

Analytical Expressions of Concentration of VOC and Oxygen in Steady-State in Biofiltration Model

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ABSTRACT

Mathematical models of steady-state biofiltration are discussed. The theoretical results are much useful for the design of biofilters. This model is based on the system of non-linear reaction/diffusion equations contains a non-linear term related to Monod kinetics, Andrews kinetics, interactive model from Monod kinetics and Andrews kinetics. Analytical expression of concentration of VOC (Volatile organic compounds) and oxygen are derived by solving the system of non-linear equations using Adomian decomposition method (ADM) method. Our analytical results are also compared with the simulation results. Satisfactory agreement is noted.

Keywords: Biofilters; Volatile Organic Compounds; Oxygen; Adomian Decomposition Method; Mathematical Modeling; Non-Linear Reaction/Diffusion Equations

1. Introduction

Biological system for elimination of volatile organics have been explored on experimental studies [1-3]. However, researches into the theoretical studies regarding biofilter models is rather limited. The pioneering contribution of Ottengraf and co-workers [1,2] of this model is based on some rather simplistic assumptions. The closed analytical expression have been used in validating-scale experimental data and actual design of pilot-scale biofilter units. Recently Zarook *et al.* [3], extended the work of Ottengraf *et al.* [1] and presented a detailed steady-state biofiltration model for single volume. Allen and Phatak [4] have extended their model to describe the biofiltration of VOC mixtures under steady-state conditions. Dehusses and Dunn [5] reports a transient biofiltration model which is based on the assumption that oxygen is in excess and the kinetics are of the Michaelis-Menten or Monod type. The steady-state model of Zarook *et al.* [3] was extended to describe the transient performance [6] of the biofilters. All the steady-state and transient biofiltration models [1-6] are based on the assumption that substrates are transported into the biofilm through diffusion. Biological systems for elimination of VOCs have been explored both on the experimental and mathematical modeling levels primarily in the Netherlands by Ottengraf *et al.* [6-8] followed by many researches even though land area requirements and lack of process con-

trol still restrict the industrial use of these systems. Several researchers [1,3,6,9-13] developed models to predict biodegradability of organic compounds in biofilters. The three general plans for biological treatment systems are biofilters, biotrickling filters and bioscrubbers. In biofilters, the porous medium is kept damp by maintaining the humidity of the incoming air and by occasional sprinkling. The reliability of biological processes and in particular of biofiltration for the treatment of waste gas streams containing VOC has been demonstrated by a very large number of experimental studies.

Recently Zarook *et al.* [14] obtained the concentration of VOC and oxygen only for the limiting cases (zero-order kinetics and first-order kinetics) for monoid kinetics. However, to the best our knowledge, no analytical expressions pertaining to the steady state concentrations of VOC, oxygen and effectiveness factor have been reported. The purpose of this paper is to derive the analytical expression of concentration of VOC and oxygen for all values of parameters and all reaction mechanisms, using the Adomian decomposition method.

2. Mathematical Formulation of the Problem

A steady-state biofiltration model (**Figure 1**) constitutes a set of mass balances within the biofilm. The mass balance equations in the biofilm are [14]:

$$D \frac{d^2 c_1}{dx^2} = \frac{X_V}{Y} \mu(c_1, c_{10}) \quad (1)$$

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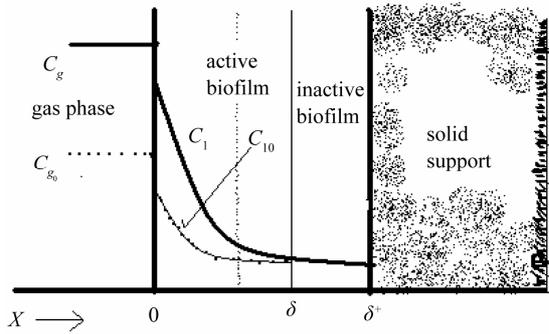


Figure 1. Biofilm concept of the biofilter.

$$D_o \frac{d^2 c_{10}}{dx^2} = \frac{X_V}{Y_o} \mu(c_1, c_{10}) \quad (2)$$

with the boundary conditions

$$c_1 = \frac{c_g}{m} \text{ and } c_{10} = \frac{c_{g0}}{m_0} \text{ at } x = 0 \quad (3)$$

$$\frac{dc_1}{dx} = 0 \text{ and } \frac{dc_{10}}{dx} = 0 \text{ at } x = \delta \quad (4)$$

where c_1 and c_{10} are the concentration of VOC and oxygen at a position x in the biofilm, D and D_o are the effective diffusion coefficient of VOC and oxygen in the biofilm. X_V denotes biofilm density, Y and Y_o is the amount of biomass produced per amount of VOC consumed and amount of biomass produced per amount of oxygen consumed. For biological systems, The growth rate $\mu(c_1, c_{10})$, for various reaction kinetics are given as follows:

Monod kinetics:

$$\mu(c_1) = \frac{\mu_m c_1}{K + c_1} \quad (5)$$

Andrews kinetics:

$$\mu(c_1) = \frac{\mu^* c_1}{K + c_1 + \frac{c_1^2}{K_I}} \quad (6)$$

When oxygen limits the biodegradation rate, the growth rate is given by interactive model. The above Equations (5) and (6) are written as follows:

Interactive model from Monod kinetics:

$$\mu(c_1, c_{10}) = \frac{\mu_m c_1}{K + c_1} \left[\frac{c_{10}}{K_0 + c_{10}} \right] \quad (7)$$

Interactive model from Andrews kinetics:

$$\mu(c_1, c_{10}) = \frac{\mu^* c_1}{K + c_1 + \frac{c_1^2}{K_I}} \left[\frac{c_{10}}{K_0 + c_{10}} \right] \quad (8)$$

In order to obtain numerical solution of model these

equations are brought in dimensionless form through the dimensionless variables and groups. We make the above non-linear partial differential Equations (1) and (2) in dimensionless form by defining the following dimensionless parameters:

$$C_1 = \frac{c_1 m}{c_g}, C_{10} = \frac{c_{10} m_0}{c_{g0}}, X = \frac{x}{\delta}, \phi^2 = \frac{X_V \mu_m \delta^2}{K D Y} \quad (9)$$

$$\theta = \frac{D Y c_g c_{g0}}{D_0 Y_o m m_0}, M = \frac{c_g}{K m}, L = \frac{c_g}{K_I m}, N = \frac{K_0 m_0}{c_{g0}}$$

where C_1 and C_{10} denotes the dimensionless concentration of VOC and oxygen, X is the dimensionless position in the biolayer. ϕ^2 represents the Thiele modulus, θ , M , L and N are dimensionless constants. By substituting the Equation (9) in Equations (1) and (2), we can obtain the following dimensionless non-linear equation for Monod kinetics:

$$\frac{d^2 C_1}{dX^2} = \phi^2 \left[\frac{C_1}{1 + M C_1} \right] \quad (10)$$

$$\frac{d^2 C_{10}}{dX^2} = \phi^2 \theta \left[\frac{C_1}{1 + M C_1} \right] \quad (11)$$

