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Investigation of the Mechanism of Tangent Bifurcation in Current Mode Controlled Boost Converter

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Abstract

Tangent bifurcation is a special bifurcation in nonlinear dynamic systems. The investigation of the mechanism of the tangent bifurcation in current mode controlled boost converters operating in continuous conduction mode (CCM) is performed. The one-dimensional discrete iterative map of the boost converter is derived. Based on the tangent bifurcation theorem, the conditions of producing the tangent bifurcation in CCM boost converters are deduced mathematically. The mechanism of the tangent bifurcation in CCM boost is exposed from the viewpoint of nonlinear dynamic systems. The tangent bifurcation in the boost converter is verified by numerical simulations such as discrete iterative maps, bifurcation map and Lyapunov exponent. The simulation results are in agreement with the theoretical analysis, thus validating the correctness of the theory.

Keywords: Tangent Bifurcation, Discrete Iterative Map, Boost Converter, Continuous Current Mode (CCM)

1. Introduction

In recent years, ones are quite interested in chaos exhibited in the field of power electronics. They are becoming the hot spots of the study in the field. DC-DC converters are a kind of strong nonlinear system. They exhibit various bifurcation and chaos behavior under some operating conditions, such as period-doubling bifurcation [1-5], Hopf bifurcation [6-8], border collision bifurcation [9-11], tangent bifurcation [12,13] and chaos behavior [14-20]. Bifurcation is a complex structure in nonlinear system. The chaos is characteristic of non-repeat, uncertainty and is extreme sensitive to initial conditions. These nonlinear phenomena make the nonlinear dynamic characteristics of DC-DC converter more complex. Deep investigation of these nonlinear phenomena is of great benefit to understanding the nonlinear behavior and practical design.

Up to now, most published papers are mainly about the period-doubling bifurcation in DC-DC converters. The tangent bifurcation, which is a special bifurcation, has been less investigated. The most studies of tangent bifurcation mainly focus on the numerical simulation modeling. The main approaches used for simulation include bifurcation diagram, Lyapunov exponent. The two methods are characteristics of simpleness and intuition, but the main shortcoming of that is large computing quantity, time consuming and blindness. The essential mechanism causing tangent bifurcation was not analyzed in these simulation methods. However, no rigorous attempts have been made to analyze formally the essential mechanism leading to the tangent bifurcation in DC-DC converters.

Boost converters are a kind of important converters with wide applications. Current mode control, being one of the most commonly used control schemes in DC-DC converters, has received much attention to power electronics engineers. Although the work in [12] gives no theoretical insights into the underlying cause of tangent bifurcation in such system, it does prompt the important question of what mechanism may give rise to tangent bifurcation behavior. This paper attempts to answer to this question in the light of the theories of nonlinear dynamic systems. The investigation of the mechanism of the tangent bifurcation in current mode controlled boost converters operating in continuous conduction mode (CCM) is deeply studied. In fact, there are strict stability criteria and the conditions leading to the tangent bifurcation in mathematics based on the theories of nonlinear dynamic systems [13,14]. Based on the tangent bifurcation theorem, the conditions leading to the tangent bifurcation in the discrete iterative model of the boost converter are demonstrated mathematically. Discrete iterative maps, bifurcation diagram, Lyapunov exponent are done to analyze the mechanism and evolution of leading to the tangent bifurcation. The simulation results are in agreement with the theoretical analysis, thus validating the correctness of the theory. The methods proposed in the paper can also be suitable to analysis of the tangent bifurcation and chaos of other kinds of converter circuits.

2. Discrete Iterative Map of a Boost Converter

In **Figure 1**, the circuit model of a boost converter is shown, which consists of a switch S, a diode D, a capacitor C, an inductor L and the load resistor R connected in parallel with the capacitor. The assumptions are made as follows:

1) The boost converter operates in continuous conduction mode.

2) All the components in the boost converter circuit are ideal, no parasitic effects are considered.

