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What Is the Natural Weight of the Current Old?*

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ABSTRACT

We consider a simple overlapping generations model with an externality à la Arrow-Romer [1,2] and a government with fiscal powers. If it wishes to maximize a criterion depending on the lifelong utility of agents, is there a natural weight for the utility of the current old? We show in a simple example that this weight depends on the specific features of the model, in particular the length of the horizon, and cannot be chosen arbitrarily. Our result has a neat economic interpretation [2].

Keywords: Overlapping Generations Model; Learning-by-Doing; Social Welfare

1. Introduction

The overlapping generations model of Allais [3], Samuelson [4,5] and Diamond [6] is ideally suited to the exploration of inter-generational issues. For a more recent and thorough presentation of these models, see the classic reference: De la Croix and Michel [7].

Most models assume that agents live for two periods; the training of the young may or may not be explicitly modeled. Steady states, temporary equilibria and intertemporal equilibria are studied and a role for government intervention appears naturally if market imperfections such as externalities are present. Public finance issues can also be considered.

As it is assumed that agents live for two periods, an objective measure of time is inherent to the model. Thus the length of one period can be taken to be around 30 years. This observation has deep consequences for the interpretation of policy recommendations.

The traditional approach of the literature on overlapping generations models has been to consider the welfare of all generations from the present onward (De La Croix and Michel [7], pp. 91-93). In such models, and with a separable utility function, the planner's criterion is

$$W_{\infty} = \sum_{t=0}^{\infty} \gamma^t \left(u(c_t) + \frac{\beta}{\gamma} u(d_t) \right),$$

where c_t and d_t refer to the consumption of young and old in period t respectively; β is their subjective rate of time preference and γ is the planner's social discount rate.

Economics is a moral science. Welfare economics should be a central part of the discipline (Atkinson [8], p. 192). Since there is usually more than one adult generation at any on time, one may reasonably ask—whose welfare function is it? Are we saying to 50-year-old that their welfare is judged by their 75-year-old parents? Or the reverse? If the reverse, when does the baton pass? The uneasiness surrounding this construction is apparent when we consider the issue of the rate at which future utility is discounted[1] (see Atkinson [7], p. 195). Bernheim [9] also mentions that many of the usual concepts of welfare are difficult to implement, since the concerned individuals do not necessarily have the opportunity to vote for them.

Here, our argument is that if government intervention is warranted, the proposed policies should be acceptable to people who are alive at the time. Policies that optimize over the very long run but lower the welfare of the current generations—compared with the status quo of no intervention at all—have little chance of being adopted. Hu [10] (p. 283) was well aware of this point. To sharpen

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[1]Note that Atkinson [8] is talking here about the discount rate applied to utility, not to the rate at which future consumption is discounted, which takes account of differences in how well-off future generations will be.

our argument, we use the simplest criterion that reflects this notion, namely the lifelong utility of people alive in period 0[2]. (The welfare function can be made to include more generations as we discuss later.) In the usual notation, it is

$$W_1 = \frac{\beta}{\gamma} u(d_0) + u(c_0) + \beta u(d_1),$$

where β and γ are exogenous parameters.

The purpose of this note is to show that, when a specific horizon is adopted, there is a natural endogenous value for β/γ (which we denote by η for simplicity of notation) that is consistent with efficient government intervention and cannot be arbitrary, contrary to the traditional approach of letting both β and γ be exogenous. *Other horizons produce different specific results but the main conclusion remains the same*: η *cannot be chosen arbitrarily*.

We begin with a standard Diamond-like model with an externality of the learning-by-doing type (Arrow [1], Sheshinski [11] and Romer [2]). The external effect is that the aggregate stock of capital has a beneficial effect on the efficiency of production by each firm. It can be internalized through government intervention in the form of fiscal policies that subsidize capital investment and are financed by a tax on labor.