Using Equation (9), in the non-linear Equations (1) and (2), we can obtain the following dimensionless non-linear equation for Andrews kinetics:

$$\frac{d^2 C_1}{dX^2} = \phi^2 \left[\frac{C_1}{1 + M C_1 + M L C_1^2} \right] \quad (12)$$

$$\frac{d^2 C_{10}}{dX^2} = \phi^2 \theta \left[\frac{C_1}{1 + M C_1 + M L C_1^2} \right] \quad (13)$$

Using Equation (9), the dimensionless non-linear equation for Interactive model of Monod kinetics becomes

$$\frac{d^2 C_1}{dX^2} = \phi^2 \left[\frac{C_1}{1 + M C_1} \right] \left[\frac{C_{10}}{N + C_{10}} \right] \quad (14)$$

$$\frac{d^2 C_{10}}{dX^2} = \phi^2 \theta \left[\frac{C_1}{1 + M C_1} \right] \left[\frac{C_{10}}{N + C_{10}} \right] \quad (15)$$

The dimensionless non-linear equation for Interactive model of Andrews kinetics of the Equations (1) and (2) is

$$\frac{d^2 C_1}{dX^2} = \phi^2 \left[\frac{C_1}{1 + M C_1 + M L C_1^2} \right] \left[\frac{C_{10}}{N + C_{10}} \right] \quad (16)$$

$$\frac{d^2 C_{10}}{dX^2} = \phi^2 \theta \left[\frac{C_1}{1 + M C_1 + M L C_1^2} \right] \left[\frac{C_{10}}{N + C_{10}} \right] \quad (17)$$

Now the boundary condition in dimensionless form may be represented as follows:

$$C_1 = 1 \text{ and } C_{10} = 1 \text{ at } X = 0 \quad (18)$$

$$\frac{dC_1}{dX} = 0 \text{ and } \frac{dC_{10}}{dX} = 0 \tag{19}$$

at $X = 1$

For all the above cases, we can obtain the relation between C_1 and C_{10} as follows:

$$\frac{d^2C_1}{dX^2} - \frac{1}{\theta} \frac{d^2C_{10}}{dX^2} = 0$$

Integrating the above equation twice and using the boundary condition (18) and (19) we get

$$C_{10} = 1 - \theta(1 - C_1) \tag{20}$$

When $\theta = 1$, the concentration of VOC (C_1) and oxygen (C_{10}) becomes equal. When $\theta = 0$, the concentration of oxygen $C_{10} = 1$. When $\theta = \infty$, the concentration of VOC (C_1) becomes one.

3. Analytical Solutions of the Concentrations Using the Adomian Decomposition Method (ADM)

Nonlinear phenomena play a crucial role in physical chemistry and biology (heat and mass transfer, filtration of liquids, diffusion in chemical reactions, etc.). Constructing a particular, exact solution for these equations remains an important problem. Finding an exact solution that has a physicochemical or biological interpretation is of fundamental importance. This model is based on a non-stationary system of diffusion equations containing a nonlinear reaction term. It is not possible to solve these equations using standard analytical techniques. The investigation of an exact solution of nonlinear equations is interesting and important. In the past several decades, many authors mainly paid attention to studying the solution of nonlinear equations by using various methods, such as the Backlund and the Darboux transformation [15,16], the inverse scattering method [17], the bilinear method [18], the tanh method [19], the variational iteration method [20], the HPM [21-25], ADM [26-30]. The ADM was successfully applied to autonomous ordinary differential equations for nonlinear polycrystalline solids and to other fields. This method has been proved by many authors to be a powerful mathematical tool for various kinds of nonlinear problems. It is a promising and evolving method. The ADM is unique in its applicability, accuracy and efficiency. In this method, the solution procedure is very simple and only few iterations lead to highly accurate solutions that are valid for the whole solution domain. Using this method (see Appendix A), the concentration of VOC and oxygen for all the four cases can be obtained. The concentration of VOC for Monod kinetics in the biofilm is,

$$C_1(X) = 1 + \left(\frac{\phi^4}{3(1+M)^3} - \frac{\phi^2}{(1+M)} \right) X + \left(\frac{\phi^2}{2(1+M)} \right) X^2 - \left(\frac{\phi^4}{6(1+M)^3} \right) X^3 + \left(\frac{\phi^4}{24(1+M)^3} \right) X^4 \tag{21}$$

By solving the Equation (12), we can obtain the concentration of VOC for Andrews kinetics as follows,

$$C_1(X) = 1 + \left(\frac{\phi^4(1+3ML)}{3(1+M+ML)^3} - \frac{\phi^2}{(1+M+ML)} \right) X + \left(\frac{\phi^2}{2(1+M+ML)} \right) X^2 - \left(\frac{\phi^4(1+3ML)}{6(1+M+ML)^3} \right) X^3 + \left(\frac{\phi^4(1+3ML)}{24(1+M+ML)^3} \right) X^4 \tag{22}$$

Solving the dimensionless form of Interactive model from Monod kinetics Equation (14), we get the concentration as,

$$C_1(X) = 1 + \left[-\frac{\phi^2}{M_o} + \frac{\phi^2 M_2}{3[M_o]^2} \right] X + \left[\frac{\phi^2}{2M_o} \right] X^2 - \left[\frac{\phi^2 M_2}{6[M_o]^2} \right] X^3 + \left[\frac{\phi^2 M_2}{24[M_o]^2} \right] X^4 \tag{23}$$

