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The $\exp\left(-\varphi\left(\xi\right)\right)$-Expansion Method and Its Application for Solving Nonlinear Evolution Equations

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Abstract

The $\exp\left(-\phi(\xi)\right)$-expansion method is used as the first time to investigate the wave solution of a nonlinear dynamical system in a new double-Chain model of DNA and a diffusive predator-prey system. The proposed method also can be used for many other nonlinear evolution equations.

Keywords

The $\exp\left(-\phi(\xi)\right)$-Expansion Method, Dynamical System in a New Double-Chain Model of DNA, A Diffusive Predator-Prey System, Traveling Wave Solutions, Solitary Wave Solutions, Kink-Anti Kink Shaped

1. Introduction

The nonlinear partial differential equations of mathematical physics are major subjects in physical science [1]. Exact solutions for these equations play an important role in many phenomena in physics such as uid mechanics, hydrodynamics, optics, plasma physics and so on. Recently many new approaches for finding these solutions have been proposed, for example, extended Jacobian Elliptic Function Expansion Method [2], the modified simple equation method [3], the tanh method [4], extended tended tanh-method [5]-[7], sine-cosine method [8]-[10], homogeneous balance method [11] [12], F-expansion method [13]-[15], exp-function method [16] [17],

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trigonometric function series method [18], $\left(\frac{G'}{G}\right)$-expansion method [19]-[22], Jacobi elliptic function method [23]-[26], the $\exp(-\varphi(\xi))$-expansion method [27]-[29] and so on.

The objective of this article is to apply the $\exp(-\varphi(\xi))$-expansion method for finding the exact traveling wave solution of dynamical system in a new double-Chain model of DNA and a diffusive predator-prey system which play an important role in biology and mathematical physics.

The rest of this paper is organized as follows: In Section 2, we give the description of the $\exp(-\varphi(\xi))$-expansion method. In Section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In Section 4, conclusions are given.

2. Description of Method

Consider the following nonlinear evolution equation

$$F(u, u_t, u_x, u_{tt}, u_{xx}, \cdots) = 0, \tag{2.1}$$

where F is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following,we give the main steps of this method:

Step 1. We use the wave transformation

$$u(x,t) = u(\xi), \quad \xi = x - ct, \tag{2.2}$$

where c is a positive constant, to reduce Equation (2.1) to the following ODE:

$$P(u, u', u'', u''', \cdots) = 0, \tag{2.3}$$

where P is a polynomial in $u(\xi)$ and its total derivatives,while $' = \frac{\mathrm{d}}{\mathrm{d}\xi}'$.

Step 2. Suppose that the solution of ODE (2.3) can be expressed by a polynomial in $\exp(-\varphi(\xi))$ as follows

$$u(\xi) = a_m \left(\exp(-\varphi(\xi))\right)^m + \cdots, \quad a_m \neq 0, \tag{2.4}$$

where $\varphi(\xi)$ satisfies the ODE in the form

$$\varphi'(\xi) = \exp(-\varphi(\xi)) + \mu \exp(\varphi(\xi)) + \lambda, \tag{2.5}$$

the solutions of ODE (2.5) are when $\lambda^2 - 4\mu > 0$, $\mu \neq 0$,

$$\varphi(\xi) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + C_1)\right) - \lambda}{2\mu}\right), \tag{2.6}$$

when $\lambda^2 - 4\mu > 0$, $\mu = 0$,

$$\varphi(\xi) = -\ln\left(\frac{\lambda}{\exp(\lambda(\xi + C_1)) - 1}\right), \tag{2.7}$$

when $\lambda^2 - 4\mu = 0$, $\mu \neq 0$, $\lambda \neq 0$,

$$\varphi(\xi) = \ln\left(-\frac{2(\lambda(\xi + C_1) + 2)}{\lambda^2(\xi + C_1)}\right), \tag{2.8}$$

when $\lambda^2 - 4\mu = 0$, $\mu = 0$, $\lambda = 0$,

$$\varphi(\xi)=\ln(\xi+C_1), \tag{2.9}$$

when $\lambda^2-4\mu<0$,

$$\varphi(\xi)=\ln\left(\frac{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C_1)\right)-\lambda}{2\mu}\right), \tag{2.10}$$

where $a_m, \cdots, \lambda, \mu$ are constants to be determined later,

Step 3. Substitute Equation (2.4) along Equation (2.5) into Equation (2.3) and collecting all the terms of the same power $\exp(-m\varphi(\xi))$, $m=0,1,2,3,\cdots$ and equating them to zero, we obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of a_i.