2. The Individuals

Individuals live for two periods: in the first period they consume, save and inelastically supply one unit of labor. In the second period they live off the revenue from their savings. There is no population growth, $L_t = L, \forall t$ The consumptions of young and old in period *t* are, respectively, c_t and d_t; s_t is savings; θ_t is the rate of tax on wage income w_t; $(1-\theta_t)w_t$ is the net wage rate and R_t is the rent of capital. The individual's subjective discount factor is $\beta < 1$. An individual born in period *t* solves the following program:

$$\max_{c_t, d_{t+1}} u(c_t) + \beta u(d_{t+1})$$

$$c_t + s_t = (1-\theta_t) w_t$$

$$d_{t+1} = R_{t+1} s_t$$

The optimality condition is

$$u'(c_t) = \beta u'(d_{t+1}). \tag{1}$$

3. The Firm

The production function exhibits constant returns to scale to the factors hired by the firm but there is an externality. The firm hires l_t units of labor and produces q_t units of good with k_t units of capital; $B(K_t)$ is a productivity factor that represents the externality where K_t is the aggregate stock of capital; however the firm is unaware of the structure of this productivity factor.

$$q_t = B(K_t) F(k_t, l_t),$$

where $F(.,.)$ is homogeneous of degree one; therefore

$$q_t = l_t B(K_t) F(k_t / l_t, 1).$$

Normalizing the size of the firm at $l_t = 1$, we have, in the usual notation:

$$\begin{aligned} q_t &= B(K_t) f(k_t) \\ &= B(Lk_t) f(k_t), \end{aligned} \tag{2}$$

The firm maximizes profit

$$\begin{aligned} \pi(k_t) = B(Lk_t) f(k_t) \\ -w_t - (R_t - \tau_t) k_t, \end{aligned}$$

where τ_t is a government subsidy designed to internalize the externality. Therefore

$$R_t = \tau_t + B(Lk_t) f'(k_t), \tag{3}$$

$$w_t = B(Lk_t)\left[f(k_t) - f'(k_t) k_t \right]. \tag{4}$$

An informed government that wishes to internalize the externality in each period *t* would choose the efficient subsidy as

$$\tau_t = LB'(Lk_t) f(k_t),$$

in order to account for the role capital plays in enhancing overall productivity.

Capital depreciates entirely in one period, hence the dynamics of the economy are given by

$$k_{t+1} = s_t.$$

4. The Government

The government is responsible for implementing the planner's policies and has fiscal authority. The government balances its budget in each period, thus the constraint on its fiscal policy (θ_t, τ_t) is,

$$\theta_t w_t = \tau_t k_t, \tag{5}$$

where θ_t is the rate of payroll tax and τ_t is the subsidy to capital. Therefore we shall be able to express one of the tax/subsidy parameters in terms of the other. Together the wage rate equation, the efficient subsidy and the budget constraint in period *t* yield an expression for the payroll tax θ_t.

$$\begin{aligned} \theta_t w_t &= \theta_t B(Lk_t)\left[f(k_t) - f'(k_t) k_t \right] = \tau_t k_t \\ &= LB'(Lk_t) f(k_t) k_t, \end{aligned}$$

[2]The utility of the young born in period -1 (hence those who are old in period 0) can be included, but treated as exogenous, therefore it is irrelevant. See for instance De la Croix and Michel [7], p. 91.

$$\begin{aligned}\theta_t &= \frac{LB'(Lk_t)f(k_t)k_t}{B(Lk_t)\left[f(k_t)-f'(k_t)k_t\right]} \\ &= \frac{K_tB'(K_t)}{B(K_t)}\frac{1}{1-f'(k_t)k_t/f(k_t)}.\end{aligned} \tag{6}$$

This expression for θ_t can be interpreted as follows. It is the ratio of the elasticity of the externality factor with respect to aggregate capital over 1 minus the elasticity of the firm's output with respect to its capital stock. Clearly, with a general CRS production-function and an arbitrary externality effect, this ratio depends on current capital stock and varies over time. The dynamic path of θ_t is nonetheless constrained.

