

Nonlinear Bending of Piezoelectric Cylindrical Shell Reinforced with BNNTs under Electro-Thermo-Mechanical Loadings

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Abstract

Under combined electro-thermo-mechanical loadings, the nonlinear bending of piezoelectric cylindrical shell reinforced with boron nitride nanotubes (BNNTs) is investigated in this paper. By employing nonlinear strains based on Donnell shell theory and utilizing piezoelectric theory including thermal effects, the constitutive relations of the piezoelectric shell reinforced with BNNTs are established. Then the governing equations of the structure are derived through variational principle and resolved by applying the finite difference method. In numerical examples, the effects of geometric nonlinear, voltage, temperature, as well as volume fraction on the deflection and bending moment of axisymmetrical piezoelectric cylindrical shell reinforced with BNNTs are discussed in detail.

Keywords

Nonlinear Bending, Piezoelectric, Cylindrical Shell, BNNT, Electro-Thermo-Mechanical Loadings

1. Introduction

BNNTs are similar to CNTs in structure and their extraordinary mechanical properties, but are different in that BNNTs possess higher temperature resistance to oxidation and stronger piezoelectric characteristics. Also, unlike CNTs, BNNTs have stable semiconducting behavior with a large band gaps regardless of radius and chirality of the structure. This property of BNNTs makes them promising candidate materials in a large variety of nanosized electronic and photonic devices. Therefore, BNNTs seem to be more suitable as reinforcement in composite structures due to their high resistance to oxidation at elevated temperatures [1], outstanding mechanical

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properties [2] and high thermal conductivity [3]. With the development of science and technology, a new sort of smart nanocomposites, with piezoelectric material as matrix and BNNTs as the reinforcement, has attracted increasing interests in both research and engineering communities. It is noted that the investigations on this new smart nanocomposites are limited in number and most discuss the linear problem. Therefore, it is necessary to do more extensive researches on the nonlinear behavior for this structure.

At present, most researches are limited to discussing the behavior of piezoelectric structure without reinforcement of BNNTs. Yao *et al.* [4] presented static behaviors of piezoelectric cantilever actuator under large electric field. Shen [5] studied the nonlinear bending for a simply supported, shear deformable cross-ply laminated plate with piezoelectric actuators subjected to a transverse uniform or sinusoidal load combined with electrical loads and in thermal environments. Shegokar *et al.* [6] deals with the stochastic nonlinear bending response of functionally graded materials beam with surface bonded piezoelectric layers subjected to thermo-electro-mechanical loadings. Narita *et al.* [7] illustrated an analytical and experimental study of nonlinear bending response and domain wall motion in piezoelectric laminated actuators under electric fields. Beldica *et al.* [8] analyzed the bending of nonlinear viscoelastic beams with small or large deformations. Yan *et al.* [9] investigated the time-dependent behavior of a simply supported, angle-ply piezoelectric laminate in cylindrical bending with viscoelastic interfaces. Narita *et al.* [10] discussed the static electromechanical displacement and polarization switching properties of piezoelectric laminated actuators under three point bending. Using a variational formulation, Li *et al.* [11] developed a size-dependent functionally graded piezoelectric beam model. Based on the local Petrov-Galerkin approach, Sladek *et al.* [12] proposed a meshless method for plate bending analysis with functionally graded piezoelectric material properties. Employing Euler-Bernoulli beam theory and the physical neutral surface concept, Fu *et al.* [13] presented the thermo-piezoelectric buckling, nonlinear free vibration and dynamic stability for the piezoelectric functionally graded beams. None of the above mentioned works have considered the behavior of piezoelectric structure reinforced with BNNTs.

Recently, some researches about the static buckling of piezoelectric shell reinforced with BNNTs have emerged. Buckling of BNNTs in a PVDF elastic medium subjected to combined electro-thermo-mechanical loadings was investigated by Salehi-Khojin and Jalili [14] who showed that applying direct and reverse voltages to BNNT changed buckling loads for any axial and circumferential wave-numbers. Mosallaie Barzoki *et al.* [15] studied torsional linear buckling of a PVDF cylindrical shell reinforced by BNNTs with an elastic core under the same loading condition as [14] were investigated, indicating that buckling strength increased substantially as harder foam cores were employed. Using virtual displacement method based on nonlocal cylindrical piezoelectricity continuum shell theory, Arani *et al.* [16] discussed the axial buckling of double-walled Boron Nitride nanotubes embedded in an elastic medium under combined electro-thermo-mechanical loadings. Up to now, to the best of authors' knowledge, the research on nonlinear bending of piezoelectric shell reinforced with BNNTs has not been reported in the open literature.