Step 4. substituting these values and the solutions of Equation (2.5) into Equation (2.3) we obtain the exact solutions of Equation (2.3).

3. Application

3.1. Example 1: Dynamical System in a New Double-Chain Model of DNA

An attractive nonlinear model for the nonlinear science in the deoxyribonucleic acid (DNA). The dynamics of DNA molecules is one of the most fascinating problems of modern biophysics because it is at the basis of life. The DNA structure has been studied during last decades. The investigation of DNA dynamics has successfully predicted the appearance of important nonlinear structures. It has been shown that the nonlinearity is responsible for forming localized waves. These localized waves are interesting because they have the capability to transport energy without dissipation [30]-[38]. In Ref. [37] [38], it is given that a new double-chain model of DNA consists of two long elastic homogeneous strands which represent two polynucleotide chains of the DNA molecule, connected with each other by an elastic membrane representing the hydrogen bonds between the base pair of the two chains. Under some appropriate approximation, the new double-chain model of DNA can be described by the following two general nonlinear dynamical system:

$$u_{tt}-c_1^2u_{xx}=\lambda_1u+\gamma_1uv+\mu_1u^3+\beta_1uv^2, \tag{3.1}$$

$$v_{tt}-c_2^2v_{xx}=\lambda_2v+\gamma_2u^2+\mu_2u^2v+\beta_2v^3+c_0, \tag{3.2}$$

where

$$\begin{gathered} c_1=\pm\frac{Y}{\rho};\quad c_2=\pm\frac{F}{\rho};\quad \lambda_1=\frac{-2\mu}{\rho\sigma h}(c-l_0); \\ \lambda_2=\frac{-2\mu}{\rho\sigma};\quad \gamma_1=2\gamma_2=\frac{2\sqrt{2}\mu l_0}{\rho\sigma h^2};\quad \mu_1=\mu_2=\frac{-2\mu l_0}{\rho\sigma h^3}; \\ \beta_1=\beta_2=\frac{4\mu l_0}{\rho\sigma h^3};\quad c_0=\frac{\sqrt{2}\mu(h-l_0)}{\rho\sigma}. \end{gathered} \tag{3.3}$$

where ρ, σ, Y and F denote respectively the mass density, the area of transverse cross-section, the Young's modulus and tension density of each strand; μ is the rigidity of the elastic membrane; h is the distance between the two strands, and l_0 is the height of the membrane in the equilibrium positive. In Equations (3.1) and (3.2), u is the difference of the longitudianl displacements of the bottom and top strands, while v is the difference of the transverse displacements of the bottom and top strands.
we first introduce the transformation

$$v=au+b, \tag{3.4}$$

where a and b are constants, to reduce Equations (3.1) and (3.2) to the following system of equations:

$$u_{tt}-c_1^2u_{xx}=u^3(\mu_1+\beta_1a^2)+u^2(2\beta_1ab+a\gamma_1)+u(\lambda_1+b\gamma_1+\beta_1b^2), \tag{3.5}$$

and

$$u_{tt}-c_2^2u_{xx}=u^3\left(\mu_2+\beta_2a^2\right)+u^2\left(\frac{\gamma_2}{a}+\frac{\mu_2b}{a}+3\beta_2ab\right)+u\left(\lambda_2+3\beta_2b^2\right)+\frac{\lambda_2b}{a}+\frac{\beta_2b^3}{a}+\frac{c_0}{a}. \tag{3.6}$$

Comparing Equations (3.5) and (3.6) and using (3.4) we deduce that $b=\frac{h}{\sqrt{2}}$ and $F=Y$. Now Equations (3.5) and (3.6) can be written as

$$u_{tt}-c_1^2u_{xx}-Au^3-Bu^2-Cu=0, \tag{3.7}$$

where

$$A=\frac{\alpha}{h^3}\left(-2+4a^2\right);\quad B=\frac{6\sqrt{2}a\alpha}{h^2};\quad C=\left(\frac{-2\alpha}{l_0}+\frac{6\alpha}{h}\right);\quad \alpha=\frac{\mu l_0}{\rho\sigma};\quad c_1^2=\frac{Y}{\rho}. \tag{3.8}$$

