

Modelling of Internal Stresses in Sheet Glass during the Cooling

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Abstract

Internal stresses in glass are generated by interactions between thermal contraction, elasticity at low temperatures, viscoelastic flow at higher temperature, and temperature gradients caused by cooling. This work intends to work out calculation program for real temperature distribution and internal stress, and to study their behaviour during the quenching through a flat plate of soda-lime glass from different temperatures.

Keywords: Modelling, Internal Stresses, Glass

1. Introduction

The internal stresses in the glass are tensions which can be transient during the thermal treatment or the permanent tensions after the treatment and that are important of the practical view point.

In this sense, the theory was not developed a long time, the first theoretical work in this sense has been done by Adams and Williamson [1], carrying on the analytic calculation of the distribution of the temperature during the cooling of the glass. After several years, models of visco-elastic properties of the glass began to appear, but the functional method of calculation takes in consideration the thermoplastic stress formation and at the same time their relaxation by the viscous out-flow that could be made since the apparition of the computer. The first work that carries on the numeric transient and permanent internal stress calculation in the glass been published by Lee, Roger and Woo [2]; at the following of several works of research done, and on the basis of these works we present this article.

2. Fundamental Equations

2.1. General Considerations

In all the following theory, which is developed for a flat plate in glass, it is supposed that the lateral dimensions

are superior to the thickness of where the normal constraints in full surface tend everywhere toward zero; by these conditions, the thermal stresses become stresses hover and uniforms in the plan y - z , but only with x [2-6].

$$\sigma_y = \sigma_z = \sigma(x, t) \quad (2.1.1)$$

$$\sigma_x = 0 \quad (2.1.2)$$

where, x , y , z are the principal axes; t is the parameter time.

2.2. Equation of the Deformation

The Equation (2.2.1) generally describes all deformation of an infinite plate of which the initial temperature T_0 at the instant $t = 0$ change to the temperature $T(x, t)$ and at the fictive temperature $T_f(x, t)$; this deformation is called free deformation and which given by the following relation [2-4,7]:

$$\varepsilon_{th}(x, t) = \alpha_g [T(x, t) - T_0] + (\alpha_l - \alpha_g) [T_f(x, t) - T_0] \quad (2.2.1)$$

where, α_g , α_l : are respectively coefficients of thermal expansion in the glass state and coefficients of thermal expansion in the liquid state in thermodynamic equilibrium; $T(x, t)$: Temperature in the x point and at the instant really determined t ; $T_f(x, t)$: fictive temperature in the x point and at the instant t ; $T(x, 0) = T_0$: the initial

temperature at the instant $t = 0$.

We suppose at the instant $t = 0$ a stabilised state, then:
 $T_f(x, 0) = T_0$.

From the conditions cited before, the plate doesn't change in the plan y - z , but independently of x , the geometry of the glass plate and the continuity of material the lengthly of the plate determines the real deformation which only depends of the time [2,3,6,7]:

$$\varepsilon(x, t) = \varepsilon(t) \neq f(x) \quad (2.2.2)$$

2.3. Stress Equations

In the interval of transition, two simultaneous effects produce, the first is the generation of stresses which is caused by the interaction between the thermal contraction (elasticity) for the low temperatures and the visco-elasticity that increase for the elevated temperatures, then the second effect that is the relaxation of these stresses.

The stress is generated by the difference between the free deformation and the real deformation [2-4,7]:

$$\sigma_g(x, t) = \frac{E}{1-\mu} [\varepsilon(t) - \varepsilon_{th}(x, t)] \quad (2.3.1)$$

where, $\sigma_g(x, t)$: Stress generated in the x point and at the instant t ; E : YOUNG modulus; μ POISSON Coefficient.

But for the glass in the state material visco-elastic, the equation of stress is expressed such as being the sum of stress generated variations with regard to the time t while considering the relaxation of these stresses that also depends of cooling time, and this equation is called visco-elastic equation which is given by [2-5,7] :

$$\sigma(x, t) = \int_0^t R(t-t') \frac{\partial \sigma_g(x, t')}{\partial t'} dt' \quad (2.3.2)$$

where, R : Stress relaxation modulus.