Motivated by the considerations, we aim to study the nonlinear bending of axisymmetrical piezoelectric shell reinforced with BNNTs under combined electro-thermo-mechanical loadings. Using the difference method in space, the whole problem is resolved by the iteration method. Numerical results in nonlinear deflection and bending moment of axisymmetrical piezoelectric cylindrical shell reinforced with BNNTs are presented for different values of voltage, temperature, volume fraction and so on.

2. Basic Equations

Consider that a piezoelectric cylindrical shell reinforced with BNNTs has midsurface radius R , thickness h , length L and mass density ρ_0 (see Figure 1). The shell is referred to the coordinate system (x, y, z) in which x and y are the axial and circumferential directions of the shell and z is in the direction of the inward normal to the middle surface. The origin of the coordinate system is located at the end of the shell on the middle plane. The shell is subjected to transverse static load q , applied voltage V and a uniform temperature rise ΔT .

2.1. Strain Displacement Relationships

Supposing that $\bar{u}, \bar{v}, \bar{w}$ denote the axial, circumferential and radial displacement of an arbitrary point on the shell, and the corresponding displacement components of middle surface are u, v and w , then the displacement components of piezoelectric cylindrical shell can be written as

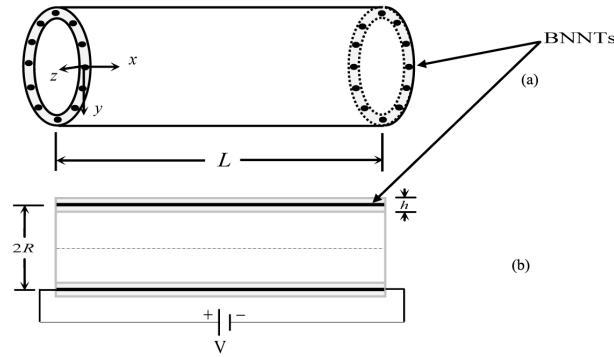


Figure 1. (a) Geometry of piezoelectric cylindrical shell reinforced with BNNTs; (b) section of cylindrical shell.

$$\begin{aligned}\bar{u}(x, y, z) &= u(x, y) - zw_{,x}(x, y); \bar{v}(x, y, z) = v(x, y) - zw_{,y}(x, y) \\ \bar{w}(x, y, z) &= w(x, y)\end{aligned}\quad (1)$$

where the inferior mark (,) denotes the partial derivative to variables coordinate.

Based on classical shell theory with von Kármán-Donnell type kinematic relations, the nonlinear strain-displacement relations can be expressed as

$$\bar{\varepsilon}_x = \varepsilon_x + z\kappa_x \quad \bar{\varepsilon}_y = \varepsilon_y + z\kappa_y \quad \bar{\varepsilon}_{xy} = \varepsilon_{xy} + z\kappa_{xy} \quad (2)$$

where $\varepsilon_x, \varepsilon_y, \varepsilon_{xy}$ are the strain components on the middle surface and $\kappa_x, \kappa_y, \kappa_{xy}$ are the change values of curvatures on the middle surface, and

$$\begin{aligned}\varepsilon_x &= u_{,x} + \frac{1}{2}w_{,x}^2, \quad \varepsilon_y = v_{,y} - \frac{w}{R} + \frac{1}{2}w_{,y}^2, \quad \varepsilon_{xy} = u_{,y} + v_{,x} + w_{,x}w_{,y} \\ \kappa_x &= -w_{,xx}, \quad \kappa_y = -w_{,yy}, \quad \kappa_{xy} = -2w_{,xy}\end{aligned}\quad (3)$$

2.2. Constitutive Equations

The constitutive relationship of a piezoelectric structure under combined mechanical, thermal and electrical loadings can be expressed as follows [15]

