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Study of Rayleigh-Bénard Magneto Convection in a Micropolar Fluid with Maxwell-Cattaneo Law

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Abstract

The effects of result from the substitution of the classical Fourier law by the non-classical Maxwell-Cattaneo law on the Rayleigh-Bénard Magneto-convection in an electrically conducting micropolar fluid is studied using the Galerkin technique. The eigenvalue is obtained for free-free, rigid-free and rigid-rigid velocity boundary combinations with isothermal or adiabatic temperature on the spin-vanishing boundaries. The influences of various micropolar fluid parameters are analyzed on the onset of convection. The classical approach predicts an infinite speed for the propagation of heat. The present non-classical theory involves a wave type heat transport (SECOND SOUND) and does not suffer from the physically unacceptable drawback of infinite heat propagation speed. It is found that the results are noteworthy at short times and the critical eigenvalues are less than the classical ones.

Keywords: Rayleigh-Bénard Magneto-Convection, Magnetic Field, Micropolar Fluid, Maxwell-Cattaneo Law

1. Introduction

The Classical Fourier law of heat conduction expresses that the heat flux within a medium is proportional to the local temperature gradient in the system. A well known consequence of this law is that heat perturbations propagates with an infinite velocity. This drawback of the classical law motivated Maxwell [1], Cattaneo [2], Lindsay and Straughan [3], Straughan and Franchi [4], Lebon and Cloot [5], Siddheshwar [6] and Pranesh [7], Dauby *et al.* [8] and Straughan [9] to adopt a non-classical heat flux Maxwell-Cattaneo law in studying Rayleigh-Bénard/Marangoni convection to get rid of this unphysical results. This Maxwell-Cattaneo equation contains an extra inertial term with respect to the Fourier law.

$$\tau \frac{d\vec{Q}}{dt} + \vec{Q} = -\chi \nabla T$$

where $\vec{Q}$ is the heat flux, τ is a relaxation time and χ is the heat conductivity. This heat conductivity equation and the conservation of energy equation introduce the hyperbolic equation, which describes heat propagation with finite speed. Puri and Jordan [10,11] and Puri and Kythe [12,13] have studied other fluid mechanics problems by employing the Maxwell-Cattaneo heat flux law.

The theory of micropolar fluid is due to Eringen (see [14-16]), whose theory allows for the presence of particles in the fluid by additionally accounting for particle motion. The motivation for the study comes from many applications involving unclean fluids wherein the clean fluid is evenly interspersed with particles, which may be dust, dirt, ice or raindrops, or other additives(see [17, 18]). This suggests geophysical or industrial convection contexts for the application of micropolar fluids. Many authors (see [19-28]) have investigated the problem of Rayleigh-Bénard convection in Eringen's micropolar fluid and concluded that the stationary convection is the preferred mode. The reported works on convection in micropolar fluid concern with classical Fourier heat flux law.

The objective of this paper is to study the Rayleigh-Bénard magneto-convection in micropolar fluid by replacing the classical Fourier law by non-classical Maxwell-Cattaneo law using Galerkin technique.

2. Mathematical Formulation

Consider an infinite horizontal layer of a Boussinesquian, electrically conducting fluid, with non-magnetic suspended particle, of depth'd'. Cartesian co-ordinate system is taken with origin in the lower boundary and z-axis vertically upwards. Let ΔT be the temperature difference between the upper and lower boundaries. (See **Figure 1**)

The governing equations for the Rayleigh-Bénard situation in a Boussinesquian fluid with suspended particles are

Continuity equation:

$$\nabla.\vec{q} = 0, \tag{1}$$

Conservation of linear momentum:

$$\left.\begin{aligned}&\rho_o\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla P - \rho g\hat{k} + \\ &(2\zeta+\eta)\nabla^2\vec{q} + \zeta\nabla\times\vec{\omega} + \mu_m\left(\vec{H}.\nabla\right)\vec{H},\end{aligned}\right\} \tag{2}$$

Conservation of angular momentum:

$$\left.\begin{aligned}&\rho_o I\left[\frac{\partial \vec{\omega}}{\partial t} + (\vec{q}.\nabla)\vec{\omega}\right] = (\lambda' + \eta')\nabla(\nabla.\vec{\omega}) + \\ &\left(\eta'\nabla^2\vec{\omega}\right) + \zeta(\nabla\times\vec{q} - 2\vec{\omega}),\end{aligned}\right\} \tag{3}$$

Conservation of energy:

$$\frac{\partial T}{\partial t} + \left(\vec{q} - \frac{\beta}{\rho_o C_v}\nabla\times\vec{\omega}\right).\nabla T = -\nabla.\vec{Q}, \tag{4}$$

Maxwell-Cattaneo heat flux law:

$$\tau\left[\dot{\vec{Q}} + \vec{\omega}_1\times\vec{Q}\right] = -\vec{Q} - \kappa\nabla T, \tag{5}$$

Magnetic induction equation:

$$\frac{\partial \vec{H}}{\partial t} + (\vec{q}.\nabla)\vec{H} = \left(\vec{H}.\nabla\right)\vec{q} + \gamma_m\nabla^2\vec{H}, \tag{6}$$

Equation of state:

$$\rho = \rho_o\left[1 - \alpha(T - T_o)\right]. \tag{7}$$

where $\vec{q}$ is the velocity, $\vec{\omega}$ is the spin, T is the temperature, P is the hydromagnetic pressure, $\vec{H}$ is the magnetic field, ρ is the density, ρ_o is the density of the fluid at reference temperature $T = T_o$, g is the acceleration due to gravity, ζ is the coupling viscosity coefficient or vortex viscosity, η is the shear kinematic viscosity coefficient, I is the moment of inertia, λ' and η' are the bulk and shear spin viscosity coefficient, β is the micropolar heat conduction coefficient, C_v is the specific heat, κ is the thermal conductivity, α is the coefficient of thermal expansion, $\gamma_m = \dfrac{1}{\mu_m\sigma_m}$ is the magnetic viscosity (σ_m: electrical conductivity and μ_m: magnetic permeability), $\vec{\omega}_1 = \frac{1}{2}\nabla\times\vec{q}$, $\vec{Q}$ is the heat flux vector and τ is the constant relation time.

