+2\delta^2 v_x u_y+4\delta^2 v_y^2\right)+\lambda_2\left(u_y^2+2\delta^2 v_x u_y+4\delta^2 v_y^2+\delta^4 v_x^2\right)\\ &\quad +2\Gamma\left(4\delta^3 v_y^3+\delta^2 u_x^2 v_y+2\delta^5 v_x^2 v_y+2\delta^3 v_x u_x u_y+4\delta u_y^2 v_y\right), \end{aligned} \tag{39}$$

$$\delta \operatorname{Re}\left[u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right]=\frac{1}{P_r}\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+E_c\delta s_{xx}\frac{\partial u}{\partial x}+E_c s_{xy}\left(\delta\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+E_c s_{yy}\frac{\partial v}{\partial y}+\frac{4}{3R_n}\frac{\partial^2\theta}{\partial y^2}, \tag{40}$$

$$\delta \operatorname{Re}\left[u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right]=\frac{1}{S_c}\left(\delta^2\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)+S_r\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)-R_c\phi, \tag{41}$$

With conditions:

$$\begin{aligned} &u=-1,\ \theta=0,\ \phi=0 \quad \text{at } y=h,\\ &u=-1,\ \theta=1,\ \phi=1 \quad \text{at } y=-h, \end{aligned} \tag{42}$$

We also note that h represents the dimensionless form of the surface of the peristaltic wall.

$$h=1+\chi\sin x \tag{43}$$

where, $\chi=\dfrac{b}{a}$ is the amplitude ratio or the occlusion

under the assumptions of long wavelength $(\delta \ll 1)$. The Equations (35)-(42) take the following form:

$$\frac{\partial p}{\partial x}=\frac{\partial s_{xy}}{\partial y}-\left(\frac{1}{D_a}+\frac{M}{1+m^2}\right)(u+1), \tag{44}$$

$$\frac{\partial p}{\partial y}=0, \tag{45}$$

$$s_{xx}=(2\lambda_1+\lambda_2)u_y^2, \tag{46}$$

$$s_{xy}=\frac{\partial u}{\partial y}+2\Gamma\left(\frac{\partial u}{\partial y}\right)^3, \tag{47}$$

$$s_{yy} = \left(2\lambda_1 + \lambda_2\right)u_y^2, \tag{48}$$

$$\left(\frac{1}{P_r} + \frac{4}{3R_n}\right)\frac{\partial^2\theta}{\partial y^2} + E_c s_{xy}\frac{\partial u}{\partial y} = 0, \tag{49}$$

$$\frac{\partial^2\phi}{\partial y^2} - S_c R_c \phi = -S_c S_r \frac{\partial^2\theta}{\partial y^2}. \tag{50}$$

Eliminating p from Equations (44) and (45), we have the following equation

$$\frac{\partial^2 s_{xy}}{\partial y^2} - \left(\frac{1}{D_a} + \frac{M}{1+m^2}\right)\frac{\partial u}{\partial y} = 0, \tag{51}$$

$\frac{\partial S_{xy}}{\partial y} - \left(\frac{1}{D_a} + \frac{M}{1+m^2}\right)u = A,$ where A is a constant.

3. Series Solution

For perturbation solution we write

$$\begin{aligned} u &= u_0 + \Gamma u_1, \\ \theta &= \theta_0 + \Gamma\theta_1, \\ \phi &= \varphi_0 + \Gamma\phi_1. \end{aligned} \tag{52}$$

Substituting (52) in the Equations (45)-(49), equating the coefficients of like powers of Γ, we get the following

Zeroth order system:

$$\frac{\partial^2 u_0}{\partial y^2} - N^2 u_0 = A, \tag{53}$$

$$\frac{\partial^2\theta_0}{\partial y^2} + E_c G\left(\frac{\partial u_0}{\partial y}\right)^2 = 0, \tag{54}$$

$$\frac{\partial^2\phi_0}{\partial y^2} - S_c R_c \phi_0 = -S_c S_r \frac{\partial^2\theta_0}{\partial y^2}, \tag{55}$$

