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Effects of Thermal Diffusion and Chemical Reaction on MHD Flow of Dusty Visco-Elastic (Walter's Liquid Model-B) Fluid

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ABSTRACT

The present note consists, the effects of thermal diffusion and chemical reaction on MHD flow of dusty viscous incompressible, electrically conducting fluid between two vertical heated, porous, parallel plates with heat source/sink. The plate temperature is raised linearly with time and concentration level near the plate to C_w. The variable temperature and uniform mass diffusion taking into account the chemical reaction of first order. The series solution method is used to solve the mathematical equations. Effects of various parameters like chemical reaction (K), thermal diffusion (S_T) and magnetic field (M) etc. on velocity profile, skin friction, concentration profile and temperature field are displayed graphically and discussed numerically for different physical parameters. The analysis developed here for thermal diffusion, bears good agreement with real life problems.

Keywords: *MHD Flow, Thermal Diffusion (Soret Effect), Heat Source/Sink, Skin Friction*

1. Introduction

Many transport processes exist in nature and industrial application in which the transfer of heat and mass occurs simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. In the last few decades several efforts have been made to solve the problems on heat and mass transfer in view of their application to astrophysics, geophysics and engineering.

Chemical reaction can be codified either heterogeneous or homogeneous processes. Its effect depends on the nature of the reaction whether the reaction is heterogeneous or homogeneous. A reaction is of order n, if the reaction rate is proportional to the n^{th} power of concentration. In particular, a reaction is of first order, if the rate of reaction is directly proportional to concentration itself. Experimental and theoretical works on MHD flow with thermal diffusion and chemical reaction have been done extensively in various areas *i.e.* sustain plasma confinement for controlled thermo nuclear fusion, liquid metal cooling of nuclear reactions and electromagnetic casting of metals. Chambre and Yang [1] have worked on thermal diffusion of a chemically reactive species in a laminar boundary layer flow. Dusty viscous and visco-elastic fluids have been discussed by Saffman [2], Micheal and Norey [3], Raptis and Perdikis [4]. Singh [5] proposed the study of free convection and mass diffusion of a dusty visco-elastic (Walter's Liquid Model-B) fluid flowing between two heated porous plates in porous media in presence of magnetic field. References [6,7] focused on the study of convective heat and mass transfer incompressible, viscous Boussinesq fluid in presence of chemical reaction of first order. References [8-10] discussed the effects of thermo diffusion (Soret effects) and diffusion-thermo (Dufour effects) on MHD mixed convection heat and mass transfer of an electrically conducting fluid. Mahantesh *et al.* [11] studied the boundary layer flow behavior and heat transfer characteristic in Walter's liquid model-B fluid flow. Sharma *et al*. [12] discussed the unsteady MHD free convection heat and mass transfer of viscous fluid flowing through a Darcian porous regime adjacent to a moving vertical semi-infinite plate under Soret and Dufour effect.

Recently Kumar and Srivastava [13] examined the effects of chemical reaction on MHD flow of dusty visco-elastic (Walters's liquid model-B) liquid with heat source/

sink. The interest of present investigation is to obtain analytical expressions for various profiles like velocity, skin friction for dusty fluid as well as dust particles and also temperature, concentration for dusty fluid. The effects of thermal diffusion parameter (Soret number), chemical reaction parameter etc. are discussed for different profiles.

2. Nomenclature

B_0: Magnetic field
m: Magnetic field Parameter
C : Species concentration in the field
P_r : Prandtl Number
C_w: Concentration of the plate
S_c : Schmidt Number
C_0: Initial uniform concentration at T_0
T_w: Plate temperature
C_p: Specific heat at constant pressure
T_0 : Initial temperature
G : Acceleration due to gravity
t : Time
G_r : Thermal Grashof number
u, v: Velocities of dusty fluid and dust particle respectively
S : Source/sink parameter respectively in the x-direction
G_m: modified Grashof number
y : Co-ordinate axes as in normal to the plate
K : dimensionless chemical reaction parameter
A : Decay factor
K_1: Chemical reaction parameter
D_T: Thermal diffusion coefficient

3. Mathematical Formulation

We consider the effects of thermal diffusion and chemical reaction on the unsteady dusty flow of an incompressible, slightly conducting, visco-elastic fluid between two heated porous infinite parallel plates (distance 2 h apart) under the influence of uniform magnetic field normal to the flow field in presence of heat source/sink. We assume x-axis along the flow in the mid-way of the plates and y-axis perpendicular to it. Let u, v be the velocities of dusty fluid and dust particles respectively in the direction of x-axis. The present analysis is based on the following assumptions:

1) The flow is in the direction of x-axis and is driven by a constant pressure $\partial p/\partial x$ with negligible body forces.