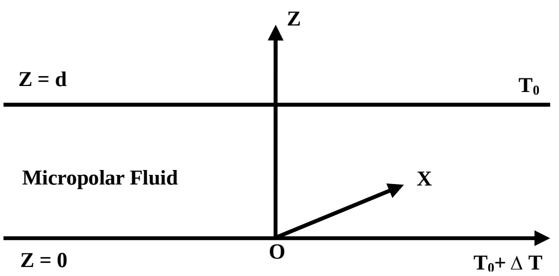


Figure 1. Schematic diagram of the Rayleigh-Bénard situation for micropolar fluid.

The basic state of the fluid being quiescent is described by

$$\left.\begin{aligned}&\vec{q}_b = 0,\ \vec{\omega}_b = 0,\ \vec{H}_b = H_o\hat{k}, \\ &P = P_b(z),\ \rho = \rho_b(z), \\ &\vec{Q} = (0, 0, Q_b(z)),\ T = T_b(z)\end{aligned}\right\} \tag{8}$$

Equations (2), (4), (5) and (7) in the basic state specified by "(8)" respectively become

$$\left.\begin{aligned}&\frac{dP_b}{dz} = -\rho_o g\hat{k}, \qquad \frac{d\vec{Q}_b}{dz} = 0, \\ &\vec{Q}_b = -\kappa\frac{dT_b}{dz}, \\ &\rho_b = \rho_o\left[1 - \alpha(T_b - T_o)\right], \\ &\frac{d^2T_b}{dz^2} = 0.\end{aligned}\right\} \tag{9}$$

Equations (1), (3) and (6) are identically satisfied by the concerned basic state variables. We now superpose infinitesimal perturbations on the quiescent basic state and study the instability.

3. Linear Stability Analysis

Let the basic state be disturbed by an infinitesimal thermal perturbation. We now have

$$\left.\begin{aligned}&\vec{q} = \vec{q}_b + \vec{q}',\ \vec{\omega} = \vec{\omega}_b + \vec{\omega}', \\ &P = P_b + P',\ \vec{Q} = \vec{Q}_b + \vec{Q}', \\ &\rho = \rho_b + \rho',\ T = T_b + T', \\ &\vec{H} = \vec{H}_b + \vec{H}'\end{aligned}\right\} \tag{10}$$

The primes indicate that the quantities are infinitesimal perturbations and subscript 'b' indicates basic state value.

Substituting "(10)" into "(1)-(7)" and using the basic state (9), we get linearised equation governing the infinitesimal perturbations in the form

$$\nabla.\vec{q}' = 0, \tag{11}$$

$$\left.\begin{aligned}&\rho_o\left[\frac{\partial \vec{q}'}{\partial t}\right] = -\nabla P - \rho' g\hat{k} + (2\zeta+\eta)\nabla^2\vec{q}' + \\ &(\zeta\nabla\times\vec{\omega}') + \mu_m\left(\vec{H}'.\nabla\right)\vec{H}',\end{aligned}\right\} \tag{12}$$

$$\left.\begin{array}{l}\rho_o I\left[\dfrac{\partial\vec{\omega}'}{\partial t}\right]=(\lambda'+\eta')\nabla(\nabla\vec{\omega}')+\\(\eta'\nabla^2\vec{\omega}')+\zeta(\nabla\times\vec{q}'-2\vec{\omega}'),\end{array}\right\}\tag{13}$$

$$\frac{\partial T'}{\partial t}=\frac{\Delta T}{d}\left[\vec{q}'-\frac{\beta}{\rho_o C_v}\nabla\times\vec{\omega}'\right]-\nabla.\vec{Q}',\tag{14}$$

$$\left[1+\tau\frac{\partial}{\partial t}\right]\vec{Q}'=-\frac{1}{2}\chi_1\frac{\Delta T}{d}\left(\frac{\partial\vec{q}'}{\partial z}-\nabla W'\right)-\kappa\,\nabla T',\tag{15}$$

$$\frac{\partial\vec{H}'}{\partial t}=\left(H_o\frac{\partial W}{\partial z}\hat{k}\right)+\gamma_m\nabla^2\vec{H}',\tag{16}$$

$$\rho'=-\alpha\rho_o T'.\tag{17}$$

Operating divergence on the "(15)" and substituting in "(14)", on using "(11)", we get

$$\left.\begin{array}{l}\left(1+\tau\dfrac{\partial}{\partial t}\right)\dfrac{\partial T'}{\partial t}=\left(1+\tau\dfrac{\partial}{\partial t}\right)\dfrac{\Delta T}{d}\\\left[W'-\dfrac{\beta}{\rho_o C_v}\Omega_z\right]+\kappa\nabla^2T'-\\\dfrac{1}{2}\chi_1\dfrac{\Delta T}{d}\left(\nabla^2W'\right),\end{array}\right\}\tag{18}$$

where $\Omega=\nabla\times\vec{\omega}'$. The perturbation "(12), (13), (16) and (18)" are non-dimensionalised using the following definition:

$$\left.\begin{array}{l}(x^*,y^*,z^*)=\dfrac{(x,y,z)}{d},W^*=\dfrac{W'}{\left(\kappa/d\right)},\\\vec{\omega}^*=\dfrac{\omega'}{\left(\kappa/d^2\right)},\ t^*=\dfrac{t}{\left(d^2/\kappa\right)},\\T^*=\dfrac{T'}{\Delta T},\ H^*=\dfrac{\vec{H}'}{H_o},\Omega^*=\dfrac{\Omega_z}{\left(\kappa/d^3\right)}\end{array}\right\}\tag{19}$$

