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On the Comparisons of PID and GI-PD Control

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Abstract

In conjunction with a second order uncertain nonlinear system, this paper makes some comparisons between PID control and general-integral-proportional-derivative (GI-PD) control; that is, by Routh's stability criterion, we demonstrate that the system matrix under GI-PD control can be stabilized more easily; by linear system theory and Lyapunov method, we demonstrate that GI-PD control can deal with the uncertain nonlinearity more effectively; by analyzing and comparing the integral control action, we demonstrate that GI-PD control has the better control performance. Design example and simulation results verify the justification of our conclusions again. All these mean that GI-PD control has the stronger robustness and higher control performance than PID control. Consequently, GI-PD control has broader application prospects than PID control.

Keywords

General Integral Control, PID control, GI-PD Control, Robust Control, Output Regulation

1. Introduction

Proportional-integral-derivative (PID) control is certainly the most widely used control strategy today. It is estimated that over 90% of control loops employ PID control [1]. Over the last half-century, a great deal of academic and industrial effort has focused on improving PID control, but the trouble, which often suffers a serious loss of performance due to integrator windup, was not resolved in principle before general integral control [2] appeared in 2009.

After that various general integral controls along with the design techniques were presented. For example, general concave integral control [3], general convex integral control [4], constructive general bounded integral control [5] and the generalization of the integrator and integral control action [6] were all developed by resorting to an ordinary control along with a known Lyapunov function; general integral control designs based on linear

system theory, sliding mode technique, feedback linearization technique, singular perturbation technique, equal ratio gain technique and power ratio gain technique were presented by [7]-[12], respectively. Although general integral control has developed rapidly in theory, its practical applications have not been reported. Therefore, in consideration of its good control performance, it is appropriate at this time to compare the simplest general integral control (GI-PD) with PID control in order to promote its applications in practice.

Motivated by the cognition above, in conjunction with a second order uncertain nonlinear system, this paper makes some comparisons between PID control and GI-PD control. The main contributions are: under GI-PD control, it is demonstrated that: 1) the system matrix can be stabilized more easily; 2) it is more effective to deal with the uncertain nonlinear actions; 3) the trouble caused by integrator windup is resolved in principle, and then it has the better control performance; 4) the harmonization of the integral control action and PD control action can be achieved. Moreover, design example and simulation results verify the justification of our conclusions again. All these mean that GI-PD control has the stronger robustness and higher control performance than PID control. Consequently, GI-PD control has broader application prospects than PID control.

Throughout this paper, we use the notation $\lambda_m(A)$ and $\lambda_M(A)$ to indicate the smallest and largest eigenvalues, respectively, of a symmetric positive-define bounded matrix $A(x)$, for any $x \in R^n$. The norm of vector x is defined as $\|x\| = \sqrt{x^{\mathrm{T}} x}$, and that of matrix A is defined as the corresponding induced norm $\|A\| = \sqrt{\lambda_M(A^{\mathrm{T}} A)}$.

The remainder of the paper is organized as follows: Section 2 describes the system under consideration, assumption, and stability analysis of the closed-loop system. Section 3 compares Hurwitz stability of the system matrix. Section 4 demonstrates the robustness against the uncertain nonlinearity. Section 5 analyzes the control action. Example and simulation are provided in Section 6. Conclusions are presented in Section 7.

2. Problem Formulation

Consider the following controllable nonlinear system,

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x, w) + g(x, w) u \end{cases} \tag{1}$$

where $x \in R^2$ is the state; $u \in R$ is the control input; $w \in R^l$ is a vector of unknown constant parameters and disturbances; the function $f(x, w)$ is the uncertain nonlinear actions, the uncertain nonlinear function $g(x, w)$ is continuous in (x, w) on the control domain $D_x \times D_w \subset R^2 \times R^l$.

Assumption 1: There is a unique pair $(0, u_0)$ that satisfies the equation,

$$0 = f(0, w) + g(0, w) u_0 \tag{2}$$

so that $x = 0$ is the desired equilibrium point and u_0 is the steady-state control that is needed to maintain equilibrium at $x = 0$.