2) The dust particles are non-conducting, solid, spherical, and equal in size, uniformly and symmetrically distributed in the flow field and their number density N_0 is constant throughout the motion.

3) There is no externally applied electric field and the induced magnetic field is negligible.

4) Initially, when $t \le 0$, the channel, walls as well as dusty fluid are assumed to be at the same temperature T_0. The foreign mass is assumed to be present at low level and it is uniformly distributed such that it is everywhere C_0.

5) When t > 0, the temperature of the walls is instantaneously raised to T_W and the species concentration is raised to C_W.

6) There exists a chemical reaction in the mixture.

Under these assumptions and Boussinesq's approximation with concentration, the equations governing the flow are:

$$\frac{\partial u}{\partial t} = g\beta(T-T_0) + g\beta^{'}(C-C_0) + \nu(1-K_0\frac{\partial}{\partial t})\frac{\partial^2 u}{\partial y^2} + \frac{KN_0}{\rho}(v-u) - \frac{\sigma}{\rho}B_0^2 u - \frac{\nu}{K}u \quad \text{….(1)}$$

$$m\frac{\partial v}{\partial t} = K^{'}(u-v) \quad (2)$$

$$\frac{\partial T}{\partial t} = \frac{K_T}{\rho C_p}\frac{\partial^2 T}{\partial^2 y} - \frac{Q}{\rho C_p}(T-T_0) \quad (3)$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial^2 y} - K_1(C-C_0) + D_T\frac{\partial^2 T}{\partial^2 y} \quad (4)$$

(where the symbols have their usual meaning), at t = 0, the temperature and concentration level changes according to the following laws:

$$T = T_0 + (T_w - T_0)(1-e^{-at})$$

$$C = C_0 + (C_w - C_0)(1-e^{-at})$$

The initial and boundary conditions relevant to the problem are:

$$t = 0: u = 0 = v, T = T_0 \quad y \in (-d, d)$$

$$t > 0: u = 0 = v, \ T = T_0 + (T_w - T_0)(1-e^{-at}),$$

$$C = C_0 + (C_w - C_0)(1-e^{-at}) \ \text{ for } y = -d$$

$$u = 0 = v, \ T = T_0 + (T_w - T_0)(1-e^{-at}),$$

$$C = C_0 + (C_w - C_0)(1-e^{-at}) \ \text{ for } y = d \quad (5)$$

We introduce the following non-dimensional quantities,

$$y^* = \frac{y}{d}, u^* = \frac{u}{d}, v^* = \frac{v}{d}, T = \frac{T-T_0}{T_w - T_0},$$

$$C^* = \frac{C-C_0}{C_w - C_0}, t^* = \frac{\nu t}{d^2}, a^* = \frac{d^2 a}{\nu}$$

Introducing these non-dimensional quantities, Equations (1), (2), (3) & (4) reduce to

$$\frac{\partial u}{\partial t} = G_r T + G_m C + (1 - E\frac{\partial}{\partial t})\frac{\partial^2 u}{\partial y^2} + \frac{\lambda}{w}(v-u) - \frac{u}{K_1} - M^2 u \tag{6}$$

$$W\frac{\partial v}{\partial t} = (u - v) \tag{7}$$

$$\frac{\partial^2 T}{\partial^2 y} - P_r\frac{\partial T}{\partial t} - ST = 0 \tag{8}$$

$$\frac{\partial^2 C}{\partial^2 y} - S_c\frac{\partial C}{\partial t} - KS_c C + S_T\frac{\partial^2 T}{\partial^2 y} = 0 \tag{9}$$