With the relation of equilibrium which is defined as being the sum of the internal stresses the lengthly of the half of the plate thickness (at the rate of symmetry) must be equal to zero; this relation is called basis equation:

$$\int_0^l \sigma(x, t) dx = 0 \quad (2.3.3)$$

2.4. Fictive Temperature

Tool [8] defines the fictive temperature T_f as being a present temperature of an equilibrium state which corresponds to give the state no equilibrium, else said, it is the proper temperature to the structure, it takes its residual value just as the relaxation eliminates itself. It is a func-

tion of the position x and the time t , which is given by the following equation [7,9,10]:

$$T_f(x, t) = T(x, t) - \int_0^t M [\xi(x, t) - \xi(x, t')] \frac{\partial T}{\partial t'} dt' \quad (2.4.1)$$

where M : structure relaxation Modulus; ξ : Reduced time.

The reduced time ξ is defines by Lee and Roger [11] as being the measured time at the low temperatures. With the superposition of the time and the temperature of which call simplicity thermo-rheological while using the shift function Φ , from which comes the notion of the reduced time.

The reduced time is in relation with the real time and the shift function Φ under the following form [3,9,12]:

$$\xi(x, t) = \int_0^t \Phi [T(x, t), T_f(x, t')] dt' \quad (2.4.2)$$

However, ξ is obtained from the real time t , which carries the shift function Φ to depend of T_f and as well of T , which is formulated by the following relation [7,12]:

$$\xi(T, T_f) = \text{Exp} \left[-\frac{H}{2R_g} \left(\frac{1}{T} + \frac{1}{T_f} - \frac{2}{T_d} \right) \right] \quad (2.4.3)$$

where H : Activation energy; R_g : Gas Constant; T_d : Minimum annealing temperature.

From Rekhson and Mazurins [13] results, the structure relaxation Modulus can be presented by the following expression:

$$M(\xi) = \text{Exp} \left[-\frac{\xi}{t_v} \right]^{0.68} \quad (2.4.4)$$

where t_v : Time of volume relaxation.

2.5. Stress Relaxation Modulus

Stress relaxation modulus R is calculated on the basis of E and G respectively elasticity modulus and sliding modulus which also depend of a time, and which given by the following equation of integral [2,7,11,14-16]:

$$\frac{4}{3k} \int_0^{\xi} G(\xi - \xi') \frac{\partial R'}{\partial \xi'} d\xi' + R'(\xi) = 2G(\xi) \quad (2.5.1)$$

where k : volume module which is in relation with a YOUNG modulus E , the sliding modulus G and a POISSON coefficient μ :

$$E = 2(1 + \mu)G = \frac{1 - 2\mu}{3k} \quad (2.5.2)$$

2.6. The Temperature Distribution

The purpose to determine the temperature distribution is to know the variation of the temperature gradient which has for consequence a variation of internal stress in a plate of glass during the cooling; this distribution is determined by the equation of heat transfer of which the general form [17]:

$$C \cdot \rho \cdot \frac{\partial T}{\partial t} = \nabla(\lambda \cdot \nabla T) \quad (2.6.1)$$

where C : Specific heat; ρ : Density; λ : Thermal conductivity.

But for the calculation of temperature at the surface of the plate, we have a heat transfer between the surrounding and the surface of the glass plate, which expressed by the following equation [7,18]:

$$\frac{\partial T_s}{\partial x} = -\frac{\alpha}{\lambda}(T_s - T_E) \quad (2.6.2)$$

where T_s : Superficial temperature of a sheet glass; T_E : Surrounding temperature; α : Coefficient of heat transfer.

3. Results and Discussions

To calculate the temperature distribution and the internal stresses in a flat plate in soda-lime glass with a thickness of 1 cm from a temperature of 650°C, we used the following constant:

- The surrounding temperature $T_E = 20^\circ\text{C}$.
- The step of the time for the calculation of fictive temperature $\Delta t = 1$ s.
- The step of the time for the calculation of real temperature $Pt = 0.25$ s.
- The step of the thickness $h = 1$ mm.
- The coefficient of heat transfer $\alpha = 198 \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$.
- The dilation coefficient of the liquid state $\alpha_l = 27 \times 10^{-6} \text{ K}^{-1}$
- The dilation coefficient of the glass state $\alpha_g = 9 \times 10^{-6} \text{ K}^{-1}$
- The elasticity modulus $E = 72000 \text{ MPa}$.
- The POISSON coefficient $\mu = 0.22$.
- The minimum annealing temperature $T_d = 477^\circ\text{C}$.
- The precision of calculation of the fictive temperature $\varepsilon_f = 0.01^\circ\text{C}$.