Solving the dimensionless form of Interactive model from Andrews kinetics Equation (16), we get the concentration as,

$$C_1(X) = 1 + \left[-\frac{\phi^2}{M_1} + \frac{\phi^2 M_3}{3[M_1]^2} \right] X + \left[\frac{\phi^2}{2[M_1]} \right] X^2 - \left[\frac{\phi^2 M_3}{6[M_1]^2} \right] X^3 + \left[\frac{\phi^2 M_3}{24[M_1]^2} \right] X^4 \tag{24}$$

where

$$M_o = (1+M)(N+1),$$

$$M_1 = (1+M+ML)(N+1),$$

$$M_2 = \left(\phi^2(1+\theta) - \frac{\phi^2\theta}{(N+1)} - \frac{M\phi^2}{(1+M)} \right), \text{ and}$$

$$M_3 = \left(\phi^2(1+\theta) - \frac{\phi^2\theta}{(N+1)} - \frac{(M+2ML)\phi^2}{(1+M+ML)} \right).$$

4. Effectiveness Factor

The effectiveness factor is defined as the ratio of actual

rate of reaction to the rate of reaction that would result if the entire biofilm was exposed to the concentration at the gas/biofilm interface. The effectiveness factor of various kinetics are as follows:

Monod kinetics

$$\eta = \frac{-(1+M)}{\phi^2} \left(\frac{dC_1}{dX} \right)_{X=0} = \left[1 - \frac{\phi^2}{3(1+M)^2} \right] \quad (25)$$

Andrews kinetics

$$\eta = \frac{-(1+M+ML)}{\phi^2} \left(\frac{dC_1}{dX} \right)_{X=0} = \left[1 - \frac{\phi^2(1+3ML)}{3(1+M+ML)^2} \right] \quad (26)$$

Interactive model from Monod kinetics

$$\eta = \frac{-M_o}{\phi^2} \left(\frac{dC_1}{dX} \right)_{X=0} = \left[1 - \frac{\phi^2((N+1)+N\theta(1+M))}{3[M_o]^2} \right] \quad (27)$$

Interactive model from Andrews kinetics

$$\eta = -\frac{M_1}{\phi^2} \left(\frac{dC_1}{dX} \right)_{X=0} = \left[1 - \frac{\phi^2}{3[M_1]^2} \left[(N+1)(1+M+ML)\phi^2(1+\theta) - (1+M+ML)\phi^2\theta - (N+1)(M+2ML)\phi^2 \right] \right] \quad (28)$$

5. Numerical Simulation

The dimensionless form of Equations (10)-(17) corresponding to the boundary conditions (18) and (19) were solved by numerical methods. We have used pdex4 to solve these equations (Pdex4 in MATLAB is a function to solve the initial-boundary value problems of differential equations. Matlab program to find the numerical solution of Equations (10) and (11) is given in the Appendix B. The numerical solution is compared with our analytical results and is shown in **Figures 2-9**. A satisfactory agreement is noticed for various values of the Thiele modulus and possible small values of reaction/ diffusion parameters.

6. Discussion

Equations (21)-(24) represents the new analytical expressions of the concentration of VOC for Monoid, Andrews, Interactive Monoid and Interactive Andrews kinetics for all values of the parameter. Using the relation (Equation (20)), we can also obtain the concentration of oxygen C_{10} for all the kinetics. Zarook *et al.* [17] obtained the

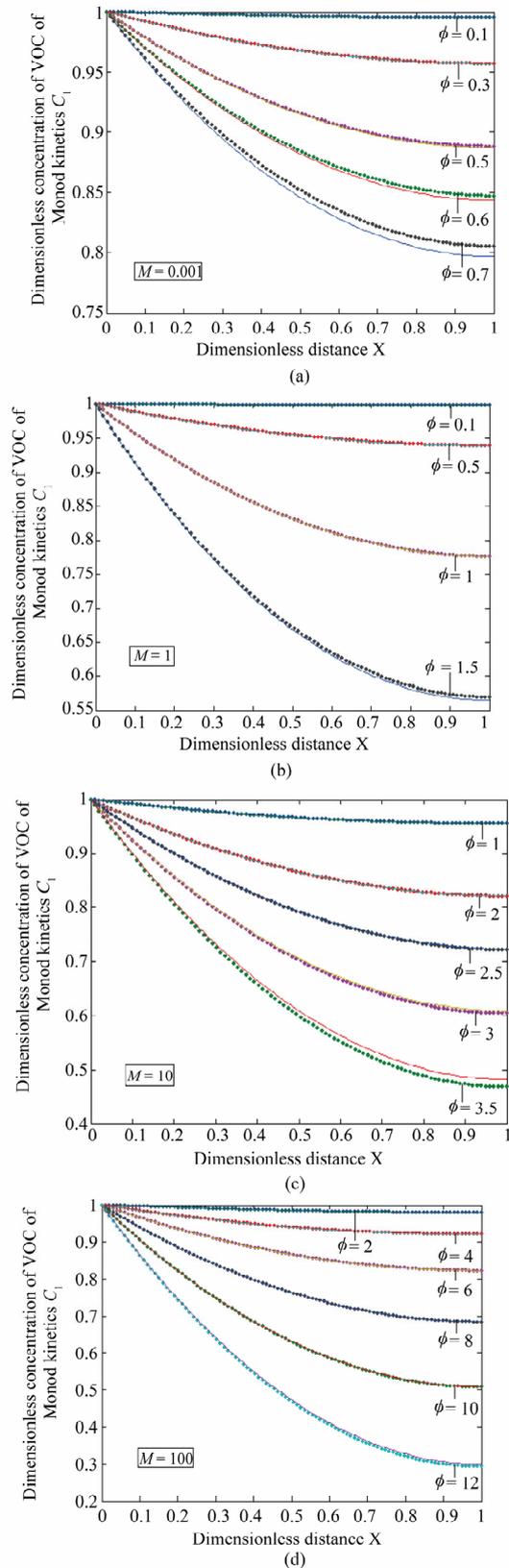
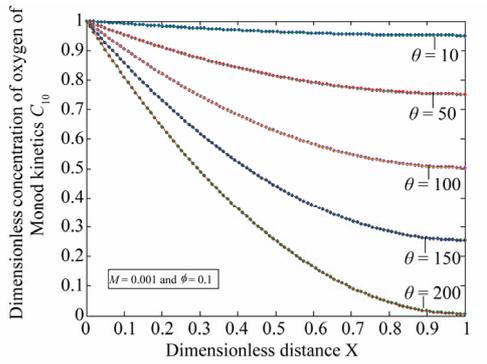
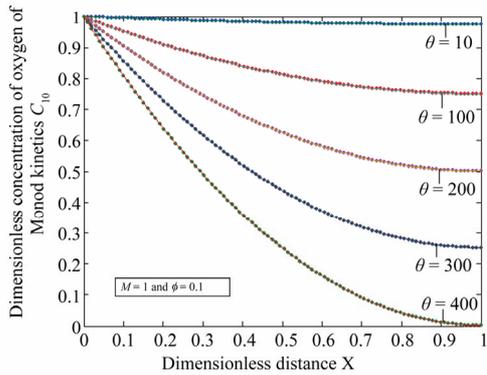