Now initial and boundary condition (5) according to new system become,

T = 0: u = 0 = v, T = 0 $y \in (-1,1)$

t > 0: u = 0 = v, T = 1 – e^{-at}, C = 1 – e^{-at}, for y = –1

u = 0 = v, T = 1 – e^{-at}, C = 1 – e^{-at}, for y = 1 (10)

where, $\lambda = \frac{mN_0}{\rho}$ (mass concentration of dust particle)

M (Hartmann number) $= B_0 d\sqrt{\frac{\sigma}{\mu}}$

W (relaxation time parameter for particles) $= \frac{m\nu}{K^{'}d^2}$

G_r (grashof number) $= \frac{g\beta d(T_w - T_0)}{\nu}$

G_m (Modified grashof number) $= \frac{g\beta^{'} d(C_w - C_0)}{\nu}$

S_c (Schmidt number) $= \frac{\nu}{D}$

S (heat source/sink parameter) $= Q\frac{d^2}{K_T}$

K (Chemical reaction parameter) $= \frac{K_1 d^2}{\nu}$

$E = \frac{K_0 \nu}{d^2}$ (Visco-elastic parameter), $K_1 = \frac{k}{d^2}$, P_r (Prandtl number) $= \frac{\mu C_p}{K_T}$

$S_T = \frac{D_T}{D}\frac{(T_w - T_0)}{(C_w - C_0)}$ (Thermal diffusion parameter)

4. Method of Solution

To solve the Equations (6) to (9) subject to the boundary conditions (10), according to I. Pop [14], we assume

$$\begin{aligned} u(y,t) &= u_0(y) + \varepsilon u_1(y,t)e^{-at} + \ldots\ldots\ldots\ldots \\ v(y,t) &= v_0(y) + \varepsilon v_1(y,t)e^{-at} + \ldots\ldots\ldots\ldots \\ T(y,t) &= T_0(y) + \varepsilon T_1(y,t)e^{-at} + \ldots\ldots\ldots\ldots \\ C(y,t) &= C_0(y) + \varepsilon C_1(y,t)e^{-at} + \ldots\ldots\ldots\ldots \end{aligned} \tag{11}$$

Substituting the equations like (11) into the Equations (6) to (9) and equating harmonic and non-harmonic terms, we get the following set of equations.

$$u_0^{"} - (\frac{1}{K_1} + M_1^2)u_0 = -G_r T_0 - G_m C_0 \tag{12}$$

$$(1 + aE)u_1^{"} + \frac{\lambda}{w}u_1 - (\frac{1}{K_1} + M_1^{\,2} - a)u_1 - \frac{\lambda}{w}v_1 = -G_r T_1 - G_m C_1 \tag{13}$$

$$u_0 = v_0 \ \& \ u_1 = v_1 (1\text{-}aw) \tag{14}$$

$$T_0^{"} - ST_0 = 0 \tag{15}$$

$$T_1^{"} - (S\text{-}aPr)\,T_1 = 0 \tag{16}$$

$$C_0^{"} - KS_c C_0 + S_T T_0^{"} = 0 \tag{17}$$

$$C_1^{"} - (K-a)S_c C_1 + S_T T_1^{"} = 0 \tag{18}$$

Where dashes represents differentiation w. r. to y.

Boundary conditions are reduced to:

$$\begin{aligned} &u_0 = v_0 = u_1 = v_1,\ T_0 = C_0 = 1, \\ &T_1 = C_1 = -\frac{1}{\varepsilon},\ at\ y = -1 \\ &u_0 = v_0 = u_1 = v_1,\ T_0 = C_0 = 1, \\ &T_1 = C_1 = -\frac{1}{\varepsilon},\ at\ y = 1 \end{aligned} \tag{19}$$