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_x \\ \bar{\varepsilon}_y \\ \bar{\varepsilon}_z \\ \bar{\varepsilon}_{yz} \\ \bar{\varepsilon}_{xz} \\ \bar{\varepsilon}_{xy} \end{Bmatrix} - \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \\ 0 \\ 0 \\ 0 \end{Bmatrix} \Delta T - \begin{bmatrix} e_{11} & 0 & 0 \\ e_{12} & 0 & 0 \\ e_{13} & 0 & 0 \\ 0 & e_{24} & 0 \\ 0 & 0 & e_{35} \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} \quad (4)$$

$$\begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{bmatrix} e_{11} & e_{12} & e_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{35} & 0 \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_x \\ \bar{\varepsilon}_y \\ \bar{\varepsilon}_z \\ \bar{\varepsilon}_{yz} \\ \bar{\varepsilon}_{xz} \\ \bar{\varepsilon}_{xy} \end{Bmatrix} - \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \\ 0 \\ 0 \\ 0 \end{Bmatrix} \Delta T - \begin{bmatrix} \varepsilon_{11}^* & 0 & 0 \\ 0 & \varepsilon_{22}^* & 0 \\ 0 & 0 & \varepsilon_{33}^* \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} \quad (5)$$

where α_κ ($\kappa = x, y, z$), ΔT and E_κ represent respectively, thermal expansion coefficient, temperature rise and electric field, and $C_{ij}, e_{ij}, \varepsilon_{ii}^*$ ($i, j = 1, \dots, 6$) are elastic constants, piezoelectric constant, dielectric constants, respectively. The material constants of the structure can be calculated using “XY (or YX) rectangle model” [17].

The closed-form formula used in “X model” (or “Y model”) expressing the mechanical, the thermal and the electrical properties of the material are as follows [15]:

$$\begin{aligned}
C_{11} &= \frac{C_{11}^r C_{11}^m}{V_f C_{11}^m + (1-V_f) C_{11}^r} \\
C_{12} &= C_{11} \left[\frac{V_f C_{12}^r}{C_{11}^r} + \frac{(1-V_f) C_{12}^m}{C_{11}^m} \right] \\
C_{13} &= C_{11} \left[\frac{V_f C_{13}^r}{C_{11}^r} + \frac{(1-V_f) C_{13}^m}{C_{11}^m} \right] \\
C_{22} &= V_f C_{22}^r + (1-V_f) C_{22}^m + \frac{C_{12}^2}{C_{11}} - \frac{V_f (C_{12}^r)^2}{C_{11}^r} - \frac{(1-V_f) (C_{12}^m)^2}{C_{11}^m} \\
C_{44} &= V_f C_{44}^r + (1-V_f) C_{44}^m \\
C_{55} &= \frac{A}{B^2 + AC} \\
C_{66} &= \frac{C_{66}^r C_{66}^m}{V_f C_{66}^m + (1-V_f) C_{66}^r} \\
e_{31} &= C_{11} \left[\frac{V_f e_{31}^r}{C_{11}^r} + \frac{(1-V_f) e_{31}^m}{C_{11}^m} \right] \\
e_{32} &= V_f e_{32}^r + (1-V_f) e_{32}^m + \frac{C_{12} e_{31}}{C_{11}} - \frac{V_f C_{12} e_{31}^r}{C_{11}^r} - \frac{(1-V_f) C_{12} e_{31}^m}{C_{11}^m} \\
e_{33} &= V_f e_{33}^r + (1-V_f) e_{33}^m + \frac{C_{13} e_{31}}{C_{11}} - \frac{V_f C_{13} e_{31}^r}{C_{11}^r} - \frac{(1-V_f) C_{13} e_{31}^m}{C_{11}^m} \\
e_{24} &= V_f e_{24}^r + (1-V_f) e_{24}^m \\
e_{15} &= \frac{B}{B^2 + AC} \\
\varepsilon_{11}^* &= \frac{C}{B^2 + AC} \\
\varepsilon_{22}^* &= V_f \varepsilon_{22}^{*r} + (1-V_f) \varepsilon_{22}^{*m} \\
\varepsilon_{33}^* &= V_f \varepsilon_{33}^{*r} + (1-V_f) \varepsilon_{33}^{*m} - \frac{e_{31}^2}{C_{11}} + \frac{V_f (e_{31}^r)^2}{C_{11}^r} + \frac{(1-V_f) (e_{31}^m)^2}{C_{11}^m}
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
A &= \frac{V_f C_{55}^r}{(e_{15}^r)^2 + C_{55}^r \varepsilon_{11}^{*r}} + \frac{(1-V_f) C_{55}^m}{(e_{15}^m)^2 + C_{55}^m \varepsilon_{11}^{*m}} \\
B &= \frac{V_f C_{15}^r}{(e_{15}^r)^2 + C_{55}^r \varepsilon_{11}^{*r}} + \frac{(1-V_f) C_{15}^m}{(e_{15}^m)^2 + C_{55}^m \varepsilon_{11}^{*m}} \\
C &= \frac{V_f \varepsilon_{11}^{*r}}{(e_{15}^r)^2 + C_{55}^r \varepsilon_{11}^{*r}} + \frac{(1-V_f) \varepsilon_{11}^{*m}}{(e_{15}^m)^2 + C_{55}^m \varepsilon_{11}^{*m}}
\end{aligned} \tag{7}$$