The subjected boundary conditions are:

$$\begin{aligned} &u_0 = -1,\ \theta_0 = 0,\ \phi_0 = 0 \quad \text{at } y = h, \\ &u_0 = -1,\ \theta_0 = 1,\ \phi_0 = 1 \quad \text{at } y = -h, \end{aligned} \tag{56}$$

First order system

$$\frac{\partial^2 u_1}{\partial y^2} + 2\frac{\partial}{\partial y}\left(\frac{\partial u_0}{\partial y}\right)^3 - N^2 u_1 = 0, \tag{57}$$

$$\frac{\partial^2\theta_1}{\partial y^2} + E_c G\left[\left(\frac{\partial u_1}{\partial y}\right)^2 + 2\left(\frac{\partial u_0}{\partial y}\right)^4\right] = 0, \tag{58}$$

$$\frac{\partial^2\phi_1}{\partial y^2} - S_c R_c \phi_1 = -S_c S_r \frac{\partial^2\theta_1}{\partial y^2}, \tag{59}$$

$$\begin{aligned} &u_1 = 0,\ \theta_1 = 0,\ \phi_1 = 0 \quad \text{at } y = h, \\ &u_1 = 0,\ \theta_1 = 0,\ \phi_1 = 0 \quad \text{at } y = -h. \end{aligned} \tag{60}$$

The solution of zero order system can be obtained analytically as

$$u_0 = W_1 \mathrm{Cosh} Ny - \frac{A}{N^2}, \tag{61}$$

$$\theta_0 = -\frac{E_c G N^2 W_1^2}{4}\left(\frac{\mathrm{Cosh}2Ny}{2N^2} - y^2\right) - \frac{y}{2h} + B_1, \tag{62}$$

$$\phi_0 = 2C_1 \mathrm{Cosh}\sqrt{S_c R_c}\, y + \frac{e^{-\sqrt{S_c R_c}\, y}}{2\mathrm{Sinh}\sqrt{S_c R_c}\, h} + \frac{S_c S_r E_c G N^2 W_1^2}{2}\left(\frac{\mathrm{Cosh}2Ny}{4N^2 - S_c R_c} + \frac{1}{S_c R_c}\right). \tag{63}$$

Also, the solution of first order system can be obtained analytically as

$$u_1 = 2W_2 \mathrm{Cosh} Ny - \frac{3}{4} N^4 W_1^3 \left(\frac{\mathrm{Cosh}3Ny}{4N^2} - \frac{y}{N}\mathrm{Sinh} Ny\right), \tag{64}$$

$$\begin{aligned}\theta_1 = -E_c G\Bigg(& W_1 W_2 N^2 \left(\frac{\mathrm{Cosh}2Ny}{2N^2} - y^2\right) - \frac{3}{2} N^5 W_1^4 \left(\frac{3}{16N^3}\frac{\mathrm{Cosh}4Ny}{8} - \frac{y\mathrm{Sinh}2Ny}{8N^2} - \frac{3\mathrm{Cosh}2Ny}{32N^3} + \frac{y^2}{4N}\right) \\ & + \frac{N^4 W_1^4}{8}\left(\frac{\mathrm{Cosh}4Ny}{8N^2} - \frac{2\mathrm{Cosh}2Ny}{N^2} + 3y^2\right)\Bigg) + B_2,\end{aligned} \tag{65}$$