Solutions of the Equations (12) to (18) under the boundary conditions (19) after substituting in (11), we have:

$$u(y,t) = (A_{11} + A_{12})\frac{CoshM_1 y}{CoshM_1} - A_{11}\frac{CoshAy}{CoshA} - A_{12}\frac{CoshA_0 y}{CoshA_0} + \left\{-(A_{13} + A_{14})\frac{CoshM_2 y}{CoshM_2} + A_{13}\frac{CoshA_1 y}{CoshA_1} + A_{14}\frac{CoshA_2 y}{CoshA_2}\right\}e^{-at}$$

$$V(y,t) = (A_{11} + A_{12})\frac{CoshM_1 y}{CoshM_1} - A_{11}\frac{CoshAy}{CoshA} - A_{12}\frac{CoshA_0 y}{CoshA_0} + \left(\frac{1}{1-aw}\right)\left\{-(A_{13} + A_{14})\frac{CoshM_2 y}{CoshM_2} + A_{13}\frac{CoshA_1 y}{CoshA_1} + A_{14}\frac{CoshA_2 y}{CoshA_2}\right\}e^{-at}$$

$$C(y,t) = (1 - A_m)\frac{CoshA_0 y}{CoshA_0} + A_m\frac{CoshAy}{CoshA} + \left\{(A_n - 1)\frac{CoshA_2 y}{CoshA_2} - A_n\frac{CoshA_1 y}{CoshA_1}\right\}e^{-at}$$

$$T(y,t) = \frac{CoshAy}{CoshA} - \frac{CoshA_1 y}{CoshA_1} e^{-at}$$

4.1. Skin Friction

Let τ_f and τ_p be the skin friction for dusty fluid and dust particles respectively then we have:

$$\tau_f = \left|\frac{\partial u}{\partial y}\right|_{y=1} = (A_{11} + A_{12}).M_1 \tanh M_1 - A_{11} A \tanh A - A_{12} A_0 \tanh A_0 + [-(A_{13} + A_{14}) M_2 \tanh M_2 + A_{13} A_1 tanh A_1 + A_{14} A_2 \tanh A_2] e^{-at}$$

$$\tau_p = \left|\frac{\partial v}{\partial y}\right|_{y=1} = (A_{11} + A_{12}) M_1 \tanh M_1 - A_{11} A \tanh A - A_{12} A_0 \tanh A_0 + \left(\frac{1}{1-aw}\right)[-(A_{13} + A_{14}) M_2 \tanh M_2 + A_{13} A_1 tanh A_1 + A_{14} A_2 \tanh A_2] e^{-at}$$

4.2. Appendix

$$A = \sqrt{S}\ , A_0 = \sqrt{KS_c}\ , A_1 = \sqrt{S - aP_r}\ , A_2 = \sqrt{(K-a)S_c}\ ,$$

$$S_T = \frac{D_T (T_W - T_0)}{D(C_W - C_0)}, M_1^2 = \left[M^2 + \frac{1}{K_1}\right],$$

$$M_2^2 = \frac{1}{1+aE}\left[M^2 + \frac{1}{K_1^2} - a - \frac{al}{1-aw}\right],$$

$$A_m = \frac{S_T.A^2}{A_1^2 - A_2^2}, A_n = \frac{S_T.A^2}{A^2 - A_0^2}, A_{11} = \frac{G_r + G_m.A_m}{A^2 - M_1^2},$$

$$A_{12} = \frac{G_m(1-A_m)}{A_0^2 - M_1^2}, A_{13} = \frac{G_r + G_m.A_n}{A_1^2 - M_1^2}\ , \quad A_{14} = \frac{G_m(1-A_n)}{A_2^2 - M_2^2}$$

5. Results & Discussion

Numerical solutions for velocity profile, skin friction for dusty fluid as well as dust particles and also temperature field, concentration profile for dusty fluid have been calculated. The values of different parameters and their effects on velocity, Temperature, concentration and skin friction have been displayed through graphs.

A temperature field has been represented in **Figure 1**, which indicates the effects of heat source/sink parameter and Prandtl number.

5.1. Temperature Field for Different Values of S and P_r (t = 1, a = 0.2)

It is observed that increasing values of heat source/sink parameter and Prandtl number decreases the temperature. Also we see that the temperature is minimum at the centre of the channel (y = 0) and increasing towards the plates.

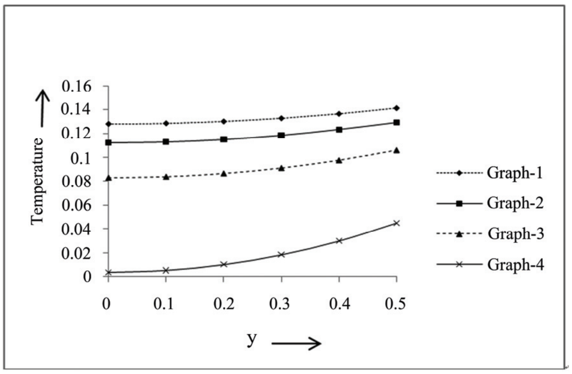

Figure 1. Temperature filed for different values of S and Pr.