Hence, there are two circuit states depending on whether S is closed or open. Assume that the circuit is at the switch state 1 when the switch S is off and diode D is on, and at the switch state 2 when S is on and D is off. The two switch states toggle periodically.

The boost converter is controlled under the current mode. Switch S is controlled by a feedback path that consists of a flip-flop and a comparator. The comparator compares the inductor current i_L with a reference current I_{ref}. The switch is triggered to ON when the clock pulse is received and is triggered to OFF when the inductor current reaches the reference current I_{ref}. Specifically, switch S is turned on at the beginning of each cycle, *i.e.* at $t=nT$, where n is an integer, T is the switching period. The inductor current i_L increases linearly while switch S is on. As i_L approaches to the value of I_{ref}, switch S is turned off, and remains off until the next cycle begins.

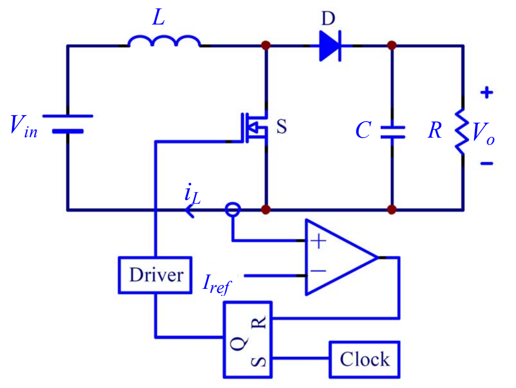

Figure 1. Circuit configuration of current-mode boost converter.

When the switch S closed, diode D is reverse biased. **Figure 2** shows the inductor current waveform. The circuit parameters of the boost converter are listed in **Table 1**.

Let x denote the state vector of the circuit, *i.e.*,

$$x = \begin{bmatrix} v_c \\ i_L \end{bmatrix} \quad (1)$$

where v_C is the voltage across the capacitor and i_L is the current through the inductor.

The state equation for the circuit in any switch state can be written in the form of

$$\dot{x} = A_i x + B_i V_{in} \quad (2)$$

where A_i and B_i are the system matrices in switch state i, and V_{in} is the input voltage. In switch state 1, we have

$$A_1 = \begin{bmatrix} \frac{1}{RC} & 0 \\ 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}$$

And in switch state 2, we have

$$A_2 = \begin{bmatrix} \frac{-1}{RC} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}$$

The switch S is turned off when the inductor current i_L reaches reference current I_{ref}. The closed-state time t_n can be obtained from (2) by integration, therefore the closed-state time t_n is calculated by the Equation (3).

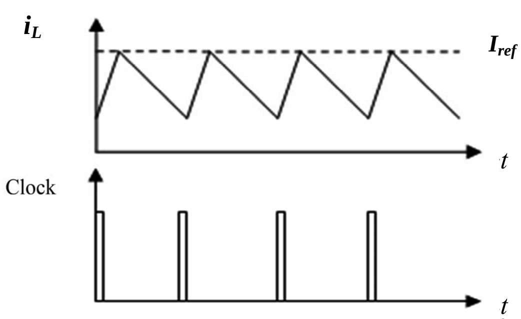

Figure 2. Inductor current waveform.

Table 1. Circuit parameters.

Circuit Components	Values
Switching period T	100 µs
Input Voltage V_{in}	10 V
Load Resistor R	20 Ω
Inductor L	1 mH
Capacitor C	12 µF
Reference Current I_{ref}	0.5~5.5 A

$$t_n = \frac{L}{V_{in}}(I_{ref} - i_n) \quad (3)$$

Subscript n denotes the value at the beginning of the nth cycle, *i.e.*, $i_n = i(nT)$, $v_n = v(nT)$.

The capacitor voltage corresponding to instant t_n is calculated by the following equation

$$v_C(t_n) = v_n e^{-\frac{t_n}{RC}} \quad (4)$$

The discrete iterative model of the boost converter can be derived as follows from the two cases, *i.e.*, $t_n \geq T$ and $t_n < T$.