Assumption 2: Suppose that the functions $f(x, w)$ and $g(x, w)$ satisfy the following inequalities,

$$\|f(x, w) - f(0, w)\| \le l_f^x \|x\| \tag{3}$$

$$0 < g_m < g(x, w) \tag{4}$$

$$\|g(x, w) - g(0, w)\| \le l_g^x \|x\| \tag{5}$$

$$\|f(0, w) g^{-1}(0, w)\| \le \gamma_g^f \tag{6}$$

for all $x \in D_x$ and $w \in D_w$, where l_f^x, l_g^x, g_m and γ_g^f are all positive constants.

For comparing PID and GI-PD control, the control law is taken as,

$$\begin{cases} u = -\alpha_1 x_1 - \alpha_2 x_2 - \alpha_3 \sigma \\ \dot{\sigma} = \beta_1 x_1 + \beta_2 x_2 \end{cases} \tag{7}$$

where α_1, α_2 and α_3 are the controller gains; β_1 and β_2 are the integrator gains.

It is worth to note that although the control law (7) is GI-PD control, it is reduced to PID control as $\beta_1 = 1$ and $\beta_2 = 0$. Thus, under GI-PD and PID control, the closed-loop system can be written as the same form, that is,

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x,w) - g(x,w)(\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 \sigma) \\ \dot{\sigma} = \beta_1 x_1 + \beta_2 x_2 \end{cases} \tag{8}$$

By assumption 1 and choosing α_3 to be large enough, and then setting $\dot{x} = 0$ and $x = 0$ of the system (8), obtain,

$$0 = f(0,w) - g(0,w)\alpha_3 \sigma_0 \tag{9}$$

Therefore, we ensure that there is a unique solution σ_0, and then $(0,\sigma_0)$ is a unique equilibrium point of the closed-loop system (8) in the domain of interest.

Now, defining $\eta = \begin{bmatrix} x_1 & x_2 & \sigma - \sigma_0 \end{bmatrix}^T$, and substituting (9) into (8), obtain,

$$\dot{\eta} = A\eta + F(x,w) \tag{10}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -\alpha_1 & -\alpha_2 & -\alpha_3 \\ \beta_1 & \beta_2 & 0 \end{bmatrix}$$

and $F(x,w)$ is a 3×1 matrix, all its elements is equal to zero except for

$$f_{21} = f(x,w) - f(0,w) - \left[g(x,w) - g(0,w) \right] f(0,w) g^{-1}(0,w).$$

Moreover, it is worthy to note that the function $g(x,w)$ is integrated into α_1, α_2 and α_3.

By linear system theory, if the matrix A is Hurwitz, and then for any given positive define symmetric matrix Q, there is a unique positive define symmetric matrix P that satisfies Lyapunov equation $PA + A^T P = -Q$. Therefore, there exists a quadratic Lyapunov function,

$$V(\eta) = \eta^T P \eta \tag{11}$$

Thus, using $V(\eta) = \eta^T P \eta$ as Lyapunov function candidate, and then its time derivative along the trajectories of the closed-loop systems (10) is,

$$\dot{V}(\eta) = \eta^T \left(PA + A^T P \right) \eta + \frac{\partial V(\eta)}{\partial \eta} F(x,w) = -\eta^T Q \eta + 2 P_2 \eta f_{21} \tag{12}$$

where $P_2 = \begin{bmatrix} p_{21} & p_{22} & p_{23} \end{bmatrix}$.

Now, using the inequalities (3), (5), (6) and definition of η, we have,

$$\| f_{21} \| \leq \kappa_f^\eta \| \eta \| \tag{13}$$

where κ_f^η is a positive constant.