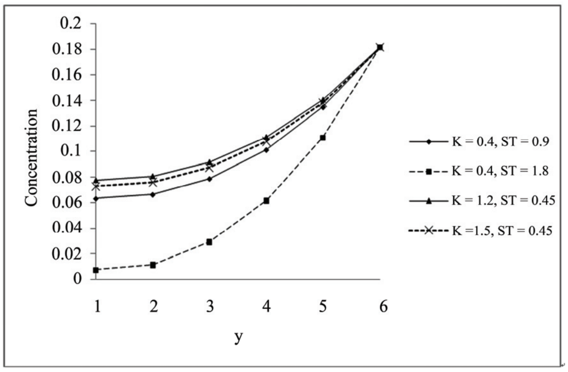

Figure 2. Concentration profile for S_T.

5.2. Concentration Profile for the Different Values of K and S_T (a = 0.2, S = 0.2,t = 1, P_r = 0.71, S_c = 0.6, D = 1)

Observation of **Figure 2** is increasing value of thermal diffusion parameter (Soret number) and chemical reaction parameter decreases the concentration. Concentration is minimum at the centre of the channel (y = 0) and increasing towards the plates.

5.3. Velocity Profile for the Different Values of K and S_T (a = 0.2, S = 0.2, t = 1, P_r = 0.71, S_c = 2, D = 1, w = 0.5, E = 1, K_1 = 10, G_m = 5, G_r = 10)

From **Figures 3 and 4** we observe that increasing value of thermal diffusion parameter (Soret number) increases the velocity of dusty fluid and dust particles while chemical reaction parameter decreases the same. Also **Figures 3 and 4** bears that the velocity is maximum at the centre of the channel and decreasing towards the plates.

5.4. Skin Friction Profile for Fluid and Dust Particles:(a = 0.2, S = 0.2, t = 1, P_r = 0.71, S_c = 2, D = 1,w = 0.5, E = 1, K_1 = 10, G_m = 5, G_r = 10)

Skin friction for different values of S_T (M = 3, K = 0.4,

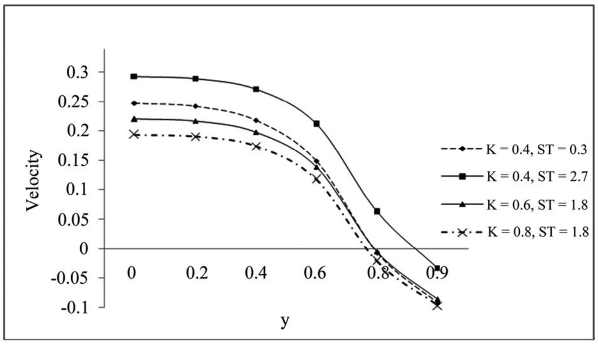

Figure 3. Velocity profile for dusty fluid.

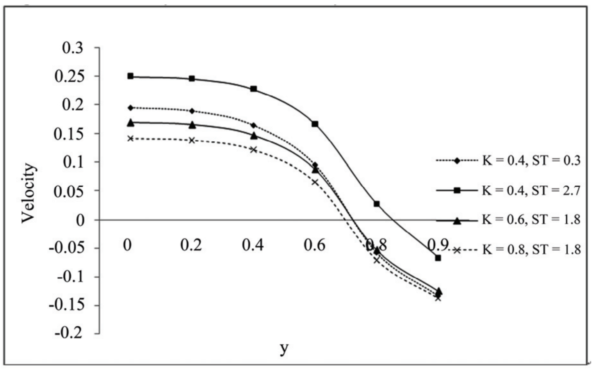

Figure 4. Velocity profile for dust particle.