Case 1. $t_n \geq T$. It means that the converter is in switch state 1 during a switching period T. The instantaneous value of i_n and v_n at next clock instant, i_{n+1} and v_{n+1}, can be calculated with i_n and v_n as initial values.

$$i_{n+1} = i_n + \frac{V_{in}}{L}T \quad (5)$$

$$v_{n+1} = v_n e^{-\frac{T}{RC}} \quad (6)$$

Case 2. $t_n < T$. It means that the converter is switched from switch state 1 to switch state 2 during a switching period T. The instantaneous value of i_n and v_n at next clock instant, i_{n+1} and v_{n+1}, can be calculated with I_{ref} and $v_n e^{-\frac{t_n}{RC}}$ as initial values.

The solution depends on the parameters of circuit values of R、L and C. From **Table 1**, we have $1 - \frac{4R^2C}{L} < 0$. In this case, the solutions of the characteristic equation corresponding to the switch state 2 are a pair of complex conjugate roots. It leads to a damped oscillatory process. Hence, the discrete iterative maps of the boost converter can be derived

$$i_{n+1} = e^{-kt_{n1}}\left[A_1 \sin\beta t_{n1} + A_2 \cos\beta t_{n1}\right] + \frac{V_{in}}{R} \quad (7)$$

$$v_{n+1} = V_{in} - Le^{-kt_{n1}}\left[B_1 \sin\beta t_{n1} + B_2 \cos\beta t_{n1}\right] \quad (8)$$

where,

$$k = \frac{1}{2RC}, t_{n1} = T\left[1 - \frac{t_n}{T}\right], \beta = \frac{1}{2RC}\sqrt{\frac{4R^2C}{L} - 1},$$

$$A_2 = I_{ref} - \frac{V_{in}}{R}, A_1 = \frac{V_{in} - v_n e^{-2kt_n} + kLA_2}{L\beta},$$

$$B_1 = \frac{kv_n e^{-2kt_n} - kV_{in} - A_2 / C}{\beta}, \quad B_2 = V_{in} - v_n e^{-2kt_n}$$

From (5-8), the discrete time values of x at $t=nT$ for all n can be obtained. The bifurcation diagram of the boost converter with reference current I_{ref} as parameter is shown in **Figure 3**, the horizontal direction is the reference current I_{ref} which is between 0.5 A and 5 A, the vertical direction is the state variable i_L which ranges from 0.5 A and 5 A. The bifurcations, subharmonics and chaotic behavior are indicated in the diagram. As shown in **Figure 3**, the boost converter goes through period-1, period-2 and eventually exhibits chaos. The period-1 solution is stable until I_{ref}= 1.7059 A whereupon a period doubling bifurcation takes place. The converter eventually goes to chaos when I_{ref} = 2.7 A. It can be interestingly observed that a small periodic window, which also exhibits period doubling cascade, is embedded in the chaos region. In the periodic widow, the converter experiences period-3 to period-6 and so on just above I_{ref} = 4.791 A. The phenomenon that system transits from chaos to period-3 is known as tangent bifurcation.

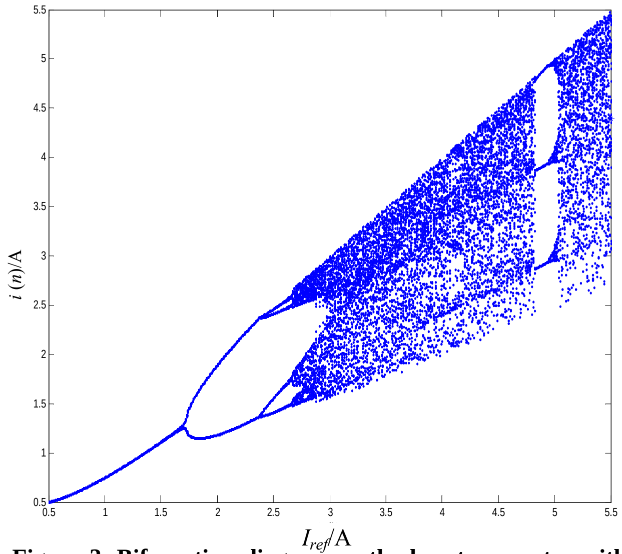

Figure 3. Bifurcation diagram of the boost converter with I_{ref} as parameter.