Substituting (13) into (12), obtain,

$$\dot{V}(\eta) \leq -\left(\lambda_m(Q) - 2\kappa_f^\eta \| P_2 \| \right) \| \eta \|^2 \tag{14}$$

It is obvious that if

$$\lambda_m(Q) > 2\kappa_f^\eta \| P_2 \| \tag{15}$$

holds, we have $\dot{V}(\eta) \le 0$.

Using the fact that Lyapunov function $V(\eta)$ is a positive define function and its time derivative is a negative define function if the inequality (15) holds, we conclude that the closed-loop system (10) is stable. In fact, $\dot{V}(\eta) = 0$ means $x = 0$ and $\sigma = \sigma_0$. By invoking LaSalle's invariance principle, it is easy to know that the closed-loop system (10) is exponentially stable.

Discussion 1: From the demonstration above, it is obvious that: for ensuring that the closed-loop system is exponentially stable, two key conditions are indispensable, that is, one is that the system matrix A is Hurwitz and another is that the inequality (15) holds. Thus, for comparing GI-PD control with PID control, the differences of two key conditions above must be demonstrated. Moreover, the analysis of PID and GI-PD control action and performance is unnecessary, too. All these are addressed in the following Sections, respectively.

3. Hurwitz Stability

The polynomials of the system matrix A under PID control and GI-PD control are,

$$s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_3 = 0 \tag{16}$$

$$s^3 + \alpha_2 s^2 + (\alpha_3 \beta_2 + \alpha_1) s + \alpha_3 \beta_1 = 0 \tag{17}$$

By Routh's stability criterion and the polynomials (16) and (17), Hurwitz stability conditions of the system matrix A under PID control and GI-PD control can be obtained as follows:

Under PID control, if α_1, α_2 and α_3 are all positive constants, and the inequality,

$$\alpha_2 \alpha_1 > \alpha_3 \tag{18}$$

holds, and then the system matrix A is Hurwitz.

Under GI-PD control, if α_2, $\alpha_3 \beta_2 + \alpha_1$ and $\alpha_3 \beta_1$ are all positive constants, and the inequality,

$$\alpha_3 \alpha_2 \beta_2 + \alpha_2 \alpha_1 > \alpha_3 \beta_1 \tag{19}$$

holds, then the system matrix A is Hurwitz.

Compared with Hurwitz stability conditions of PID control, the one of GI-PD control has the following features:

1) The striking feature is that the role of gain β_2 manifests itself in two aspects: one is that the gain β_2 produces a special term $\alpha_3 \beta_2$ such that the gain α_1 is enhanced, and then for achieving Hurwitz stability, it is not necessary to increase the value of α_1, even α_1 can be taken as a negative constant; another is that the gain β_2 educes another special term $\alpha_2 \alpha_3 \beta_2$ such that it makes the inequality (19) holds more easily, and then for achieving Hurwitz stability, it is unnecessary to increase α_1 and/or α_2, or decrease α_3.

2) As $\beta_1 = 1$, if the system matrix A with PID control is Hurwitz, and then the one with GI-PD control and $\beta_2 > 0$ must be Hurwitz.

3) The gain β_1 is indispensable. For ensuring Hurwitz stability, $\beta_1 > 1$ seems to be unfavorable, but $\beta_1 < 1$ is absolutely favorable.

4) There are two additional gains β_2 and β_1 in GI-PD control law. Therefore, more information can be exploited to stabilize the system matrix A than PID control.

All these means that the system matrix A under GI-PD control can be stabilized more easily than PID control.

4. Robustness against Uncertain Nonlinear Actions

For comparing PID control and GI-PD control robustness against uncertain nonlinear actions, we need to solve the Lyapunov equation $PA + A^{\mathrm{T}}P = -Q$ with any given positive define symmetric matrix Q to obtain the solution of the matrix P_2.