Superscripts r and m refer to the reinforced and matrix components of the composite, respectively. V_f is the vol% of the reinforced BNNTs in matrix.

2.3. Governing Equations

For the piezoelectric cylindrical shell reinforced with BNNTs, the total potential energy Π can be written as

$$\Pi = -U + W \quad (8)$$

where U represents the strain energy and W represents the work done by the transverse load.

The expression of the strain energy is

$$U = \frac{1}{2} \iiint_V \sigma_i \bar{\varepsilon}_i dV - \frac{1}{2} \iiint_V E_i D_i dV \quad (9)$$

Considering Equations (4) and (5), as well as the zigzag structure for BNNTs employed here, and the longitudinal arrangement of strips in matrix, makes $E_y = E_z = 0$. Hence, Equation (9) becomes:

$$U = \frac{1}{2} \iiint_V \left\{ \bar{\varepsilon}_x - \alpha_x \Delta T, \bar{\varepsilon}_y - \alpha_y \Delta T, \bar{\varepsilon}_{xy}, -E_x \right\} \begin{bmatrix} C_{11} & C_{12} & 0 & e_{11} \\ C_{12} & C_{22} & 0 & e_{12} \\ 0 & 0 & C_{66} & 0 \\ e_{11} & e_{12} & 0 & -\varepsilon_{11}^* \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_x - \alpha_x \Delta T \\ \bar{\varepsilon}_y - \alpha_y \Delta T \\ \bar{\varepsilon}_{xy} \\ -E_x \end{Bmatrix} \quad (10)$$

Letting V is the voltage applied on both ends of shell, then

$$E_x = V/L \quad (11)$$

The work done by the transverse load $q(x, y)$ is

$$W = \iint_A q(x, y) w dx dy \quad (12)$$

Applying the variational principle ($\delta \Pi = 0$), the nonlinear governing equations of piezoelectric cylindrical shell reinforced with BNNTs can be derived as

$$\begin{aligned} N_{x,x} + N_{xy,y} &= 0 \\ N_{xy,x} + N_{y,y} &= 0 \\ M_{x,xx} + 2M_{xy,xy} + M_{y,yy} + N_y/R + N_x w_{,xx} + 2N_{xy} w_{,xy} + N_y w_{,yy} + q &= 0 \end{aligned} \quad (13)$$

where

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} - \begin{Bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{Bmatrix} - \begin{Bmatrix} N_x^p \\ N_y^p \\ N_{xy}^p \end{Bmatrix}, \quad \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} - \begin{Bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{Bmatrix} - \begin{Bmatrix} M_x^p \\ M_y^p \\ M_{xy}^p \end{Bmatrix} \quad (14)$$

in which

$$\begin{Bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \alpha_x \\ \alpha_y \\ 0 \end{Bmatrix} (1, z) \Delta T dz, \quad \begin{Bmatrix} N_x^p \\ N_y^p \\ N_{xy}^p \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_x \\ 0 \\ 0 \end{Bmatrix} (1, z) dz \quad (15)$$