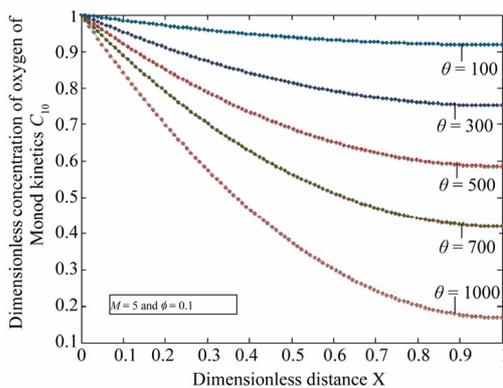
Figure 2. The dimensionless concentration C_1 versus dimensionless distance X for various values of Thiele modulus ϕ and M using Equation (21).



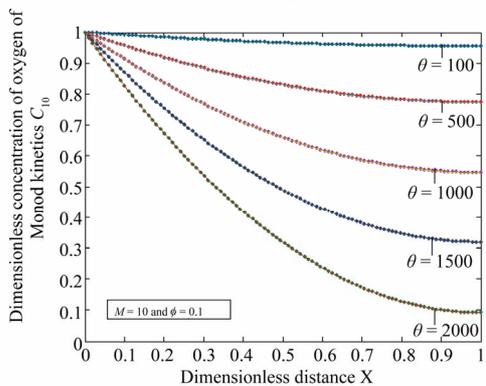
(a)



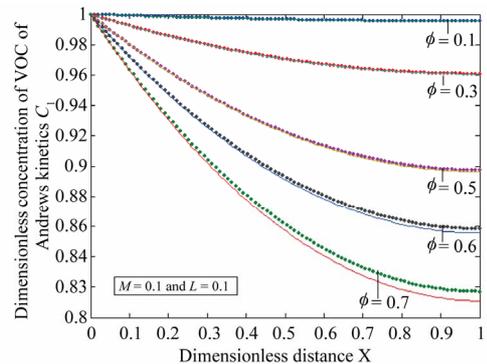
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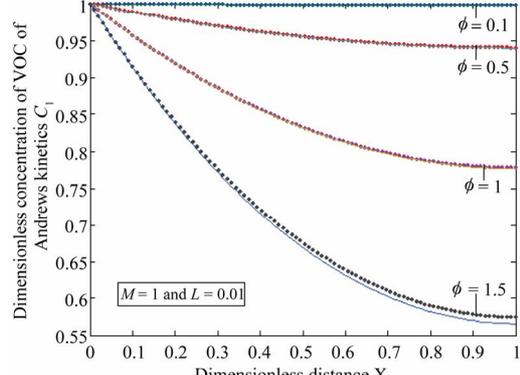
(c)



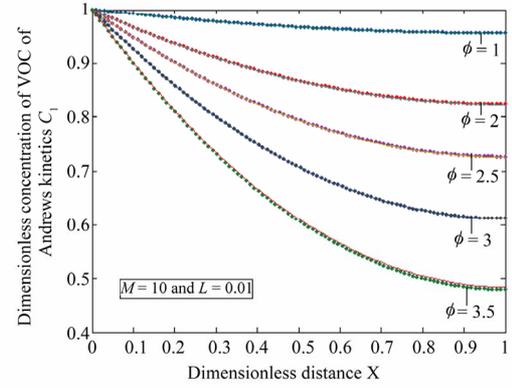
(d)



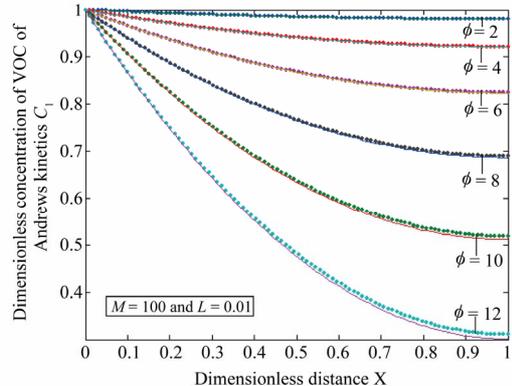
(a)



(b)



(c)



(d)

Figure 3. The dimensionless concentration C_{10} versus dimensionless distance X for various values of dimensionless quantity θ , M and ϕ using Equation (20).

Figure 4. The dimensionless concentration C_1 versus dimensionless distance X for various values of Thiele modulus ϕ , M and L using Equation (22).

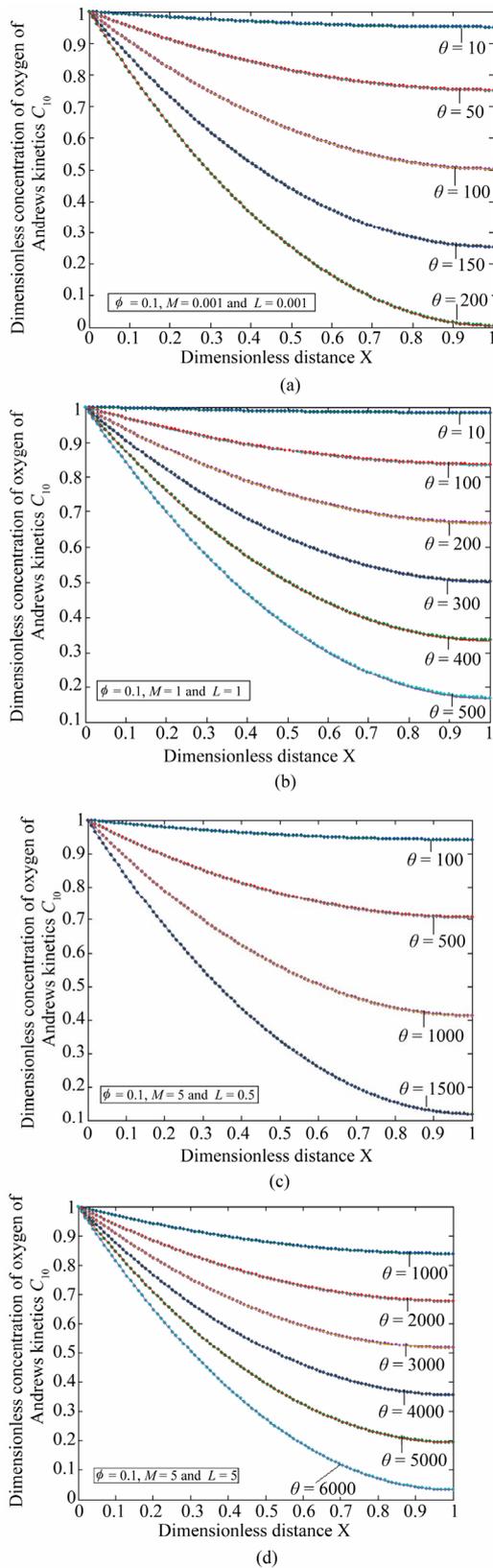


Figure 5. The dimensionless concentration C_{10} versus dimensionless distance X for various values of dimensionless quantity θ, M, L and ϕ using Equation (20).