$$\begin{aligned}\phi_1 = S_c S_r E_c G\Bigg(& W_1 W_2 N^2 \left(\frac{\mathrm{Cosh}2Ny}{2N^2} - y^2\right) - \frac{3N^5 W_1^4}{2}\Bigg(\frac{3}{8N^3}\frac{\mathrm{Cosh}4Ny}{16} - \frac{y\mathrm{Sinh}2Ny}{N} \\ & + \left(\frac{1}{2N^2} + \frac{11}{32N^3}\right)\mathrm{Cosh}2Ny - \frac{y^2}{2N}\Bigg)\Bigg) + \frac{N^4 W_1^4}{4}\left(\frac{\mathrm{Cosh}4Ny}{16N^2} - \frac{\mathrm{Cosh}2Ny}{N^2} + \frac{3}{2}y^2\right) + C_2.\end{aligned} \tag{66}$$

where

$$N^2 = \frac{1}{D_a} + \frac{M}{1+m^2},$$

$$W_1 = \frac{1}{\mathrm{Cosh} Nh}\left(\frac{A}{N^2} - 1\right),$$

$$W_2 = \frac{3N^4 W_1^3}{8\mathrm{Cosh} Nh}\left(\frac{\mathrm{Cosh}3Nh}{4N^2} - \frac{h}{N}\mathrm{Sinh} Nh\right),$$

$$B_1 = \frac{E_c G N^2 W_1^2}{4}\left(\frac{\mathrm{Cosh}2Nh}{2N^2} - h^2\right) + \frac{1}{2},$$

$$\begin{aligned}B_2 = E_c G\Bigg(& W_1 W_2 N^2 \left(\frac{\mathrm{Cosh}2Nh}{2N^2} - h^2\right) - \frac{3}{2} N^5 W_1^4 \Bigg(\frac{3}{16N^3}\frac{\mathrm{Cosh}4Nh}{8} - \frac{h\mathrm{Sinh}2Nh}{8N^2} \\ & - \frac{3\mathrm{Cosh}2Nh}{32N^3} + \frac{h^2}{4N}\Bigg) + \frac{N^4 W_1^4}{8}\left(\frac{\mathrm{Cosh}4Nh}{8N^2} - \frac{2\mathrm{Cosh}2Nh}{N^2} + 3h^2\right)\Bigg),\end{aligned}$$

$$C_1 = \frac{-1}{2\mathrm{Cosh}\sqrt{S_c R_c}\, h}\left(\frac{\mathrm{e}^{-\sqrt{S_c R_c}\, h}}{2\mathrm{Sinh}\sqrt{S_c R_c}\, h} + \frac{S_c S_r E_c G N^2 A_1^2}{2}\left(\frac{\mathrm{Cosh}2Nh}{4N^2 - S_c R_c} + \frac{1}{S_c R_c}\right)\right).$$

4. Results and Discussion

In order to obtain the physical insight of the problem, velocity, temperature and concentration are computed numerically for different values of the emerging parameters, viz., Darcy number is D_a, M is the Hartman number, P_r is the Prandtl number, E_c is the Eckert number and R_n is the Radiation parameter using Mathematica and are presented in **Figures 2-10**.

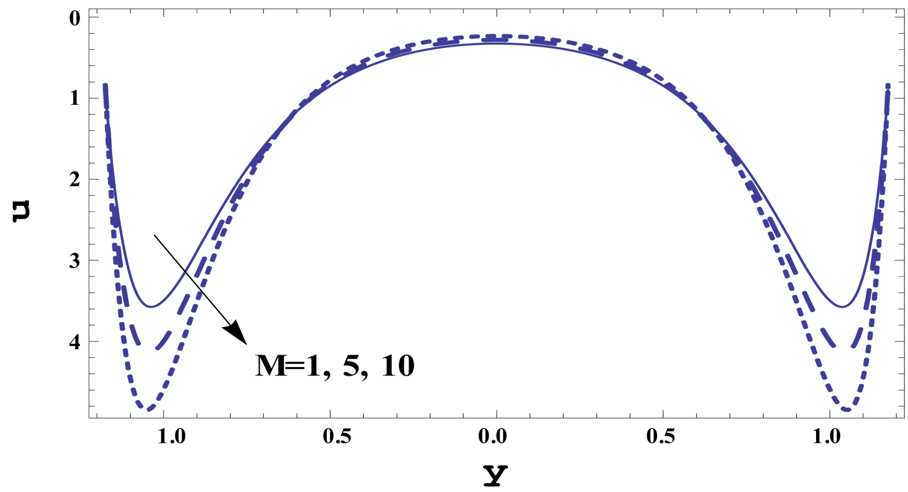

Figure 2. Velocity profiles $u(y)$ for varying values of M.