Using "(17)" in "(12)", Operating curl twice on the resulting equation, operating curl once on "(13)" and non-dimensionalising the two resulting equations and also "(16)-(18)".

$$\left.\begin{array}{l}\dfrac{1}{\Pr}\dfrac{\partial}{\partial t}\left(\nabla^2W\right)=R\nabla_1^2T+\\(1+N_1)\nabla^4W+N_1\nabla^2\Omega_z+\\Q\dfrac{\Pr}{Pm}\nabla^2\left(\dfrac{\partial H_z}{\partial z}\right)\end{array}\right\}\tag{20}$$

$$\frac{N_2}{\Pr}\frac{\partial}{\partial t}(\Omega_z)=N_3\nabla^2\Omega_z-N_1\nabla^2W-2N_1\Omega_z\tag{21}$$

$$\frac{\partial H_z}{\partial z}=\frac{\partial W}{\partial z}+\frac{\Pr}{Pm}\nabla^2H_z\tag{22}$$

$$\left.\begin{array}{l}\left(1+2C\dfrac{\partial}{\partial t}\right)\dfrac{\partial T}{\partial t}=\left(1+2C\dfrac{\partial}{\partial t}\right)W-\\\left(1+2C\dfrac{\partial}{\partial t}\right)N_5\Omega_z+\nabla^2T-C\nabla^2W\end{array}\right\}\tag{23}$$

where the asterisks have been dropped for simplicity and the non-dimensional parameters N_1, N_3, N_5, R, Q, Pr, Pm and C are as defined as

$N_1=\dfrac{\zeta}{\zeta+\eta}$ (Coupling Parameter)

$N_3=\dfrac{\eta'}{(\zeta+\eta)d^2}$ (Couple Stress Parameter)

$N_5=\dfrac{\beta}{\rho_o C_v d^2}$ (Micropolar Heat Conduction Parameter)

$\Pr=\dfrac{\zeta+\eta}{\rho_o\kappa}$ (Prandtl Number)

$Pm=\dfrac{\zeta+\eta}{\gamma_m}$ (Magnetic Prandtl Number)

$R=\dfrac{\rho_o\alpha g\Delta Td^3}{\kappa(\zeta+\eta)}$ (Rayleigh Number)

$Q=\dfrac{\mu_m\vec{H}_o d^2}{\gamma_m(\zeta+\eta)}$ (Chandrasekhar Number)

$C=\dfrac{\tau\kappa}{2d^2}$ (Cattaneo Number)

The infinitesimal perturbation W,Ω_z and T are assumed to be periodic waves (see Chandrasekhar 1961) and hence these permit a normal mode solution in the form

$$\begin{bmatrix}W\\\Omega_z\\H_z\\T\end{bmatrix}=\begin{bmatrix}W(z).e^{i(lx+my)}\\G(z).e^{i(lx+my)}\\H_z(z).e^{i(lx+my)}\\T(z).e^{i(lx+my)}\end{bmatrix}\tag{24}$$

where l and m are horizontal components of the wave number $\vec{a}$, Substituting "(24)" into "(20)-(23)", we get

$$\left.\begin{array}{l}(1+N_1)\left(D^2-a^2\right)^2W-\\Ra^2T+N_1\left(D^2-a^2\right)G+\\Q\dfrac{\Pr}{Pm}\left(D^2-a^2\right)H_z\end{array}\right\}=0\tag{25}$$

$$\left.\begin{array}{r} N_3\left(D^2-a^2\right)G- \\ N_1\left(D^2-a^2\right)W-2N_1G \end{array}\right\}=0 \quad (26)$$

$$\left.DW+\frac{\Pr}{Pm}\left(D^2-a^2\right)H_z\right\}=0 \quad (27)$$

$$\left.\begin{array}{r} W-N_5G+\left(D^2-a^2\right)T- \\ C\left(D^2-a^2\right)W \end{array}\right\}=0 \quad (28)$$

where $D=\dfrac{d}{dz}$.

The set of ordinary differential "(25)-(28)" are approximations based on physical considerations to the system of partial differential "(20)-(23)". Although the relationship between the solutions of the governing partial differential equations and the corresponding ordinary differential equations has not been established, these linear models reproduce qualitatively the convective phenomena observable through the full system.

Eliminating H_z between "(25) and (27)", we get

$$\left.\begin{array}{r} \left(1+N_1\right)\left(D^2-a^2\right)^2W-Ra^2T+ \\ N_1\left(D^2-a^2\right)G-QD^2W \end{array}\right\}=0 \quad (29)$$

We now apply the Galerkin method to "(26), (28) and (29)" that gives general results on the eigen value of the problem using simple, polynomial, trial functions for the lowest eigen value. We obtain an approximate solution of the differential equations with the given boundary conditions by choosing trial functions for velocity, microrotation and temperature perturbations that satisfy the boundary conditions but may not exactly satisfy the differential equations. This leads to residuals when the trial functions are substituted into the differential equations. The Galerkin method requires the residual to be orthogonal to each individual trial function.

In the Galerkin procedure, we expand the velocity, microrotation and temperature by,

$$\left.\begin{array}{l} W(z,t)=\sum A_i(t)W_i(z), \\ G(z,t)=\sum B_i(t)G_i(z), \\ T(z,t)=\sum E_i(t)T_i(z) \end{array}\right\}$$

where $W_i(z), G_i(z)$ *and* $T_i(z)$ are polynomials in z that generally have to satisfy the given boundary conditions.