The wave transformation $u(x,t)=u(\xi)$, $\xi=kx+\omega t$, reduce Equation (3.7) to the following ODE:

$$\left(\omega^2-k^2c_1^2\right)u''-Au^3-Bu^2-Cu=0, \tag{3.9}$$

where $\omega^2-k^2c_1^2\neq 0$. Balancing u'' and u^3 yields, $N+2=3N\to N=1$. Consequently, we have the formal solution:

$$u=a_0+a_1\exp(-\varphi), \tag{3.10}$$

where a_0 and a_1 are constants to be determined, such that $a_1\neq 0$. It is easy to see that

$$u''=2a_1\exp(-3\varphi)+3\lambda a_1\exp(-2\varphi)+a_1\left(\lambda^2+2\mu\right)\exp(-\varphi)+a_1\lambda\mu, \tag{3.11}$$

substituting Equation (3.10) and its derivatives in Equation (3.9) and equating the coefficient of different power's of $\exp(-\varphi(\xi))$ to zero, we get

$$2a_1\left(w^2-c_1^2k^2\right)-Aa_1^3=0, \tag{3.12}$$

$$3\lambda a_1\left(w^2-c_1^2k^2\right)-3Aa_0a_1^2-Ba_1^2=0, \tag{3.13}$$

$$a_1\left(\lambda^2+2\mu\right)\left(w^2-c_1^2k^2\right)-3Aa_0^2a_1-2Ba_0a_1-Ca_1=0, \tag{3.14}$$

$$a_1\lambda\mu\left(w^2-c_1^2k^2\right)-Aa_0^3-Ba_0^2-Ca_0=0. \tag{3.15}$$

Equations (3.12)-(3.15) yields

$$a_0=a_0,\quad a_1=\sqrt{-\frac{2\left(-w^2+c_1^2k^2\right)}{A}},\quad \lambda=\lambda,\quad \mu=\frac{a_0\left(a_1\lambda-a_0\right)}{a_1^2},$$

$$C=-\frac{4a_0a_1\lambda w^2-4a_0a_1\lambda c_1^2k^2-4a_0^2w^2+4a_0^2c_1^2k^2-\lambda^2w^2a_1^2+\lambda^2c_1^2k^2a_1^2}{a_1^2},$$

$$B=-\frac{3\left(-a_1\lambda w^2+a_1\lambda c_1^2k^2+2a_0w^2-2a_0c_1^2k^2\right)}{a_1^2},\quad A=A.$$

Thus the solution is

$$u=a_0\pm\sqrt{-\frac{2\left(-w^2+c_1^2k^2\right)}{A}}\exp(-\varphi) \tag{3.16}$$

Let us now discuse the following case:

Case 1. if $\lambda^2-4\mu>0,\ \mu\neq 0.$

$$u(\xi) = a_0 \pm \sqrt{\frac{-2\left(w^2 + c_1^2 k^2\right)}{A}} \frac{2\mu}{\left[-\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + c_1)\right) - \lambda\right]} \tag{3.17}$$

Case 2. if $\lambda^2 - 4\mu > 0,\ \mu = 0.$

$$u(\xi) = a_0 \pm \sqrt{\frac{-2\left(w^2 + c_1^2 k^2\right)}{A}} \frac{\lambda}{\exp\left(\lambda(\xi + c_1)\right) - 1}. \tag{3.18}$$

Case 3. if $\lambda^2 - 4\mu = 0,\ \mu \neq 0,\ \lambda \neq 0.$

$$u(\xi) = a_0 \mp \sqrt{\frac{-2\left(w^2 + c_1^2 k^2\right)}{A}} \frac{\lambda^2 (\xi + c_1)}{2\left[\lambda(\xi + c_1) + 2\right]}. \tag{3.19}$$

Case 4. if $\lambda^2 - 4\mu = 0,\ \mu = 0,\ \lambda = 0.$

$$u(\xi) = a_0 \pm \sqrt{\frac{-2\left(w^2 + c_1^2 k^2\right)}{A}} \frac{1}{[\xi + c_1]}. \tag{3.20}$$