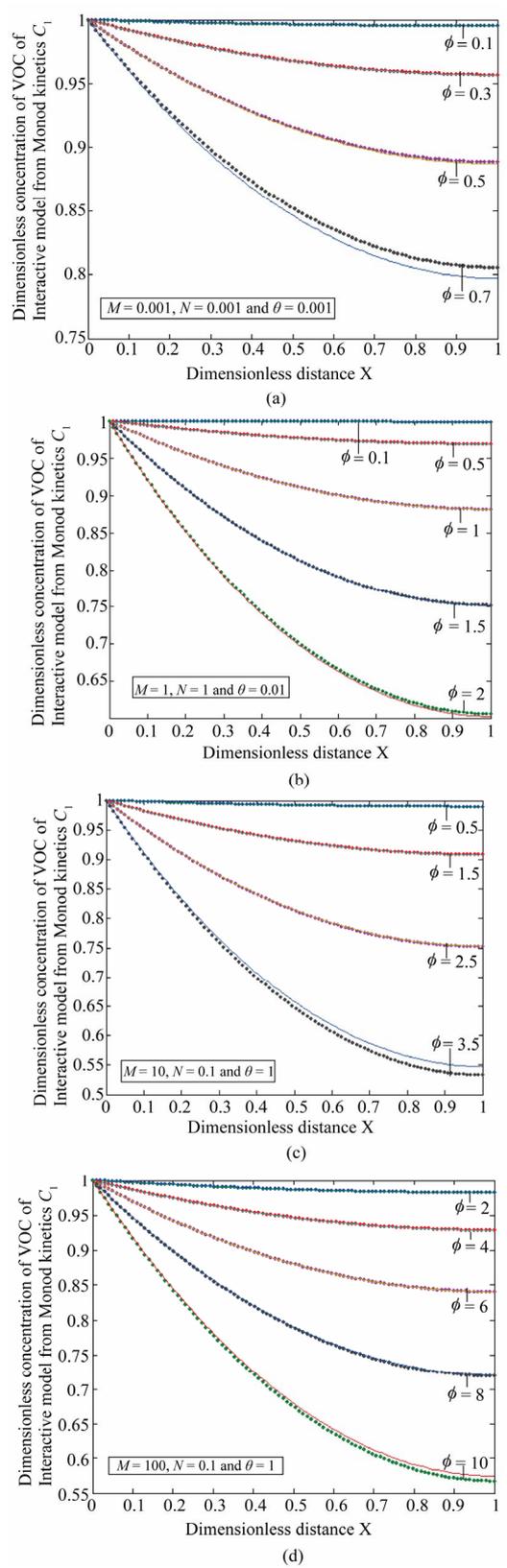


Figure 6. The dimensionless concentration C_1 versus dimensionless distance X for various values of Thiele modulus ϕ, M, N and θ using Equation (23).

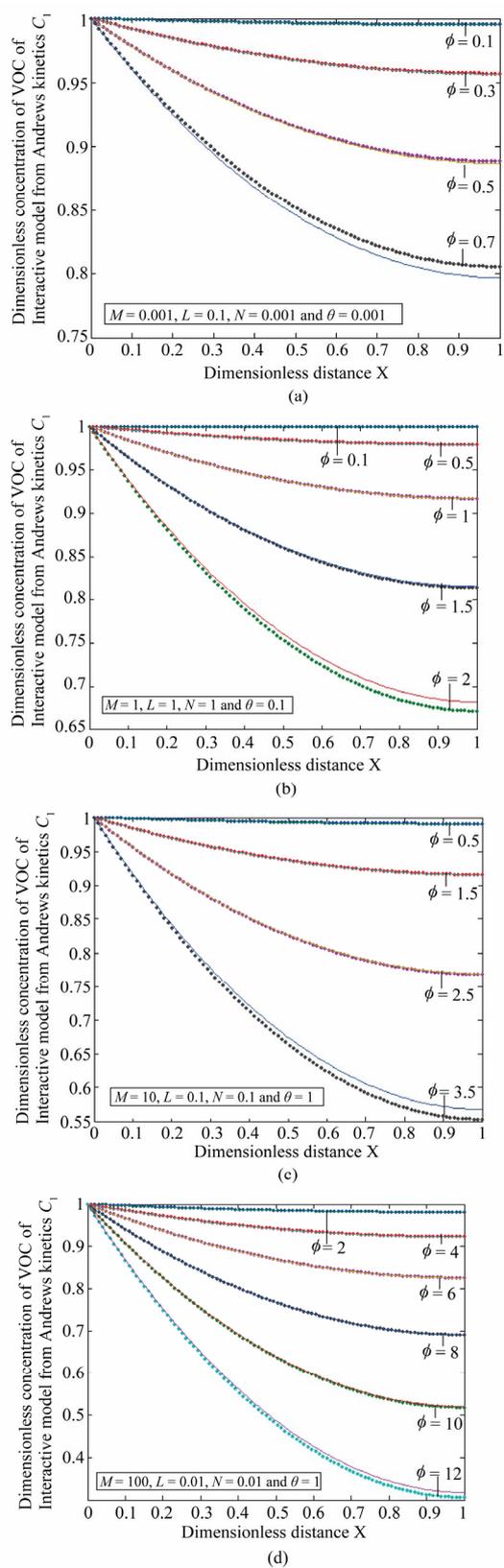
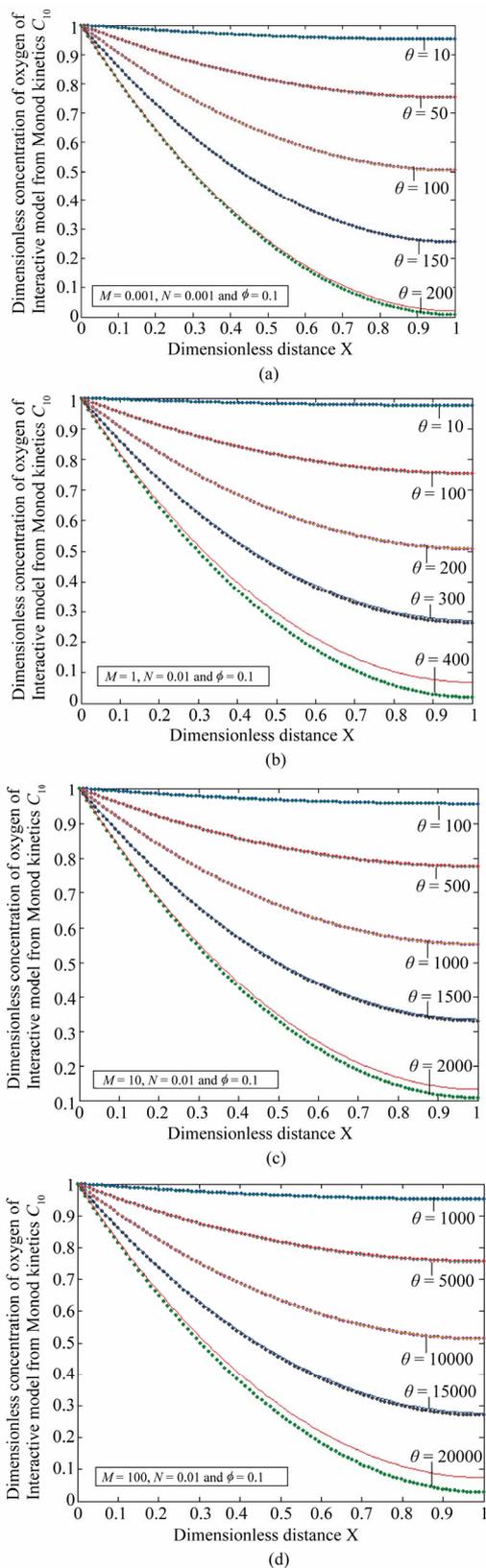