Figure 1 shows us, in the beginning of the cooling, real temperature T and fictive temperature T_f nearly had the same values during the first three seconds of cooling, next the difference appears and begin to increase until the total cooling.

Since the first second, the real temperature gradient between the one of the surface and the one of the center appears, after it increases until it marks its maximum

between the instant $t = 20$ s (0.3 min) and $t = 60$ s (1 min) of the cooling; from this moment, it begins to decrease until it becomes nil, thus the two temperatures of surface and center takes the same value (total cooling).

But for the fictive temperature, it begins to decrease until the instant $t = 40$ s (0.7 min) and then it stabilises; at this instant the relaxation of the structure terminates itself.

Results obtained for the internal stress are regrouped in a **Figure 2** under profiles form.

To draw internal stress profiles, one has need of the specific difference of light march polarised X which is the consequence of the photo-elastic measures of stresses.

The stress σ and the march difference X are linked by the photo-elastic constant (called BREWSTER constant) B by the following relation:

$$X = \sigma * B \quad (3.1)$$

In the beginning of cooling for the first five seconds, internal stress profiles are given by the **Figure 2(a)**, but those of 6 seconds until 20 seconds are not presented because they keep the same pace as the one of 5 seconds but sizes differ.

One remarks that for the first 3 seconds, the surface is under the influence of the traction stress, whereas normally the superficial layer must compress itself if it was free; this contraction is caused by the continuity of matter because the superficial layer will have tendency to compress but the other inner layers don't let it and which are under the influence of the compression stress.

Since the fourth second, the surface will become under the influence of the compression and the traction stress transfers to the second layer of the surface (to **Figure 2(a)**), it is caused by the continuity of matter, this transfer continuous from a layer to another one until the central layer (to See **Figure 2**).

After 20 seconds of cooling, the surface will be under

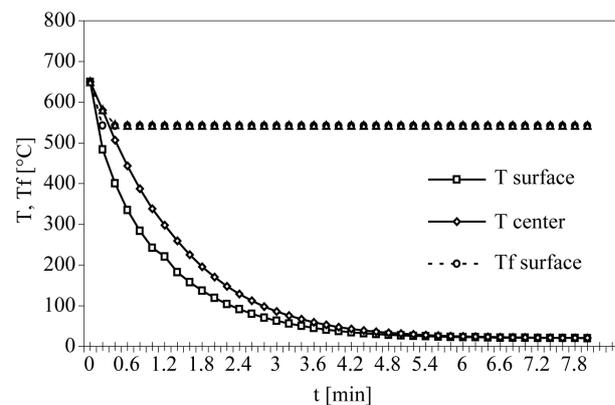


Figure 1. Temperature (T) and fictive temperature (T_f) variations during 8 minutes of cooling through a flat plate in soda-lime glass.

the influence of the compression stress, but the center under the traction stress of which sizes increase progressively until the moment where they take their residual values (total cooling) (to see **Figures 2(b), (c)**).

From the 30 seconds, profiles of stresses have the same form but sizes which differ.

For the internal stress variation (to see **Figure 3**); in the beginning of cooling the stress begins to increase until the instant $t = 20$ s where it stabilises lightly during 10 seconds whose value is -18.08 Mpa for the surface, and of 4.99 Mpa for the center whose marches differences are respectively -452 nm/cm and 125 nm/cm.

Next, compression at the surface increases until the value -80 Mpa ($X = -2000$ nm/cm); but traction at the center increases until 32.9 Mpa ($X = 882.5$ nm/cm), where the two stresses stabilise.

After the total cooling, the profile of the stress has a parabolic form having a bigger slope than 2 what corresponds to the generally known experimental fact (example [19]); the ratio of the absolute values of the stress in

the center and at the surface is more or less equal to 2.

The quantitative numeric value comparison can be done better with the permanent internal stress for which we have enough applied values. In the **Figure 4**; we have the comparison of calculated values of march difference and those measured by photo-elasticimetry by transmission, for a flat plate in soda-lime glass of 0.61 cm thickness from a initial temperature $T_0 = 738^\circ\text{C}$.