In **Figure 4**, the larger of the Lyapunov exponents is plotted as a function of the parameter I_{ref} over the same range as in **Figure 3**. It is well known that the presence of chaos is signaled by positive Lyapunov exponent. A negative Lyapunov exponent is characteristic of dissipative (non-conservative) systems, which exhibit point stability. A Lyapunov exponent of zero is characteristic of a cycle-stable system. In this case, the orbits maintain their separation. The tangent bifurcation will be happened when the Lyapunov exponent is changed from the started positive value to zero then to negative value. At $I_{ref} = 1.7059\,\text{A}$, where the fixed point changes from attracting to repelling and an attracting periodic orbit is born, the Lyapunov exponent is 0. Just above $I_{ref} = 2.7\,\text{A}$, the Lyapunov exponent is positive, which means that the system is chaotic. This is the same range in which the bifurcation diagram given in **Figure 3** showed a whole interval. For larger values of I_{ref}, above 4.791 A, there is another short parameter interval in which there is an attracting period-3 orbit and the Lyapunov exponent is negative. Therefore, the tangent bifurcation will be happened.

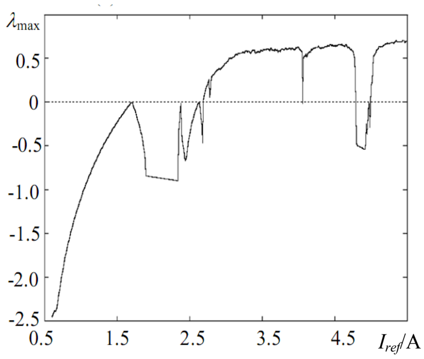

Figure 4. Larger Lyapunov exponent diagram.

3. The Conditions Leading to Tangent Bifurcation

3.1. A Theorem of Tangent Bifurcation

The theorem of tangent bifurcation is briefly reviewed in this section.

Consider the discrete-time nonlinear system

$$x = f(x, \mu) \tag{9}$$

where x is the system variable and μ is a parameter.

A point x^* is called a fixed point or a stationary point if $x^* = f(x^*, \mu^*)$.

It is convenient to have a notation for these functions. We write $f^0(x) = x$ for the 0th iterate that is the identity, $f^1(x)$ for $f(x)$, and $f^2(x)$ for the composition of f with f, that is $f^2(x) = f(f(x))$. Continuing by induction, we obtain $f^{(n)}(\mu, x) = f(f^{n-1}(x))$, is the composition of f with itself n times. Using this notation, for the initial condition x_0, $x_1 = f(x_0)$, $x_2 = f^2(x_0)$, and $x_n = f^n(x_0)$.

Theorem 1 [13,14] (Tangent Bifurcation). Assume that f is a C^2 function from R^2 to R. We write $f(x, \mu) = f_\mu(x)$. Assume that there is a bifurcation value μ^* that has a fixed point x^* with derivative equal to one

1). $f(x^*, \mu^*) = x^*$

2). $f'_{\mu^*}(x^*) = 1$

3). The second derivative $f''_{\mu^*}(x^*) \neq 0$, so the graph of $f_{\mu*}$ lies on one side of the diagonal for x near x^*.