In the above equations, the A_{ij}, D_{ij} are the tensile and bending rigidity and they can be defined as

$$(A_{ij}, D_{ij}) = \int_{-h/2}^{h/2} C_{ij} (1, z^2) dz \quad (i, j = 1, 2, 6) \quad (16)$$

Under the axisymmetrical circumstances, the circumferential displacement $v = 0$ and u, w is only the function of coordinate x . Hence, the second equation of Equation (12) is automatic balance and it can be omitted. Then by Equations (3), (14) and introducing the following dimensionless parameters,

$$\xi = \frac{x}{L}, \quad W = \frac{w}{h}, \quad U = \frac{u}{L}, \quad Q = \frac{qL^4}{A_{22}h^3}, \quad M_\xi = \frac{L^2 M_x}{A_{22}h^3}, \quad H = \frac{L}{h}, \quad K = \frac{L^2}{Rh} \quad (17)$$

the nonlinear governing equations of axisymmetrical piezoelectric shell reinforced with BNNTs under electro-thermo-mechanical loadings can be reduced as

$$\begin{aligned} S_{11A}U_{,\xi\xi} - S_{12A}\frac{K}{H^2}W_{,\xi} + S_{11A}\frac{1}{H^2}W_{,\xi}W_{,\xi\xi} = 0 \\ -\frac{1}{12}S_{11D}W_{,\xi\xi\xi\xi} + \left(S_{11A}H^2U_{,\xi} + \frac{S_{11A}}{2}W_{,\xi}^2 - S_{12A}KW - H^2(S_{11A}\alpha_x + S_{12A}\alpha_y)\Delta T - H^2e_{11}E_x \right)W_{i,\xi\xi} \\ + S_{12A}KH^2U_{,\xi} + \frac{S_{12A}K}{2}W_{,\xi}^2 - S_{22A}K^2W - H^2K(S_{12A}\alpha_x + S_{22A}\alpha_y)\Delta T - H^2Ke_{12}E_x + S_{22A}Q = 0 \end{aligned} \quad (18)$$

where, $S_{ijA} = A_{ij}/h$, $S_{ijD} = D_{ij}/(h^3/12)$.

Supposing the both ends of the shell are clamped, then the dimensionless boundary conditions are respectively as follows:

$$\begin{aligned} N_{\xi}(0) = 0, \quad W(0) = 0, \quad W_{,\xi}(0) = 0 \\ N_{\xi}(1) = 0, \quad W(1) = 0, \quad W_{,\xi}(1) = 0 \end{aligned} \quad (19)$$

where

$$N_{\xi} = S_{11A}\left(U_{,\xi} + \frac{1}{2H^2}W_{,\xi}^2\right) - S_{12A}\frac{K}{H^2}W - (S_{11A}\alpha_x + S_{12A}\alpha_y)\Delta T - e_{11}E_x \quad (20)$$

3. Solution Methodology

For seeking the solution of differential Equation (18) with boundary condition (19), the dimensionless displacement functions W and U are dispersed in time-space domain to obtain their approximate solution.

Difference method is adopted in space domain. For the disposal of linear item, taking $W_{,\xi\xi}$ as example, we have

$$W_{,\xi\xi} = \frac{1}{(\Delta\xi)^2} [W(i+1, j) - 2W(i, j) + W(i-1, j)] \quad (21)$$

Referring to difference scheme, the difference expressions of the other linear items in governing equation can be easily achieved.

Then the nonlinear items of governing equations are linearized and can be written as follows [18],

$$(x \cdot y)_j = (x)_j (y)_{jp} \quad (22)$$

in which $(y)_{jp}$ is the value of the former iterative. For the primary iteration, secondary extrapolation method is introduced to obtain the value of $(y)_{jp}$, that is

$$(y)_{jp} = \Delta_1(y)_{j-1} + \Delta_2(y)_{j-2} + \Delta_3(y)_{j-3} \quad (23)$$

As for different iterations, the coefficients Δ_1 , Δ_2 and Δ_3 are decided as follows:

$$\begin{aligned} j=1: \Delta_1=1, \quad \Delta_2=0, \quad \Delta_3=0 \\ j=2: \Delta_1=2, \quad \Delta_2=-1, \quad \Delta_3=0 \\ j \geq 3: \Delta_1=3, \quad \Delta_2=-3, \quad \Delta_3=1 \end{aligned} \quad (24)$$

After the equations and conditions are linearized and disposed by using the finite difference method, the nonlinear partial differential equations are transformed into linear algebraical equations expressed by difference schemes. These algebraic equations are solved by using the iteration method. For every step, the iterative lasts until the difference of the present value and the former is smaller than 0.01%, then continues the calculation of the next step.