Under PID control, $P_2^I = \begin{bmatrix} p_{21}^I & p_{22}^I & p_{23}^I \end{bmatrix}$ is,

$$p_{21}^I = \frac{q_{11}\alpha_3\alpha_2 + q_{22}\alpha_3\alpha_3 + q_{33}\alpha_2\alpha_2 - 2q_{23}\alpha_3\alpha_2}{2(\alpha_2\alpha_1 - \alpha_3)\alpha_3}$$

$$p_{22}^{I} = \frac{q_{11}\alpha_3 - 2q_{23}\alpha_3 + q_{22}\alpha_3\alpha_1 + q_{33}\alpha_2}{2(\alpha_2\alpha_1 - \alpha_3)\alpha_3}$$

$$p_{23}^{I} = \frac{q_{33}}{2\alpha_3}$$

Under GI-PD control, $P_2^{GI} = \begin{bmatrix} p_{21}^{GI} & p_{22}^{GI} & p_{23}^{GI} \end{bmatrix}$ is,

$$p_{21}^{GI} = \frac{\left(q_{11}\alpha_3\alpha_2 + q_{22}\alpha_3\alpha_3\beta_1 + q_{33}\alpha_2\alpha_2\beta_1 - 2q_{23}\alpha_3\alpha_2\beta_1 - q_{33}(\alpha_1\alpha_2 - \alpha_3\beta_1)\beta_2 + 2q_{13}\alpha_3\alpha_2\beta_2\right)}{2(\alpha_2\alpha_1 - \alpha_3\beta_1)\alpha_3 + 2\alpha_3\alpha_3\alpha_2\beta_2}$$

$$p_{22}^{GI} = \frac{\left(q_{11}\alpha_3 - 2q_{23}\alpha_3\beta_1 + q_{22}\alpha_3\alpha_1 + q_{33}\alpha_2\beta_1 + 2q_{13}\alpha_3\beta_2 + q_{22}\alpha_3\alpha_3\beta_2 + q_{33}\alpha_3\beta_2\beta_2\right)}{2(\alpha_2\alpha_1 - \alpha_3\beta_1)\alpha_3 + 2\alpha_3\alpha_3\alpha_2\beta_2}$$

$$p_{23}^{GI} = \frac{q_{33}}{2\alpha_3}$$

For the sake of simplicity, we just consider the case of $\beta_1 = 1$ and $Q = I$. Thus, by comparing p_{22}^{I} with p_{22}^{GI}, we have,

$$p_{22}^{I} > p_{22}^{GI} \quad \text{as} \quad 0 < \beta_2 < \beta_2^*$$

and then by $p_{21}^{I} > 0$ and $p_{21}^{GI} = 0$ as $\beta_2 = \beta_2^*$, we obtain,

$$p_{21}^{I} > p_{21}^{GI} > 0 \quad \text{as} \quad 0 < \beta_2 < \beta_2^*$$

where

$$\beta_2^* = \frac{\alpha_3\alpha_2 + \alpha_2\alpha_2 + \alpha_3\alpha_3}{\alpha_2\alpha_1 - \alpha_3}$$

It is easy to see that there exists β_2^* such that $\left\| P_2^{GI} \right\| < \left\| P_2^{I} \right\|$ holds for all $0 < \beta_2 < \beta_2^*$, and then by the inequality (15), we can conclude that GI-PD control is more effective to deal with the uncertain nonlinear actions than PID control. This means that under the case of the same gains α_3, α_2, α_1 and $\beta_1 = 1$ along with moderately choosing β_2 and Q, GI-PD control can be designed to have the stronger robustness against the uncertain nonlinear actions than PID control.

Discussion 2: Although the demonstration above aims at a special case, it is not hard to conclude that by synthesizing all the gains α_3, α_2, α_1, β_1 and β_2, GI-PD control can be designed to have the stronger robustness with respect to the uncertain nonlinear actions than PID control since more information can be used to decrease the value of $\left\| P_2^{GI} \right\|$.

5. Analysis of Control Action

No matter PID control or GI-PD control, Proportional and Derivative control actions are all identical, that is:

Proportional control action is proportional to the error. If the error is small, its corrective effect is small, and vice versa.

Derivative control action is proportional to the rate at which the error is changing. Its corrective effect attempts to anticipate a large error and prevent this future error.