4). The graph of f_μ is moving up or down as the parameter μ varies, or more specifically,

$$\frac{\partial f}{\partial \mu}(x^*, \mu^*) \neq 0$$

The tangent bifurcation takes place in the nonlinear system at the fixed point (x^*, μ^*),

3.2. Derivation of One-Dimensional Discrete Iterative Map

The research of tangent bifurcation should be start from one-dimensional discrete iterative map [12,13]. With one state vector be fixed, reduction of dimension can be done in the boost converter so that the boost converter is transformed into one-dimensional dynamic system. In this study, the capacitor voltage is taken as the state variable needing to be fixed, and the inductor current is chosen as the state variable. The capacitor voltage v_c is assumed to be a constant V_{CO}, then, the inductor current increases and decreases linearly during any period. The following one-dimensional discrete iterative map can be derived by substituting of $v_c = V_{CO}$ into (5-8),

Case 3. $t_n \geq T$.

$$i_{n+1} = f(i_n) = i_n + \frac{V_{in}}{L}T \tag{10}$$

Case 4. $t_n < T$.

$$\begin{aligned} i_{n+1} &= f(i_n) \\ &= e^{-kt_{n1}}\left[A_3 \sin\beta t_{n1} + A_2 \cos\beta t_{n1}\right] + \frac{V_{in}}{R} \end{aligned} \tag{11}$$

where $A_3 = \dfrac{V_{in} - V_{co}e^{-2kt_n} + kLA_2}{L\beta}$

From (10) and (11), $f^2(i_n)$ is obtained

Case 5. $t'_{n2} \geq T$.

$$i_{n+2} = f(i_{n+1}) = f^2(i_n) = i_n + 2 \cdot \frac{V_{in}}{L}T \tag{12}$$

Case 6. $t'_{n2} < T$.

$$\begin{aligned} i_{n+2} &= f(i_{n+1}) = f^{(2)}(i_n) \\ &= e^{-kt_{n2}}\left[A_4 \sin\beta t_{n2} + A_2 \cos\beta t_{n2}\right] + \frac{V_{in}}{R} \end{aligned} \tag{13}$$

where, $t_{n2} = T\left[1 - \dfrac{t'_{n2}}{T}\right], t'_{n2} = \dfrac{L}{V_{in}}(I_{ref} - i_{n+1})$,

$$A_4 = \frac{V_{in} - V_{co}e^{-2kt'_{n2}} + kLA_2}{L\beta}$$

Similarly, $f^3(i_n)$ is obtained

Case 7. $t'_{n3} \geq T$.

$$i_{n+3} = f(i_{n+2}) = f^3(i_n) = i_n + 3 \cdot \frac{V_{in}}{L}T \tag{14}$$

Case 8. $t'_{n3} < T$.

$$\begin{aligned} i_{n+3} &= f(i_{n+2}) = f^{(3)}(i_n) \\ &= e^{-kt_{n3}}\left[A_5 \sin\beta t_{n3} + A_2 \cos\beta t_{n3}\right] + \frac{V_{in}}{R} \end{aligned} \tag{15}$$

where, $t_{n3} = T\left[1 - \dfrac{t'_{n3}}{T}\right], t'_{n3} = \dfrac{L}{V_{in}}(\pi I_{ref} - i_{n+2})$,

$$A_5 = \frac{V_{in} - V_{co}e^{-2kt'_{n3}} + kLA_2}{L\beta}$$

The graph of $f(i_n)$ and the diagonal is shown in **Figure 5**, and the graph of $f^3(i_n)$ and the diagonal is shown in **Figure 6**, in which the parameters are same as those in [12], that is, $V_{CO} = 17.2\text{ V}, I_{ref} = 4.7915\text{ A}$, $i_n \in [2\text{ A}, 5\text{ A}]$.

Compared with [12], the discrete iterative map of $f(i_n)$ is different at the interval of [4.75, 5], and that of $f^3(i_n)$ is different at the interval of [4.85, 5]. But the difference has no effect on the analysis of the equilibrium point. These results testify the validity and practicality of the proposed discrete iterative map method of $f(i_n)$ and $f^3(i_n)$.

3.3. The Conditions Leading to Tangent Bifurcation

Definition 1. The graph of a function *f* is the set of points $\{(x, f(x))\}$. The diagonal, denoted by △, is the graph of the identity function that takes *x* to *x*: $\triangle = \{(x, x)\}$

Obviously, a point *p* is fixed for a function *f* if and only if $(p, f(p))$ is on the diagonal △.