4. Numerical Results and Discussion

The nonlinear bending of piezoelectric shell reinforced with BNNTs under electro-thermo-mechanical loadings

is investigated in the following calculations. The geometrical parameter of the shell is $L/R = 5/3$, $R/h = 30$. The material used for matrix is PVDF and the reinforced material is BNNT. The material constants are listed in **Table 1**. In the following figures (**Figures 2-5**), the vertical ordinate W and M_x are the dimensionless deflection and bending moment of each point on shell along x .

The effects of geometric nonlinear on the bending of piezoelectric shell reinforced with BNNTs are presented in **Figure 2(a)** and **Figure 2(b)**. The volume fraction $V_f = 0.6$, the voltage $V = 0$ and the temperature rise $\Delta T = 0$.

From the two figures, it can be noticed that the dimensionless deflection and bending moment of the shell in linear case is greater than that in nonlinear case, and this phenomenon becomes more evident when the transverse load Q increases. As we know, the linearity case is based on the limited deformation consumption, and the higher order item in the geometric relations is neglected while it is in consideration for the nonlinear case. So in some sense it can be concluded that the linear lowly predicts the stiffness of the structure. In order to reflect the property of the piezoelectric shell reinforced with BNNTs accurately, the consideration of the nonlinear effect is very necessary.

Figure 3 shows the effect of positive and negative voltage on the nonlinear bending of piezoelectric shell reinforced with BNNTs. The volume fraction $V_f = 0.6$, the temperature rise $\Delta T = 0$ and the mechanical load is taken as $Q = 150$. From the figure, it is observed that applying negative voltage to BNNT decreases the deflection and bending moment. This is due to the fact that applying negative voltage creates polarization in the BNNT in the longitudinal direction, and leads to its contraction. This makes the structure of BNNT more compact and strong, and correspondingly increases the structure's stiffness. Therefore, the deflection and bending moment of the structure decrease. **Figure 3** also depicts the results of deflection and bending moment when applying positive voltage. As expected, the deflection and bending moment increase compared to normal situation, and the results can be explained using the similar concept as mentioned above.

The effects of temperature on the nonlinear bending of the shell are presented in **Figure 4**. The volume fraction $V_f = 0.6$, the voltage $V = 0$ and the mechanical load is taken as $Q = 150$ in the figure. As can be seen, the deflection as well as the bending moment increases when the temperature increases.

The effect of volume fraction on the nonlinear bending of piezoelectric shell reinforced with BNNTs is discussed in **Figure 5**. The voltage is $V = 0$, the temperature rise is $\Delta T = 0$ and the mechanical load is $Q = 150$.

Table 1. Mechanical, electrical and thermal properties of PVDF and BNNT.

PVDF	$C_{11} = 238.24(\text{GPa})$, $C_{22} = 23.6(\text{GPa})$, $C_{12} = 3.98(\text{GPa})$, $C_{66} = 6.43(\text{GPa})$, $e_{11} = -0.135(\text{C/m}^2)$ $e_{12} = -0.145(\text{C/m}^2)$, $\varepsilon_{11}^* = 1.168 \times 10^{-8}(\text{F/m})$, $\alpha_x = 7.1 \times 10^{-5}(1/\text{K})$, $\alpha_y = 7.1 \times 10^{-5}(1/\text{K})$
BNNT	$E = 1.8(\text{TPa})$, $\nu = 0.34$, $e_{11} = 0.95(\text{C/m}^2)$, $\alpha_x = 1.2 \times 10^{-6}(1/\text{K})$, $\alpha_y = 0.6 \times 10^{-6}(1/\text{K})$