Figure 7. The dimensionless concentration C_{10} versus dimensionless distance X for various values of dimensionless quantity θ, M, L and ϕ using Equation (20).

Figure 8. The dimensionless concentration C_1 versus dimensionless distance X for various values of dimensionless quantity ϕ, M, L, N and θ using Equation (24).

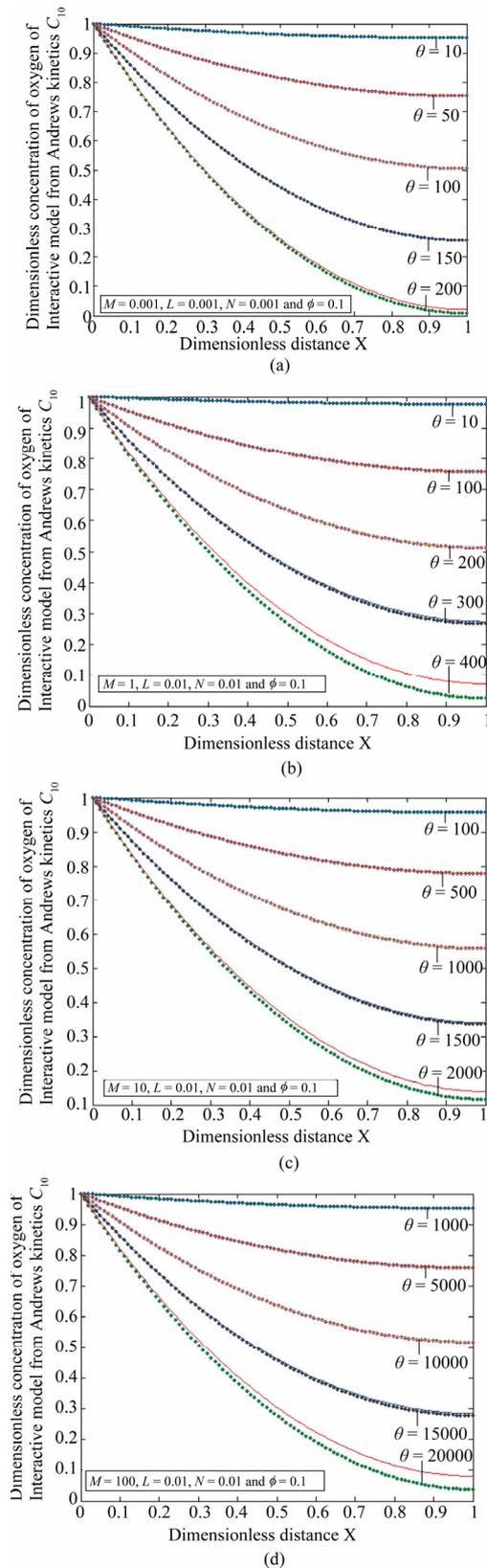


Figure 9. The dimensionless concentration C_{10} versus dimensionless distance X for various values of dimensionless quantity θ, M, L, N and ϕ using Equation (20).

analytical expressions of concentration of VOC and oxygen only for the limiting cases (Zero kinetics and First-order kinetics). Concentration of VOC (C_1) and oxygen (C_{10}) depends upon the value of parameters ϕ, M, L, N and θ .

6.1. The Thiele Modulus

The Thiele module $\phi^2 = X_V \mu_m \delta^2 / K D Y$, essentially compares biodegradation rate $X_V \mu_m / K Y$ with diffusion rate (δ^2 / D). We observe the rise and downfall of concentration profiles in two cases. 1) If Thiele modulus is small ($\phi^2 < 1$), then enzyme kinetics predominate. The overall kinetics is governed by the total amount of active enzyme; 2) The response is under diffusion control, if the Thiele module is large ($\phi^2 > 1$), which is observed at high catalytic activity and active membrane thickness or at low reaction kinetic constant (K) or diffusion coefficient values D .

6.2. Monod Kinetics

Equation (21) represents the concentration of VOC for Monod kinetics in the biofilm.