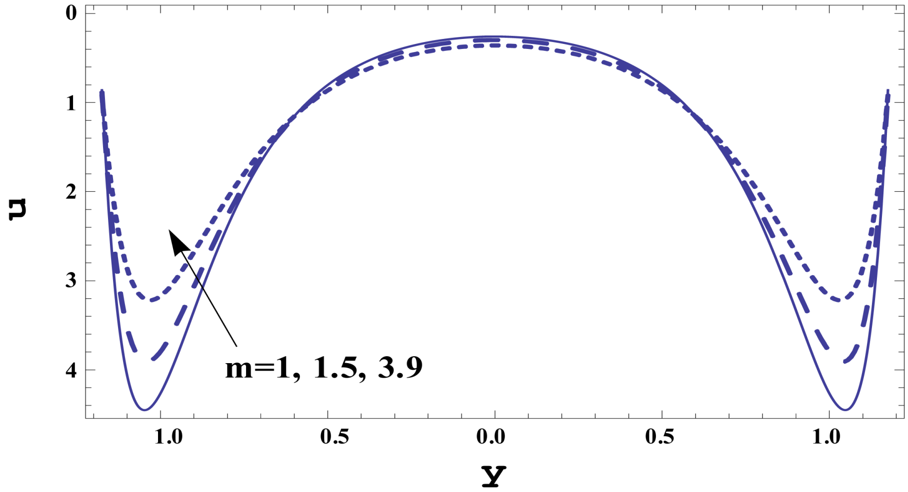

Figure 3. Velocity profiles $u(y)$ for varying values of m.

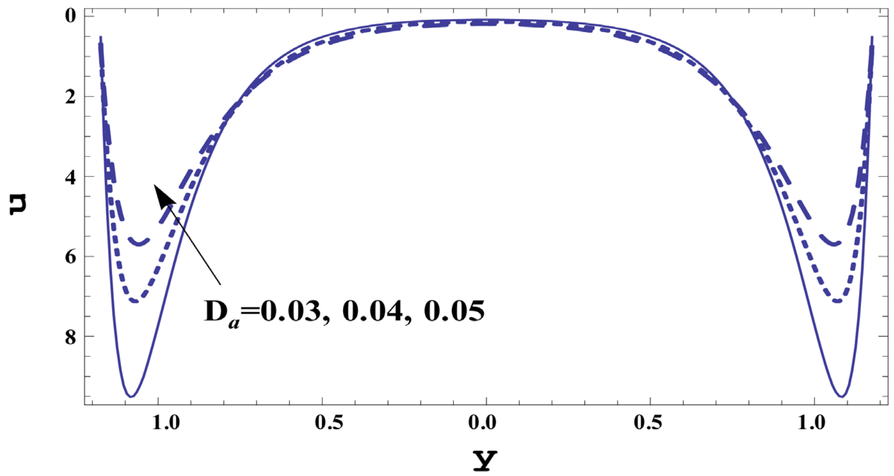

Figure 4. Velocity profiles $u(y)$ for varying values of D_a.