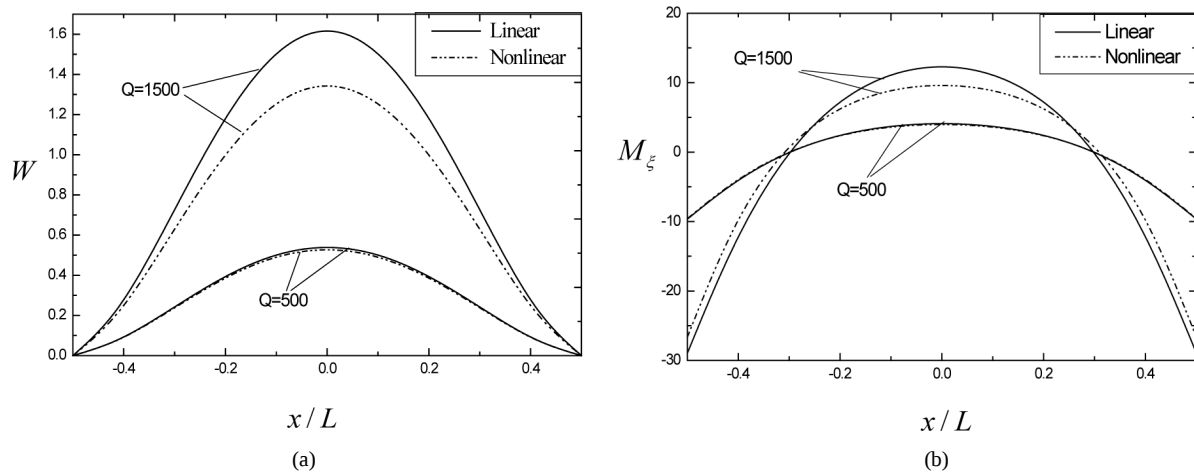


Figure 2. Effect of geometric nonlinear on bending of piezoelectric shell reinforced with BNNTs; (a) Deflection of each point along x ; (b) Bending moment of each point along x .

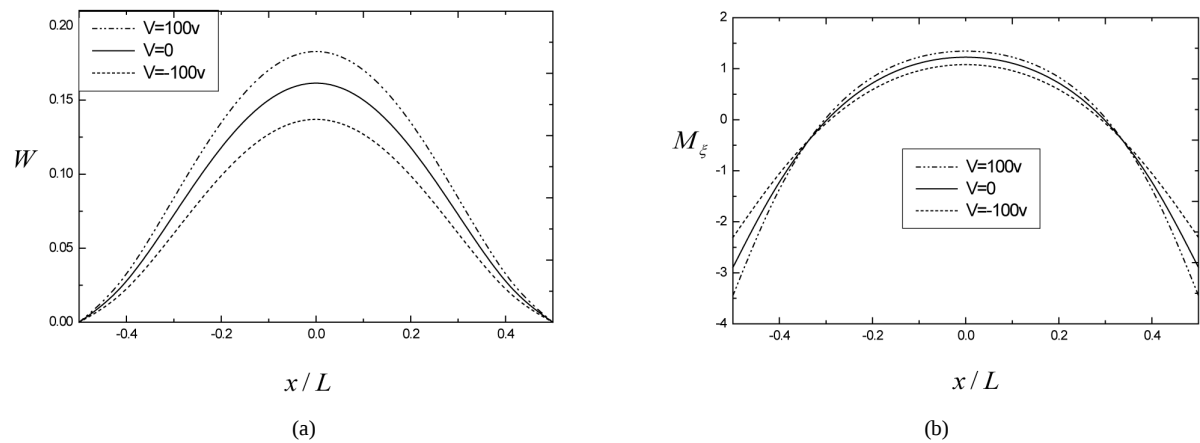


Figure 3. Effect of voltage on nonlinear bending of piezoelectric shell reinforced with BNNTs; (a) Deflection of each point along x ; (b) Bending moment of each point along x .