In theorem 1, a fixed point is requested according to condition (a). The condition (b) indicates that the iterative map function lose the stability in the instability boundary, in other words, the tangent bifurcation will happen in the instability boundary. Form **Figure 6**, it can be seen that there are four fixed points, *i.e.*, $f^{(3)}(2.82,\ 4.7515) = 2.82$, $f^{(3)}(3.82,\ 4.7515) = 3.82$, $f^{(3)}(4.25, 4.7515) = 4.25$, $f^{(3)}(4.79,\ 4.7515) = 4.79$, thus satisfying the condition (a) of theorem 1.

Three fixed points $(i^{*1}_n = 2.82, i^{*2}_n = 3.82, i^{*4}_n = 4.79)$ are tangent to the diagonal that the slopes of them are +1,

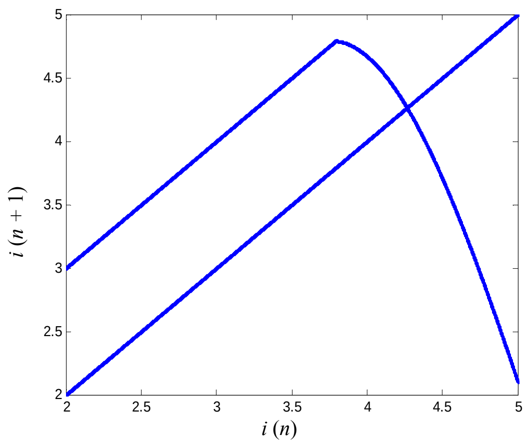

Figure 5. Graph of $f(i_n)$.

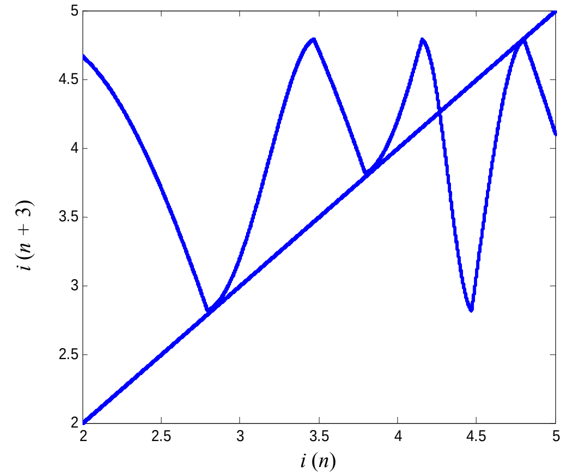

Figure 6. Graph of $f^3(i_n)$.

and the slope of the fixed point $(i^{*3}_n = 4.25)$ is –2. It means that

$$\frac{\partial}{\partial i_n} f^{(3)}(i_n, I_{ref},)\Big|_{i_n=2.82, I_{ref}=4.7915} = +1$$

$$\frac{\partial}{\partial i_n} f^{(3)}(i_n, I_{ref},)\Big|_{i_n=3.82, I_{ref}=4.7915} = +1$$

$$\frac{\partial}{\partial i_n} f^{(3)}(i_n, I_{ref},)\Big|_{i_n=4.79, I_{ref}=4.7915} = +1$$

It satisfies the condition (b) of theorem 1.

From (14) and (15), $\partial f^{(3)}(i_n)/\partial I_{ref}$ can be worked out,

Case 7. $t'_{n3} \geq T$.

$$\partial f^{(3)}(i_n)/\partial I_{ref} = 0 \tag{16}$$

Case 8. $t'_{n3} < T$.

$$\frac{\partial}{\partial i_{ref}} f^{(3)}(i_n, I_{ref}) =$$
$$\frac{de^{-ktn_3}}{dI_{ref}}(A_5 \sin\beta t_{n3} + A_2 \cos\beta t_{n3}) + e^{-ktn_3}$$
$$\left(\frac{dA_5}{dI_{ref}}\sin\beta t_{n3} + A_5\frac{d\sin\beta t_{n3}}{dI_{ref}} + \frac{dA_2}{dI_{ref}}\cos\beta t_{n3} + A_2\frac{d\cos\beta t_{n3}}{dI_{ref}}\right) \tag{17}$$