One sees that the calculated values are smaller than the experimental values (at the surface 37.36% and in the center of the sample 41.21%), the principal causes of this difference is the calculation of the real temperature because the equation of heat transfer doesn't take in consideration the radiation component; then If we compare the evolution of the calculated temperature with values found by Narayanaswamy [4] who used another equation of which the radiation component taken in consideration (to see **Table 1**).

We notice that the calculated ΔT are smaller than those of Narayanaswamy [4]; because values of the permanent

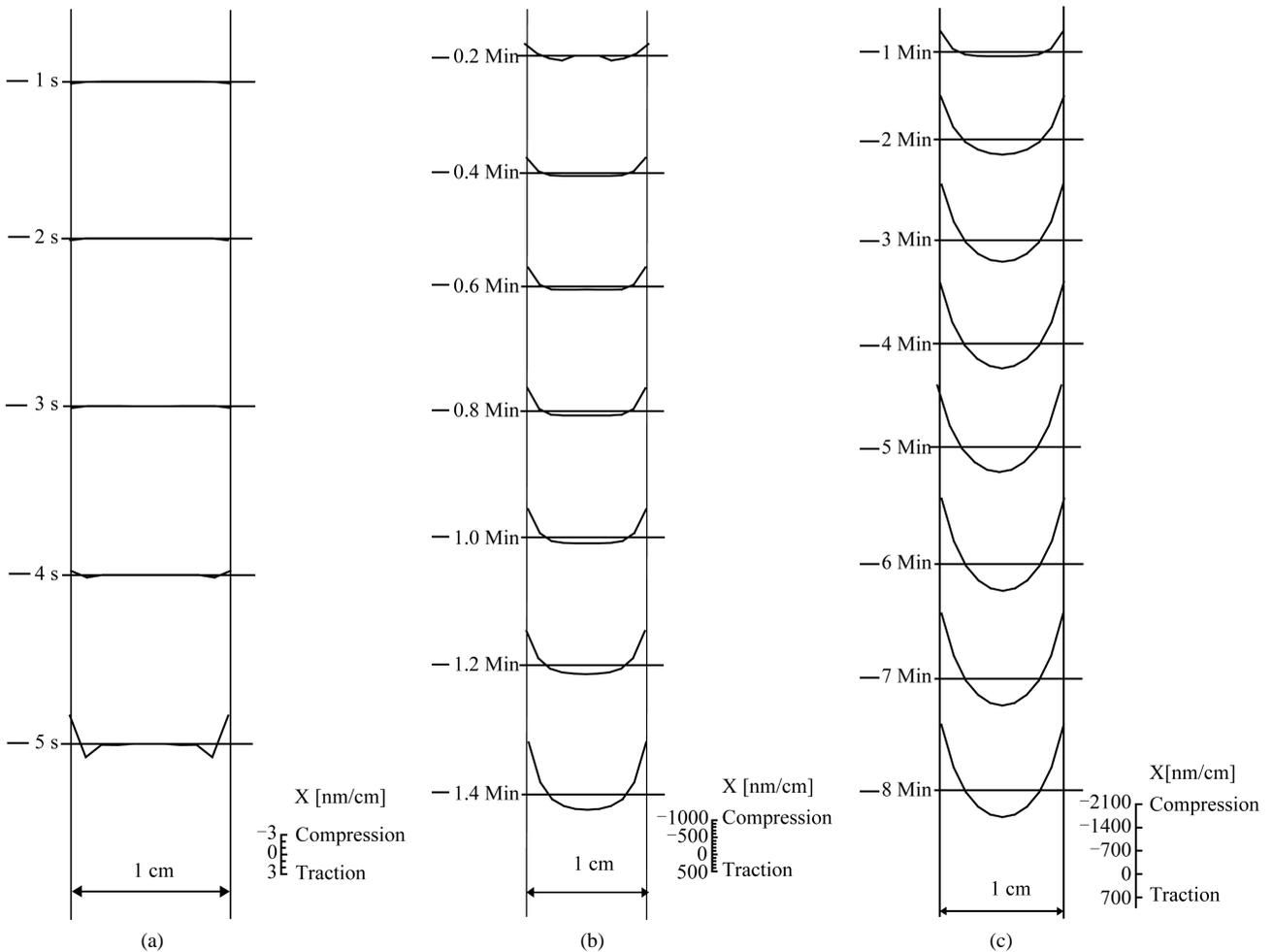


Figure 2. Stress distributions through a flat plate in soda-lime glass during cooling; (a) During 5 seconds of cooling; (b) During 1.4 minute of cooling; (c) During 8 minutes of cooling.