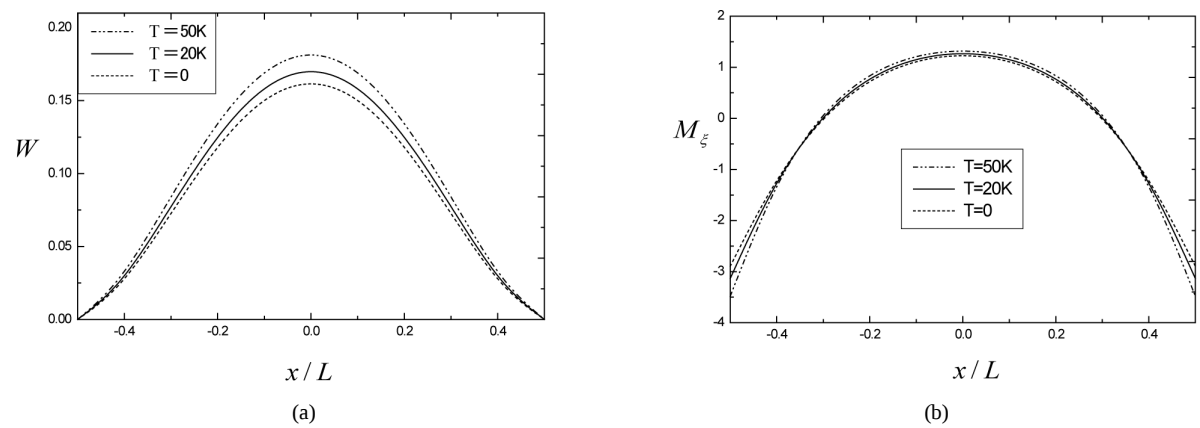


Figure 4. Effect of temperature on nonlinear bending of piezoelectric shell reinforced with BNNT; (a) Deflection of each point along x ; (b) Bending moment of each point along x .

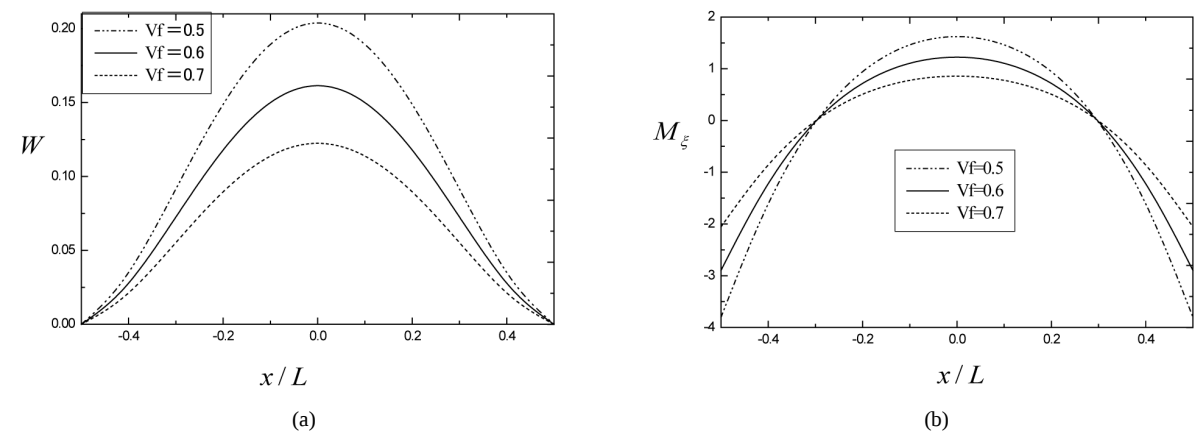


Figure 5. Effect of volume fraction on nonlinear bending of piezoelectric shell reinforced with BNNT. (a) Deflection of each point along x ; (b) Bending moment of each point along x .

in this figure. It can be noticed that the deflection and the bending moment decrease when the volume fraction of BNNT in matrix (V_f) increases. That is due to the fact that the increase of volume fraction would increase the stiffness of the structure, and thus the deflection decreases.

5. Conclusions

In present study, the governing equations of nonlinear bending are presented for piezoelectric cylindrical shell reinforced with BNNTs under combined electro-thermo-mechanical loadings. Results indicate that some parameters, including geometric nonlinear, voltage, temperature, volume fraction and so on, have significant influence on the deflection and bending moment of the shell. The following conclusions may be drawn from the present work:

- 1) The deflection and bending moment of the shell in linear case is greater than that in nonlinear case, and the nonlinear effect enhances when the transverse load Q increases.
- 2) Applying positive and negative voltage to BNNT leads to increase and decrease of the deflection and bending moment.
- 3) The deflection as well as the bending moment increases with the increase of temperature, and decreases when the volume fraction of BNNT in matrix increases.

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