For the single term Galerkin expansion technique we take $i = j = 1$. Multiplying "(29)" by W, "(26)" by G and "(28)" by T, integrating the resulting equation with respect to z from 0 to 1 and taking $W = AW_1$, $G = BG_1$ and $T = ET_1$ in which A, B and E are constants with W_1, G_1 and T_1 are trial functions. This procedure yields the following equation for the Rayleigh number R:

$$R=\frac{\left\langle T_1\left(D^2-a^2\right)T_1\right\rangle\left[Y_1.Y_2+N_1^2.Y_3\right]}{a^2\left\langle W_1T_1\right\rangle Y_4} \quad (30)$$

where

$$Y_1=\left(1+N_1\right)\left\langle W_1\left(D^2-a^2\right)^2W_1\right\rangle-$$
$$Q\left\langle W_1D^2W_1\right\rangle,$$
$$Y_2=N_3\left\langle G_1\left(D^2-a^2\right)G_1\right\rangle-2N_1\left\langle G_1^2\right\rangle,$$
$$Y_3=\left\langle G_1\left(D^2-a^2\right)W_1\right\rangle\left\langle W_1\left(D^2-a^2\right)G_1\right\rangle,$$
$$Y_4=N_1N_5\left\langle G_1\left(D^2-a^2\right)W_1\right\rangle\left\langle T_1G_1\right\rangle+$$
$$Y_2\left[C.\left\langle T_1\left(D^2-a^2\right)W_1\right\rangle-\left\langle T_1W_1\right\rangle\right]$$

In the "(30)", $\langle---\rangle$ denotes integration with respect to z between $z=0$ and $z=1$. We note here that R in equation (30) is a functional and the Euler-Lagrange equations for the extremisation of R are "(26), (28) and (29)".

The value of critical Rayleigh number depends on the boundaries. In this paper we consider various boundary combinations and these are discussed below.

1) Free – Free isothermal/adiabatic, no spin.
2) Rigid – Rigid isothermal/adiabatic, no spin.
3) Rigid – Free isothermal/adiabatic, no spin.

Critical Rayleigh number for free-free isothermal boundaries, No spin:

The boundary conditions are

$$W=D^2W=T=G=0,\quad at\quad z=0,1. \quad (31)$$

The trial functions satisfying "(30)" are

$$\left.\begin{array}{l} W_1=z^4-2z^3+z, \\ T_1=z\left(1-z\right), \\ G_1=z\left(1-z\right) \end{array}\right\} \quad (32)$$

Substituting "(32)" in "(30)" and performing the integration, we get

$$R=\frac{\left(10+a^2\right)M_1}{51a^2} \quad (33)$$

where

$$M_1=\frac{28.K_2\left[\left(1+N_1\right)K_1+306Q\right]-3N_1^2.K_3^2}{K_2\left[C.K_3+17\right]-N_1.N_5.K_3},$$
$$K_1=3024+612a^2+31a^4,$$
$$K_2=N_3\left(10+a^2\right)+2N_1,$$
$$K_3=168+17a^2.$$

R attains its minimum value R_c at $a = a_c$.

Critical Rayleigh number for rigid-rigid isothermal boundaries, No spin:

The boundary conditions are

$$W = DW = T = G = 0,\ at\ z = 0,1. \quad (34)$$

The trial functions satisfying "(34)" are

$$\left.\begin{aligned} W_1 &= z^4 - 2z^3 + z^2, \\ T_1 &= z(1-z), \\ G_1 &= z(1-z) \end{aligned}\right\} \quad (35)$$

Substituting "(35)" in "(30)" and performing the integration, we get

$$R = \frac{(10+a^2)M_1}{9a^2} \quad (36)$$

where

$$M_1 = \frac{28.K_2\left[(1+N_1)K_1 + 12Q\right] - 3N_1^2.K_3^2}{K_2\left[C.K_3 + 3\right] - N_1.N_5.K_3},$$

$$K_1 = 504 + 24a^2 + a^4,$$

$$K_2 = N_3(10+a^2) + 2N_1,$$

$$K_3 = 28 + 3a^2.$$

R attains its minimum value R_c at $a = a_c$.

***Critical Rayleigh number for rigid-free isothermal boundaries, No spin*:**

The boundary conditions are

$$\left.\begin{aligned} W = DW = T = G = 0, \quad at \quad z = 0 \\ W = D^2W = T = G = 0, \quad at \quad z = 1 \end{aligned}\right\} \quad (37)$$

The trial functions satisfying "(37)" are

$$\left.\begin{aligned} W_1 &= 2z^4 - 5z^3 + 3z^2, \\ T_1 &= z(1-z), \\ G_1 &= z(1-z) \end{aligned}\right\} \quad (38)$$

Substituting "(38)" in "(30)" and performing the integration, we get

$$R = \frac{(10+a^2)M_1}{39a^2} \quad (39)$$

where

$$M_1 = \frac{28.K_2\left[(1+N_1)K_1 + 216Q\right] - 3N_1^2.K_3^2}{K_2\left[C.K_3 + 13\right] - N_1.N_5.K_3},$$

$$K_1 = 4536 + 432a^2 + 19a^4,$$

$$K_2 = N_3(10+a^2) + 2N_1,$$

$$K_3 = 126 + 13a^2.$$

R attains its minimum value R_c at $a = a_c$.

***Critical Rayleigh number for free-free adiabatic boundaries, No spin*:**

The boundary conditions are

$$W = D^2W = DT = G = 0,\ at\ z = 0,1. \quad (40)$$

The trial functions satisfying "(40)" are

$$\left.\begin{aligned} W_1 &= z^4 - 2z^3 + z, \\ T_1 &= 1, \\ G_1 &= z(1-z) \end{aligned}\right\} \quad (41)$$

Substituting "(41)" in "(30)" and performing the integration, we get

$$R = \left(\frac{5\,M_1}{42}\right) \quad (42)$$

where

$$M_1 = \frac{28.K_2\left[(1+N_1)K_1 + 306Q\right] - 3N_1^2.K_3^2}{84K_2\left[C.(10+a^2) + 1\right] - 5N_1.N_5.K_3},$$

$$K_1 = 3024 + 612a^2 + 31a^4,$$

$$K_2 = N_3(10+a^2) + 2N_1,$$

$$K_3 = 168 + 17a^2.$$

R attains its minimum value R_c at $a = a_c$.