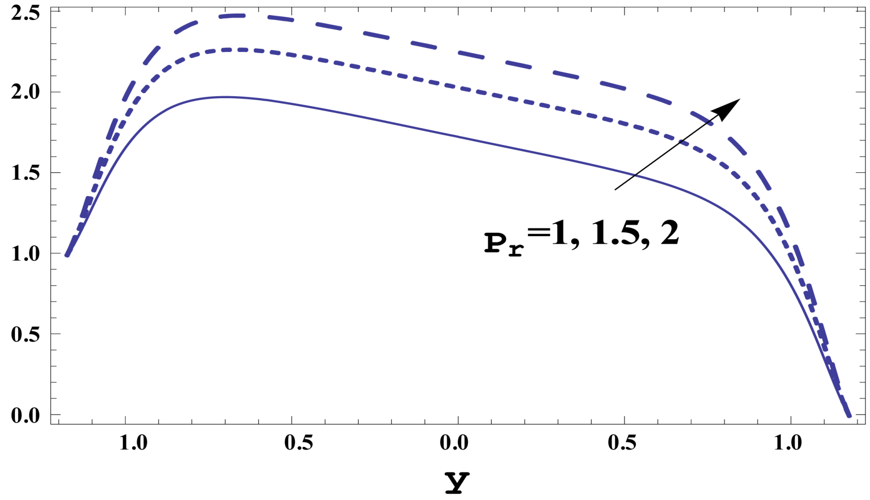

Figure 5. Temperature profiles $\theta(y)$ for varying values of P_r.

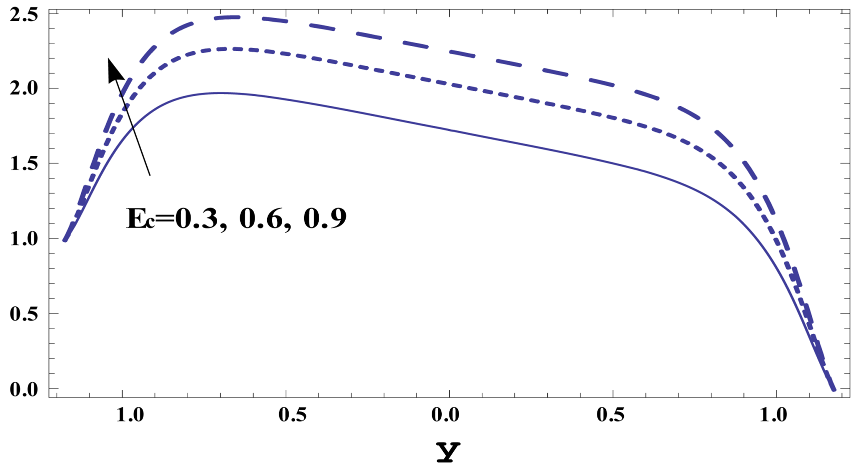

Figure 6. Temperature profiles $\theta(y)$ for varying values of E_c.

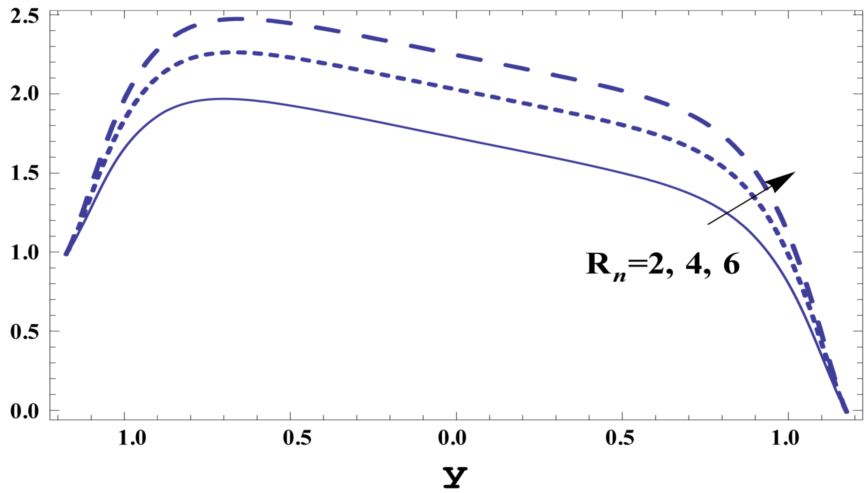

Figure 7. Temperature profiles $\theta(y)$ for varying values of R_n.