At this stage we must emphasize the following point in order to show the essential nature of our argument: *The logic consequence of three elements*, *a competitive wage*, *an efficient capital subsidy and a balanced budget in every period*, *by itself determines the dynamic structure of fiscal policies. This is done without any reference to the planner*'*s objective.*

In this extensive literature (Samuelson, [4,5], Lerner, [12,13], De La Croix and Michel, [7]) the criterion is often an infinite horizon welfare function or sometimes, more simply, the steady state outcome. These views have the advantage of supplying clear answers to real problems in an abstract world. However, steady state criteria and infinite horizon optimal paths suffer from an implementation problem, namely that policies designed to maximize such criteria may entail losses of utility for several generations, including those alive at the time of planning (See Gaumont& Leonard, [14], for some compelling evidence). We argue that such policies stand little chance of being implemented *and therefore we look for a more feasible criterion*.

Here we assume that the government objective is simply to maximize the lifelong utility of people who are alive at the time (see footnote 2). Therefore, it takes into account the utility of the old and young in period zero, plus the utility of those who will be old in period one. In the traditional formulation the exogenous weight of the old is $\eta \equiv \beta/\gamma$, the ratio of the subjective rate of time preference of households and the planner's social discount factor. Our purpose is to show that, for the optimal choice of θ_t to be consistent with an efficient fiscal policy, the weight of the current old must take on a natural value that depends on the specific features of the model and the length of the horizon.

In order to make our argument simple and concise we show by counterexample that the value of η cannot be exogenous, even in a very standard version of the model.

5. A Simple Case

We use a logarithmic utility and a Cobb-Douglas production with $B(K_t)=K_t^\sigma$ where $0<\sigma<1$ measures the strength of the externality[3]. Therefore conditions (1), (2), (3), and (4) become, respectively,

$$d_{t+1}=\beta R_{t+1}c_t,$$

$$q_t = L^\sigma k_t^\sigma k_t^\alpha,$$

$$R_t=\tau_t+\alpha L^\sigma k_t^{\sigma+\alpha-1},$$

$$w_t=(1-\alpha)L^\sigma k_t^{\sigma+\alpha}.$$

We use (5) to eliminate τ_t from our calculations and obtain

$$d_0=\left[\alpha+(1-\alpha)\theta_0\right]L^\sigma k_0^{\sigma+\alpha}, \tag{7}$$

$$c_0=\frac{1-\alpha}{1+\beta}(1-\theta_0)L^\sigma k_0^{\sigma+\alpha},$$

$$d_1=\beta R_1 c_0,$$

$$d_1=\frac{\beta R_1}{1+\beta}(1-\theta_0)(1-\alpha)L^\sigma k_0^{\sigma+\alpha},$$

with

$$\begin{aligned}R_1 &= \left[\alpha+(1-\alpha)\theta_1\right] \\ &\cdot L^\sigma\left[\frac{\beta}{1+\beta}(1-\theta_0)(1-\alpha)L^\sigma k_0^{\sigma+\alpha}\right]^{\alpha+\sigma-1}\end{aligned} \tag{8}$$

The planner's objective is to maximize a welfare function that incorporates the utility of the current old and the lifetime utility of the current young,

$$W=\eta\ln d_0+\ln c_0+\beta\ln d_1 \tag{9}$$

where η is, for the time being, treated as exogenous.

For the chosen production function the equation that dictates the dynamicstructure of fiscal policies (6) simplifies to

$$\theta_t=\frac{\sigma}{1-\sigma}\equiv\theta., \tag{10}$$

Therefore in this specific model an efficient θ_t is constant over time. This is because the two elasticities are constant due to the Cobb-Douglas production function and the power function form of the externality factor. *This is so*, *irrespective of the planner*'*s choice of welfare function*[4].

Turning now to the planner's choice of an optimal fiscal policy to characterize the θ value that maximizes (9) and using the solved consumption Equations (7) and (8), the first-order condition is:

[3]This is a very simplified version of Gaumont and Leonard [14], which addresses the question of knowledge transmission among generations.