Case 5. if $\lambda^2 - 4\mu < 0,$

$$u(\xi) = a_0 \pm \sqrt{\frac{-2\left(w^2 + c_1^2 k^2\right)}{A}} \frac{2\mu}{\left[\sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2}(\xi + c_1)\right) - \lambda\right]}. \tag{3.21}$$

3.2. Example 2. A Diffusive Predator-Prey System

Consider a system of two coupled nonlinear partial differential equations describing the spatio-temporal dynamics of a predator-prey system [39],

$$\begin{aligned} u_t &= u_{xx} - \beta u + (1 + \beta)u^2 - u^3 - uv, \\ v_t &= v_{xx} + \kappa uv - mv - \delta v^3. \end{aligned} \tag{3.22}$$

where κ, δ, m and β are positive parameters. The solutions of predator-prey system have been studied in various aspects [39]-[41]. The dynamics of the diffusive predator-prey system have assumed the following relations between the parameters, namely $m = \beta$ and $\kappa + \frac{1}{\sqrt{\delta}} = \beta + 1$. Under there assumptions, Equation (3.22) can be rewritten in the form:

$$\begin{aligned} u_t &= u_{xx} - \beta u + \left(\kappa + \frac{1}{\sqrt{\delta}}\right)u^2 - u^3 - uv, \\ v_t &= v_{xx} + \kappa uv - \beta v - \delta v^3. \end{aligned} \tag{3.23}$$

We use the wave transformation $u(x,t) = u(\xi),\ \xi = x - ct$ to reduce Equation (3.23) to the following nonlinear system of ordinary differential equations:

$$\begin{cases} u'' + cu' - \beta u + \left(\kappa + \frac{1}{\sqrt{\delta}}\right)u^2 - u^3 - uv = 0, \\ v'' + cv' + \kappa uv - \beta v - \delta v^3 = 0, \end{cases} \tag{3.24}$$

where c is a nonzero constant.

In order to solve Equation (3.24), let us consider the following transformation

$$v = \frac{1}{\sqrt{\delta}} u \tag{3.25}$$

Substituting the transformation (3.25) into Equation (3.24), we get

$$u'' + cu' - \beta u + \kappa u^2 - u^3 = 0 \tag{3.26}$$

Balancing u'' with u^3 in Equation (3.26) yields, $N + 2 = 3N \Rightarrow N = 1$. Consequently, we get the same formal solution (3.10). Substituting Equation (3.10) and its derivatives in Equation (3.26) and equating the coefficient of different power's of $\exp(-\varphi(\xi))$ to zero, we get

$$2a_1 - a_1^3 = 0 \tag{3.27}$$

$$3a_1\lambda - ca_1 + \kappa a_1^2 - 3a_0 a_1^2 = 0 \tag{3.28}$$

$$2a_1\mu + a_1\lambda^2 - ca_1\lambda - \beta a_1 + 2\kappa a_0 a_1 - 3a_0^2 a_1 = 0 \tag{3.29}$$

$$a_1\lambda\mu - ca_1\mu - \beta a_0 + \kappa a_0^3 - a_0^3 = 0 \tag{3.30}$$

Equations (3.27)-(3.30) yields

Case 1.

$$c = \mp\frac{\sqrt{2}}{2}\kappa, \quad \beta = 2\mu - \frac{\lambda^2}{2} + \frac{\kappa^2}{4}, \quad \lambda = \lambda$$

$$a_0 = \pm\frac{\sqrt{2}}{2}\lambda + \frac{1}{2}\kappa, \quad a_1 = \pm\sqrt{2}$$

Case 2.

$$c = \mp\frac{\sqrt{2}}{2a_0}\left(-3a_0^2 + 6\mu + 2\kappa a_0\right), \quad \beta = -4\mu - \kappa a_0 + a_0^2 + \frac{4\mu^2}{a_0^2} + \frac{2\kappa\mu}{a_0}$$

$$\lambda = \pm\frac{\sqrt{2}}{2a_0}\left(a_0^2 + 2\mu\right), \quad a_0 = a_0, \quad a_1 = \pm\sqrt{2}$$

Thus the solution is

Case 1.

$$u = \pm\frac{\sqrt{2}}{2}\lambda + \frac{1}{2}\kappa \pm \sqrt{2}\exp(-\varphi) \tag{3.31}$$

Case 2.

$$u = a_0 \pm \sqrt{2}\exp(-\varphi) \tag{3.32}$$

Let us now discuss the following cases:

Case 1.