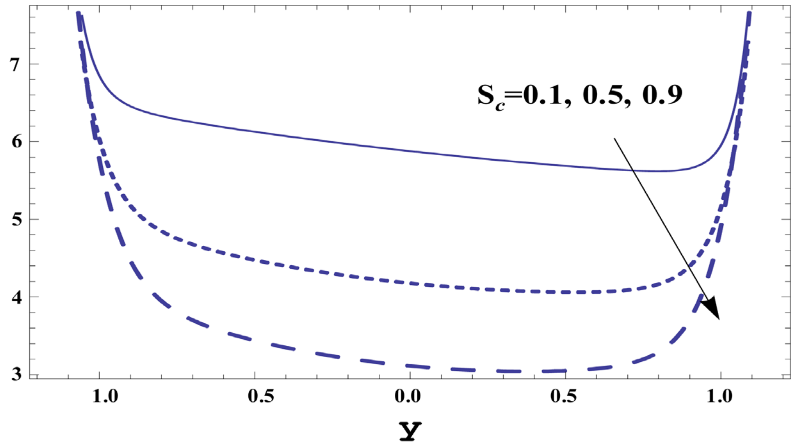

Figure 8. Concentration profiles $\phi(y)$ for varying values of R_n.

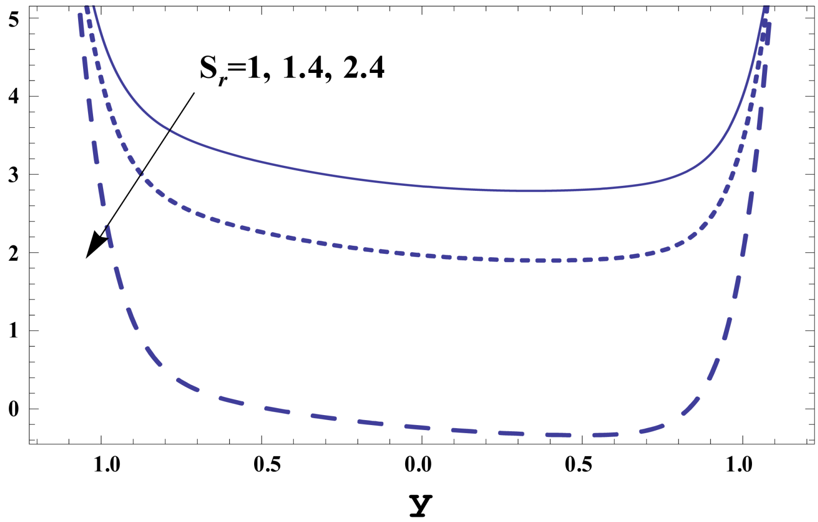

Figure 9. Concentration profiles $\phi(y)$ for varying values of S_r.

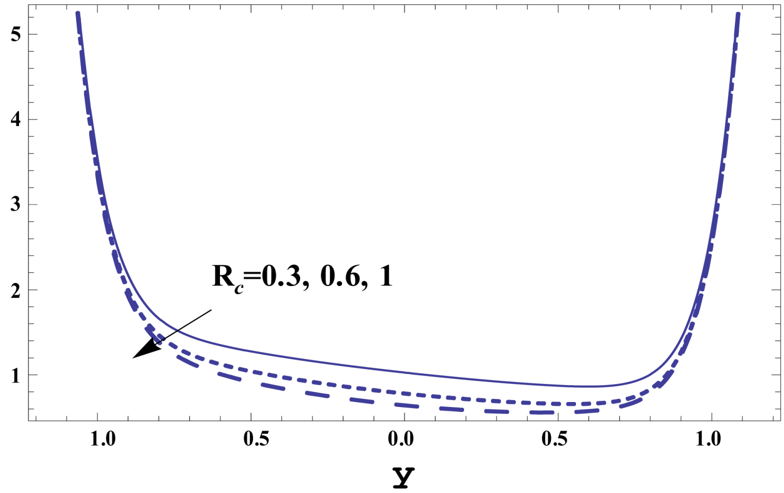

Figure 10. Concentration profiles $\phi(y)$ for varying values of R_c.