Table 1. The difference of temperature between the surface and the center ΔT during the cooling of a flat plate in soda-lime glass of 7.3 mm thickness from $T_0 = 677^\circ\text{C}$.

	t [s]	4	6	8	10	12	14	16
ΔT [$^\circ\text{C}$]	Calculated	83	84	86	86	87	87	86
	Naray [4]	137	152	162	161	157	152	150

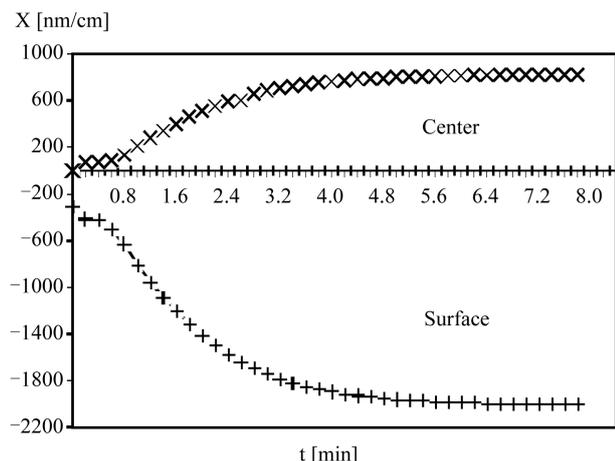


Figure 3. Internal stress variation during 8 minute of cooling through a flat plate in soda-lime glass.

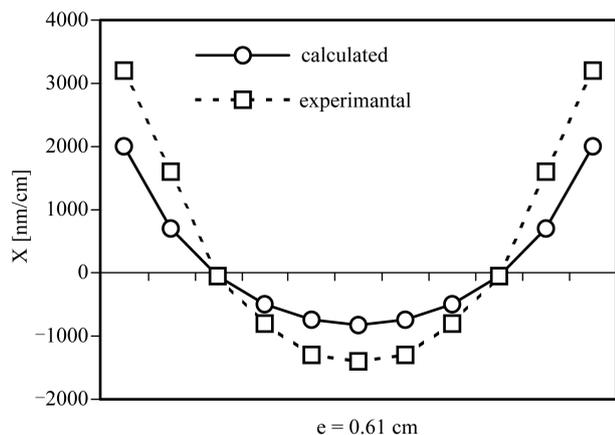


Figure 4. Comparison between calculated and measured internal stress differences for a flat plate in soda-lime glass of thickness $e = 0.61$ cm.

internal stress are influenced in the first by the value of the temperature gradient created during the cooling; therefore it is obvious that values of the permanent internal stress that we calculated must be smaller.

4. Conclusions

The numeric calculation method of the internal stress in a flat plate in soda-lime glass, its cooling has been developed on the basis of knowledge of the distribution of the calculated real temperature by Equations (2.6.1) and

(2.6.2); the reduced time and the distribution of the fictive temperature by iteration method with Equations (2.4.1), (2.4.2) and (2.4.3); the normalised relaxation modulus of stress which is used then with the reduced time, the real and fictive temperature to calculate the internal stress by Equations (2.3.1), (2.3.2) and (2.3.3).

For the precision satisfying of the internal stress values, it is sufficient to calculate the normalised relaxation modulus of a stress with a step of the time logarithmic $D = 0.25$; for the calculation of a stress we have to our disposition 35 values of this module.

During the calculation of the fictive temperature with the precision 0.01°C , it is sufficient to calculate the integral of the Equation (2.4.1) with the precision 0.1; the sufficient time step in the principal program is one second.

The calculation program proposed gives the evolution of the temporary and permanent internal stress whose concept is qualitatively exact; but quantitatively, the comparison shows that values of the calculated internal stress are smaller than the applied values, the main reason of this difference is the insufficiency of the real temperature (deficiency of the radiation component), what has for consequence the obtaining of a small temperature gradient so the internal stress.

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