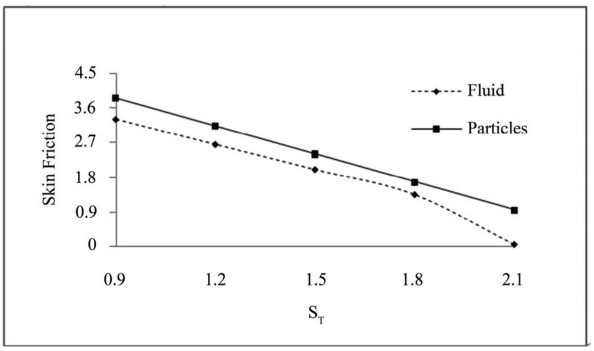

Figure 5.1. Skin friction for S_T.

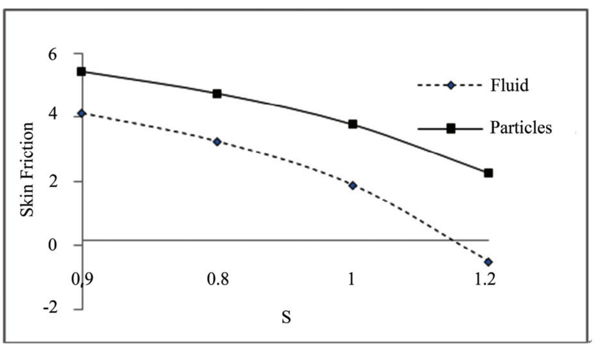

Figure 5.2. Skin friction for S.

S = 0.3):

Skin friction for different values of S, (M = 3, K = 1, S_T = 1.4):

Skin friction for different values of K (M = 3, S_T = 2.7.4, S = 0.3)**:** Skin friction for different values of M (S_T = 1.4, K = 1, S = 0.4):

The results displayed in **Figures 5.1-5.4** are as, the increasing value of thermal diffusion parameter and heat source/sink parameter decreases the skin friction of dusty fluid and dust particles. Increasing value of chemical reaction parameter and magnetic field parameter increases the skin friction of dusty fluid and dust particles.

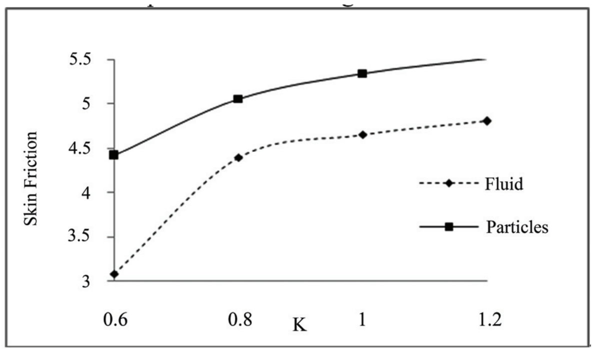

Figure 5.3. Skin friction for K.

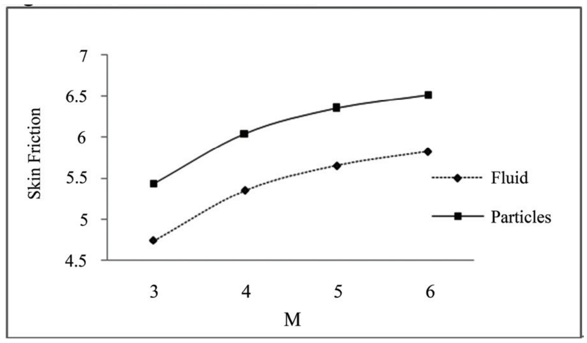

Figure 5.4. Skin friction for M.

6. Conclusions

The theoretical and numerical solutions are obtained for different profiles. From graphical representations, we have the following observations:

1) Velocity and skin friction of the dust particles behaves same as dusty fluid.

2) Increasing value of y increases the temperature, concentration while decreases the velocity of dusty fluid and dust particles.

3) Velocity of dust particles is less than velocity of dusty fluid and skin friction of dust particles is greater than that of dusty fluid.

4) Increasing values of thermal diffusion parameter (Soret number) decreases the concentration, skin friction

while increases the velocity of dust particles and dusty fluid.

5) Increasing values of magnetic field parameter increases the skin friction for both the dusty fluid and dust particles.

6) Increasing values of heat source/sink parameter decreases the skin friction for the dusty fluid as well as dust particles.

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