***Critical Rayleigh number for rigid-rigid adiabatic boundaries, No spin*:**

The boundary conditions are

$$W = DW = DT = G = 0,\ at\ z = 0,1. \quad (43)$$

The trial functions satisfying "(43)" are

$$\left.\begin{aligned} W_1 &= z^4 - 2z^3 + z^2, \\ T_1 &= 1, \\ G_1 &= z(1-z) \end{aligned}\right\} \quad (44)$$

Substituting "(44)" in "(30)" and performing the integration, we get

$$R = \left(\frac{30\,M_1}{42}\right) \quad (45)$$

where

$$M_1 = \frac{28.K_2\left[(1+N_1)K_1 + 12Q\right] - 3N_1^2.K_3^2}{14K_2\left[C.a^2 + 1\right] - 5N_1.N_5.K_3},$$

$$K_1 = 504 + 24a^2 + a^4,$$

$$K_2 = N_3(10+a^2) + 2N_1,$$

$$K_3 = 28 + 3a^2.$$

R attains its minimum value R_c at $a = a_c$.

***Critical Rayleigh number for rigid-free adiabatic boundaries, No spin*:**

The boundary conditions are

$$\left.\begin{aligned} W = DW = DT = G = 0, \quad at \quad z = 0 \\ W = D^2W = DT = G = 0, \quad at \quad z = 1 \end{aligned}\right\} \quad (46)$$

The trial functions satisfying "(46)" are

$$\left.\begin{aligned} W_1 &= 2z^4 - 5z^3 + 3z^2, \\ T_1 &= 1, \\ G_1 &= z(1-z) \end{aligned}\right\} \quad (47)$$

Substituting "(47)" in "(30)" and performing the integration, we get

$$R = \left(\frac{20\ M_1}{126}\right) \quad (48)$$

where

$$M_1 = \frac{28.K_2\left[(1+N_1)K_1 + 216Q\right] - 3N_1^2.K_3^2}{21K_2\left[C.(20+3a^2)+1\right] - 5N_1.N_5.K_3},$$

$$K_1 = 4536 + 432a^2 + 19a^4,$$

$$K_2 = N_3(10+a^2) + 2N_1,$$

$$K_3 = 126 + 13a^2.$$

R attains its minimum value R_c at $a = a_c$.

4. Results and Discussions

In this paper we study the onset of Rayleigh-Bénard Magneto Convection in a micropolar fluid by replacing the classical Fourier heat flux law by non-classical Maxwell-Cattaneo law.

Figures 2 (a)-(c) is the plot of critical Rayleigh number R_c versus coupling parameter N_1, Couple stress parameter N_3 and micropolar heat conduction parameter N_5 respectively for different values of Cattaneo number C and Chandrasekhar number Q for free-free isothermal, no- spin boundary conditions. It is observed that as N_1 and N_5 increases, R_c increases, that is an increase in N_1 and N_5 is to stabilize the system. The increase in N_3 decreases R_c, that is increase in N_3 destabilizes the system. Why and how micropolar parameters N_1, N_3 and N_5 stabilizes/destabilizes the system are given in the **Table 1**. **Figures 3-4** are the plot of R_c versus N_1, N_3 and N_5 for rigid-rigid and rigid-free velocity boundary combination with isothermal temperature and spin vanishing boundaries. **Figures 5-7** are the plot of R_c verses N_1, N_3 and N_5 for free-free, rigid-rigid and rigid-free velocity boundary combination respectively, with adiabatic temperature and spin vanishing boundaries. The results of these graphs, *i.e.*, the effects of N_1, N_3 and N_5 on the onset of convection are qualitatively similar to free-free isothermal case.

It is also observed from the above figures that the increase in Q increases the R_c, from this we conclude that

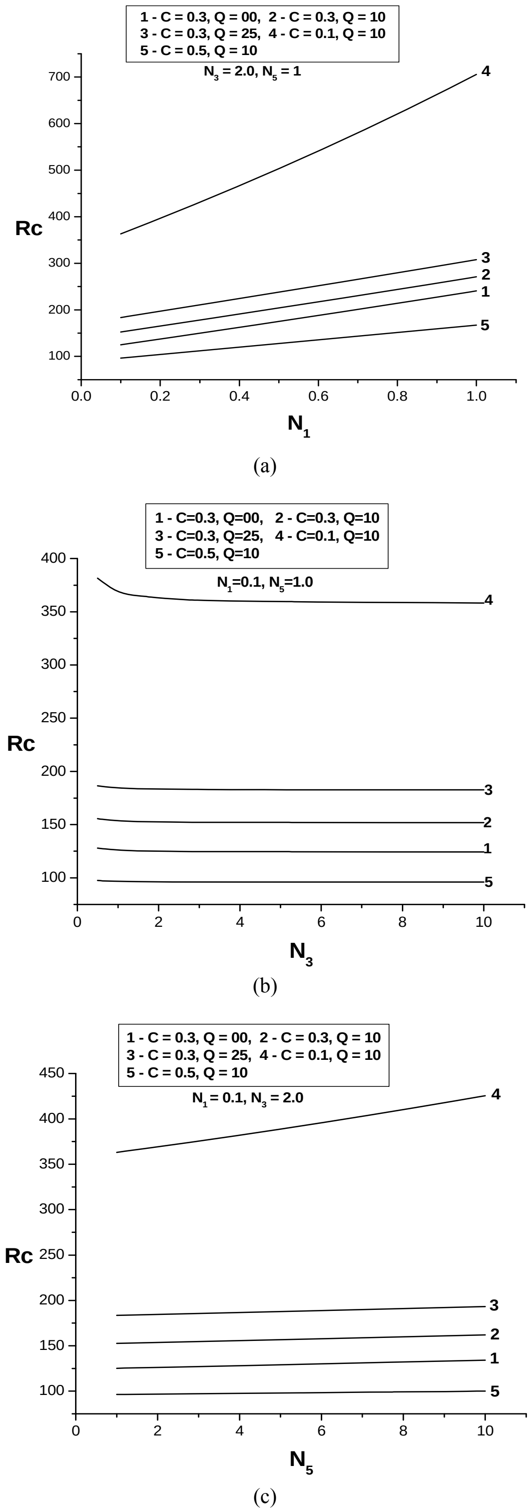
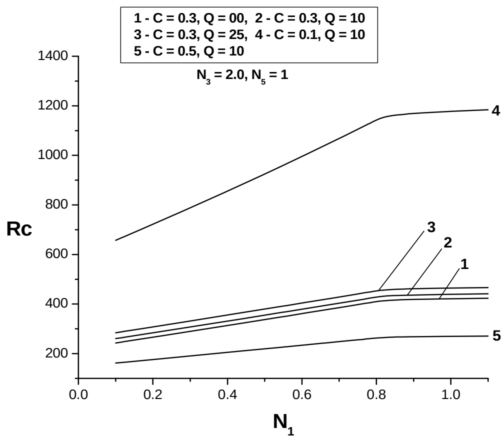
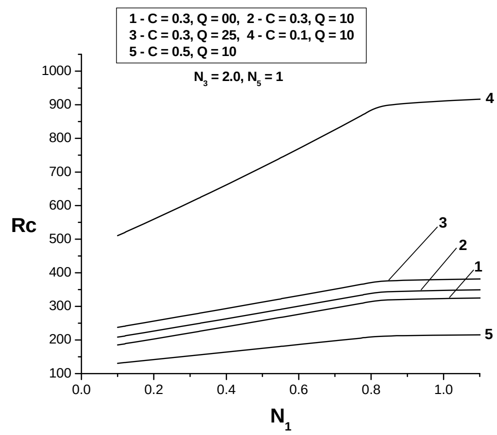