Case (1.1). if $\lambda^2 - 4\mu > 0,\ \mu \neq 0$

$$u = \pm\frac{\sqrt{2}}{2}\lambda + \frac{1}{2}\kappa \pm \frac{2\sqrt{2}\mu}{-\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + C_1)\right) - \lambda} \tag{3.33}$$

Case (1.2). if $\lambda^2 - 4\mu > 0,\ \mu = 0$

$$u = \pm\frac{\sqrt{2}}{2}\lambda + \frac{1}{2}\kappa \pm \frac{\sqrt{2}\lambda}{\exp(\lambda(\xi + C_1)) - 1} \tag{3.34}$$

Case (1.3). if $\lambda^2 - 4\mu = 0,\ \mu \neq 0,\ \lambda \neq 0$

$$u = \pm\frac{\sqrt{2}}{2}\lambda + \frac{1}{2}\kappa \mp \frac{\sqrt{2}\lambda^2(\xi + C_1)}{2(\lambda(\xi + C_1) + 2)} \tag{3.35}$$

Case (1.4). if $\lambda^2 - 4\mu = 0,\ \mu = 0,\ \lambda = 0$

$$u = \pm\frac{\sqrt{2}}{2}\lambda + \frac{1}{2}\kappa \pm \frac{\sqrt{2}}{\xi + C_1} \tag{3.36}$$

Case (1.5). if $\lambda^2 - 4\mu < 0$

$$u = \pm\frac{\sqrt{2}}{2}\lambda + \frac{1}{2}\kappa \pm \frac{2\sqrt{2}\mu}{\sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2}(\xi + C_1)\right) - \lambda} \tag{3.37}$$

Case 2.

Case (2.1). if $\lambda^2 - 4\mu > 0,\ \mu \neq 0$

$$u = a_0 \pm \frac{2\sqrt{2}\mu}{-\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + C_1)\right) - \lambda} \tag{3.38}$$

Case (2.2). if $\lambda^2 - 4\mu > 0,\ \mu = 0$

$$u = a_0 \pm \frac{\sqrt{2}\lambda}{\exp(\lambda(\xi + C_1)) - 1} \tag{3.39}$$

Case (2.3). if $\lambda^2 - 4\mu = 0,\ \mu \neq 0,\ \lambda \neq 0$

$$u = a_0 \mp \frac{\sqrt{2}\lambda^2(\xi + C_1)}{2(\lambda(\xi + C_1) + 2)} \tag{3.40}$$