Figure 2 presents the effect of Hartman number M on the velocity. It is noted that the velocity increases by increasing the Hartman number in the interval [−0.6, 0.6] and vice versa in the other intervals.

Figure 3 shows the effect of the Hall parameter m on the velocity. It is observed that as m increases the velocity decreases in the interval [−0.6, 0.6] and vice versa in the other intervals.

Figure 4 shows the effect of Darcy parameter D_a against the velocity. It is found that the velocity decreases by the increasing of D_a in the interval [−0.6, 0.6] and vice versa in the other intervals.

Figures 5-7 describe the effect of different parameters on the temperature distribution θ. It is found that the temperature increases as the Prandtle number P_r increases this is shown in **Figure 5**, also in **Figure 6** it is observed that the temperature increases as the Eckert number E_c increases. In **Figure 7** the temperature increases as the Radiation parameter R_n increases.

Figures 8-10 display results for the concentration ϕ profiles. It is clear that the concentration decreases as the Schmidt number S_c increases this is shown in **Figure 8**, also in **Figure 9** and **Figure 10** the concentration decreases as the Soret number S_r, Chemical reaction parameter R_C respectively.

5. Conclusions

In this paper, we studied the effects of the physical parameters of the considered problem on peristaltic transport in a tube, filled with an incompressible non-Newtonian (Third order) fluid, and considered the effects of hall current, body temperature and concentration. The system is solved analytically by perturbation technique. The effects of various emerging parameters on the flow, the temperature and the concentration distributions are shown and discussed with the help of graphs. The main findings can be summarized as follows.

1) The velocity decreases in the interval [−0.6, 0.6] and vice versa in the other intervals with the increase of each of m and D_a, whereas it increases as M increase.

2) The temperature T increases with the increase of each of as the Prandtle number P_r, the Eckert number and the Radiation parameter R_n.

3) The concentration decreases as the Schmidt number S_c, the Soret number S_r and Chemical reaction parameter R_C increases.

Caption of Figures

Figure 2 the velocity profiles are plotted versus y for different values of M for a system have the particulars $\phi = 0.2, m = 1, D_a = 0.08, x = \frac{\pi}{3}$.

Figure 3 the velocity profiles are plotted versus y for different values of m for a system have the particulars $\phi = 0.2, M = 10, D_a = 0.09, x = \frac{\pi}{3}$.

Figure 4 the velocity profiles are plotted versus y for different values of D_a for a system have the particulars $M = 1, \phi = 0.2, m = 1, x = \frac{\pi}{3}$.

Figure 5 the temperature profiles are plotted versus y for different values of P_r for a system have the particulars $M = 5, \phi = 0.2, m = 1, D_a = 0.08, x = \frac{\pi}{3}, E_c = 2$.

Figure 6 the temperature profiles are plotted versus y for different values of E_c for a system have the particulars $M = 5, \phi = 0.2, m = 1, D_a = 0.08, x = \frac{\pi}{3}, P_r = 1, R_n = 2$.

Figure 7 the temperature profiles are plotted versus y for different values of R_n for a system have the particulars $M = 5, \phi = 0.2, m = 1, D_a = 0.08, x = \frac{\pi}{3}, P_r = 1, E_c = 0.3$.

Figure 8 The concentration profiles are plotted versus y for different values of S_c for a system have the particulars $M = 5, \phi = 0.2, m = 1, D_a = 0.08, x = \frac{\pi}{3}, P_r = 1.5, E_c = 0.9, R_n = 2, S_r = 1, R_c = 1$.