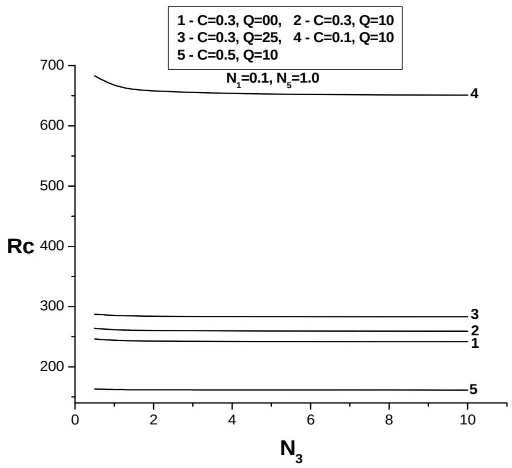
Figure 2. Plot of critical Rayleigh number R_c Vs. (a) coupling parameter N_1, (b) couple stress parameter N_3, (c) coupling parameter N_5 with respect to free-free isothermal no spin boundary condition for different values Chandrasekhar number Q and different values of Cattaneo number C.



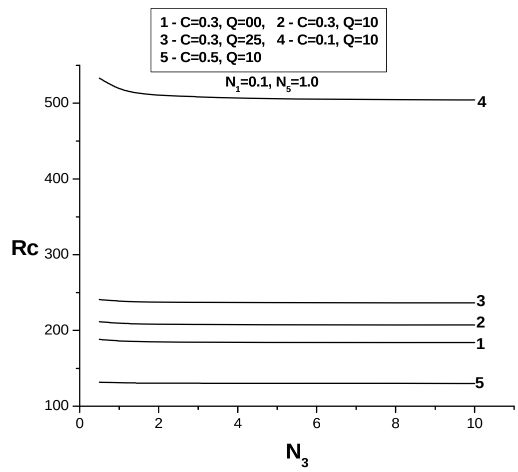
(a)



(a)



(b)



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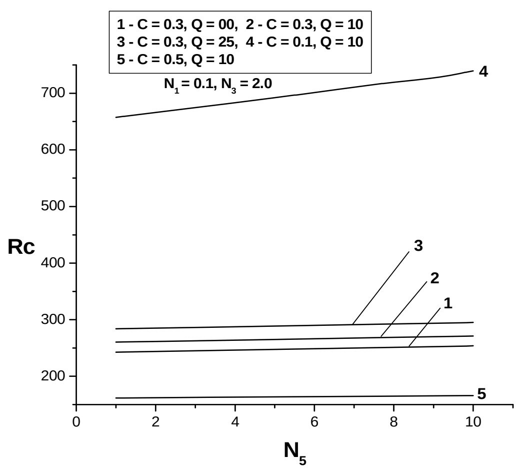

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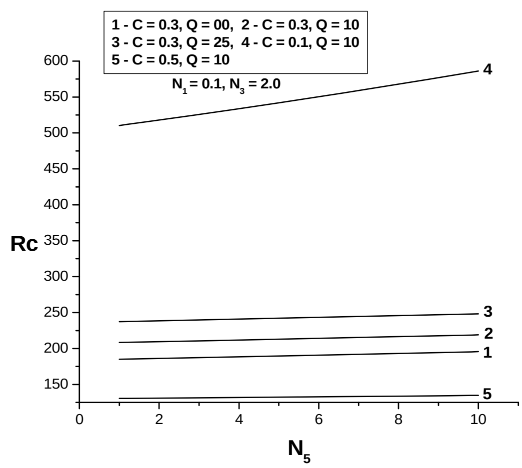

(c)

Figure 3. Plot of critical Rayleigh number R_c Vs. (a) coupling parameter N_1, (b) couple stress parameter N_3, (c) coupling parameter N_5 with respect to rigid-rigid isothermal no spin boundary condition for different values Chandrasekhar number Q and different values of Cattaneo number C.