Compared with PID control, the main difference of GI-PD control is the integrator, that is, the error derivative is introduced into the integrator. This lead to an important change of the integral control action, that is,

Under PID control, the integrator is $\dot{\sigma} = x_1$. Obviously, the integral control action continues to increase unless the error passes through zero, and then for making the integral control action tends to a constant, the error is usually needed to pass through zero repeatedly. Just the stubborn increase of integral control action results in the

integrator windup.

Under GI-PD control, the integrator is $\dot{\sigma} = \beta_1 x_1 + \beta_2 x_2$. Thus, as $\dot{\sigma} = \beta_1 x_1 + \beta_2 x_2 = 0$, the integral control action does not increased and remains a constant; if the integral control action is large, x_2 increases, and the integral control action instantly decreases, and vice versa. This shows that the effect of $\beta_2 x_2$ is an attempt to anticipate and prevent an excess integral control action, and then integrator windup can be removed completely. Moreover, as $\alpha_2 \beta_1 = \alpha_1 \beta_2$, the integral control action is equivalent to the accumulation of PD control action. This means that the harmonization of the integral control action and PD control action can be achieved. All these means that GI-PD control has the better control performance than PID control.

6. Example and Simulation

Consider the pendulum system [13] described by,

$$\ddot{\theta} = -a\sin(\theta) - b\dot{\theta} + cT$$

where $a, b, c > 0$, θ is the angle subtended by the rod and the vertical axis, and T is the torque applied to the pendulum. View T as the control input and suppose we want to regulate θ to r. Now, taking $x_1 = \theta - r$, $\dot{x}_2 = \dot{\theta}$, the pendulum system can be written as,

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -a\sin(x_1 + r) - bx_2 + cu \end{cases}$$

and then it can be verified that $u_0 = a\sin(r)/c$ is the steady-state control that is needed to maintain equilibrium at the origin.

GI-PD control law is,

$$\begin{cases} u = -\alpha_1 x_1 - \alpha_2 x_2 - \alpha_3 \sigma \\ \dot{\sigma} = x_1 + \beta_2 x_2 \end{cases}$$

It is worth to note that as $\beta_2 = 0$, the control law above is PID control law. Thus, the closed-loop system can be written as,

$$\dot{\eta} = A\eta + F(x, w)$$

where

$$\eta = \begin{bmatrix} x_1 & x_2 & \sigma - \sigma_0 \end{bmatrix}^{\mathrm{T}},$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -c\alpha_1 & -c(\alpha_2 + c^{-1}b) & -c\alpha_3 \\ 1 & \beta_2 & 0 \end{bmatrix},$$

and

$$F(x, w) = \begin{bmatrix} 0 & a\sin(r) - a\sin(x_1 + r) & 0 \end{bmatrix}^{\mathrm{T}}.$$

The normal parameters are $a = c = 10$ and $b = 2$, and in the perturbed case, b and c are reduced to 1 and 5, respectively, corresponding to double the mass. Thus, we have,

$$\|a\sin(r) - a\sin(x_1 + r)\| \le 10\|\eta\| \tag{20}$$

Now, taking $\alpha_1 = 8$, $\alpha_2 = 5$, $\alpha_3 = 8$, $\beta_2 = \alpha_2 \alpha_1^{-1}$, $a = 10$, $c = 5$ and $b = 1$, and using Routh's stability criterion, we have,

$$c\alpha_3 (c\alpha_2 + b)\beta_2 + (c\alpha_2 + b)c\alpha_1 - c\alpha_3 = 1650 \tag{21}$$

$$c(c\alpha_2 + 1)\alpha_1 - c\alpha_3 = 1000 \tag{22}$$

and then the system matrix A under GI-PD control and PID control is Hurwitz. Thus, solving Lyapunov equation $PA + A^T P = -I$, we obtain $\left\| P_2^{GI} \right\| \approx 0.031$ and $\left\| P_2^I \right\| \approx 0.048$, and then by the inequality (15) and the bound condition (20), we have,