[4]Note that this is a convenient result as it guarantees inter-temporal consistency, were the planner to redo this exercise in the next period and thereafter.

$$\frac{(\beta+\eta)(1-\alpha)}{\alpha+(1-\alpha)\theta}=\frac{1+\beta(\sigma+\alpha)}{1-\theta}.$$

Therefore the welfare-maximizing θ value is

$$\theta^*=\frac{\beta\left[1-\alpha(1+\alpha+\sigma)\right]-\alpha(1+\eta)}{(1-\alpha)\left[1+\beta(\alpha+\sigma)+\beta+\eta\right]}. \quad (11)$$

This welfare-maximizing tax depends on the values of the parameters of the problem, including the weight of the old generation, η, which has so far been treated as exogenous. Given θ^*, all the consumption variables can be calculated and a complete solution obtained.

Finally, combining the dynamic fiscal structure (10) and the planner's choice of policy (11) we find that there is only one value of η that is consistent with both. It is therefore endogenous to the model and depends, among other things, on the strength of the externality, σ. We denote it by η_σ:

$$\eta_\sigma=\frac{(\alpha+\sigma)\left(1+(\alpha+\sigma)\beta\right)}{1-\alpha-\sigma}-\beta. \quad (12)$$

We insist that our argument does not depend on the existence of an externality, although it can accommodate it. In order to make our point sharper, we now look at the special case of no externality when there is no need for government intervention as (10) makes clear, and the natural η value is

$$\eta_0=\frac{\alpha(1+\alpha\beta)}{1-\alpha}\beta. \quad (13)$$

The expression in (13) is always less than 1 for sensible values of $\alpha\in[0,1/2]$. There is a similar result for (12) but $\alpha+\sigma\in[0,1/2]$ cannot be assumed.

Proposition: *The natural weight of the current old cannot be exogenous but depends on the specific features of the model* (*including the length of the planning horizon*).

With discount rates of 1% - 2% p.a. compounded over a generation and a capital share of around 1/3, the values of η_0 are around 10%. When, and if, there is an externality, $\sigma>0$, a government that wishes to optimize (9) and impose an efficient tax/subsidy scheme in a competitive economy must choose a weight for the old of the current generation as given by (12). This weight, and only this weight, will insure that welfare is maximized and that the tax/subsidy scheme is efficient.

When there is no externality, there is no role for government, tax or subsidy. Therefore, η_0—which is the only weight consistent with no taxes—is the weight which the central planner must give to the current old if its plans are to coincide with the competitive outcome. The value of η (or, equivalently γ) cannot be determined exogenously.

This makes clear the meaning of our result in the simplest terms: a short-term horizon, coupled with an efficient tax/subsidy fiscal policy, cannot use an arbitrary exogenous weight for the old generation.

The expressions in (12) and (13) clearly depend on the features of the model as well as on the length of the horizon selected by the government. Possible extensions and alternatives have been explored. Additional calculations using (12), and in the simplest case(13), shows that the natural value of η can vary enormously. Cases when W also includes terms such as $\ln(c_1)$ and $\ln(d_2)$ yield more complicated expressions. Another type of production function such as $q_t=Ak_t+B_tl_t$, with $B_t=L^\sigma k_t^\sigma$ also yields simple results. A model with a different production function might require more complex calculations but the constraint on the dynamic structure of θ_t given by (6) would still need to be accommodated with the optimal choice of θ_t.

6. Conclusion

In a simple two-period overlapping generations model with an externality (*à la* Arrow-Romer [1,2]), when the government has the power to tax the wage of the young, we have shown that the "natural" value of the weight of the current old—the value of the weight that reconciles the maximization of the chosen welfare function with the use of the efficient externality-correcting fiscal policy—is endogenous to the model and depends on the strength of the externality as well as on the government's chosen criterion. It is also true when the external effects are non-existent. It is not possible to choose both the subjective rate of time preference of households and the planner's social discount factor arbitrarily. The choice of the value of the weight of the current old crucially depends on the length of the social planner's horizon.

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