Figure 4. Plot of critical Rayleigh number R_c Vs. (a) coupling parameter N_1, (b) couple stress parameter N_3, (c) coupling parameter N_5 with respect to rigid-free isothermal no spin boundary condition for different values Chandrasekhar number Q and different values of Cattaneo number C.

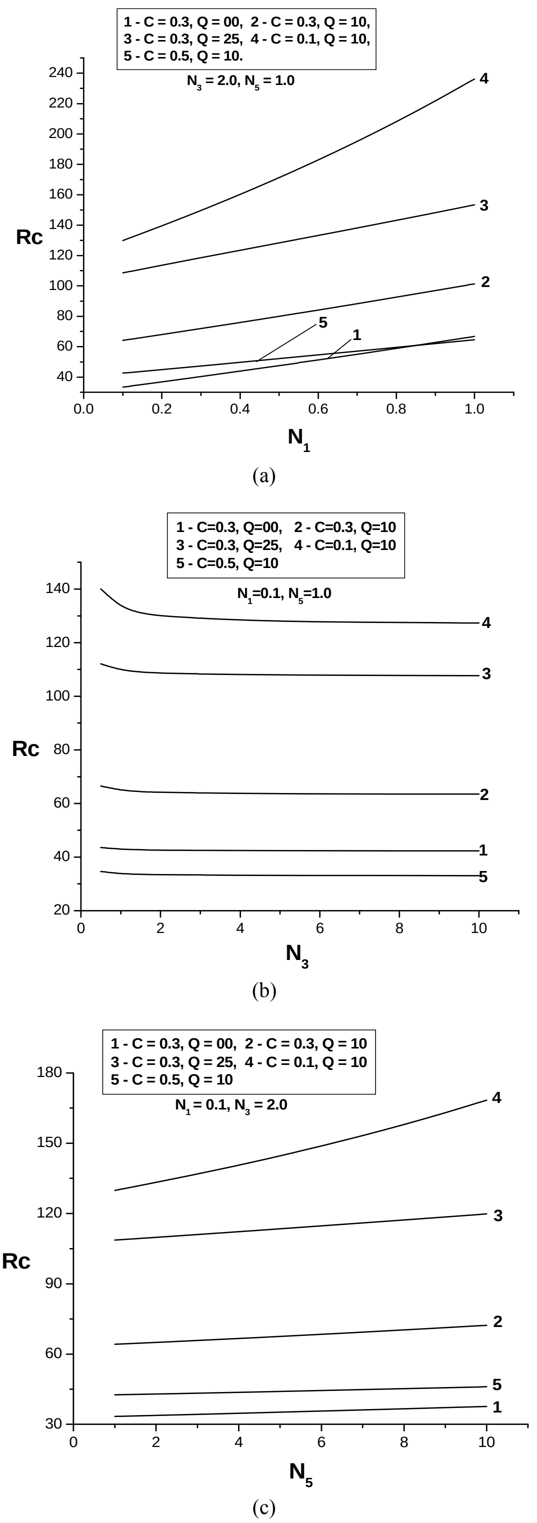

Figure 5. Plot of critical Rayleigh number R_c Vs. (a) coupling parameter N_1, (b) couple stress parameter N_3, (c) coupling parameter N_5 with respect to free-free adiabatic no spin boundary condition for different values Chandrasekhar number Q and different values of Cattaneo number C.

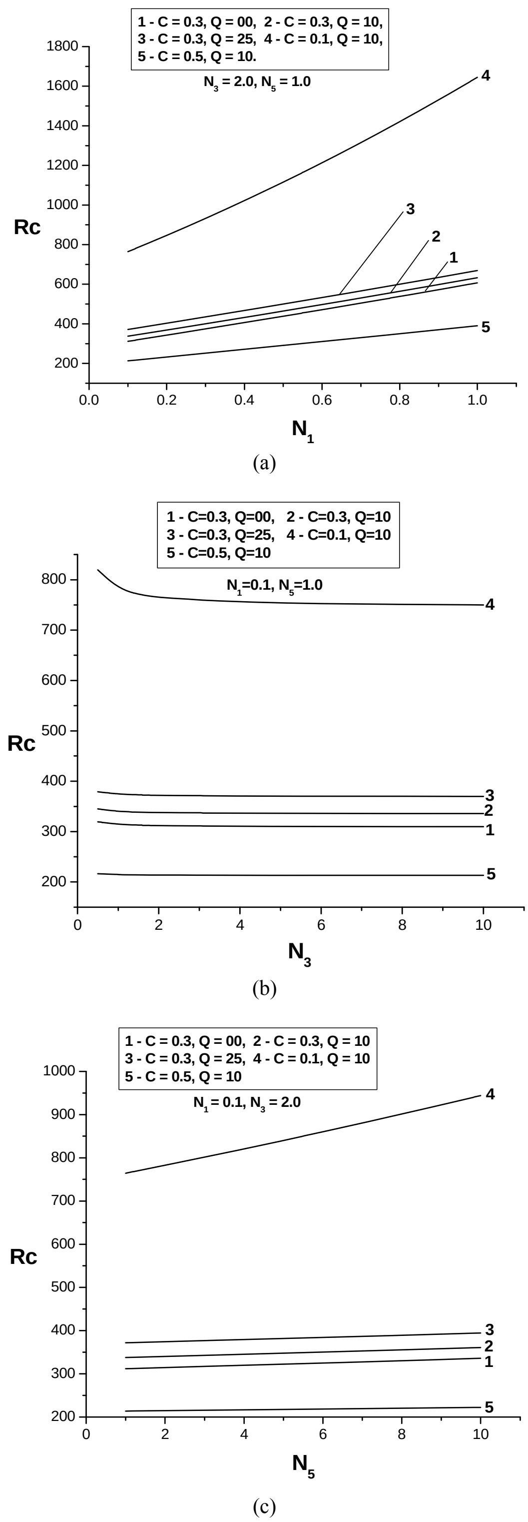

Figure 6. Plot of critical Rayleigh number R_c Vs. (a) coupling parameter N_1, (b) couple stress parameter N_3, (c) coupling parameter N_5 with respect to rigid-rigid adiabatic no spin boundary condition for different values Chandrasekhar number Q and different values of Cattaneo number C.