Case (2.4). if $\lambda^2 - 4\mu = 0,\ \mu = 0,\ \lambda = 0$

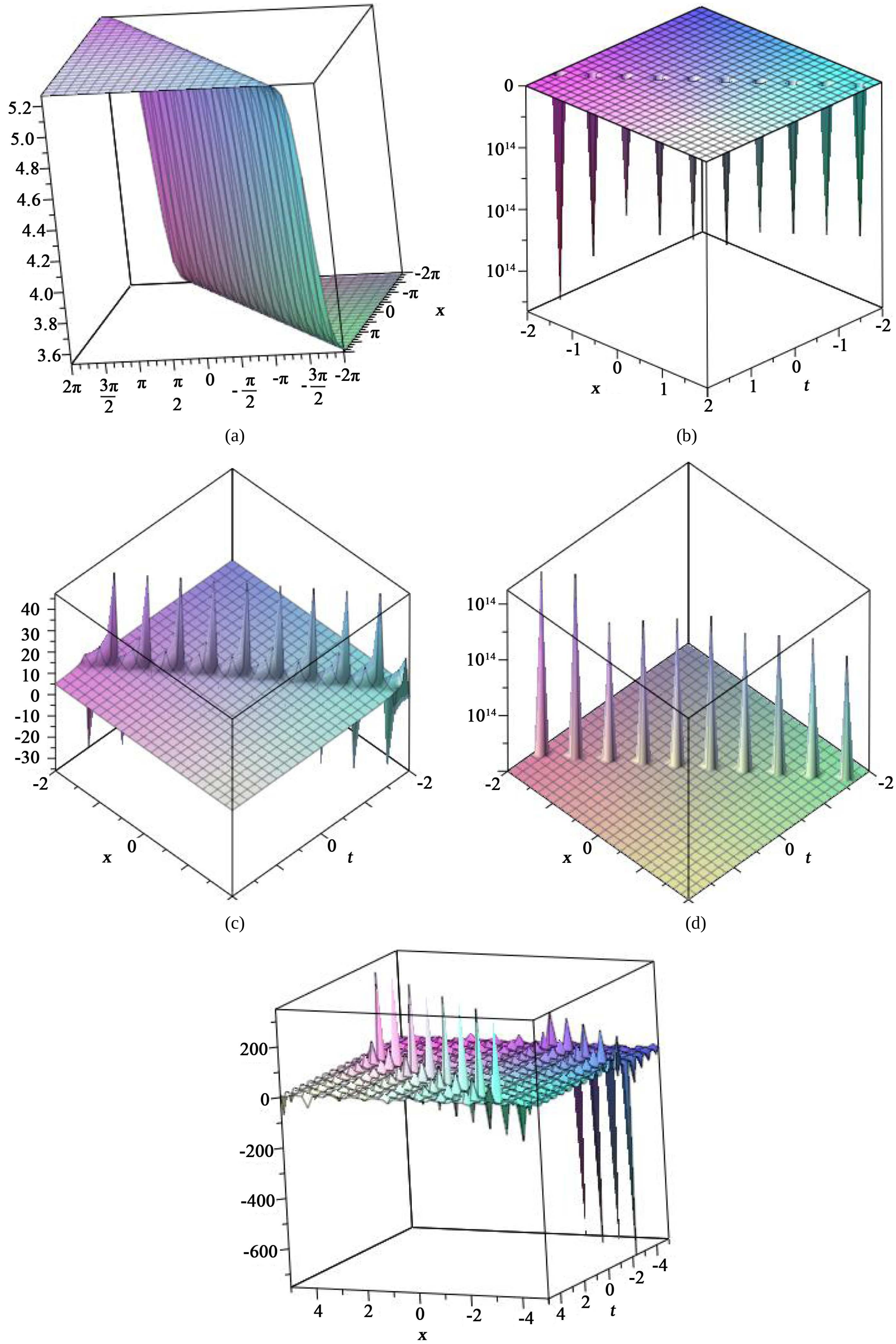
$$u = a_0 \pm \frac{\sqrt{2}}{\xi + C_1} \tag{3.41}$$

Case (2.5). if $\lambda^2 - 4\mu < 0$

$$u = a_0 \pm \frac{2\sqrt{2}\mu}{\sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2}(\xi + C_1)\right) - \lambda} \tag{3.42}$$

4. Conclusion

We establish exact solutions for the dynamics of DNA molecules is one of the most fascinating problems of modern biophysics because it is at the basis of life. The DNA structure has been studied during last decades. The investigation of DNA dynamics has successfully predicted the appearance of important nonlinear structures and a system of two coupled nonlinear partial differential equations describing the spatio-temporal dynamics of a predator-prey system where the prey per capita growth rate is subject to the Allee effect. The $\exp(-\varphi(\xi))$-expansion method has been successfully used to find the exact traveling wave solutions of some nonlinear evolution equations. As an application, the traveling wave solutions for Dynamical system in a new Double-Chain Model of DNA and a diffuusive predator-prey system, which have been constructed using the $\exp(-\varphi(\xi))$-expansion method. Let us compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of Dynamical system in a new Double-Chain Model of DNA and a diffusive predator-prey system, are new and different from those obtained in [37]-[41] and **Figure 1** and **Figure 2** show the solitary traveling wave solution of Dynamical system in a new



(a) (b) (c) (d) (e)

Figure 1. Solution of Equations (3.17)-(3.21). (a) Equations (3.17); (b) Equations (3.18); (c) Equations (3.19); (d) Equations (3.20); (e) Equations (3.21).