Figure 9 the concentration profiles are plotted versus y for different values of S_r for a system have the particulars $M=5, \phi=0.2, m=1, D_a=0.03, x=\frac{\pi}{3}, P_r=1.5, E_c=0.9, R_n=2, S_c=1, R_c=1$.

Figure 10 the concentration profiles are plotted versus y for different values of R_c for a system have the particulars $M=5, \phi=0.2, m=1, D_a=0.03, x=\frac{\pi}{3}, P_r=1.5, E_c=0.9, R_n=2, S_c=1, S_r=2$.

References

[1] Latham, T.W. (1966) Fluid Motions in a Peristaltic Pump, M.S. Thesis, M.I.T., Cambridge.

[2] Haroun, M.H. (2007) Effect of Deborah Number and Phase Difference on Peristaltic Transport of a Third-Order Fluid in an Asymmetric Channel. *Communications in Nonlinear Science and Numerical Simulation*, **12**, 1464-1480. http://dx.doi.org/10.1016/j.cnsns.2006.03.002

[3] Eldabe, N.T.M., El-Sayed, M.F., Ghaly, A.Y. and Sayed, H.M. (2007) Peristaltically Induced Transport of a MHD Bi-viscosity Fluid in a Non-Uniform Tube. *Physica A*, **383**, 253-266. http://dx.doi.org/10.1016/j.physa.2007.05.027

[4] Hayat, T., Afsar, A., Khan, M. and Asghar, S. (2007) Peristaltic Transport of a Third Order Fluid under the Effect of a Magnetic Field. *Computers and Mathematics with Applications*, **53**, 1074-1087. http://dx.doi.org/10.1016/j.camwa.2006.12.014

[5] Srinivas, S. and Kothandapani, M. (2009) The Influence of Heat and Mass Transfer on MHD Peristaltic Flow through a Porous Space with Compliant Walls. *Applied Mathematics and Computation*, **213**, 197-208. http://dx.doi.org/10.1016/j.amc.2009.02.054

[6] Siddiqui, A.M., Rana, M.A. and Ahmed, N. (2008) Effects of Hall Current and Heat Transfer on MHD Flow of a Burgers' Fluid Due to a Pull of Eccentric Rotating Disks. *Communications in Nonlinear Science and Numerical Simulation*, **13**, 1554-1570. http://dx.doi.org/10.1016/j.cnsns.2006.10.005

[7] Hayat, T., Abbas, Z. and Asghar, S. (2008) Effects of Hall Current and Heat Transfer on Rotating Flow of a Second Grade Fluid through a Porous Medium. *Communications in Nonlinear Science and Numerical Simulation*, **13**, 2177-2192. http://dx.doi.org/10.1016/j.cnsns.2007.03.033

[8] Nowar, K. (2014) Peristaltic Flow of a Nanofluid under the Effect of Hall Current and Porous Medium. *Mathematical Problems in Engineering*, **2014**, 1-15. http://dx.doi.org/10.1155/2014/389581

[9] Eldabe, N.T.M., Fouad, A. and Hussein, M.M. (2010) Magnetohydrodynamic Flow and Heat Transfer for a Peristaltic Motion of Carreau Fluid through a Porous Medium. *Journal of Mathematics*, **42**, 1-16.

[10] Eldabe, N.T.M., Agoor, M. and Alame, H. (2014) Peristaltic Motion of Non-Newtonian Fluid with Heat and Mass Transfer through a Porous Medium in Channel under Uniform Magnetic Field. *Journal of Fluids*, **2014**, 1-12. http://dx.doi.org/10.1155/2014/525769

[11] Eldabe, N.T.M., Sallam, N.S., Mohamed, M.A.A, Abo Zaid, M.Y. and Abd-Elmonem, A. (2011) Magnetohydrodynamic Peristaltic Motion with Heat and Mass Transfer of a Jeffery Fluid in a Tube through Porous Medium. *Innovative Systems Design and Engineering*, 264-275.