Substituting of circuit parameters and the values of V_{CO}, I_{ref} into (16) and (17), gives

$$\frac{\partial}{\partial I_{ref}} f^{(3)}(i_n, I_{ref})\Big|_{i_n=2.82, I_{ref}=4.7915} = -0.1952 \neq 0$$

$$\frac{\partial}{\partial I_{ref}} f^{(3)}(i_n, I_{ref})\Big|_{i_n=3.82, I_{ref}=4.7915} = -0.1952 \neq 0$$

$$\frac{\partial}{\partial i_{ref}} f^{(3)}\left(i_{ref}, x\right)\Big|_{i_{ref}=4.7915, i_n=4.79} = 0.1639 \neq 0$$

There is no question that it satisfies condition (c) of theorem 1.

The secondary partial derivative $\frac{\partial^2}{\partial^2 i_n} f^{(3)}\left(i_n, I_{ref}\right)$ can be also obtained according to (14) and (15), which is as follows

Case 7. $t'_{n3} \geq T$.

$$\frac{\partial^2}{\partial^2 i_n} f^{(3)}\left(i_n, I_{ref}\right) = 0 \tag{18}$$

Case 8. $t'_{n3} < T$.

$$\frac{\partial^2}{\partial^2 i_n} f^{(3)}\left(i_n, I_{ref},\right) = \frac{\partial^2 e^{-kt_{n3}}}{\partial^2 i_n}\left(A_5 \sin \beta t_{n3} + A_2 \cos \beta t_{n3}\right) + e^{-kt_{n3}} \frac{\partial^2}{\partial^2 i_n}\left(A_5 \sin \beta t_{n3} + A_2 \cos \beta t_{n3}\right) \tag{19}$$

Similarly, substituting of the parameters values into (18) and (19), gives

$$\frac{\partial^2}{\partial^2 i_n} f^{(3)}\left(i_n, I_{ref}\right)\Big|_{i_n=2.82, I_{ref}=4.7915} = 14.4706 \neq 0$$

$$\frac{\partial^2}{\partial^2 i_n} f^{(3)}\left(i_n, I_{ref}\right)\Big|_{i_n=3.82, I_{ref}=4.7915} = 14.4706 \neq 0$$

$$\frac{\partial^2}{\partial^2 i_n} f^{(3)}\left(i_n, I_{ref}\right)\Big|_{i_n=4.79, I_{ref}=4.7915} = -47.7344 \neq 0$$

Without question, it satisfies condition (d) of theorem 1.

In summary, the current mode controlled boost converter operating in CCM satisfies the hypothesis of theorem 1. Therefore, the discrete iterative map of $f^3(i_n)$ undergoes the tangent bifurcation at the fixed point, and the tangent bifurcation behavior occurs in this system.

4. Conclusions

The mechanism of tangent bifurcation in the current mode controlled boost converter operating in CCM is explored in this paper. Based on the discrete iterative map of the boost converter, by taking the capacitor voltage as a constant, and choosing the inductor current as the state variable, the one-dimensional discrete iterative maps of $f\left(i_n\right)$ and $f^{(3)}\left(i_n\right)$ have been derived. It is demonstrated in mechanism that the tangent bifurcation will happen inevitably in the boost converter according to the tangent bifurcation theorem. The computer simulations, such as discrete iterative maps, bifurcation diagram with reference current I_{ref} as parameter, Lyapunov exponent are used to verify the phenomenon. It has been shown that tangent bifurcation does exist for this system. The method presented in the paper provides the theoretical basics for analyzing the tangent bifurcation and chaos. It has generality and can be also used to analyze the tangent bifurcation of other kinds of DC-DC converters.

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