Figures 2(a)-(d) is the plot of dimensionless concentration (C_1) versus dimensionless distance X for various values of Thiele modulus ϕ and the dimensionless quantity M using Equation (21). From this Figure it is inferred that, the concentration of VOC, at $X=1$, increase when the value of ϕ or bio-filter thickness decreases. Also the concentration is uniform when $\phi \leq 0.1$ and all values of M .

Figures 3(a)-(d) is the plot of dimensionless concentration (C_{10}) for various values of dimensionless quantity θ, M and the Thiele modulus ϕ . From this figure it is observed that the concentration of oxygen is decreases when θ increases.

6.3. Andrews Kinetics

Equation (22) is the concentration of VOC for Andrews-type kinetics.

Figures 4(a)-(d) is the plot of dimensionless concentration (C_{10}) versus dimensionless distance X for various values of dimensionless parameters. From this figure, it is noted that the concentration of VOC decreases when ϕ, L, M increases.

Figures 5(a)-(d) represents the dimensionless concentration (C_{10}) for various values of dimensionless quantity θ, M, L and ϕ .

6.4. Interactive Monod Kinetics

Equation (23) represents the concentration of VOC of Interactive model from Monod kinetics.

Figures 6(a)-(d) is the dimensionless concentration (C_1) versus dimensionless distance X for various values

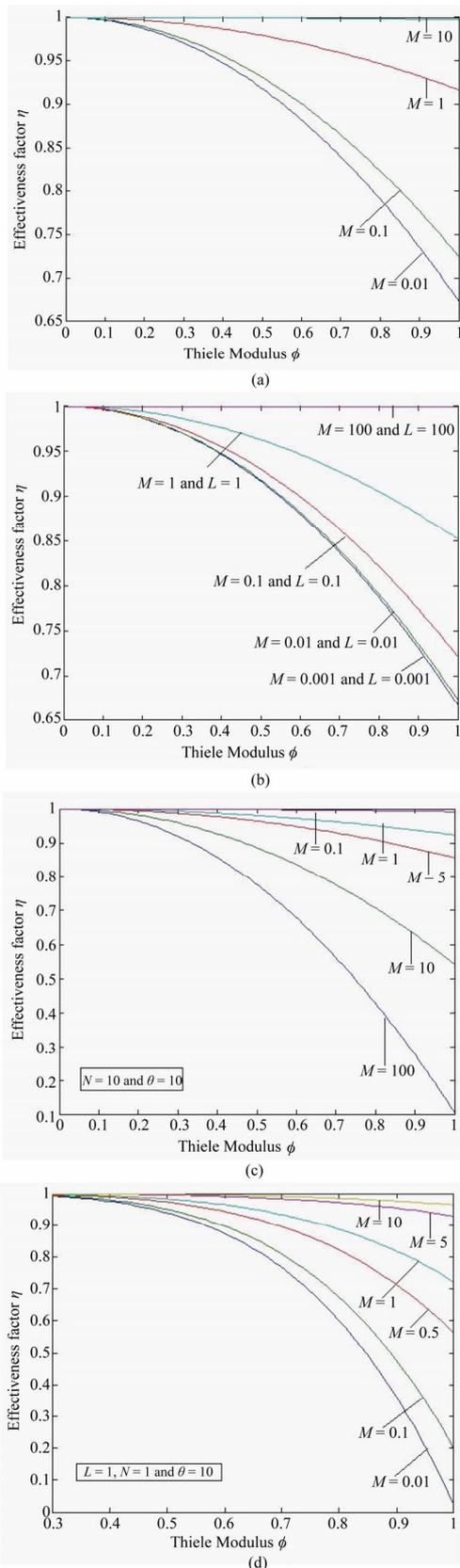


Figure 10. Effectiveness factor η versus the Thiele modulus ϕ using the Equations (25)-(28) respectively.

of parameters using Equation (23). From this Figure it is noted that, the concentration of (C_{10}) is uniform when $\theta \leq 10$. Also (C_{10}) decreases when θ increases whereas the **Figures 7(a)-(d)** for the dimensionless concentration (C_{10}) for various values of dimensionless quantity θ , M , N and ϕ . From this figure it is inferred that the concentration of VOC (C_1) is equal to one when $\phi \leq 0.5$ for all values of parameters.

6.5. Interactive Andrews Kinetics

Equation (24) is the concentration of VOC of Interactive model from Andrews kinetics.

Figures 8(a)-(d) is the dimensionless concentration (C_1) versus dimensionless distance X for various values of Thiele modulus ϕ , M , L , N and θ using Equation (24).

Figures 9(a)-(d) is the dimensionless concentration (C_{10}) for various values of dimensionless parameters. From this figure it is inferred that the concentration of VOC (C_1) is constant when bio-filter thickness ϕ decreases.

6.6. Effectiveness Factor

Figures 10(a)-(d) represent the effectiveness factor η versus the Thiele modulus ϕ using Equations (25)-(28). From this figure it is observed that the effectiveness factor = 1 when $\phi \leq 0.2$ for all mechanisms. Also the effectiveness factor decreases when ϕ increases and the values of parameters M and L decreases.

7. Conclusion

The non-linear differential equations in biofilter models have been solved analytically for various kinetics using the Adomian decomposition method. Analytical expression of concentration of VOC and oxygen and corresponding effectiveness factor have been obtained for Monoid, Andrews, Interactive Monoid and Andrews kinetics and for all values of parameters. These analytical reactions very much useful for designing or scaling-up of biofilters.

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Appendix A

Analytical Solution of Non-Linear (Equation (16)) Using The Adomian Decomposition Method

In the operator form, Equation (16) becomes

$$L(C_1(X)) = \phi^2 N(C_1(X)) \tag{B1}$$

where

$$L = \frac{d^2}{dX^2}$$

and

$$N(C_1(X)) = \left[\frac{C_1}{1 + MC_1 + MLC_1^2} \right] \left[\frac{C_{10}}{N + C_{10}} \right] \tag{B2}$$

Applying L^{-1} to both sides of (B1) yields

$$C_1(X) = aX + b + \phi^2 L^{-1} [N(C_1(X))] \tag{B3}$$

where a and b are constants of integration. To solve (B3) by the Adomian method, we get

$$C_1(X) = \sum_{n=0}^{\infty} C_n(X) \tag{B4}$$

$$N(C_1(X)) = \sum_{n=0}^{\infty} A_n \tag{B5}$$

In view of the Equations (B4)-(B5), Equation (B3) gives

$$\sum_{n=0}^{\infty} C_n(X) = aX + b + \phi^2 L^{-1} \left[\sum_{n=0}^{\infty} A_n \right] \tag{B6}$$

we identify the zeroth component as

$$C_{01}(X) = aX + b \tag{B7}$$

and the remaining components as the recurrence relation,

$$C_{(n+1)1}(X) = \phi^2 L^{-1} [A_n], \quad n \geq 0 \tag{B8}$$

where A_n are the Adomian polynomials that represent the non-linear term in (B8).