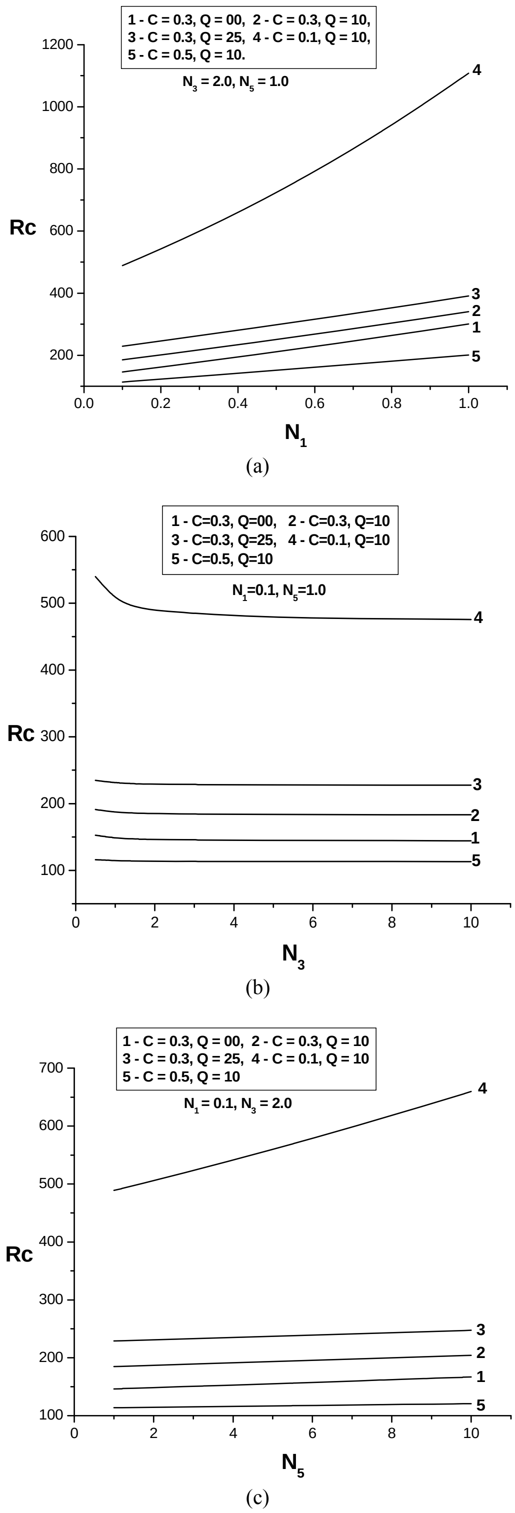

Figure 7. Plot of critical Rayleigh number R_c Vs. (a) coupling parameter N_1, (b) couple stress parameter N_3, (c) coupling parameter N_5 with respect to rigid-free adiabatic no spin boundary condition for different values Chandrasekhar number Q and different values of Cattaneo number C.

Table 1. Why and how of the stabilizing/destabilizing effects of the micropolar fluid parameters N_1 N_3 N_5.

Parameter	Nature of effect	Physical reason
N_1 $0 \le N_1 \le 1$	Stabilizing (as N_1 increases)	Increase in N_1 indicates the increase in the concentration of microelements. These elements consume the greater part of the energy of the system in developing the gyrational velocities of the fluid and as a result the onset of convection is delayed.
N_3 $0 \le N_3 \le$ m (m: finite, real)	Destabilizing (as N_3 increases)	Increase in N_3, decreases the couple stress of the fluid which causes a decrease in microrotation and hence makes the system more unstable.
N_5 $0 \le N_5 \le$ n (n: finite, real)	Stabilizing (as N_5 increases)	When N_5 increases, the heat induced into the fluid due to these microelements also increases, thus reducing the heat transfer from bottom to top.
C $C \in [0, 1]$	Destabilizing (as C increases)	It is a scaled relaxation time and hence it accelerates the onset of convection.

the Q has stabilizing effect on the system. When the magnetic field strength permeating the medium is considerably strong, it induces viscosity into the fluid, and the magnetic lines are distorted by convection. Then these magnetic lines hinder the growth of disturbances, leading to the delay in the onset of instability. However, the viscosity produced by the magnetic field lessens the rotation of the fluid particles, thus controlling the stabilizing effect of N_1.

From the figures it is observed that C which represents Cattaneo number has a destabilizing influence. Increase in Cattaneo number leads to narrowing of the convection cells and thus lowering of the critical Rayleigh number. It is also observed from the figures that influence of Cattaneo number is dominant for small values because the convection cells have fixed aspect ratio.

5. Conclusions

Following conclusions are drawn from the problem:

1) Rayleigh-Bénard convection in Newtonian fluids may be delayed by adding micron sized suspended particles.

2) By adjusting the Chandrasekhar number Q we can control the convection.

3) We can conclude the following for stationary convection in micropolar fluids

$$R_c^{HHE} < R_c^{PHE},$$

where HHE – Hyperbolic heat equation and PHE – Parabolic heat equation.

4) We also find that

$$R_c^{RR} > R_c^{RF} > R_c^{FF}$$

where the superscripts correspond to the three different velocity boundary combinations. The above qualitative results are true for both isothermal and adiabatic boundaries.

5) The non-classical Maxwell-Cattaneo heat flux law involves a hyperbolic type heat transport equation that predicts finite speeds of heat wave propagation (see [29]). Hence it does not suffer from the physically unacceptable draw back of infinite heat propagation speed predicted by the parabolic heat equation. The classical Fourier flux law overpredicts the critical Rayleigh number compared to that predicted by the non-classical law.

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