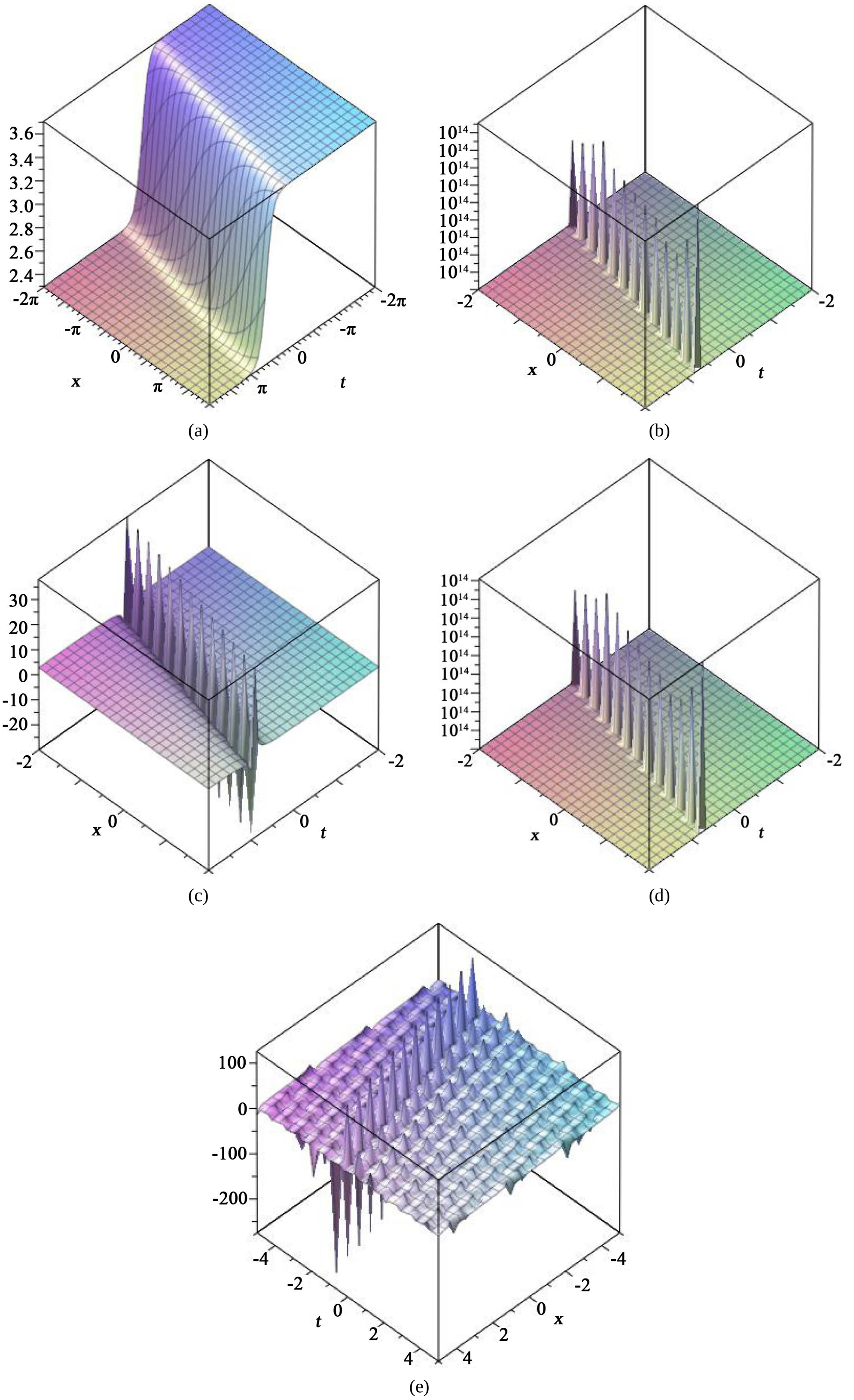


Figure 2. Solution of Equations (3.38)-(3.42). (a) Equations (3.38); (b) Equations (3.39); (c) Equations (3.40); (d) Equations (3.41); (e) Equations (3.42).

Double-Chain Model of DNA and a diffusive predator-prey system. It can be concluded that this method is reliable and proposes a variety of exact solutions NPDEs. The performance of this method is effective and can be applied to many other nonlinear evolution equations.

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