$$\sum_{n=0}^{\infty} A_n(\chi) = \frac{1}{n!} \left[\frac{d^n}{d\lambda^n} N \left(\sum_{i=0}^{\infty} [\lambda^i y_i(\chi)] \right) \right]_{\lambda=0} \tag{B9}$$

Using (B9) we can find the first few A_n as follows:

$$A_0 = \left[\frac{u_0 v_0}{(1 + Mu_0 + MLu_0^2)(N + v_0)} \right] \tag{B10}$$

$$A_1 = \frac{M_1(v_1 + u_1)}{M_1^2} - \frac{(v_1(1 + M + ML) + u_1 M(N + 1)(1 + 2L))}{M_1^2} \tag{B11}$$

The remaining polynomials can be generated easily.

The corresponding boundary condition becomes

$$C_1 = 1 \text{ when } X^* = 0 \quad \text{and} \quad \frac{dC_1}{dX} = 0 \text{ when } X^* = 1 \tag{B12}$$

$$C_1 = 0 \text{ when } X^* = 0 \quad \text{and} \quad \frac{dC_1}{dX} = 0 \text{ when } X^* = 1$$

Substitution of (B10) and (B11) in (B8) and operating with L^{-1} in conjunction with the boundary conditions (B12) in each case separately, we obtain

$$u_0 = AX + B = 1 \tag{B13}$$

$$u_1 = \phi^2 L^{-1} [A_0] = \frac{\phi^2}{M_1} \left(\frac{X^2}{2} - X \right) \tag{B14}$$

$$u_2 = \phi^2 L^{-1} [A_1] = \frac{\phi^2}{[M_1]^2} \left[\phi^2(1 + \theta) - \frac{(1 + M + ML)\phi^2\theta}{M_1} \right] \tag{B15}$$

$$- \frac{(N + 1)(M + 2ML)\phi^2}{M_1} \left] \left(\frac{X^4}{24} - \frac{X^3}{6} + \frac{X}{3} \right) \right.$$

$$v_0 = AX + B = 1 \tag{B16}$$

$$v_1 = \phi^2 L^{-1} [A_0] = \frac{\phi^2\theta}{M_1} \left(\frac{X^2}{2} - X \right) \tag{B17}$$

Substituting the Equations (B13)-(B15) in

$$u = \text{It}_{p \rightarrow 1} (u_0 + pu_1 + p^2u_2 + \dots) \tag{B18}$$

we can obtain the Equation (24) in the text. Similarly, applying the above same procedure, we obtain the Equations (21)-(23).

Appendix B

Matlab/Scilab program to find the numerical solution of the Equations (10)-(11).

```
function pdex4
m = 0;
x = linspace(0,1);
t=linspace(0,10);
sol = pdepe(m,@pdex4pde,@pdex4ic,@pdex4bc,x,t);
u1 = sol(:,:,1);
u2 = sol(:,:,2);
figure
plot(x,u1(end,:))
title('u1(x,t)')
xlabel('Distance x')
ylabel('u1(x,2)')
%-----
figure
plot(x,u2(end,:))
title('u2(x,t)')
xlabel('Distance x')
```

```

ylabel('u2(x,2)')
% -----
function [c,f,s] = pdex4pde(x,t,u,DuDx)
c = [1; 1];
f = [1; 1] .* DuDx;
phi=10;
M=1;
theta=1;
F=-((phi^2)*u(1))/(1+M*u(1));
F1=-(theta*(phi^2)*u(1))/(1+M*u(1));
s=[F; F1];

% -----
function u0 = pdex4ic(x);
%create a initial conditions
u0 = [1; 1];
% -----
function [pl,ql,pr,qr]=pdex4bc(xl,ul,xr,ur,t)
%create a boundary conditions
pl = [ul(1)-1; ul(2)-1];
ql = [0; 0];
pr = [0; 0];
qr = [1; 1];

```

Nomenclature and Units

Symbol	Explanation	Usual Dimension
c_g	Concentration of VOC in the gas phase at a height, h , along the column.	$\text{kg} \cdot \text{m}^{-3}$
c_{g0}	Concentration of oxygen in the gas phase at a height, h , along the column.	$\text{kg} \cdot \text{m}^{-3}$
C_1	Dimensionless concentration of VOC at a position x in the biofilm	No unit
C_{10}	Dimensionless concentration of oxygen at a position x in the biofilm	No unit
D	Effective diffusion coefficient of VOC in the biofilm	$\text{m}^2 \cdot \text{s}^{-1}$
D_0	Effective diffusion coefficient of oxygen in the biofilm	$\text{m}^2 \cdot \text{s}^{-1}$
K	Kinetic constant	$\text{kg} \cdot \text{m}^{-3}$
K_0	Kinetic constant	$\text{kg} \cdot \text{m}^{-3}$
K_t	Kinetic constant	$\text{kg} \cdot \text{m}^{-3}$
m	Air/biofilm distribution coefficient for the VOC as dictated by Henry's law	No unit
m_0	Air/biofilm distribution coefficient for the oxygen as dictated by Henry's law	No unit
x	Distance in the biofilm	m
X_v	Biofilm density defined as the dry weight of cell per volume of biofilm	$\text{kg} \cdot \text{m}^{-3}$
Y	Amount of biomass produced per amount of VOC consumed	$\text{kg} \cdot \text{kg}^{-1}$
Y_0	Amount of biomass produced per amount of oxygen consumed	$\text{kg} \cdot \text{kg}^{-1}$
X	Dimensionless position in the biolayer	No unit
M	Dimensionless quantity	No unit
L	Dimensionless quantity	No unit
N	Dimensionless quantity	No unit
θ	Dimensionless quantity	No unit
Greek letters		
δ	Active biofilm thickness	m
δ^*	Active biofilm thickness	m
η	Effectiveness factor	No unit
ϕ^2	Square of Thiele modulus based on methanol	No unit
μ	Specific growth rate of the biomass on VOC	h^{-1}
μ_m	Maximum specific growth rate	h^{-1}
μ^*	Kinetic constant	h^{-1}