$$1 - 20 \left\| P_2^{GI} \right\| = 0.38 \tag{23}$$

$$1 - 20 \left\| P_2^I \right\| = 0.04 \tag{24}$$

Thus, Under PID and GI-PD control, the asymptotical stability of the whole closed-loop system can all be ensured. Consequently, the simulations are implemented under the normal and perturbed cases, respectively. Moreover, in the perturbed case, we consider an additive impulse-like disturbance $d(t)$ of magnitude 60 acting on the system input between 30s and 31s.

Figure 1 and **Figure 2** showed the simulation results under normal and perturbed cases. From the simulation results and design procedure, the following observations can be made: 1) by Hurwitz stability conditions (21) and (22), stability margin of the system matrix A under GI-PD control is larger than the one of PID control; 2) by stability conditions (23) and (24), GI-PD control has the stronger robustness with respect to the uncertain nonlinear action than PID control; 3) by **Figure 1** and **Figure 2**, under GI-PD control, no matter normal case or perturbed case, the optimum responses can all be achieved in the whole control domain. However, under PID control, the overshoot is proportional to the initial error and the settling time is long. Due to the above experimental results, it could be concluded that GI-PD control has more broad application prospects than PID control.
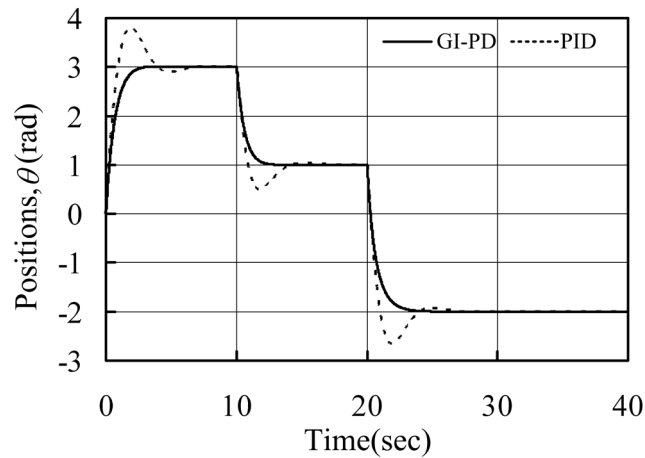
Figure 1. System output under the normal case.
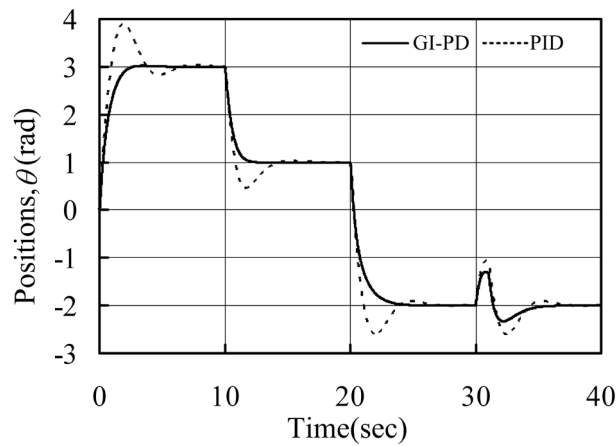
Figure 2. System output under the perturbed cases.

7. Conclusion

In conjunction with a second order uncertain nonlinear system, this paper makes some comparisons between PID control and GI-PD control. The main contributions are: under GI-PD control, it is demonstrated that: 1) the system matrix can be stabilized more easily; 2) it is more effective to deal with the uncertain nonlinear actions; 3) the trouble caused by integrator windup is resolved in principle, and then it has the better control performance; 4) the harmonization of the integral control action and PD control action can be achieved. Moreover, design example and simulation results verify the justification of our conclusions again. All these means that GI-PD control has the stronger robustness and higher control performance than PID control. Consequently, GI-PD control has broader application prospects than PID control.

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